1 Helix equation fit

The points will lay on a helix, or (for our purposes) on a circle (we will neglect the z axis). The error ε_i of the measurement point i will be given by:

$$\varepsilon_i = \frac{1}{2}\rho r_i^2 - (1+\rho d) r_i \sin(\phi_i - \phi_0) + \frac{1}{2}\rho d^2 + d - y_i \tag{1}$$

there r_i is the layer radius, ϕ_0 is the initial track's polar angle in the transverse plane, ϕ_i is the polar angle of each hit, ρ is the curvature of the track ($\rho = 1/R$, with R radius of curvature) and d is the distance of closest approach of the track to the z axis. The curvature ρ is related to the transverse momentum accoding to

$$\rho(\mathrm{m}) = \frac{0.3B}{p_T} \tag{2}$$

assuming a single-charged particle, with B measured in Tesla and p_T in GeV/c. Assuming the CMS magnet field of 3.8 T, this becomes

$$\rho(\mathrm{mm}) = \frac{1.14 \times 10^3}{p_T} \tag{3}$$

We have a set of measurement points, where the only error is baically on the y position of hit $y_i = d\sin(\phi - \phi_0)$. For momenta high enough, fitting the helix reduces to minimzing the χ^2 defined as follows

$$\chi^2 = \sum_{i,j} \varepsilon_j C_{i,j}^{-1} \varepsilon_i \tag{4}$$

with $C_{i,j}$ the correlation between the measurement points. The weight matrix W will be:

$$W_{k,l} = \sum_{i,j} \frac{\partial \varepsilon_i}{\partial \alpha_k} C_{i,j}^{-1} \frac{\partial \varepsilon_j}{\partial \alpha_l} \tag{5}$$

with $\alpha_1 = \rho$, $\alpha_2 = \phi$, $\alpha_3 = d$. The covariance matrix S is given by $S = W^{-1}$, and thus the measurement errors are:

$$\begin{aligned} \Delta\rho &= \sqrt{W_{1,1}^{-1}} \\ \Delta\phi &= \sqrt{W_{2,2}^{-1}} \\ \Delta d &= \sqrt{W_{3,3}^{-1}} \end{aligned}$$

which can be also written as

$$\begin{aligned} \sigma_\rho^2 &= S_{1,1} \\ \sigma_\phi^2 &= S_{2,2} \\ \sigma_d^2 &= S_{3,3} \\ \sigma_{\rho,\phi} &= S_{1,2} \\ &\ldots \end{aligned}$$

to evidentiate the covariances. The derivatives of (1) are:

$$\begin{aligned} \frac{\partial \varepsilon_i}{\partial \alpha_1} = \frac{\partial \varepsilon_i}{\partial \rho} &= \frac{1}{2} r_i^2 + d(d + y_i) \\ \frac{\partial \varepsilon_i}{\partial \alpha_2} = \frac{\partial \varepsilon_i}{\partial \phi} &= -x_i(1+\rho d) \\ \frac{\partial \varepsilon_i}{\partial \alpha_3} = \frac{\partial \varepsilon_i}{\partial d} &= 1 + \rho(d - y_i) \end{aligned}$$

If we take into account that $d \ll r_i$, $d \ll 1/\rho$, $y_i \ll 1/\rho$, $r_i \simeq x_i$ we can approximate the previous set of equations as

$$\begin{aligned}\frac{\partial\varepsilon_i}{\partial\alpha_1} = \frac{\partial\varepsilon_i}{\partial\rho} &= \frac{1}{2}r_i^2 \\ \frac{\partial\varepsilon_i}{\partial\alpha_2} = \frac{\partial\varepsilon_i}{\partial\phi} &= -r_i \\ \frac{\partial\varepsilon_i}{\partial\alpha_3} = \frac{\partial\varepsilon_i}{\partial d} &= 1\end{aligned}$$

If we have M measurement points, the partial derivatives matrix D will be $M \times 3$ and defined by

$$D_{i,j} = \frac{\partial\varepsilon_i}{\partial\alpha_j} \tag{6}$$

and the 3×3 weight matrix W will be (in matrix notation) $W = D^T C^{-1} D$.

2 Error estimate

2.1 Track parameters

Given the layer radii $x_n = x_1, x_2, \ldots, x_N$ with scattering angles $\theta_1, \theta_2, \ldots, \theta_3$, then the deviation from the ideal path y_n is

$$y_n = \sum_{i=1}^{n-1} (x_n - x_i)\,\theta_i \tag{7}$$

The angles θ_i are distributed as a Gaussian, with r.m.s. such that

$$\langle\theta^2\rangle = \left(\frac{13.6\,\mathrm{MeV}}{p}\right)^2 \frac{x}{X_0}\left[1 + 0.038\log\left(\frac{x}{X_0}\right)\right]^2 \tag{8}$$

The correlation between two deviations y_n, y_m is (we will assume without loss of generality that $m \geq n$)

$$a_{n,m}\,\langle y_n y_m\rangle = \left\langle \sum_{i=1}^{m-1} (x_m - x_i)\,\theta_i \times \sum_{j=1}^{n-1} (x_n - x_j)\,\theta_j \right\rangle \tag{9}$$

Since the angles θ_i are uncorrelated, any term containing in $\langle\theta_i\theta_j\rangle$ with $i \neq j$ will be zero, thus

$$\begin{aligned}a_{n,m} = \langle y_n y_m\rangle &= \left\langle \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} (x_m - x_i)(x_n - x_j)\,\theta_i\theta_j\delta_{i,j} \right\rangle \\ a_{n,m} = \langle y_n y_m\rangle &= \sum_{i=1}^{n-1} (x_m - x_i)(x_n - x_i)\,\langle\theta_i^2\rangle \end{aligned} \tag{10}$$

The measurement "error" depends both on the scattering of the real track with respect to the ideal case and also on the intrinsic measurement error σ_i, which depends approximately on the strip pitch p_i according to $\sigma_i = p_i/\sqrt{12}$, or $\sigma_i^2 = p_i^2/12$, thus the covariance matrix $b_{n,m}$ is

$$b_{n,m} = \begin{cases} \sum_{i=1}^{n-1} (x_m - x_i)(x_n - x_i)\,\langle\theta_i^2\rangle & n < m \\ p_n^2/12 + \sum_{i=1}^{n-1} (x_n - x_i)^2\,\langle\theta_i^2\rangle & n = m \\ b_{m,n} & n > m \end{cases} \tag{11}$$

Let's suppose we have N hits, but in these N only M are measurement points and $N - M$ are hits on inactive surfaces. In this matrix $b_{n,m}$ is computed exactly in the same way, but the rows and columns corresponding to the inactive hits are removed. We thus start from a $N \times N$ square matrix of correlations $b_{n,m}$ and we end up with a $M \times M$ measurement point covariance matrix $C_{n,m}$, or in matrix notation C.

3 Impact parameter quality

We assume that the primary vertex is known with a much better precision than the impact paramteter, so we place our reference system with the origin in the primary vertex.

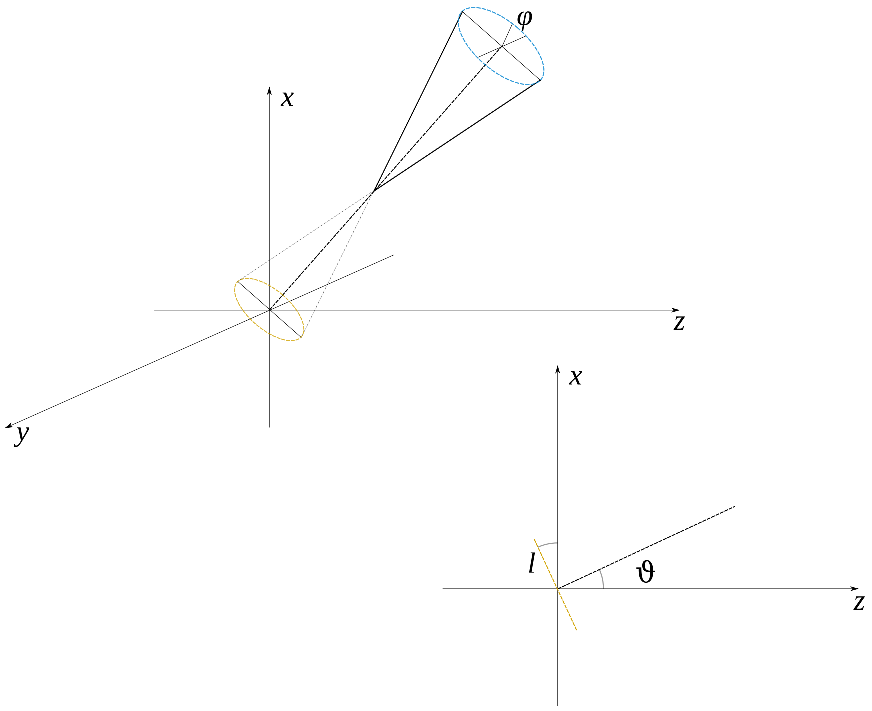

Figure 1: Impact parameter

For symmetry reasons we can assume that the meson we want to identify is created in the $\{x, z\}$ plane. The angle between the direction of flight and the z axis is ϑ.

The decay happens after a distance f, and the angle between the tracked decay product and the original meson is α.

The projection of the track on the plane containing the origin and perpendicular to the meson line of flight will be on a circle of radius $l = f\ \sin(\alpha)$. It can be noted that $f = \gamma\, c\, \tau$ and $\alpha \simeq 1/\gamma$, thus

$$l \simeq c\, \tau \tag{12}$$

Said plane will have axes $\vec{e_{x'}}$ and $\vec{e_y}$, with

$$\vec{e_{x'}} = \vec{e_x}\ \cos(\vartheta) - \vec{e_z}\ \sin(\vartheta) \tag{13}$$

If we call ϕ the angle running on this circle, the projection point has equation

$$\left\{ \begin{array}{ccl} x' & = & l\ \sin(\phi) \\ y & = & l\ \cos(\phi) \\ z' & = & 0 \end{array} \right. \tag{14}$$

that is

$$\left\{ \begin{array}{ccl} x & = & l\ \sin(\phi)\ \cos(\vartheta) \\ y & = & l\ \cos(\phi) \\ z & = & -l\ \sin(\phi)\ \sin(\vartheta) \end{array} \right. \tag{15}$$

Assuming we can measure the transverse impact parameter with resolution σ_{d0} and the longitudinal impact parameter with resolution σ_{z0}, then we want to combine these two measurements into one parameter t. In the $\{d_0, z_0\}$ plane the origin is $\frac{d_0}{\sigma_{d0}}$ standard deviations away along one axis and $\frac{z_0}{\sigma_{z0}}$ standard deviations away along the other, so a reasonable discriminant is

$$t^2 = \left(\frac{r}{\sigma_{d0}}\right)^2 + \left(\frac{z}{\sigma_{z0}}\right)^2 \tag{16}$$

with $r^2 = x^2 + y^2$ which leads to:

$$t^2 = \frac{l^2\,\cos^2(\vartheta)\,\sin^2(\phi) + l^2\,\cos^2(\phi)}{\sigma_{d0}^2} + \frac{l^2\,\sin^2(\vartheta)\,\sin^2(\phi)}{\sigma_{z0}^2} \tag{17}$$

Averaging on ϕ gives:

$$t^2 = \frac{l^2}{2}\left[\frac{\cos^2(\vartheta)+1}{\sigma_{d0}^2} + \frac{\sin^2(\vartheta)}{\sigma_{z0}^2}\right] \tag{18}$$

The significance t can thus be interpreted as the proper decay length l divided by its measurement error:

$$\frac{l}{\sigma_l} = l\sqrt{\frac{1}{2}\left[\frac{\cos^2(\vartheta)+1}{\sigma_{d0}^2} + \frac{\sin^2(\vartheta)}{\sigma_{z0}^2}\right]} \tag{19}$$

and

$$\sigma_l = \frac{1}{\sqrt{\frac{1}{2}\left[\frac{\cos^2(\vartheta)+1}{\sigma_{d0}^2} + \frac{\sin^2(\vartheta)}{\sigma_{z0}^2}\right]}} \tag{20}$$

and finally:

$$\sigma_l = \sqrt{\frac{2}{\frac{\cos^2(\vartheta)+1}{\sigma_{d0}^2} + \frac{\sin^2(\vartheta)}{\sigma_{z0}^2}}} \tag{21}$$

considering that $\gamma \gg 1$, then $\alpha \ll 1$ and thus the track angle $\theta \simeq \vartheta$.

$$\sigma_{c\tau} = \sqrt{\frac{2}{\frac{\cos^2(\theta)+1}{\sigma_{d0}^2} + \frac{\sin^2(\theta)}{\sigma_{z0}^2}}} \tag{22}$$

When looking at possible optimizations, one possible measurement is the relative increase of $\sigma_{c\tau}$ for an increase of σ_{d0} and σ_{z0}

$$F(\theta, \sigma_{d0}, \sigma_{z0}) = \frac{\frac{\partial\sigma_{c\tau}}{\partial\sigma_{d0}}}{\frac{\partial\sigma_{c\tau}}{\partial\sigma_{z0}}} = \frac{(\cos^2(\theta)+1)\,\sigma_{z0}^3}{\sin^2(\theta)\,\sigma_{d0}^3} \tag{23}$$

This function F could tell the relative gain of increasing σ_{d0} or σ_{z0}. One easy way of representing it is $\beta = \arctan(F(\theta, \sigma_{d0}, \sigma_{z0}))$. When $\beta \simeq 0$ the resolution is dominated by σ_{z0} and when $\beta \simeq \pi/2$ the resolution is dominated by σ_{d0}.

Another parameter for optimization comes from the observation that, to the first order, the resolution is proportional to the pixel size and that for a fixed channel count, one can double the resolution on one axis by halfening it on the other. In these terms it makes sense to express the resolutions as a function of their product $k = \sigma_{z0} \times \sigma_{d0}$ (which can be taken as a constraint) and their ratio $x = \sigma_{z0}/\sigma_{d0}$, which can be considered our optimization free variable (higher x means longer pixels). We want to maximize $B = 1/\sigma_l^2$, that is

$$B = \frac{1}{2}\left[\frac{\cos^2(\theta)+1}{k/x} + \frac{\sin^2(\theta)}{k\,x}\right] \tag{24}$$

$$\frac{\partial B}{\partial x} = \frac{\cos^2(\theta)+1+\frac{1}{x}\sin^2(\theta)}{2\,k} \tag{25}$$

As usual we can express this as an angle $\Omega = \arctan(\partial B/\partial x)$. For $\Omega = \pi/2$ it is better to have longer pixels and for $\Omega = -\pi/2$ it is better to have shorter (and wider) pixels. The optimal compromise is reached for $\Omega \simeq 0$.