

Given the layer radii $x_n = x_1, x_2, \dots, x_N$ with scattering angles $\theta_1, \theta_2, \dots, \theta_3$, then the deviation from the ideal path y_n is

$$y_n = \sum_{i=1}^{n-1} (x_n - x_i) \theta_i \quad (1)$$

The angles θ_i are distributed as a Gaussian, with r.m.s. such that

$$\langle \theta^2 \rangle = \left(\frac{13.6 \text{ MeV}}{p} \right)^2 \frac{x}{X_0} \left[1 + 0.038 \log \left(\frac{x}{X_0} \right) \right]^2 \quad (2)$$

The correlation between two deviations y_n, y_m is (we will assume without loss of generality that $m \geq n$)

$$C_{n,m} \langle y_n y_m \rangle = \left\langle \sum_{i=1}^{m-1} (x_m - x_i) \theta_i \times \sum_{i=1}^{n-1} (x_n - x_i) \theta_i \right\rangle \quad (3)$$

Since the angles θ_i are uncorrelated, any term containing in $\langle \theta_i \theta_j \rangle$ with $i \neq j$ will be zero, thus

$$\begin{aligned} C_{n,m} \langle y_n y_m \rangle &= \left\langle \sum_{i=1}^{m-1} \sum_{i=1}^{n-1} (x_m - x_i) (x_n - x_i) \theta_i \theta_j \delta_{i,j} \right\rangle \\ &= \sum_{i=1}^{n-1} (x_m - x_i) (x_n - x_i) \langle \theta_i^2 \rangle \end{aligned} \quad (4)$$

The measurement “error” depends both on the scattering of the real track with respect to the ideal case and also on the intrinsic measurement error σ_i , which depends approximately on the strip pitch p_i according to $\sigma_i = p_i/\sqrt{12}$, or $\sigma_i^2 = p_i^2/12$, thus the covariance matrix $c_{n,m}$ is

$$c_{n,m} = \begin{cases} \sum_{i=1}^{n-1} (x_m - x_i) (x_n - x_i) \langle \theta_i^2 \rangle & n < m \\ p_i^2/12 + \sum_{i=1}^{n-1} (x_n - x_i)^2 \langle \theta_i^2 \rangle & n = m \\ c_{m,n} & n > m \end{cases} \quad (5)$$