Subject:Commitator

$$[AB+CD,EF+GH]=[AB,EF]+[AB,GH]+[CD,EF]+[CD,GH]$$

$$(1)[AB-CD,EF-GH]=[AB,EF]-[AB,GH]-[CD,EF]+[CD,GH]$$

The top four sections of each opened section are:

$$(2)[AB,CD]=AB\cdot CD-CD\cdot AB$$

$$(3)[AB,CD]=A[B,CD]+[A,CD]B$$

$$(4)[B,CD]=[B,C]D+C[B,D]$$

$$(5)[A,CD]=[A,C]D+C[A,D]$$

$$\boxed{(6)[AB,CD]=A[B,C]D+AC[B,D]+[A,C]DB+C[A,D]B}$$

The sixth mode is for one of the four modes.
Example:

$$\vec{L}=\vec{r}\times\vec{p}$$

$$\vec{L}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ p_x & p_y & p_z\end{vmatrix}=\hat{i}(yp_z-zp_y)-\hat{j}(xp_z-zp_x)+\hat{k}(xp_y-yp_x)$$

use 1 , 2 , 3 , 4 and 5 This was only for one of the four episodes above.

$$L_z=xp_y-yp_x$$

$$\hat{L}_x=\hat{y}\,\hat{p}_z-\hat{z}\,\hat{p}_y,\qquad \hat{L}_y=\hat{z}\,\hat{p}_x-\hat{x}\,\hat{p}_z$$

use (1)

$$[\hat{L}_x,\hat{L}_y]=i\hbar\,\hat{L}_z$$

$$\hat{L}^2=\hat{L}_x^2+\hat{L}_y^2+\hat{L}_z^2$$

$$[\hat{L}^2,\hat{L}_x]=[\hat{L}_x^2,\hat{L}_x]+[\hat{L}_y^2,\hat{L}_x]+[\hat{L}_z^2,\hat{L}_x]$$

$$[\hat{L}_x^2,\hat{L}_x]=0$$

$$[A^2,B]=A[A,B]+[A,B]A$$

$$[AA,B]$$

with $A=\hat{L}_y$, $B=\hat{L}_x$:

$$[\hat{L}_y^2,\hat{L}_x]=\hat{L}_y[\hat{L}_y,\hat{L}_x]+[\hat{L}_y,\hat{L}_x]\hat{L}_y$$

$[\hat{L}_y,\hat{L}_x]=-i\hbar\,\hat{L}_z$:

$$[\hat{L}_y^2,\hat{L}_x]=-i\hbar(\hat{L}_y\hat{L}_z+\hat{L}_z\hat{L}_y)$$

$$[\hat{L}_z^2,\hat{L}_x]=\hat{L}_z[\hat{L}_z,\hat{L}_x]+[\hat{L}_z,\hat{L}_x]\hat{L}_z$$

Using $[\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y$:

$$[\hat{L}_z^2, \hat{L}_x] = i\hbar(\hat{L}_z\hat{L}_y + \hat{L}_y\hat{L}_z)$$

$$-i\hbar(\hat{L}_y\hat{L}_z + \hat{L}_z\hat{L}_y) + i\hbar(\hat{L}_z\hat{L}_y + \hat{L}_y\hat{L}_z) = 0$$

$$\boxed{[\hat{L}^2, \hat{L}_x] = 0}$$

$$\hat{L}_+ = \hat{L}_x + i\hat{L}_y$$
$$\hat{L}_- = \hat{L}_x - i\hat{L}_y$$

$$\hat{L}_z\,|\ell, m\rangle = \hbar m\,|\ell, m\rangle$$
$$\hat{L}_\pm\,|\ell, m\rangle = \hbar\sqrt{\ell(\ell+1) - m(m\pm 1)}\,|\ell, m\pm 1\rangle$$

$$-\ell \leq m \leq \ell$$

$$\hat{L}_+|\ell, \ell\rangle = 0, \quad \hat{L}_-|\ell, -\ell\rangle = 0$$

Consider the state $|\ell = 1, m = 0\rangle$.
Applying the raising operator $\hat{L}_+$:

$$\hat{L}_+|\ell = 1, m = 0\rangle = \hbar\sqrt{1\cdot(1+1) - 0\cdot(0+1)}\,|\ell = 1, m = 1\rangle = \hbar\sqrt{2}\,|\ell = 1, m = 1\rangle$$

Similarly, applying the lowering operator $\hat{L}_-$ on $|\ell = 1, m = 0\rangle$:

$$\hat{L}_-|\ell = 1, m = 0\rangle = \hbar\sqrt{1\cdot(1+1) - 0\cdot(0-1)}\,|\ell = 1, m = -1\rangle = \hbar\sqrt{2}\,|\ell = 1, m = -1\rangle$$

Here, the size does not increase or decrease with the up and down arrows, only their direction is changed.