Quantum Exercise Book

Abbas Abedini

September 15, 2025

Contents

1 Probability

$$|C_n|^2 \;=\; Probabilityoffindingthesysteminstaten$$

(1) [[Problem 8.2 of Griffith's book]]A particle of mass m in an infinite square well (of width a) is initially located in the left half of the well and (at time t = 0) is found with equal probability at any point in that region.

(a) What is the initial wave function (x,O)? (Assume that this function is real. Don't forget to normalize it).

(b) What is the probability that an energy measurement will yield a value of n2h2/2ma2?

$$(a) \quad \Psi(x,0) = \{\, A \,, 0 < x < a2, 0, otherwise,$$

$$1 = \int_{-\infty}^{\infty} |\Psi(x,0)|^2 dx = \int_0^{a/2} A^2 \, dx = A^2 \left(\frac{a}{2}\right),$$

$$\Rightarrow \quad A = \sqrt{\frac{2}{a}}.$$

—

$$(b) \quad c_1 = \int_0^a \Psi(x,0)\, \phi_1(x)\, dx, \qquad \phi_1(x) = \sqrt{2a}\sin\left(\pi a x\right).$$

$$c_1 \;=\; \sqrt{\frac{2}{a}} \int_0^{a/2} \sqrt{\frac{2}{a}} \sin\left(\frac{\pi}{a}x\right)\, dx = \frac{2}{a}\int_0^{a/2} \sin\left(\frac{\pi}{a}x\right)\, dx.$$

$$\int_0^{a/2} \sin\left(\frac{\pi}{a}x\right)\, dx = \left[-\frac{a}{\pi}\cos\left(\frac{\pi}{a}x\right)\right]_0^{a/2} = -\frac{a}{\pi}\left[\cos\left(\pi 2\right) - \cos(0)\right].$$

$$= -\frac{a}{\pi}(0-1) = \frac{a}{\pi}.$$

$$\Rightarrow \quad c_1 = \frac{2}{a}\cdot\frac{a}{\pi} = \frac{2}{\pi}.$$

—

$$P_1 = |c_1|^2 = \left(\frac{2}{\pi}\right)^2 \approx 0.4053.$$

We add one thing to it and solve for the other case.

$$c_2 = \int_0^a \Psi(x,0)\,\phi_2(x)\,dx, \qquad \phi_2(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{2\pi}{a}x\right)$$

$$c_2 = \sqrt{\frac{2}{a}}\int_0^{a/2}\sqrt{\frac{2}{a}}\sin\left(\frac{2\pi}{a}x\right)dx = \frac{2}{a}\int_0^{a/2}\sin\left(\frac{2\pi}{a}x\right)dx$$

$$\int_0^{a/2}\sin\left(\frac{2\pi}{a}x\right)dx = \left[-\frac{a}{2\pi}\cos\left(\frac{2\pi}{a}x\right)\right]_0^{a/2} = -\frac{a}{2\pi}\Big[\cos(\pi)-\cos(0)\Big]$$

$$= -\frac{a}{2\pi}\Big[-1-1\Big] = \frac{a}{\pi}$$

$$\Rightarrow \quad c_2 = \frac{2}{a}\cdot\frac{a}{\pi} = \frac{2}{\pi}$$

$$P_2 = |c_2|^2 = \left(\frac{2}{\pi}\right)^2 \approx 0.4053$$

General state of the function:

$$C_n = \sum_{n=1}^{\infty}\frac{a}{n\pi}\big(1-\cos(n\pi/2)\big)$$

$$\psi(x) = \sum_{n=1}^{\infty} C_n\psi_n(x) = \sum_{n=1}^{\infty} n\pi a\left(1-\cos\left(\frac{n\pi}{2}\right)\right)\sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi x}{L}\right)$$

Look at the top, you'll notice that the energy is for the first state, so we calculated the probability for the first state.

$$V(x) = \{\,0\ ,0<x<a,\infty, otherwise,$$

$$\phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right), \quad n = 1,2,3,\ldots$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad n = 1,2,3,\ldots$$

---

(2) n=1 for problem 1)
if n=2 use c2
[Exercise number nine, chapter two, Gasiurovich's book
]
The wave function of a particle placed in the left half of a box whose walls are at 2/r=a is as follows:
Does the valley remain in place at later times? (b) What is the probability that the measurement of the valley energy will yield the ground state energy? What about the energy of the first excited state? Initial wave function:

$$\Psi(x,0) = \{ \ A \ , 0 < x < \frac{a}{2}, 0, \frac{a}{2} < x < a$$

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \phi_n(x) e^{-iE_n t/\hbar}, \quad \phi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right), \quad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$c_n = \int_0^a \Psi(x,0)\phi_n(x)\,dx = \int_0^{a/2} A\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) dx = \frac{2}{n\pi}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]$$

For the ground state (n=1):

$$c_1 = \frac{2}{\pi}(1 - \cos(\pi/2)) = \frac{2}{\pi}, \quad P_1 = |c_1|^2 \approx 0.4053$$

For the first excited state (n=2):

$$c_2 = \frac{2}{2\pi}(1 - \cos(\pi)) = \frac{2}{\pi}, \quad P_2 = |c_2|^2 \approx 0.4053$$

---

(3) [[Exercise number seven, chapter two of Griffith's book]]7.2 A particle in an infinite square well has an initial wave function of the form:

$$y(x,0) = \{ \ A \ x, 0 < x < \frac{a}{2}, A(a-x), \frac{a}{2} < x < a, 0, otherwise.$$

What is the probability that the energy measurement will give the value E1?
[12pt]article amsmath, amssymb
Normalization condition:

$$\int_0^a |y(x,0)|^2\,dx = 1$$

$$\int_0^{a/2} (Ax)^2\,dx + \int_{a/2}^a [A(a-x)]^2\,dx = 1$$

$$A^2\int_0^{a/2} x^2\,dx + A^2\int_{a/2}^{a}(a-x)^2\,dx = A^2\left[\frac{(a/2)^3}{3}+\frac{(a/2)^3}{3}\right] = A^2\frac{a^3}{12}$$

Solve for A:

$$A^2\frac{a^3}{12} = 1 \quad\Rightarrow\quad A = \frac{2\sqrt{3}}{a^{3/2}}$$

$$We want to calculate: \quad \int_0^{a/2} x\sin\left(\frac{\pi x}{a}\right)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$$u = x \quad\Rightarrow\quad du = dx, \quad dv = \sin\left(\frac{\pi x}{a}\right)dx \quad\Rightarrow\quad v = -\frac{a}{\pi}\cos\left(\frac{\pi x}{a}\right)$$

$$\int x\sin\left(\frac{\pi x}{a}\right)dx = -\frac{a}{\pi}x\cos\left(\frac{\pi x}{a}\right)+\frac{a}{\pi}\int\cos\left(\frac{\pi x}{a}\right)dx = -\frac{a}{\pi}x\cos\left(\frac{\pi x}{a}\right)+\frac{a^2}{\pi^2}\sin\left(\frac{\pi x}{a}\right)$$

$$x = a/2: \quad -\frac{a}{\pi}\cdot\frac{a}{2}\cos\left(\frac{\pi}{2}\right)+\frac{a^2}{\pi^2}\sin\left(\frac{\pi}{2}\right) = \frac{a^2}{\pi^2}$$

$$x = 0: \quad -\frac{a}{\pi}\cdot 0\cdot\cos(0)+\frac{a^2}{\pi^2}\sin(0) = 0$$

$$\int_0^{a/2} x\sin\left(\frac{\pi x}{a}\right)dx = \frac{a^2}{\pi^2}$$

$$We want to calculate: \quad \int_{a/2}^{a}\sin\left(\frac{\pi x}{a}\right)(a-x)\,dx$$

$$\int_{a/2}^{a}\sin\left(\frac{\pi x}{a}\right)(a-x)\,dx = \int_{a/2}^{a} a\sin\left(\frac{\pi x}{a}\right)dx - \int_{a/2}^{a} x\sin\left(\frac{\pi x}{a}\right)dx$$

$$\int_{a/2}^{a} a\sin\left(\frac{\pi x}{a}\right)dx = a\int_{a/2}^{a}\sin\left(\frac{\pi x}{a}\right)dx = a\left[-\frac{a}{\pi}\cos\left(\frac{\pi x}{a}\right)\right]_{a/2}^{a} = -\frac{a^2}{\pi}\left(\cos(\pi)-\cos\left(\frac{\pi}{2}\right)\right) = \frac{a^2}{\pi}$$

$$\int_{a/2}^{a} x\sin\left(\frac{\pi x}{a}\right)dx$$

Let $u = x, \quad dv = \sin\left(\frac{\pi x}{a}\right) dx$
Then $du = dx, \quad v = -\frac{a}{\pi}\cos\left(\frac{\pi x}{a}\right)$

$$\int x \sin\left(\frac{\pi x}{a}\right) dx = -\frac{a}{\pi} x \cos\left(\frac{\pi x}{a}\right) + \frac{a}{\pi}\int \cos\left(\frac{\pi x}{a}\right) dx = -\frac{a}{\pi} x \cos\left(\frac{\pi x}{a}\right) + \frac{a^2}{\pi^2}\sin\left(\frac{\pi x}{a}\right)$$

Evaluate from $x = a/2$ to $x = a$:

$$\left[-\frac{a}{\pi} x \cos\left(\frac{\pi x}{a}\right) + \frac{a^2}{\pi^2}\sin\left(\frac{\pi x}{a}\right)\right]_{a/2}^{a} = \frac{a^2}{\pi} - \frac{a^2}{\pi^2} = \frac{a^2(\pi - 1)}{\pi^2}$$

$$\int_{a/2}^{a} \sin\left(\frac{\pi x}{a}\right)(a - x)\, dx = \frac{a^2}{\pi} - \frac{a^2(\pi - 1)}{\pi^2} = \frac{a^2}{\pi^2}$$

$$\frac{a^2}{\pi^2}$$

$$\sqrt{\frac{2}{a}} \times \sqrt{\frac{12}{a^3}} \times \left(\frac{a^2}{\pi^2} + \frac{a^2}{\pi^2}\right)$$

$$P_1 = 2\frac{\sqrt{24}}{\pi^2}$$

$$P_1 = |c_1|^2$$

Here we only wrote for one of the cases. I want to include the general case in the equation bout =¿
This is the general situation:

$$\frac{a^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) + \left[\frac{a^2}{n\pi}\cos(n\pi) - \frac{a^2}{n\pi}\cos\left(\frac{n\pi}{2}\right)\right]$$

$$-\left[\frac{a}{n\pi}\cos(n\pi) - \frac{a}{2}\cdot\frac{a}{n\pi}\cos\left(\frac{n\pi}{2}\right)\right] + \left[\frac{a^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) + \frac{a^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

---

(4)

[[Exercise number twelve, chapter two, Gasiurovich's book]]

A particle is initially in an infinite potential well with walls at $x = 0$ and $x = a$. Suddenly, the right wall is moved to $x = b$ ($b > a$, for example $b = 3a$). We want to find the probability that the particle is found in the ground state of the new well. What is the probability that it will be found in the first excited state? The latter has a simple answer. What is the answer?

Previously, in special cases, we examined special states in the well, but in the general case, we must choose a state that is without nodes, meaning that the probability at a point should not be zero.

$$cos(\frac{\pi x}{2a})$$

Send me an email so I can send you the reason.

`abbasabedini88@gmail.com`

$$I \ = 1/2 \int_{-a}^{a} \cos\left(\frac{\pi x(b-a)}{2ab}\right) dx \ + \ \int_{-a}^{a} \cos\left(\frac{\pi x(b+a)}{2ab}\right) dx$$

$$k_1 = \frac{\pi(b-a)}{2ab}, \qquad k_2 = \frac{\pi(b+a)}{2ab}$$

$$I = \int_{-a}^{a} \cos(k_1 x)\, dx + \int_{-a}^{a} \cos(k_2 x)\, dx$$

$$\int_{-a}^{a} \cos(kx)\, dx = 2\int_{0}^{a} \cos(kx)\, dx = \frac{2}{k}\sin(ka)$$

$$I = \frac{2}{k_1}\sin(k_1 a) + \frac{2}{k_2}\sin(k_2 a)$$

$$I = \frac{4ab}{\pi(b-a)}\sin\left(\frac{\pi(b-a)}{2b}\right) + \frac{4ab}{\pi(b+a)}\sin\left(\frac{\pi(b+a)}{2b}\right)$$

$$I = \frac{4ab}{\pi(b^2-a^2)}\left[(b+a)\sin\left(\frac{\pi(b-a)}{2b}\right) + (b-a)\sin\left(\frac{\pi(b+a)}{2b}\right)\right]$$

$$\alpha = \frac{\pi(b-a)}{2b}, \qquad \beta = \frac{\pi(b+a)}{2b}$$

$$\sin(\alpha) = \sin\Big(\pi 2 - \pi a 2b\Big) = \cos\Big(\pi a 2b\Big),$$

$$\sin(\beta) = \sin\Big(\pi 2 + \pi a 2b\Big) = \cos\Big(\pi a 2b\Big)$$

$$(b+a)\sin(\alpha) + (b-a)\sin(\beta) = 2b\cos\Big(\pi a 2b\Big)$$

$$I = \frac{4ab}{\pi(b^2 - a^2)} \cdot 2b\cos\Big(\pi a 2b\Big)$$

$$I = 1/2 * \frac{8ab^2}{\pi(b^2 - a^2)} \cos\Big(\pi a 2b\Big)$$

normailztion :

$$Normalization: \quad A = \frac{1}{\sqrt{ab}}$$

$$C_1 = \frac{4\sqrt{ab}\, b}{\pi(b^2 - a^2)}$$

This section has additional explanations. Please email to send.:

—-

(5) Finding Probability in an Infinite Well

I made a pattern:

$$P = \left[\frac{b-a}{L} - \frac{1}{2n\pi}\left(\sin\left(\frac{2n\pi b}{L}\right) - \sin\left(\frac{2n\pi a}{L}\right)\right)\right]$$

For the range 0.25 to 0.75 for the base case:

$$P = \left[\frac{0.25 - 0.75}{L} - \frac{1}{2 \cdot 1 \cdot \pi}\left(\sin\left(\frac{2 \cdot 1 \cdot \pi \cdot 0.75}{L}\right) - \sin\left(\frac{2 \cdot 1 \cdot \pi \cdot 0.25}{L}\right)\right)\right]$$

=¿ 0.81

$Angle()$	$\sin\theta$	$\cos\theta$
0	0	1
45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
90	1	0
180	0	-1
270	-1	0
360	0	1

Let's better tell where this pattern came from.

$$P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} |\psi_n(x)|^2 \, dx$$

Since I am investigating in an infinite well, I write the function for an infinite potential well.

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \quad n = 1, 2, 3, \ldots$$