Electric Power Markets **Spring 2023** **ENV 717**

Nicholas School of the Environment Duke University

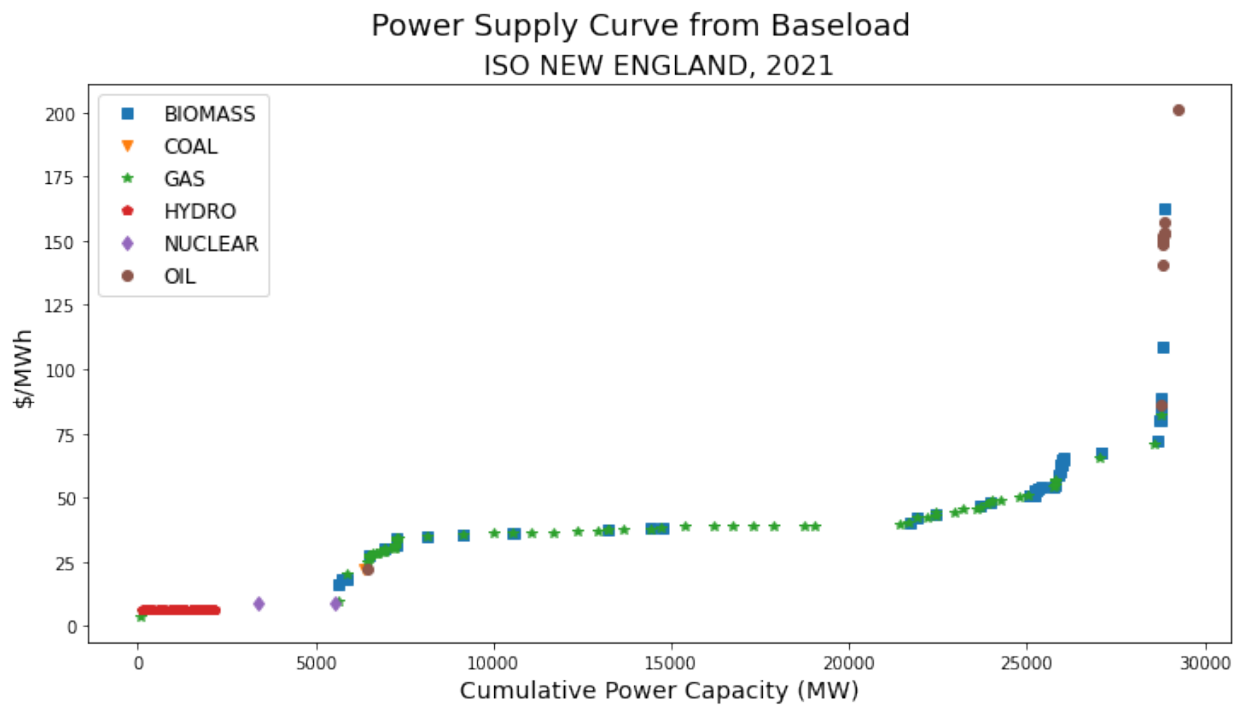
Assignment #4
****Submit a word document with your answers together with an excel or python file for problem 1***

1. (25%)
Extreme weather conditions could cause outages on power plants and transmission lines across New England. Use eGrid2021 data and data on peak electricity consumption to analyze the composition of ISO New England's power generation assets and their adequacy.

a. Summarize the total power generation capacity by fuel and the total electricity generation in 2021. Present totals and percentages as shown in the table below.

Fuel Type	Installed capacity in ISONE in 2021		Generation in 2021	
	MW	% of total installed capacity in ISONE	MWh	% of total generation capacity in ISONE
Biomass	1,405.40	3.54	6,940,133.26	6.77
Coal	496.40	1.25	285,553.00	0.28
Gas	22,289.70	56.13	56,149,245.02	54.77
Geothermal	0	0	0	0
Hydro	3,837.10	9.66	5,790,572.89	5.65
Nuclear	3,404.90	8.57	27,072,626.00	26.41
Oil	4,483.80	11.29	158,024.59	0.15
Other fossil fuel	0	0	0	0
Solar	2,228.00	5.61	2,456,018.00	2.40
Wind	1,523.00	3.84	3,662,787.00	3.57
Other fuel	42.50	0.11	-844.00	0.00

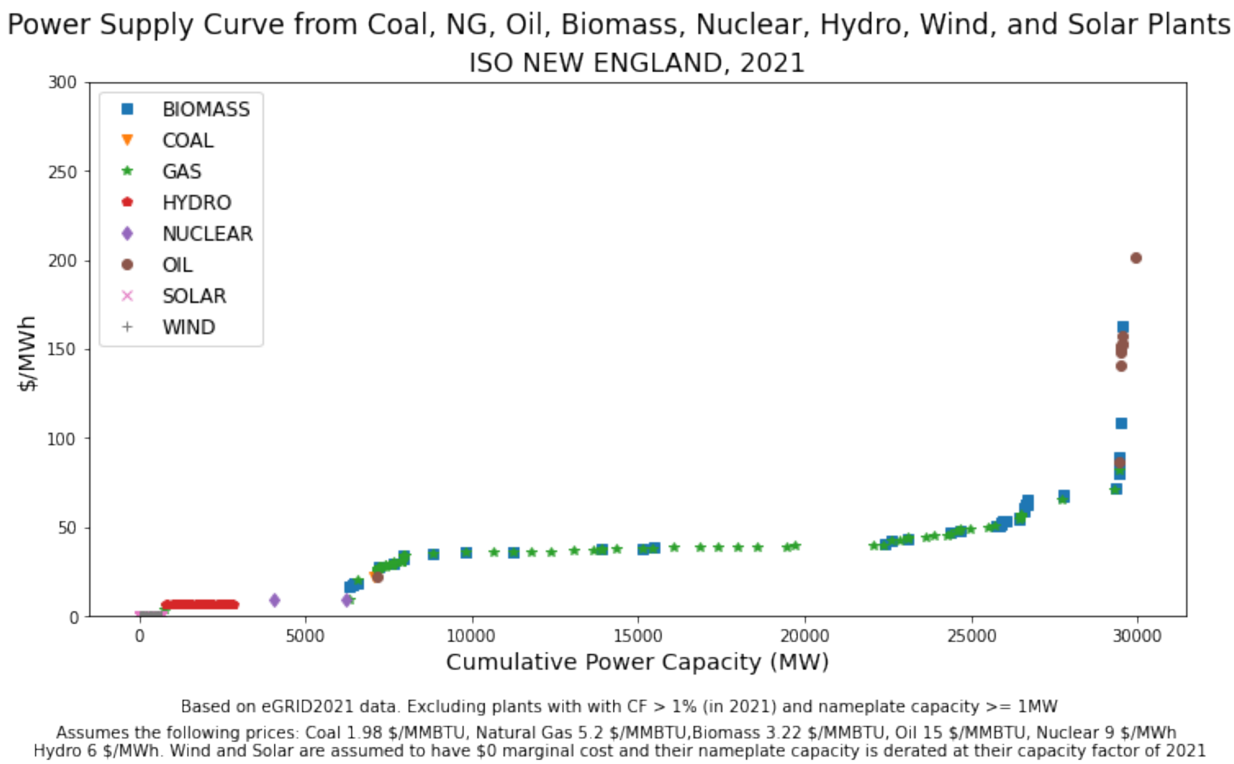
b. Build an electric power supply curve using baseload power plants that have a capacity factor above 1% and a nameplate capacity above 1MW. Assume a competitive wholesale electricity market and make any necessary assumptions for fuel prices. Respond: what is the highest demand the system can serve with these resources?



Based on eGRID2021 data. Excluding plants with with CF > 1% (in 2021) and nameplate capacity >= 1MW
Assumes the following prices: Coal 1.98 $/MMBTU, Natural Gas 5.2 $/MMBTUBiomass 3.22 $/MMBTU, Oil 15 $/MMBTU,Nuclear 9 $/MWh, Hydro 6 $/MWh

The maximum demand that can be served in ISO New England with baseload resources that have an installed capacity of more than 1MW and in 2021 had a capacity factor of more than 1% is: 29,352.70 MW.

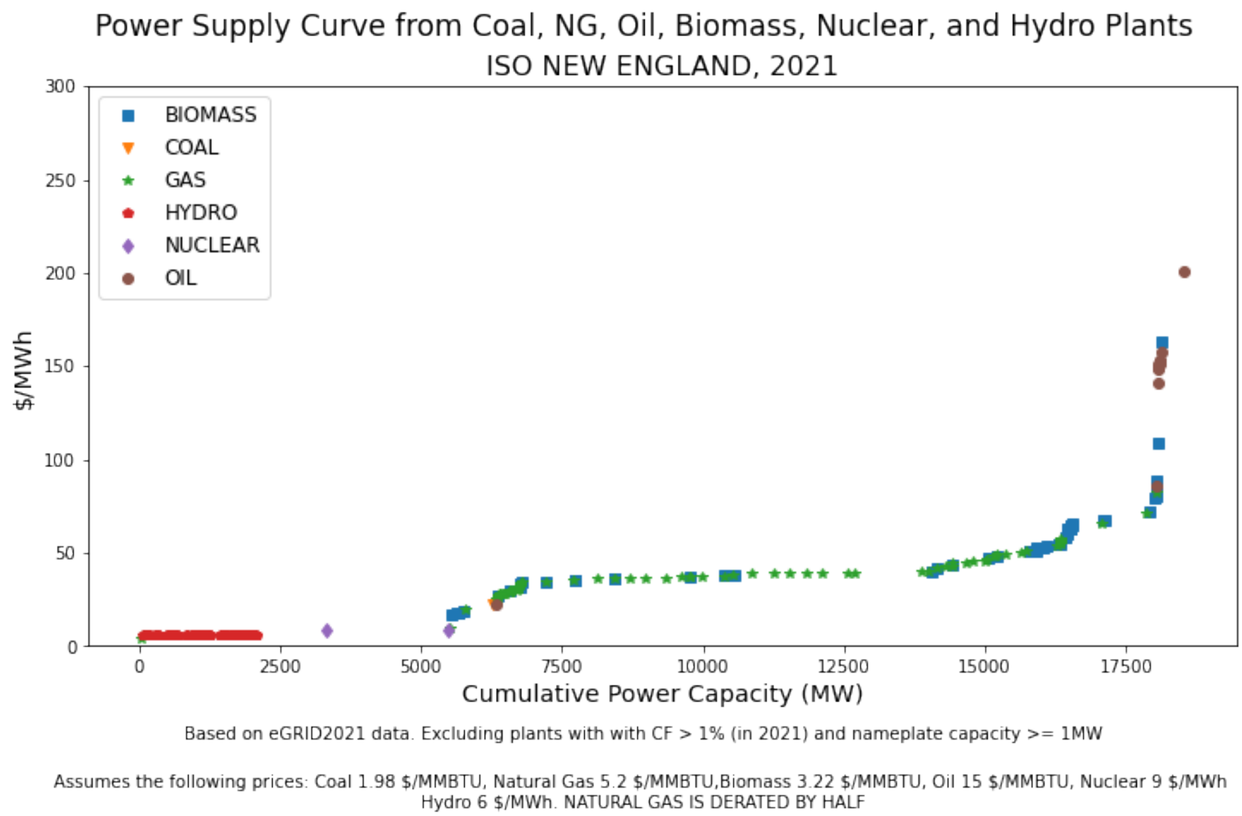
c. Repeat b. adding solar and wind power generation resources. Of course, solar and wind resources cannot be counted at full nameplate capacity. Assume that wind and solar resources capacity is equal to their nameplate capacity, de-rated by their average 2021 capacity factor. Respond: what is the highest demand the system can serve with these resources? How much more compared to b.?



The maximum demand that can be served in ISO New England with baseload resources PLUS SOLAR AND WIND that have an installed capacity of more than 1MW and in 2021 had a capacity factor of more than 1% is: 30,037.79 MW.

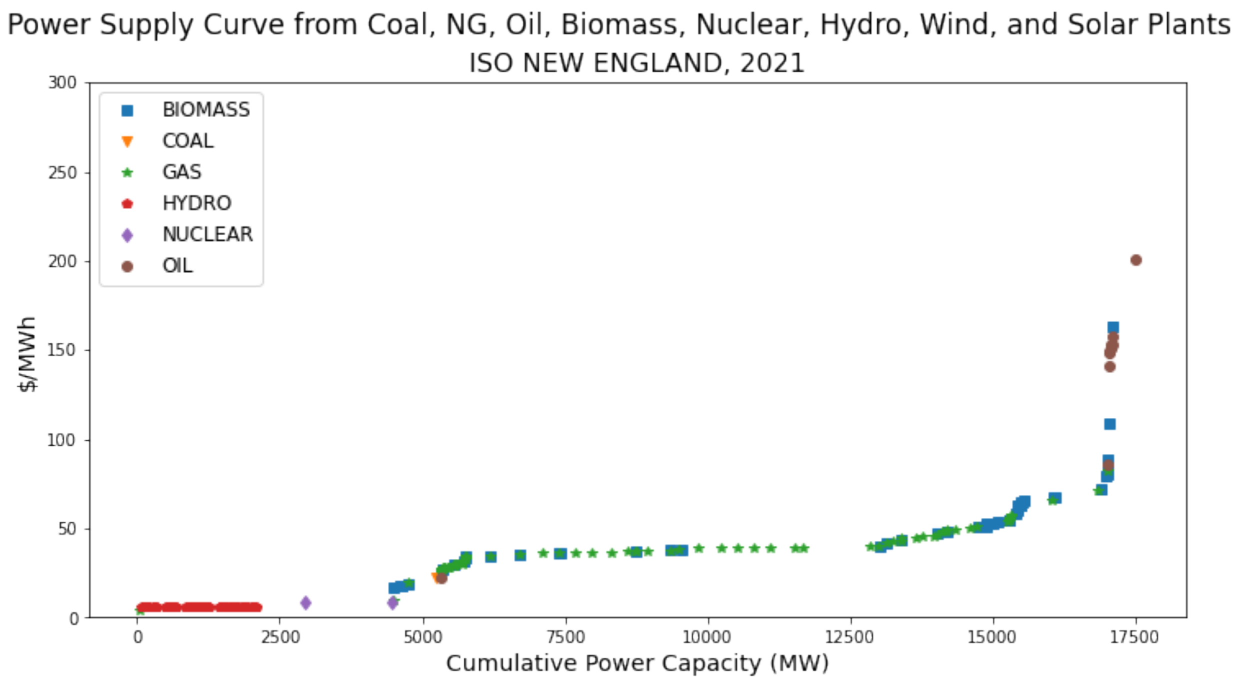
This is a total of 685.09 MW more than if just baseload plants are considered.

d. What would happen if natural gas demand to heat homes disrupted the supply of fuel to power generators? Repeat b but this time assume that only half of the natural gas capacity is available due to fuel supply disruptions. What is the highest electricity demand that can be served under these conditions?



The maximum demand that can be served in ISO New England with the power plants contained in the PLNT21 tab of eGRID2021, that had a capacity of more than 1 MW and a CF of more than 1%, derating the capacity of NATURAL GAS plants by half is 18,588.85 MW.

e. Repeat d. but consider the situation when supply from nuclear power plants is reduced to 70% of the nameplate capacity. What is the highest electricity demand that can be served under these conditions?



Based on eGRID2021 data. Excluding plants with with CF > 1% (in 2021) and nameplate capacity >= 1MW

Assumes the following prices: Coal 1.98 $/MMBTU, Natural Gas 5.2 $/MMBTU,Biomass 3.22 $/MMBTU, Oil 15 $/MMBTU, Nuclear 9 $/MWh Hydro 6 $/MWh. NUCLEAR CAPACITY IS DERATED TO 70% OF NAMEPLATEand NATURAL GAS IS DERATED BY HALF

The maximum demand that can be served in ISO New England with the power plants contained in the PLNT21 tab of eGRID2021, that had a capacity of more than 1 MW and a CF of more than 1%, and derating the capacity of NUCLEAR plants to 70% of nameplate is 17,567.38 MW.

f. The highest demand observed on a winter day was 22,818 MW (on Thursday 01/15/2014[1]). Is ISONE power supply adequate to meet the peak demand plus reserves?

The maximum demand that can be served in ISO New England with the power plants contained in the PLNT21 tab of eGRID21, that had a capacity of more than 1 MW and a CF of more than 1%, with solar and wind plants derated at their capacity factor of 2021 is 30,037.79 (if solar and wind resources are not included). So, in theory the ISONE could satisfy this demand plus reserves. Nevertheless, a disruption of some of its plants could affect its ability to satisfy this demand, for example, a derating in the capacity of Natural Gas plants by 50% would reduce its capacity to 21,929.05 MW (18,588.85 MW if solar and wind plants are not included). If these kinds of de-ratings of capacity occurred during the time of high demand, ISONE would not be able to meet demand plus reserves, and outages would occur.

[1] See [Electricity Use (iso-ne.com)]

2. (50%) Unit commitment problem[2]:

A balancing authority needs to solve the UC problem for a 3-hours planning horizon. Three thermal generating units are used to supply demands of 160MW, 500MW and 400MW in time periods 1, 2 and 3, respectively. Data on technical limits and economic parameters of the generating units is provided in Table 1. Required spinning reserves during each of the three periods are 160MW, 50MW, and 40MW but we will ignore them in the formulation of this UC problem.

Generating Unit	C_g^F (1)	C_g^{SD} (2)	C_g^{SU} (3)	C_g^V (4)	$P_g^{\max}$ (5)	$P_g^{\min}$ (6)	R_g^D (7)	R_g^{SD} (8)	R_g^{SU} (9)	R_g^U (10)
1	5	0.5	20	0.1	350	50	300	300	200	200
2	7	0.3	18	0.125	200	80	150	150	100	100
3	6	1	5	0.15	140	40	100	100	100	100

Table 1: Technical characteristics of generators. (1) Online (fixed) cost of generating unit g ($/h). (2) shut-down cost of generating unit g ($). (3) Start-up cost of generating unit g ($). (4) Variable cost of generating unit g ($/MWh). (5) Power generation capacity of unit g (MW). (6) minimum power output of unit g (MW). (7) ramping-down limit of generating unit g (MW/h). (8) shut-down ramping limit of generating unit g (MW/h). (9) start-up ramping limit of generating unit g(MW/h). (10) ramping-up limit of generating unit g (MW/h).

Assume that at time t=0, units #1 and # 2 are off-line while generating unit# 3 is online and producing 100MW.

Formulate the unit commitment (UC) problem spelling out all the values and constraints.

a) (2%) Write down a list of all the decision variables, spelling out each of them (i.e., do not use the indices to refer to types of decision variables, but rather list them all). The first decision variable in your list is: $p_{1,1}$=power produced by generator # 1 in hour1. The second element in the list is: $p_{1,1}$=power produced by generator # 1 in hour 2, and so on.

$p_{1,1}$ Power produced by generator 1 in hour 1
$p_{1,2}$ Power produced by generator 1 in hour 2
$p_{1,3}$ Power produced by generator 1 in hour 3
$p_{2,1}$ Power produced by generator 2 in hour 1
$p_{2,2}$ Power produced by generator 2 in hour 2
$p_{2,3}$ Power produced by generator 2 in hour 3
$p_{3,1}$ Power produced by generator 3 in hour 1
$p_{3,2}$ Power produced by generator 3 in hour 2
$p_{3,3}$ Power produced by generator 3 in hour 3
$u_{1,1}$ On/Off status for generator # 1 in hour 1
$u_{1,2}$ On/Off status for generator # 1 in hour 2
$u_{1,3}$ On/Off status for generator # 1 in hour 3
$u_{2,1}$ On/Off status for generator # 2 in hour 1
$u_{2,2}$ On/Off status for generator # 2 in hour 2
$u_{2,3}$ On/Off status for generator # 2 in hour 3
$u_{3,1}$ On/Off status for generator # 3 in hour 1

[2] For this problem and the next one you can submit handwritten formulation -inserted in your word document- as long as everything is neat and clear.

$u_{3,2}$ On/Off status for generator # 3 in hour 2
$u_{3,3}$ On/Off status for generator # 3 in hour 3
$y_{1,1}$ Start-up status for generator # 1 in hour 1
$y_{1,2}$ Start-up status for generator # 1 in hour 2
$y_{1,3}$ Start-up status for generator # 1 in hour 3
$y_{2,1}$ Start-up status for generator # 2 in hour 1
$y_{2,2}$ Start-up status for generator # 2 in hour 2
$y_{2,3}$ Start-up status for generator # 2 in hour 3
$y_{3,1}$ Start-up status for generator # 3 in hour 1
$y_{3,2}$ Start-up status for generator # 3 in hour 2
$y_{3,3}$ Start-up status for generator # 3 in hour 3
$z_{1,1}$ Shut-down status for generator # 1 in hour 1
$z_{1,2}$ Shut-down status for generator # 1 in hour 2
$z_{1,3}$ Shut-down status for generator # 1 in hour 3
$z_{2,1}$ Shut-down status for generator # 2 in hour 1
$z_{2,2}$ Shut-down status for generator # 2 in hour 2
$z_{2,3}$ Shut-down status for generator # 2 in hour 3
$z_{3,1}$ Shut-down status for generator # 3 in hour 1
$z_{3,2}$ Shut-down status for generator # 3 in hour 2
$z_{3,3}$ Shut-down status for generator # 3 in hour 3

b) (5%) Write down the objective function spelling out all the terms and writing down the values of all the parameters. Explain this function in words

$$\begin{aligned}Min\ [&C_1^V(p_{1,1}+p_{1,2}+p_{1,3})+C_2^V(p_{2,1}+p_{2,2}+p_{2,3})+C_3^V(p_{3,1}+p_{3,2}+p_{3,3})+C_1^F(u_{1,1}+u_{1,2}+u_{1,3})\\&+C_2^F(u_{2,1}+u_{2,2}+u_{2,3})+C_3^F(u_{3,1}+u_{3,2}+u_{3,3})+C_1^{SU}(y_{1,1}+y_{1,2}+y_{1,3})\\&+C_2^{SU}(y_{2,1}+y_{2,2}+y_{2,3})+C_3^{SU}(y_{3,1}+y_{3,2}+y_{3,3})+C_1^{SD}(z_{1,1}+z_{1,2}+z_{1,3})\\&+C_2^{SD}(z_{2,1}+z_{2,2}+z_{2,3})+C_3^{SD}(z_{3,1}+z_{3,2}+z_{3,3})]\end{aligned}$$

$$\begin{aligned}Min\ [&0.1(p_{1,1}+p_{1,2}+p_{1,3})+0.125(p_{2,1}+p_{2,2}+p_{2,3})+0.15(p_{3,1}+p_{3,2}+p_{3,3})\\&+5(u_{1,1}+u_{1,2}+u_{1,3})+7(u_{2,1}+u_{2,2}+u_{2,3})+6(u_{3,1}+u_{3,2}+u_{3,3})\\&+20(y_{1,1}+y_{1,2}+y_{1,3})+18(y_{2,1}+y_{2,2}+y_{2,3})+5(y_{3,1}+y_{3,2}+y_{3,3})\\&+0.5(z_{1,1}+z_{1,2}+z_{1,3})+0.3(z_{2,1}+z_{2,2}+z_{2,3})+1(z_{3,1}+z_{3,2}+z_{3,3})]\end{aligned}$$

This function minimizes total costs for all generators and time periods. Total costs include the costs of turning on and off, costs of being online, and costs per unit of electricity generated.

c) (7%) Write down all the constraints that relate the operating status of the plants with the indicator decision variables for shutting down and starting up. These are 18 equations with logical conditions that state that any generator that is online can be shut-down but not started-up and that any generator that is off-line can be started –up but not shut-down.

Constraints related the operating status

$$y_{1,1} - z_{1,1} = u_{1,1} - u_{1,0}$$
$$y_{1,2} - z_{1,2} = u_{1,2} - u_{1,1}$$
$$y_{1,3} - z_{1,3} = u_{1,3} - u_{1,2}$$
$$y_{2,1} - z_{2,1} = u_{2,1} - u_{2,0}$$
$$y_{2,2} - z_{2,2} = u_{2,2} - u_{2,1}$$
$$y_{2,3} - z_{2,3} = u_{2,3} - u_{2,2}$$
$$y_{3,1} - z_{3,1} = u_{3,1} - u_{3,0}$$
$$y_{3,2} - z_{3,2} = u_{3,2} - u_{3,1}$$
$$y_{3,3} - z_{3,3} = u_{3,3} - u_{3,2}$$

$$y_{1,1} + z_{1,1} \leq 1$$
$$y_{1,2} + z_{1,2} \leq 1$$
$$y_{1,3} + z_{1,3} \leq 1$$
$$y_{2,1} + z_{2,1} \leq 1$$
$$y_{2,2} + z_{2,2} \leq 1$$
$$y_{2,3} + z_{2,3} \leq 1$$
$$y_{3,1} + z_{3,1} \leq 1$$
$$y_{3,2} + z_{3,2} \leq 1$$
$$y_{3,3} + z_{3,3} \leq 1$$

$$U_{1,0} = 0$$
$$U_{2,0} = 0$$
$$U_{3,0} = 1$$

d) (2%) Write down all the equations that ensure that binary decision variables take the value 0 or 1.

$$\begin{array}{lll}
u_{1,1} \in \{0,1\} & y_{1,1} \in \{0,1\} & z_{1,1} \in \{0,1\} \\
u_{1,2} \in \{0,1\} & y_{1,2} \in \{0,1\} & z_{1,2} \in \{0,1\} \\
u_{1,3} \in \{0,1\} & y_{1,3} \in \{0,1\} & z_{1,3} \in \{0,1\} \\
u_{2,1} \in \{0,1\} & y_{2,1} \in \{0,1\} & z_{2,1} \in \{0,1\} \\
u_{2,2} \in \{0,1\} & y_{2,2} \in \{0,1\} & z_{2,2} \in \{0,1\} \\
u_{2,3} \in \{0,1\} & y_{2,3} \in \{0,1\} & z_{2,3} \in \{0,1\} \\
u_{3,1} \in \{0,1\} & y_{3,1} \in \{0,1\} & z_{3,1} \in \{0,1\} \\
u_{3,2} \in \{0,1\} & y_{3,2} \in \{0,1\} & z_{3,2} \in \{0,1\} \\
u_{3,3} \in \{0,1\} & y_{3,3} \in \{0,1\} & z_{3,3} \in \{0,1\}
\end{array}$$

e) (3%) Write down all the power limit constraints (i.e., constrain power generation to be within the min and the maximum technical limits of the generator). Please write one constraint for the minimum power generation and another for the maximum power generation.

Generator 1	**Generator 2**	**Generator 3**
$p_{1,1} \leq P_1^{max} * u_{1,1}$	$p_{2,1} \leq P_2^{max} * u_{2,1}$	$p_{3,1} \leq P_3^{max} * u_{3,1}$

$$p_{1,2} \leq P_1^{max} * u_{1,2} \qquad p_{2,2} \leq P_2^{max} * u_{2,2} \qquad p_{3,2} \leq P_3^{max} * u_{3,2}$$
$$p_{1,3} \leq P_1^{max} * u_{1,3} \qquad p_{2,3} \leq P_2^{max} * u_{2,3} \qquad p_{3,3} \leq P_3^{max} * u_{3,3}$$
$$p_{1,1} \geq P_1^{min} * u_{1,1} \qquad p_{2,1} \geq P_2^{min} * u_{2,1} \qquad p_{3,1} \geq P_3^{min} * u_{3,1}$$
$$p_{1,2} \geq P_1^{min} * u_{1,2} \qquad p_{2,2} \geq P_2^{min} * u_{2,2} \qquad p_{3,2} \geq P_3^{min} * u_{3,2}$$
$$p_{1,3} \geq P_1^{min} * u_{1,3} \qquad p_{2,3} \geq P_2^{min} * u_{2,3} \qquad p_{3,3} \geq P_3^{min} * u_{3,3}$$

Replacing the constants with their values:

Generator 1	Generator 2	Generator 3
$p_{1,1} \leq 350 * u_{1,1}$	$p_{2,1} \leq 200 * u_{2,1}$	$p_{3,1} \leq 140 * u_{3,1}$
$p_{1,2} \leq 350 * u_{1,2}$	$p_{2,2} \leq 200 * u_{2,2}$	$p_{3,2} \leq 140 * u_{3,2}$
$p_{1,3} \leq 350 * u_{1,3}$	$p_{2,3} \leq 200 * u_{2,3}$	$p_{3,3} \leq 140 * u_{3,3}$
$p_{1,1} \geq 50 * u_{1,1}$	$p_{2,1} \geq 80 * u_{2,1}$	$p_{3,1} \geq 40 * u_{3,1}$
$p_{1,2} \geq 50 * u_{1,2}$	$p_{2,2} \geq 80 * u_{2,2}$	$p_{3,2} \geq 40 * u_{3,2}$
$p_{1,3} \geq 50 * u_{1,3}$	$p_{2,3} \geq 80 * u_{2,3}$	$p_{3,3} \geq 40 * u_{3,3}$

f) (8%) Write down all the ramping limits constraints.

Up-ramp constraints: (note that $Ug,0$ is a parameter and not a decision variable) We are assuming that at time t=0, units #1 and # 2 are off-line while generating unit# 3 is online and producing 100MW. Hence, $U_{1,0} = 0$, $U2,0 = 0$,$U3,0 = 1$. Also, P is a parameter, where $P1,0 = 0$, $P2,0 = 0$, $P3,0 = 100$

Generator 1	Generator 2	Generator 3
$p_{1,1} - p_{1,0} \leq R_1^U u_{1,0} + R_1^{SU} y_{1,1}$	$p_{2,1} - p_{2,0} \leq R_2^U u_{2,0} + R_2^{SU} y_{2,1}$	$p_{3,1} - p_{3,0} \leq R_3^U u_{3,0} + R_3^{SU} y_{3,1}$
$p_{1,2} - p_{1,1} \leq R_1^U u_{1,1} + R_1^{SU} y_{1,2}$	$p_{2,2} - p_{2,1} \leq R_2^U u_{2,1} + R_2^{SU} y_{2,2}$	$p_{3,2} - p_{3,1} \leq R_3^U u_{3,1} + R_3^{SU} y_{3,2}$
$p_{1,3} - p_{1,2} \leq R_1^U u_{1,2} + R_1^{SU} y_{1,3}$	$p_{2,3} - p_{2,2} \leq R_2^U u_{2,2} + R_2^{SU} y_{2,3}$	$p_{3,3} - p_{3,2} \leq R_3^U u_{3,2} + R_3^{SU} y_{3,3}$
$p_{1,0} - p_{1,1} \leq R_1^D u_{1,1} + R_1^{SD} z_{1,1}$	$p_{2,0} - p_{2,1} \leq R_2^D u_{2,1} + R_2^{SD} z_{2,1}$	$p_{3,0} - p_{3,1} \leq R_3^D u_{3,1} + R_3^{SD} z_{3,1}$
$p_{1,1} - p_{1,2} \leq R_1^D u_{1,2} + R_1^{SD} z_{1,2}$	$p_{2,1} - p_{2,2} \leq R_2^D u_{2,2} + R_2^{SD} z_{2,2}$	$p_{3,1} - p_{3,2} \leq R_3^D u_{3,2} + R_3^{SD} z_{3,2}$
$p_{1,2} - p_{1,3} \leq R_1^D u_{1,3} + R_1^{SD} z_{1,3}$	$p_{2,2} - p_{2,3} \leq R_2^D u_{2,3} + R_2^{SD} z_{2,3}$	$p_{3,2} - p_{3,3} \leq R_3^D u_{3,3} + R_3^{SD} z_{3,3}$
$P_{1,0} = 0$	$P_{2,0} = 0$	$P_{3,0} = 100$

Replacing the constants with their values:

Generator 1	Generator 2	Generator 3
$p_{1,1} - p_{1,0} \leq 200u_{1,0} + 200y_{1,1}$	$p_{2,1} - p_{2,0} \leq 100u_{2,0} + 100y_{2,1}$	$p_{3,1} - p_{3,0} \leq 100u_{3,0} + 100y_{3,1}$
$p_{1,2} - p_{1,1} \leq 200u_{1,1} + 200y_{1,2}$	$p_{2,2} - p_{2,1} \leq 100u_{2,1} + 100y_{2,2}$	$p_{3,2} - p_{3,1} \leq 100u_{3,1} + 100y_{3,2}$
$p_{1,3} - p_{1,2} \leq 200u_{1,2} + 200y_{1,3}$	$p_{2,3} - p_{2,2} \leq 100u_{2,2} + 100y_{2,3}$	$p_{3,3} - p_{3,2} \leq 100u_{3,2} + 100y_{3,3}$
$p_{1,0} - p_{1,1} \leq 300u_{1,1} + 300z_{1,1}$	$p_{2,0} - p_{2,1} \leq 150u_{2,1} + 150z_{2,1}$	$p_{3,0} - p_{3,1} \leq 100u_{3,1} + 100z_{3,1}$
$p_{1,1} - p_{1,2} \leq 300u_{1,2} + 300z_{1,2}$	$p_{2,1} - p_{2,2} \leq 150u_{2,2} + 150z_{2,2}$	$p_{3,1} - p_{3,2} \leq 100u_{3,2} + 100z_{3,2}$
$p_{1,2} - p_{1,3} \leq 300u_{1,3} + 300z_{1,3}$	$p_{2,2} - p_{2,3} \leq 150u_{2,3} + 150z_{2,3}$	$p_{3,2} - p_{3,3} \leq 100u_{3,3} + 100z_{3,3}$
$P_{1,0} = 0$	$P_{2,0} = 0$	$P_{3,0} = 100$

g) (3%) Write down the power balance constraints.

$$p_{1,1} + p_{2,1} + p_{3,1} = 160$$
$$p_{1,2} + p_{2,2} + p_{3,2} = 500$$
$$p_{1,3} + p_{2,3} + p_{3,3} = 400$$

h) (10%) Now **re-write the formulation** in the most succinct and clear way. Start by listing all the indices and sets, then list all the parameters, then list the decision variables (referring to the indices), then write expressions that represent several constraints by using the sigma and forall operators.

Please note that to be rigorous, for the ramping constraints we need to write one expression for t=1 and another for t=2 and t=3.

Here is the constraint of t=1 for ramping up. (You need to write an expression for the ramp-up constraint for t=2,3, and two expressions for the ramp-down constraints.

$$p_{g,1} - P_{g,0} \leq R_g^U U_{g,0} + R_g^{SU} y_{g,1} \quad \forall g$$

where $P_{g,0}, U_{g,0}$ are parameters representing initial conditions

Indices:

g: $generators, in\ the\ set$: {1,2,3)

t: $time\ period, in\ the\ set$: {1,2,3)

Decision variables:

pg,t : $power\ generation\ from\ generator\ g\ at\ time\ t$ ug,t : $on - off\ status\ of\ generator\ g\ at\ time\ t$ yg,t : $start - up\ status\ of\ generator\ g\ at\ time\ t$ zg,t : $shut - down\ status\ of\ generator\ g\ at\ time\ t$

Indices and Sets

$t = 1,2,3$	*time period, in the set*: {1,2,3)
$g = 1,2,3$	*generators, in the set*: {1,2,3)

Decision Variables

$p_{g,t}$	Power *generation from generator g at time t*
$u_{g,t}$	*on – off status of generator g at time t*
$y_{g,t}$	*start – up status of generator g at time t*
$z_{g,t}$	*shut – down status of generator g at time t*

Parameters

C_g^V	*Variable cost of generator g ($/MWh)*
C_g^F	*Online (fixed) cost of generator g ($/h)*
C_g^{SU}	*Start-up cost of generator g ($)*
C_g^{SD}	*Shut-down cost of generator g ($)*
P_g^{max}	*Power generation capacity generator g (MW)*
P_g^{min}	*Minimum power output of generator g (MW)*
R_g^D	*Ramping-down limit of generator g (MW/h)*
R_g^{SD}	*Shut-down ramping limit of generator g (MW/h)*
R_g^U	*Ramping-up limit of generator g (MW/h)*
R_g^{SU}	*Start-up ramping limit of generator g (MW/h)*
D_t	*Total demand in hour t*

Objective function:

$$\min \sum_{t=1}^{3}\sum_{g=1}^{3} C_g^V p_{g,t} + C_g^F u_{g,t} + C_g^{SU} y_{g,t} + C_g^{SD} z_{g,t}$$

Subject to:

Min/max power generation constraints

$$p_{g,t} \le P_g^{max} u_{g,t} \qquad \forall g, \forall t$$

$$p_{g,t} \ge P_g^{min} u_{g,t} \qquad \forall g, \forall t$$

Power balance constraints

$$\sum_{g=1}^{3} p_{g,t} = D_t \qquad \forall t$$

Constraints relating on/off status, start-up, and shutdown

$$y_{g,t} - z_{g,t} = u_{g,t} - u_{g,t-1} \qquad \forall g, \forall t$$

$$y_{g,t} + z_{g,t} \le 1 \qquad \forall g, \forall t$$

Constraints limiting ramp-up in power generation (note the difference between U and u, and between P and p)

$$p_{g,1} - P_{g,0} \le R_g^U U_{g,0} + R_g^{SU} y_{g,1} \qquad \forall g$$

$$p_{g,t} - p_{g,t-1} \le R_g^U u_{g,t-1} + R_g^{SU} y_{g,t} \qquad \forall g, \quad for\ t = 2,3$$

Constraints limiting ramp-down in power generation

$$P_{g,0} - p_{g,1} \le R_g^D u_{g,1} + R_g^{SD} z_{g,1} \qquad \forall g$$

$$p_{g,t-1} - p_{g,t} \le R_g^D u_{g,t} + R_g^{SD} z_{g,t} \qquad \forall g, \quad for\ \ t = 2,3$$

Binary constraint

$$u_{g,t}, y_{g,t}, z_{g,t} \in \{0,1\} \qquad \forall g, \forall t$$

3. (10%) For each of the following solutions of the UC problem described in 2, determine feasibility. If the solution is determined to be not feasible then explain which constraints are being violated

a) $p_{1,1} = 0, p_{2,1} = 160, p_{3,1} = 0$

This is not a feasible solution. We are assuming that at time t=0, units #1 and # 2 are off-line while generating unit# 3 is online and produces 100MW. Hence, $U_{1,0} = 0$,$U_{2,0} = 0$, $U_{3,0} = 1$. Also, P is a parameter, where $P_{1,0} = 0$, $P_{2,0} = 0$, $P_{3,0} = 100$. This solution violates the up-ramp constraint for generator 2. This is because at time 0, generator 2 is off-line, and its ramping capability Rg SU at start-up is 100 MW, which means that it cannot ramp to 160 MW at time t=1.

Ramp constraint is being violated:

$$p_{g,t} - p_{g,t-1} \leq R_g^U u_{g,t-1} + R_g^{SU} y_{g,t}$$
$$p_{2,1} - p_{2,0} \leq R_2^U u_{2,0} + 100 y_{2,1}$$
$$160 - 0 \leq 0 * u_{2,0} + 100 * 1$$
$$160 \leq 100 \quad \text{is a contradiction}$$

b)

$$p_{1,1} = 160,\ p_{2,1} = 0,\ p_{3,1} = 0,\ p_{1,2} = 350,\ p_{2,2} = 50,\ p_{3,2} = 100,$$
$$u_{1,1} = 1,\ u_{2,1} = 0,\ u_{3,1} = 0,\ u_{1,2} = 1,\ u_{2,2} = 1,\ u_{3,2} = 1$$
$$y_{1,1} = 1,\ y_{2,1} = 0,\ y_{3,1} = 0,\ y_{1,2} = 0,\ y_{2,2} = 1,\ y_{3,2} = 1$$
$$z_{1,1} = 0,\ z_{2,1} = 0,\ z_{3,1} = 1,\ z_{1,2} = 0,\ z_{2,2} = 0,\ z_{3,2} = 0$$

This is NOT a feasible solution because $p_{2,2}$ violates the minimum generation constraint

c)

$$p_{1,1} = 160,\ p_{2,1} = 0,\ p_{3,1} = 0,\ p_{1,2} = 350,\ p_{2,2} = 50,\ p_{3,2} = 100,$$
$$u_{1,1} = 1,\ u_{2,1} = 0,\ u_{3,1} = 0,\ u_{1,2} = 1,\ u_{2,2} = 1,\ u_{3,2} = 1$$
$$y_{1,1} = 1,\ y_{2,1} = 0,\ y_{3,1} = 1,\ y_{1,2} = 0,\ y_{2,2} = 1,\ y_{3,2} = 1$$
$$z_{1,1} = 0,\ z_{2,1} = 0,\ z_{3,1} = 0,\ z_{1,2} = 0,\ z_{2,2} = 0,\ z_{3,2} = 0$$

This is NOT a feasible solution. Unit 3 cannot start-up in time period 1 and still be offline at that time period.

$$y_{g,t} - z_{g,t} = u_{g,t} - u_{g,t-1}$$
$$y_{3,1} - z_{3,1} = u_{3,1} - u_{3,0}$$
$$1 - 0 = 0 - 1$$
$$1 = -1 \text{ is a contradiction}$$

d)

$$p_{1,1} = 60,\ p_{2,1} = 10,\ p_{3,1} = 90,\ p_{1,2} = 300,\ p_{2,2} = 110,\ p_{3,2} = 90,$$
$$u_{1,1} = 1,\ u_{2,1} = 1,\ u_{3,1} = 1,\ u_{1,2} = 1,\ u_{2,2} = 1,\ u_{3,2} = 1$$
$$y_{1,1} = 1,\ y_{2,1} = 1,\ y_{3,1} = 0,\ y_{1,2} = 0,\ y_{2,2} = 1,\ y_{3,2} = 1$$
$$z_{1,1} = 0,\ z_{2,1} = 0,\ z_{3,1} = 0,\ z_{1,2} = 0,\ z_{2,2} = 0,\ z_{3,2} = 0$$

This is NOT a feasible solution. The ramp-up capability constraint for unit 1 is violated. Also, unit 2 cannot start up at time 2 because it started at time 1.
$p_{1,2}$ = 300 MW cannot be. At time t=1, generator 1 is producing 60 MW. To reach the output of $p_{1,2}$ = 300 MW, its ramp-up R_g^U should be 240 MW, but we know R_g^U is 100 MW.

$$p_{g,t} - p_{g,t-1} \leq R_g^U u_{g,t-1} + R_g^{SU} y_{g,t}$$
$$p_{1,2} - p_{1,1} \leq R_1^U u_{1,1} + R_1^{SU} y_{1,2}$$
$$300 - 60 \leq 200 * 1 + R_1^{SU} * 0$$
$$240 \leq 200 \text{ is a contradiction}$$

Also, unit 2 cannot start up at time 2 because it was online at time t=1.

$$y_{g,t} - z_{g,t} = u_{g,t} - u_{g,t-1}$$
$$y_{2,2} - z_{2,2} = u_{2,2} - u_{2,1}$$
$$1 - 0 = 1 - 1$$
$$1 = 0 \text{ is a contradiction}$$

Also, unit 3 cannot start up at time 2 because it was online at time t=1.

$$y_{g,t} - z_{g,t} = u_{g,t} - u_{g,t-1}$$
$$y_{3,2} - z_{3,2} = u_{3,2} - u_{3,1}$$
$$1 - 0 = 1 - 1$$
$$1 = 0 \text{ is a contradiction}$$

3. (25%) Self-scheduling problem:
A power producer owns two power generating resources with characteristics as described in Table 2.

Generating Unit	C_g^F (1)	C_g^{SD} (2)	C_g^{SU} (3)	C_g^V (4)	$P_g^{\max}$ (5)	$P_g^{\min}$ (6)	R_g^D (7)	R_g^{SD} (8)	R_g^{SU} (9)	R_g^U (10)
1	5	400	10,700	24.0	300	30	150	180	200	100
2	5	300	10,000	22.9	150	50	50	25	75	25

Table 2: Technical characteristics of generators. (1) Online (fixed) cost of generating unit g ($/h). (2) shut-down cost of generating unit g ($). (3) Start-up cost of generating unit g ($). (4) Variable cost of generating unit g ($/MWh). (5) Power generation capacity of unit g (MW). (6) minimum power output of unit g (MW). (7) ramping-down limit of generating unit g (MW/h). (8) shut-down ramping limit of generating unit g (MW/h). (9) start-up ramping limit of generating unit g(MW/h). (10) ramping-up limit of generating unit g (MW/h).

Formulate the optimization problem for finding the optimal scheduling of the generating units to maximize profits during the next 5 hours (estimated prices given in table below) (There is no need to find a solution to this problem. <u>Please ignore</u> the constraints for the minimum uptime and minimum down time, and assume that the generating units are off-line and hence have an initial power output of zero).

t (hour)	λ_t Forecast market Price ($/MWh)
1	27
2	30
3	35
4	33

5	26

Table 2: Forecasted prices

a) (5%) Write the decision variables of this problem. Feel free to define indices and sets

Indices and Sets

t — *time periods, in the set*: $\{1,2,3,4,5\}$

g — *generators, in the set*: $\{1,2\}$

Decision Variables

$p_{g,t}$ — *power generation from generator g at time*

$u_{g,t}$ — *on − off status of generator g at time t*

$y_{g,t}$ — *startup status of generator g at time t*

$z_{g,t}$ — *shutdown status of generator g at time t*

Other Decision Variables

$p_{g,0}$ — *power generation from generator g at time t* = 0

$u_{g,0}$ — *on-off status of generator g at time t* = 0

Parameters that are not in the table (and we will set the two types of decision variables for t=0, to be equal to these parameters):

$P_{g,t}$ — *Power generation from generator g at time t* = 0

$U_{g,t}$ — *All generators are off at time t* = 0

b) (5%) Write the objective function (Feel free to use sigma notation).

$$\sum_{t=1}^{5}\sum_{g=1}^{2}\left((\lambda_t p_{g,t}) - (C_g^V p_{g,t} + C_g^F u_{g,t} + C_g^{SU} y_{g,t} + C_g^{SD} z_{g,t})\right)$$

c) (20%) Write down all the constraints of this optimization problem. Write all constraints in canonical form* (i.e., have all decision variables in the left hand-side LHS of the constraint and any constant on the right hand-side RHS). Writing constraints in canonical form for a maximization problem means that all decision variables need to be to the left of a "<-" or "=" sign. Please write the maximum and minimum power output constraints and the ramp rate constraints for each generator using for all t (but spell out one set of constraints for each generator). For the rest of the constraints use sigma notation and for all).

Constraints:

Max power constraints

$$p_{1,t} - P_1^{max} u_{1,t} \leq 0 \quad \forall t$$
$$p_{2,t} - P_2^{max} u_{2,t} \leq 0 \quad \forall t$$

Min power constraints

$$-p_{1,t} + P_1^{min} u_{1,t} \leq 0 \quad \forall t$$
$$-p_{2,t} + P_2^{min} u_{2,t} \leq 0 \quad \forall t$$

Ramping constraints

Up-ramp

$$p_{1,t} - p_{1,t-1} - R_1^U u_{1,t-1} - R_1^{SU} y_{1,t} \leq 0 \qquad \forall t$$
$$p_{2,t} - p_{2,t-1} - R_2^U u_{2,t-1} - R_2^{SU} y_{2,t} \leq 0 \qquad \forall t$$

Down ramp

$$p_{1,t-1} - p_{1,t} - R_1^D u_{1,t} - R_1^{SD} z_{1,t} \leq 0 \qquad \forall t$$
$$p_{2,t-1} - p_{2,t} - R_2^D u_{2,t} - R_2^{SD} z_{2,t} \leq 0 \qquad \forall t$$

Constraints relating on/off status, start-up, and shutdown

$$y_{g,t} - z_{g,t} - u_{g,t} + u_{g,t-1} = 0 \qquad \forall t, \forall g$$
$$y_{g,t} + z_{g,t} \leq 1 \qquad \forall g, \forall t$$

Initial Conditions

$$U_{1,0} = 0;\ U_{2,0} = 0$$
$$P_{1,0} = 0;\ P_{2,0} = 0$$

Binary Constraints

$$u_{g,t}, y_{g,t}, z_{g,t} \in \{0,1\} \qquad \forall g, \forall t$$

d) (5%) What is the difference between this model and the unit commitment model? Please explain who runs these models, how these models are used, what are the uncertainties faced and how are these uncertainties tackled.

	UC model run by Balancing Authority	**UC model run by a power producer that wants to self-schedule generation**
Objective function	**Minimize costs**	**Maximize profits**
Demand-balance constraints	Included. This is the job of the BA; to schedule resources so that demand is met.	Not included. The power producer is not responsible for meeting system demand
Reserve requirements	Included. This is the job of the BA; to schedule resources so that the system operates in a reliable way, and it is prepared for contingencies	Not included. The power producer is not responsible for ensuring system reliability