Electric Power Markets   Spring 2023   ENV 717

Nicholas School of the Environment   Duke University

Assignment #6

****Submit a word document with your answers to all problems and an html file with code for the last problem in this document. Make sure you run your code <u>before</u> converting into an html file so we can see the results.***

1. (50%) Unit commitment problem:

Consider all the information provided for problem 1 in A5 but this time consider the need to include constraints that ensure meeting **reserve requirements**.

Assume that reserve requirements for the three periods considered are 16MW, 50MW and 40MW.

==============Here is the information from A5 for your reference ===========

A balancing authority needs to solve the UC problem for a 3-hours planning horizon. Three thermal generating units are used to supply demands of 160MW, 500MW and 400MW in time periods 1, 2 and 3, respectively. Data on technical limits and economic parameters of the generating units is provided in Table 1.

Generating Unit	C_g^F (1)	C_g^{SD} (2)	C_g^{SU} (3)	C_g^V (4)	$P_g^{\max}$ (5)	$P_g^{\min}$ (6)	R_g^D (7)	R_g^{SD} (8)	R_g^{SU} (9)	R_g^U (10)
1	5	0.5	20	0.1	350	50	300	300	200	200
2	7	0.3	18	0.125	200	80	150	150	100	100
3	6	1	5	0.15	140	40	100	100	100	100

Table 1: Technical characteristics of generators. (1) Online (fixed) cost of generating unit g ($/h). (2) shut-down cost of generating unit g ($). (3) Start-up cost of generating unit g ($). (4) Variable cost of generating unit g ($/MWh). (5) Power generation capacity of unit g (MW). (6) minimum power output of unit g (MW). (7) ramping-down limit of generating unit g (MW/h). (8) shut-down ramping limit of generating unit g (MW/h). (9) start-up ramping limit of generating unit g(MW/h). (10) ramping-up limit of generating unit g (MW/h).

Assume that at time t=0, units #1 and # 2 are off-line while generating unit# 3 is online and producing 100MW.

Formulate the unit commitment (UC) problem spelling out all the values and constraints.

a) (3%) Write down a list of all the decision variables, spelling out each of them (i.e., do not use the indices to refer to types of decision variables, but rather list them all). The first decision variable in your list is: $p_{1,1}$=power produced by generator # 1 in hour1. The second element in the list is: $p_{1,1}$=power produced by generator # 1 in hour 2, and so on.

$p_{1,1}$ Power produced by generator 1 in hour 1
$p_{1,2}$ Power produced by generator 1 in hour 2
$p_{1,3}$ Power produced by generator 1 in hour 3
$p_{2,1}$ Power produced by generator 2 in hour 1
$p_{2,2}$ Power produced by generator 2 in hour 2
$p_{2,3}$ Power produced by generator 2 in hour 3

$p_{3,1}$ Power produced by generator 3 in hour 1
$p_{3,2}$ Power produced by generator 3 in hour 2
$p_{3,3}$ Power produced by generator 3 in hour 3
$\underline{p}_{1,1}$ Available power generation of generator 1 at hour 1
$\underline{p}_{1,2}$ Available power generation of generator 1 at hour 2
$\underline{p}_{1,3}$ Available power generation of generator 1 at hour 3
$\underline{p}_{2,1}$ Available power generation of generator 2 at hour 1
$\underline{p}_{2,2}$ Available power generation of generator 2 at hour 2
$\underline{p}_{2,3}$ Available power generation of generator 2 at hour 3
$\underline{p}_{3,1}$ Available power generation of generator 3 at hour 1
$\underline{p}_{3,2}$ Available power generation of generator 3 at hour 2
$\underline{p}_{3,3}$ Available power generation of generator 3 at hour 3
$u_{1,1}$ On/Off status for generator 1 in hour 1
$u_{1,2}$ On/Off status for generator 1 in hour 2
$u_{1,3}$ On/Off status for generator 1 in hour 3
$u_{2,1}$ On/Off status for generator 2 in hour 1
$u_{2,2}$ On/Off status for generator 2 in hour 2
$u_{2,3}$ On/Off status for generator 2 in hour 3
$u_{3,1}$ On/Off status for generator 3 in hour 1
$u_{3,2}$ On/Off status for generator 3 in hour 2
$u_{3,3}$ On/Off status for generator 3 in hour 3
$y_{1,1}$ Start-up status for generator 1 in hour 1
$y_{1,2}$ Start-up status for generator 1 in hour 2
$y_{1,3}$ Start-up status for generator 1 in hour 3
$y_{2,1}$ Start-up status for generator 2 in hour 1
$y_{2,2}$ Start-up status for generator 2 in hour 2
$y_{2,3}$ Start-up status for generator 2 in hour 3
$y_{3,1}$ Start-up status for generator 3 in hour 1
$y_{3,2}$ Start-up status for generator 3 in hour 2
$y_{3,3}$ Start-up status for generator 3 in hour 3
$z_{1,1}$ Shut-down status for generator 1 in hour 1
$z_{1,2}$ Shut-down status for generator 1 in hour 2
$z_{1,3}$ Shut-down status for generator 1 in hour 3
$z_{2,1}$ Shut-down status for generator 2 in hour 1
$z_{2,2}$ Shut-down status for generator 2 in hour 2
$z_{2,3}$ Shut-down status for generator 2 in hour 3
$z_{3,1}$ Shut-down status for generator 3 in hour 1
$z_{3,2}$ Shut-down status for generator 3 in hour 2
$z_{3,3}$ Shut-down status for generator 3 in hour 3

b) (3%) Write down the objective function spelling out all the terms and writing down the values of all the parameters. Why does this objective function not include pbar?

$$\begin{gathered}
Min\ [C_1^V(p_{1,1}+p_{1,2}+p_{1,3}) + C_2^V(p_{2,1}+p_{2,2}+p_{2,3}) + C_3^V(p_{3,1}+p_{3,2}+p_{3,3}) + \\
C_1^F(u_{1,1}+u_{1,2}+u_{1,3}) + C_2^F(u_{2,1}+u_{2,2}+u_{2,3}) + C_3^F(u_{3,1}+u_{3,2}+u_{3,3}) + \\
C_1^{SU}(y_{1,1}+y_{1,2}+y_{1,3}) + C_2^{SU}(y_{2,1}+y_{2,2}+y_{2,3}) + C_3^{SU}(y_{3,1}+y_{3,2}+y_{3,3}) + \\
C_1^{SD}(z_{1,1}+z_{1,2}+z_{1,3}) + C_2^{SD}(z_{2,1}+z_{2,2}+z_{2,3}) + C_3^{SD}(z_{3,1}+z_{3,2}+z_{3,3})]
\end{gathered}$$

Writing down the values of all the parameters:

$$\begin{aligned} Min\ [0.1&(p_{1,1}+p_{1,2}+p_{1,3})+0.125(p_{2,1}+p_{2,2}+p_{2,3})+0.15(p_{3,1}+p_{3,2}+p_{3,3}) \\ &+5(u_{1,1}+u_{1,2}+u_{1,3})+7(u_{2,1}+u_{2,2}+u_{2,3})+6(u_{3,1}+u_{3,2}+u_{3,3}) \\ &+20(y_{1,1}+y_{1,2}+y_{1,3})+18(y_{2,1}+y_{2,2}+y_{2,3})+5(y_{3,1}+y_{3,2}+y_{3,3}) \\ &+0.5(z_{1,1}+z_{1,2}+z_{1,3})+0.3(z_{2,1}+z_{2,2}+z_{2,3})+1(z_{3,1}+z_{3,2}+z_{3,3})] \end{aligned}$$

$\underline{p}_{g,t}$ (feasible power generation capacity at generator g at time t in MW) is not included in the objective function because it doesn't have an associated cost. So far, it is assumed that that there is not cost for having available reserves in the system, so $\underline{p}_{g,t}$ does not affect the goal of the objective function: minimizing the cost of the system.

c) (34%) Write down all the other constraints. For each group of constraints include a brief explanation (write something like: "These constraints ensure that all the power production from all the units, at all times, is within the limits of the ramp-feasible available generation")

Power limit constraints:

These constraints ensure that the available power generation is within the minimum and the maximum technical limits of the generator

Generator 1	Generator 2	Generator 3
$\underline{p}_{1,1} \le P_1^{max} * u_{1,1}$	$\underline{p}_{2,1} \le P_2^{max} * u_{2,1}$	$\underline{p}_{3,1} \le P_3^{max} * u_{3,1}$
$\underline{p}_{1,2} \le P_1^{max} * u_{1,2}$	$\underline{p}_{2,2} \le P_2^{max} * u_{2,2}$	$\underline{p}_{3,2} \le P_3^{max} * u_{3,2}$
$\underline{p}_{1,3} \le P_1^{max} * u_{1,3}$	$\underline{p}_{2,3} \le P_2^{max} * u_{2,3}$	$\underline{p}_{3,3} \le P_3^{max} * u_{3,3}$
$\underline{p}_{1,1} \ge P_1^{min} * u_{1,1}$	$\underline{p}_{2,1} \ge P_2^{min} * u_{2,1}$	$\underline{p}_{3,1} \ge P_3^{min} * u_{3,1}$
$\underline{p}_{1,2} \ge P_1^{min} * u_{1,2}$	$\underline{p}_{2,2} \ge P_2^{min} * u_{2,2}$	$\underline{p}_{3,2} \ge P_3^{min} * u_{3,2}$
$\underline{p}_{1,3} \ge P_1^{min} * u_{1,3}$	$\underline{p}_{2,3} \ge P_2^{min} * u_{2,3}$	$\underline{p}_{3,3} \ge P_3^{min} * u_{3,3}$

Replacing the constants with their values:

Generator 1	Generator 2	Generator 3
$\underline{p}_{1,1} \le 350 * u_{1,1}$	$\underline{p}_{2,1} \le 200 * u_{2,1}$	$\underline{p}_{3,1} \le 140 * u_{3,1}$
$\underline{p}_{1,2} \le 350 * u_{1,2}$	$\underline{p}_{2,2} \le 200 * u_{2,2}$	$\underline{p}_{3,2} \le 140 * u_{3,2}$
$\underline{p}_{1,3} \le 350 * u_{1,3}$	$\underline{p}_{2,3} \le 200 * u_{2,3}$	$\underline{p}_{3,3} \le 140 * u_{3,3}$
$\underline{p}_{1,1} \ge 50 * u_{1,1}$	$\underline{p}_{2,1} \ge 80 * u_{2,1}$	$\underline{p}_{3,1} \ge 40 * u_{3,1}$
$\underline{p}_{1,2} \ge 50 * u_{1,2}$	$\underline{p}_{2,2} \ge 80 * u_{2,2}$	$\underline{p}_{3,2} \ge 40 * u_{3,2}$
$\underline{p}_{1,3} \ge 50 * u_{1,3}$	$\underline{p}_{2,3} \ge 80 * u_{2,3}$	$\underline{p}_{3,3} \ge 40 * u_{3,3}$

Ramping Limit Constraints:

These constraints ensure that the power units can reduce or increase their generation from time t to time t+1 in order to handle the expected changes in the demand.

Generator 1	Generator 2	Generator 3
$\underline{p}_{1,1} - P_{1,0} \le R_1^U U_{1,0} + R_1^{SU} y_{1,1}$	$\underline{p}_{2,1} - P_{2,0} \le R_2^U U_{2,0} + R_2^{SU} y_{2,1}$	$\underline{p}_{3,1} - P_{3,0} \le R_3^U U_{3,0} + R_3^{SU} y_{3,1}$
$\underline{p}_{1,2} - p_{1,1} \le R_1^U u_{1,1} + R_1^{SU} y_{1,2}$	$\underline{p}_{2,2} - p_{2,1} \le R_2^U u_{2,1} + R_2^{SU} y_{2,2}$	$\underline{p}_{3,2} - p_{3,1} \le R_3^U u_{3,1} + R_3^{SU} y_{3,2}$

$\underline{p}_{1,3} - p_{1,2} \leq R_1^U u_{1,2} + R_1^{SU} y_{1,3}$	$\underline{p}_{2,3} - p_{2,2} \leq R_2^U u_{2,2} + R_2^{SU} y_{2,3}$	$\underline{p}_{3,3} - p_{3,2} \leq R_3^U u_{3,2} + R_3^{SU} y_{3,3}$
$P_{1,0} - \underline{p}_{1,1} \leq R_1^D u_{1,1} + R_1^{SD} z_{1,1}$	$P_{2,0} - \underline{p}_{2,1} \leq R_2^D u_{2,1} + R_2^{SD} z_{2,1}$	$P_{3,0} - \underline{p}_{3,1} \leq R_3^D u_{3,1} + R_3^{SD} z_{3,1}$
$p_{1,1} - \underline{p}_{1,2} \leq R_1^D u_{1,2} + R_1^{SD} z_{1,2}$	$p_{2,1} - \underline{p}_{2,2} \leq R_2^D u_{2,2} + R_2^{SD} z_{2,2}$	$p_{3,1} - \underline{p}_{3,2} \leq R_3^D u_{3,2} + R_3^{SD} z_{3,2}$
$p_{1,2} - \underline{p}_{1,3} \leq R_1^D u_{1,3} + R_1^{SD} z_{1,3}$	$p_{2,2} - \underline{p}_{2,3} \leq R_2^D u_{2,3} + R_2^{SD} z_{2,3}$	$p_{3,2} - \underline{p}_{3,3} \leq R_3^D u_{3,3} + R_3^{SD} z_{3,3}$

Replacing the constants with their values:

Generator 1	**Generator 2**	**Generator 3**
$\underline{p}_{1,1} - P_{1,0} \leq 200U_{1,0} + 200y_{1,1}$	$\underline{p}_{2,1} - P_{2,0} \leq 100U_{2,0} + 100y_{2,1}$	$\underline{p}_{3,1} - P_{3,0} \leq 100U_{3,0} + 100y_{3,1}$
$\underline{p}_{1,2} - p_{1,1} \leq 200u_{1,1} + 200y_{1,2}$	$\underline{p}_{2,2} - p_{2,1} \leq 100u_{2,1} + 100y_{2,2}$	$\underline{p}_{3,2} - p_{3,1} \leq 100u_{3,1} + 100y_{3,2}$
$\underline{p}_{1,3} - p_{1,2} \leq 200u_{1,2} + 200y_{1,3}$	$\underline{p}_{2,3} - p_{2,2} \leq 100u_{2,2} + 100y_{2,3}$	$\underline{p}_{3,3} - p_{3,2} \leq 100u_{3,2} + 100y_{3,3}$
$P_{1,0} - \underline{p}_{1,1} \leq 300u_{1,1} + 300z_{1,1}$	$P_{2,0} - \underline{p}_{2,1} \leq 150u_{2,1} + 150z_{2,1}$	$P_{3,0} - \underline{p}_{3,1} \leq 100u_{3,1} + 100z_{3,1}$
$p_{1,1} - \underline{p}_{1,2} \leq 300u_{1,2} + 300z_{1,2}$	$p_{2,1} - \underline{p}_{2,2} \leq 150u_{2,2} + 150z_{2,2}$	$p_{3,1} - \underline{p}_{3,2} \leq 100u_{3,2} + 100z_{3,2}$
$p_{1,2} - \underline{p}_{1,3} \leq 300u_{1,3} + 300z_{1,3}$	$p_{2,2} - \underline{p}_{2,3} \leq 150u_{2,3} + 150z_{2,3}$	$p_{3,2} - \underline{p}_{3,3} \leq 100u_{3,3} + 100z_{3,3}$

Power Balance Constraints:

These constraints ensure that the aggregated power generation of the power units matches the demand in each hour.

$$p_{1,1} + p_{2,1} + p_{3,1} = 160$$
$$p_{1,2} + p_{2,2} + p_{3,2} = 500$$
$$p_{1,3} + p_{2,3} + p_{3,3} = 400$$

Reliability Constraints:

These constraints ensure that the aggregated available generation of all the units is enough to satisfy the demand and the reserves requirements at each hour.

$$\underline{p}_{1,1} + \underline{p}_{2,1} + \underline{p}_{3,1} \geq P_1^D + P_1^R$$
$$\underline{p}_{1,2} + \underline{p}_{2,2} + \underline{p}_{3,2} \geq P_2^D + P_2^R$$
$$\underline{p}_{1,3} + \underline{p}_{2,3} + \underline{p}_{3,3} \geq P_3^D + P_3^R$$

Replacing the constants with their values:

$$\underline{p}_{1,1} + \underline{p}_{2,1} + \underline{p}_{3,1} \geq 160 + 16$$
$$\underline{p}_{1,2} + \underline{p}_{2,2} + \underline{p}_{3,2} \geq 500 + 50$$
$$\underline{p}_{1,3} + \underline{p}_{2,3} + \underline{p}_{3,3} \geq 400 + 40$$

Additional reliability constraints:

These constraints ensure that the available generation from each generator at each hour is greater or equal to its power produced.

Generator 1	**Generator 2**	**Generator 3**
$p_{1,1} \leq \underline{p}_{1,1}$	$p_{2,1} \leq \underline{p}_{1,1}$	$p_{3,1} \leq \underline{p}_{1,1}$
$p_{1,2} \leq \underline{p}_{1,2}$	$p_{2,2} \leq \underline{p}_{1,2}$	$p_{3,2} \leq \underline{p}_{1,2}$

$$p_{1,3} \le \underline{p}_{1,3} \qquad p_{2,3} \le \underline{p}_{1,3} \qquad p_{3,3} \le \underline{p}_{1,3}$$

Feasible Power Generation Constraints:

These constraints ensure that all the power production from all the units, at all times, is within the limits of the ramp-feasible available generation

Generator 1	Generator 2	Generator 3
$\underline{p}_{1,1} \le P_1^{max} * u_{1,1}$	$\underline{p}_{2,1} \le P_2^{max} * u_{2,1}$	$\underline{p}_{3,1} \le P_3^{max} * u_{3,1}$
$\underline{p}_{1,2} \le P_1^{max} * u_{1,2}$	$\underline{p}_{2,2} \le P_2^{max} * u_{2,2}$	$\underline{p}_{3,2} \le P_3^{max} * u_{3,2}$
$\underline{p}_{1,3} \le P_1^{max} * u_{1,3}$	$\underline{p}_{2,3} \le P_2^{max} * u_{2,3}$	$\underline{p}_{3,3} \le P_3^{max} * u_{3,3}$
$\underline{p}_{1,1} \ge P_1^{min} * u_{1,1}$	$\underline{p}_{2,1} \ge P_2^{min} * u_{2,1}$	$\underline{p}_{3,1} \ge P_3^{min} * u_{3,1}$
$\underline{p}_{1,2} \ge P_1^{min} * u_{1,2}$	$\underline{p}_{2,2} \ge P_2^{min} * u_{2,2}$	$\underline{p}_{3,2} \ge P_3^{min} * u_{3,2}$
$\underline{p}_{1,3} \ge P_1^{min} * u_{1,3}$	$\underline{p}_{2,3} \ge P_2^{min} * u_{2,3}$	$\underline{p}_{3,3} \ge P_3^{min} * u_{3,3}$

Replacing the constants with their values:

Generator 1	Generator 2	Generator 3
$\underline{p}_{1,1} \le 350 * u_{1,1}$	$\underline{p}_{2,1} \le 200 * u_{2,1}$	$\underline{p}_{3,1} \le 140 * u_{3,1}$
$\underline{p}_{1,2} \le 350 * u_{1,2}$	$\underline{p}_{2,2} \le 200 * u_{2,2}$	$\underline{p}_{3,2} \le 140 * u_{3,2}$
$\underline{p}_{1,3} \le 350 * u_{1,3}$	$\underline{p}_{2,3} \le 200 * u_{2,3}$	$\underline{p}_{3,3} \le 140 * u_{3,3}$
$\underline{p}_{1,1} \ge 50 * u_{1,1}$	$\underline{p}_{2,1} \ge 80 * u_{2,1}$	$\underline{p}_{3,1} \ge 40 * u_{3,1}$
$\underline{p}_{1,2} \ge 50 * u_{1,2}$	$\underline{p}_{2,2} \ge 80 * u_{2,2}$	$\underline{p}_{3,2} \ge 40 * u_{3,2}$
$\underline{p}_{1,3} \ge 50 * u_{1,3}$	$\underline{p}_{2,3} \ge 80 * u_{2,3}$	$\underline{p}_{3,3} \ge 40 * u_{3,3}$

Generator 1	Generator 2	Generator 3
$\underline{p}_{1,1} - P_{1,0} \le R_1^U U_{1,0} + R_1^{SU} y_{1,1}$	$\underline{p}_{2,1} - P_{2,0} \le R_2^U U_{2,0} + R_2^{SU} y_{2,1}$	$\underline{p}_{3,1} - P_{3,0} \le R_3^U U_{3,0} + R_3^{SU} y_{3,1}$
$\underline{p}_{1,2} - p_{1,1} \le R_1^U u_{1,1} + R_1^{SU} y_{1,2}$	$\underline{p}_{2,2} - p_{2,1} \le R_2^U u_{2,1} + R_2^{SU} y_{2,2}$	$\underline{p}_{3,2} - p_{3,1} \le R_3^U u_{3,1} + R_3^{SU} y_{3,2}$
$\underline{p}_{1,3} - p_{1,2} \le R_1^U u_{1,2} + R_1^{SU} y_{1,3}$	$\underline{p}_{2,3} - p_{2,2} \le R_2^U u_{2,2} + R_2^{SU} y_{2,3}$	$\underline{p}_{3,3} - p_{3,2} \le R_3^U u_{3,2} + R_3^{SU} y_{3,3}$

Replacing the constants with their values:

Generator 1	Generator 2	Generator 3
$\underline{p}_{1,1} - 0 \le 200 * 0 + 200 y_{1,1}$	$\underline{p}_{2,1} - 0 \le 100 * 0 + 100 y_{2,1}$	$\underline{p}_{3,1} - 100 \le 100 * 1 + 100 y_{3,1}$
$\underline{p}_{1,2} - p_{1,1} \le 200 u_{1,1} + 200 y_{1,2}$	$\underline{p}_{2,2} - p_{2,1} \le 100 u_{2,1} + 100 y_{2,2}$	$\underline{p}_{3,2} - p_{3,1} \le 100 u_{3,1} + 100 y_{3,2}$
$\underline{p}_{1,3} - p_{1,2} \le 200 u_{1,2} + 200 y_{1,3}$	$\underline{p}_{2,3} - p_{2,2} \le 100 u_{2,2} + 100 y_{2,3}$	$\underline{p}_{3,3} - p_{3,2} \le 100 u_{3,2} + 100 y_{3,3}$

Constraints related the operating status

These constraints ensure that the logic to start up and shutdown the units work properly when the commitment binaries are modified. Thse constraints also ensure that units are (1) not turned on and turned off at the same time, (2) off and not shutting down, (3) on are not starting up.

$$y_{1,1} - z_{1,1} = u_{1,1} - u_{1,0}$$
$$y_{1,2} - z_{1,2} = u_{1,2} - u_{1,1}$$
$$y_{1,3} - z_{1,3} = u_{1,3} - u_{1,2}$$
$$y_{2,1} - z_{2,1} = u_{2,1} - u_{2,0}$$
$$y_{2,2} - z_{2,2} = u_{2,2} - u_{2,1}$$
$$y_{2,3} - z_{2,3} = u_{2,3} - u_{2,2}$$
$$y_{3,1} - z_{3,1} = u_{3,1} - u_{3,0}$$
$$y_{3,2} - z_{3,2} = u_{3,2} - u_{3,1}$$
$$y_{3,3} - z_{3,3} = u_{3,3} - u_{3,2}$$

$$y_{1,1} + z_{1,1} \leq 1$$
$$y_{1,2} + z_{1,2} \leq 1$$
$$y_{1,3} + z_{1,3} \leq 1$$
$$y_{2,1} + z_{2,1} \leq 1$$
$$y_{2,2} + z_{2,2} \leq 1$$
$$y_{2,3} + z_{2,3} \leq 1$$
$$y_{3,1} + z_{3,1} \leq 1$$
$$y_{3,2} + z_{3,2} \leq 1$$
$$y_{3,3} + z_{3,3} \leq 1$$

$$U_{1,0} = 0$$
$$U_{2,0} = 0$$
$$U_{3,0} = 1$$

Binary Constraints

These constraints ensure that all the on/off, startup and shutdown status variable take only binary values.

$$u_{1,1} \in \{0,1\} \qquad y_{1,1} \in \{0,1\} \qquad z_{1,1} \in \{0,1\}$$
$$u_{1,2} \in \{0,1\} \qquad y_{1,2} \in \{0,1\} \qquad z_{1,2} \in \{0,1\}$$
$$u_{1,3} \in \{0,1\} \qquad y_{1,3} \in \{0,1\} \qquad z_{1,3} \in \{0,1\}$$
$$u_{2,1} \in \{0,1\} \qquad y_{2,1} \in \{0,1\} \qquad z_{2,1} \in \{0,1\}$$
$$u_{2,2} \in \{0,1\} \qquad y_{2,2} \in \{0,1\} \qquad z_{2,2} \in \{0,1\}$$
$$u_{2,3} \in \{0,1\} \qquad y_{2,3} \in \{0,1\} \qquad z_{2,3} \in \{0,1\}$$
$$u_{3,1} \in \{0,1\} \qquad y_{3,1} \in \{0,1\} \qquad z_{3,1} \in \{0,1\}$$
$$u_{3,2} \in \{0,1\} \qquad y_{3,2} \in \{0,1\} \qquad z_{3,2} \in \{0,1\}$$
$$u_{3,3} \in \{0,1\} \qquad y_{3,3} \in \{0,1\} \qquad z_{3,3} \in \{0,1\}$$

2. (30%) For the problem above, consider the minimum up-time and minimum-down time requirements of the power generating units, as presented in the table below.

Generating Unit	V_g^U (1)	V_g^D (2)
1	5	2
2	7	3
3	6	4

(1) Minimum number of periods the unit needs to remain online after starting up
(2) Minimum number of periods the unit needs to remain off-line after shutting down

Define M_g^U M_g^D as we defined them in lesson 10. Find the value of M_g^U and M_g^D based on the information provided.

M_g^U is the minimum uptime requirement and represents the required number of periods that generator g needs to be online at the beginning of the planning horizon. This is the minimum number of consecutive hours starting at times t=1.

M_g^D is the minimum downtime requirement and represents the required number of periods that generator g needs to be offline at the beginning of the planning horizon. This is the number of consecutive periods that generator g needs to be offline starting at time t=1.

a) We are solving the UC problem for t=1, 2,,24. We know the following to be true from our observations of the system in the prior day:

$u_{1,-2} = 0,\ u_{1,-1} = 0,\ u_{1,0} = 1$

Find the value of the parameter M_1^U

Since the minimum number of periods the generator 1 needs to remain online after starting up is 5 $(V_1^U = 5)$ and it became online at t=0, generator 1 should be online at least until t=4. As the planning horizon starts at t=1, generator 1 should be online for 4 hours after the beginning of planning horizon. So, $M_1^U = 4$

b) Write the minimum up-time constraints for t=1, t=2, t=3, t=4 and t=5, for g=1, assuming the information on initial conditions provided in a) above

$$u_{1,1} + u_{1,2} + u_{1,3} + u_{1,4} = 4$$
$$u_{1,5} \geq y_{1,5}$$
$$u_{1,2} + u_{1,3} + u_{1,4} + u_{1,5} \geq 4y_{1,2}$$
$$u_{1,3} + u_{1,4} + u_{1,5} \geq 3y_{1,3}$$
$$u_{1,4} + u_{1,5} \geq 2y_{1,4}$$

Note: The last three constraints are redundant as the unit should be online and cannot start up at t=2, 3 or 4.

c) We are solving the UC problem for t=1, 2,,24. We know the following to be true from our observations of the system in the prior day:

$u_{1,-4} = 1, u_{1,-3} = 1, u_{1,-2} = 1,\ u_{1,-1} = 1,\ u_{1,0} = 1$

Find the value of the parameter M_1^U

As generator 1 was online since t= -4, it was already online for 5 hours when the planning horizon starts at t=1. Since $V_1^U = 5$, there is no restriction to keep generator 1 online at t=1, so $M_1^U = 0$.

d) Write the minimum up-time constraints for t=1, t=2, t=3, t=4 and t=5, for g=1, assuming the information on initial conditions provided in c) above

$$u_{1,1} + u_{1,2} + u_{1,3} + u_{1,4} + u_{1,5} \geq 5 \times y_{1,1}$$
$$u_{1,2} + u_{1,3} + u_{1,4} + u_{1,5} \geq 4y_{1,2}$$
$$u_{1,3} + u_{1,4} + u_{1,5} \geq 3y_{1,3}$$
$$u_{1,4} + u_{1,5} \geq 2y_{1,4}$$
$$u_{1,5} \geq y_{1,5}$$

e) We are solving the UC problem for t=1, 2,,24. We know the following to be true from our observations of the system in the prior day:

$u_{3,-4} = 1, u_{3,-3} = 1, u_{3,-2} = 0,\ u_{3,-1} = 1,\ u_{3,0} = 1$

Find the value of the parameter M_3^U

As generator 3 was online since t= -1, it was already online for 2 hours when the planning horizon starts at t=1. Since $V_3^U = 6$, generator 3 should be online at least until t=4. As the planning horizon starts at t=1, generator 3 should be online for 4 hours after the beginning of planning horizon. So, $M_3^U = 4$.

f) Write the minimum up-time constraints for t=1, t=2, t=3, t=4 and t=5, for g=3, assuming the information on initial conditions provided in e) above

$$u_{3,1} + u_{3,2} + u_{3,3} + u_{3,4} = 4$$
$$u_{3,5} \geq y_{3,5}$$
$$u_{3,2} + u_{3,3} + u_{3,4} + u_{3,5} \geq 4y_{3,2}$$
$$u_{3,3} + u_{3,4} + u_{3,5} \geq 3y_{3,3}$$
$$u_{3,4} + u_{3,5} \geq 2y_{3,4}$$

Note: The last three constraints are redundant as the unit should be online and cannot start up at t=2, 3, 4 anyway.

g) We are solving the UC problem for t=1, 2,,24. We know the following to be true from our observations of the system in the prior day:

$u_{3,-4} = 1, u_{3,-3} = 1, u_{3,-2} = 1,\ u_{3,-1} = 0,\ u_{3,0} = 0$

Find the value of the parameter M_3^D

As generator 3 was offline since t= -1, it was already offline for 2 hours when the planning horizon starts at t=1. Since $V_3^D = 4$, generator 3 should be offline at least until t=2. As the planning horizon starts at t=1, generator 3 should be offline for 2 hours after the beginning of planning horizon. So $M_3^D = 2$.

Write the minimum **down-time** constraints for t=1, t=2, t=3, t=4 and t=5, for g=3, assuming the information on initial conditions provided in g) above

For formulating these constraints, we use the general expressions:

Eq. 1: $$f_g = \min\{T, M_g^D\}$$

Eg.2: $$\sum_{t=1}^{f_g} u_{g,t} = 0$$

Eq.3: $$u_{g,t} + \sum_{j=t-V_g^D+1, j\geq 1}^{t} z_{g,j} \leq 1 \quad \forall\ t = f_g + 1,..,T \quad \forall\ \mathrm{g}$$

As calculated in g) above, $M_3^D = 2$. Therefore from eq.1:

$$f_g = \min\{5,2\} = 2$$

From eq.2: $\sum_{t=1}^{2} u_{3,t} = 0 \rightarrow u_{3,1} + u_{3,2} = 0$

From eq.3 we know there will be constraints for $t = f_g + 1$. So, we can write constraints for t=3, t=4, t=5, etc.. using the expression $u_{g,t} + \sum_{j=t-V_g^D+1, j\geq 1}^{t} z_{g,j} \leq 1$

For t=3:

$$j = \max(t - V_g^D + 1, 1)$$

$$j = \max(3 - 4 + 1, 1) = 1$$

$$u_{3,3} + \sum_{j=1}^{3} z_{3,j} \leq 1$$

For t=4:

$$j = \max(t - V_g^D + 1, 1)$$

$$j = \max(4 - 4 + 1, 1) = 1$$

$$u_{3,4} + z_{3,1} + z_{3,2} + z_{3,3} + z_{3,4} \leq 1$$

For t=5:

$$j = \max(t - V_g^D + 1, 1)$$

$$j = \max(5 - 4 + 1, 1) = 2$$

$u_{3,5} + z_{3,2} + z_{3,3} + z_{3,4} + z_{3,5} \le 1$

An alternative way to write these constraints for t=3, t=4, t=5 is:

For t=3:

$u_{3,3} + u_{3,4} + u_{3,5} \le -3z_{3,3} + 3$ or

$u_{3,3} + u_{3,4} + u_{3,5} \le 3(1 - z_{3,3})$

For t=4:

$u_{3,4} + u_{3,5} \le -2z_{3,4} + 2$

Or

$u_{3,4} + u_{3,5} \le 2(1 - z_{3,4})$

For t=5:

$u_{3,5} \le -z_{3,5} + 1$

Or

$u_{3,5} \le 1 - z_{3,5}$

h) We are solving the UC problem for t=25, 26,,48. We know the following to be true from our observations of the system in the prior day:

$u_{3,20} = 1, u_{3,21} = 1, u_{3,22} = 1,\ u_{3,23} = 1,\ u_{3,24} = 0$

Find the value of the parameter M_3^D

As generator 3 was offline since t= 24, it was already offline for 1 hour when the planning horizon starts at t=25. Since $V_3^D = 4$, generator 3 should be offline at least until t=27. As the planning horizon starts at t=24, generator 3 should be offline for 3 hours after the beginning of planning horizon. So $M_3^D = 3$.

i) Write the minimum **down-time** constraints for t=24, t=25, t=26, t=27 and t=28, for g=3, assuming the information on initial conditions provided in i) above

For formulating these constraints, we can use again the general expressions:

Eq. 1: $f_g = \min\{T, M_g^D\}$

Eg.2: $\sum_{t=1}^{f_g} u_{g,t} = 0 \quad \rightarrow \sum_{t=1}^{2} u_{g,t} = 0 \ \rightarrow u_{g,1} + u_{g,2}$

$$u_{g,t} + \sum_{j=t-V_g^D+1, j\ge 1}^{t} z_{g,j} \le 1 \quad \forall\ t = f_g + 1,..,T \quad \forall \text{ g}$$

Eq.3:

BUT WE NEED TO BE CAREFULL TO REMEMBER THAT OUR HORIZON STARTS AT t=25.

As calculated in i) above, $M_3^D = 3$. We have to be careful when applying equation 1: , $f_g = \min\{T,3\} = 3$ because, this would mean that we need to write equations for t=4, t=5, t=6.. etc.. but this would be incorrect because t=4, t=5,…..t=23, t=24 are all in the PAST; our planning horizon starts at t=25. Therefore $M_3^D = 3$ means that we need to make sure the unit remains OFF for t=25, t=26, t=27.
Knowing this, we can write the first constraint:

$\sum_{t=25}^{27} u_{3,t} = 0$ which is the same as: $u_{3,25} + u_{3,26} + u_{3,27} = 0$

Before we found f_g with equation 1 and knew that the summation of equation 2 would go from t to f_g. But we just wrote that equation above, which constraints the u variables for t=25, t=26 and t=27. That means that f_g=27. If we use this value of f_g we can use equation 3 to find the other constraints.
Using $f_g = 27$, From eq.3 we know there will be several constraints, with the first one being for $t = f_g + 1$. We again need to be careful and remember that our planning horizon starts at t=25 and $f_g = 27$. So, we can write constraints for t=28, t=29, t=30, ..t=48, using the expression $u_{g,t} + \sum_{j=t-V_g^D+1, j\geq 1}^{t} z_{g,j} \leq 1$ However, we are only asked to write these constraints for t=24, ..t=28. We already constrained t=25, t=26, and t=27 (and we said t=24 is in the past, so no need to write a constraint for it). So, the only remaining constraint is for t=28

For t=28:

$$j = \max(t - V_g^D + 1, 1)$$

$$j = \max(28 - 4 + 1, 1) = 25$$

$$u_{3,28} + \sum_{j=25}^{28} z_{3,j} \leq 1$$

$$u_{3,28} + z_{3,25} + z_{3,26} + z_{3,27} + z_{3,28} \leq 1$$

Another alternative to write these equations without having to think too much about f_g, would be to reset the counter of hours and realize that t=24 is t=0 of the second day, and hence, t=25 is t=1 of the second day. In that case $f_g = \min\{T,3\} = 3$ would be correct.

An alternative way to write these minimum down time constraints we found above is:

$$u_{3,25} + u_{3,26} + u_{3,27} = 0$$
$$u_{3,27} + u_{3,28} \leq 2\,(1 - z_{3,27})*$$
$$u_{3,28} \leq 1 - z_{3,28}$$

Or

$$u_{3,25} + u_{3,26} + u_{3,27} = 0$$
$$u_{3,27} + u_{3,28} \leq -2z_{3,27} + 2\ *$$
$$u_{3,28} \leq -z_{3,28} + 1$$

*(Note this one is redundant since we know, from the equation above that the unit needs to be off at t=27 and hence, it will not shutdown at t=27)

3. (20%) Consider a small balancing authority that needs to schedule three thermal generators for the next six hours. Formulate and **solve** the Unit Commitment problem, disregarding the ramp-feasibility of reserves and min up time and min down time requirements, and assuming that:

The generators parameters are as follows:

Generating Unit	C_g^F (1)	C_g^{SD} (2)	C_g^{SU} (3)	C_g^V (4)	$P_g^{\max}$ (5)	$P_g^{\min}$ (6)	R_g^D (7)	R_g^{SD} (8)	R_g^{SU} (9)	R_g^U (10)	V_g^U (11)	V_g^D (12)
1	0	0	800	5	300	80	30	30	100	50	3	2
2	0	0	500	15	200	50	40	40	70	60	2	2
3	0	0	250	30	100	30	50	50	40	70	1	2

Table 1: Technical characteristics of generators. (1) Online (fixed) cost of generating unit g ($/h). (2) shut-down cost of generating unit g ($). (3) Start-up cost of generating unit g ($). (4) Variable cost of generating unit g ($/MWh). (5) Power generation capacity of unit g (MW). (6) minimum power output of unit g (MW). (7) ramping-down limit of generating unit g (MW/h). (8) shut-down ramping limit of generating unit g (MW/h). (9) start-up ramping limit of generating unit g(MW/h). (10) ramping-up limit of generating unit g (MW/h). (11) Minimum number of time periods the unit must run after starting up. (12) Minimum number of periods the unit must be off after shutting down.

The initial operating conditions are:

Generating Unit	$u_{g,o}$ (1)	$p_{g.0}$ (2)	M_g^U (3)	M_g^D (4)
1	1	130	2	0
2	0	0	0	0
3	0	0	0	0

(1) On/off status of generator g at time t=0. (2) Power generation of generator g at time t=0 (MW). (3) Number of time periods that generator g is required to be online at the start of the planning horizon. (4) Number of time periods that generator g is required to be offline at the start of the planning horizon.

The demand and reserve requirements are (MW):

Time period t □	1	2	3	4	5	6
p_t^D	240	250	200	170	230	190
p_t^R	10	10	10	10	10	10

a. Present a table with your solution specifying for all generators and time periods the following: 1. on-off status, 2. start-up status, 3. Shutdown status, 4. Power generation,

Generator	Hour	On-Off	Startup	Shutdown	Power Generation
1	1	1	0	0	180
1	2	1	0	0	180
1	3	1	0	0	150
1	4	1	0	0	120
1	5	1	0	0	170
1	6	1	0	0	140
2	1	1	1	0	60
2	2	1	0	0	70
2	3	1	0	0	50
2	4	1	0	0	50
2	5	1	0	0	60
2	6	1	0	0	50
3	1	0	0	0	0
3	2	0	0	0	0
3	3	0	0	0	0
3	4	0	0	0	0
3	5	0	0	0	0
3	6	0	0	0	0

b. Using the solution of this UC calculate the reserves provided at each time period from each generator. In this calculation make sure to consider all of the generator's constraints -including ramping limits-. i.e., you do not need to enforce ramp-feasibility of reserves in the UC formulation or solution, but account for this when calculating the reserves. For which time periods are reserves provided above the reserve requirements? Should we always enforce ramp-feasibility of reserve requirements in the UC formulation and solution?

Generator	Hour	On-Off	Startup	Shutdown	Power Generation	Reserves Provided
1	1	1	0	0	180	50
1	2	1	0	0	180	50
1	3	1	0	0	150	50
1	4	1	0	0	120	50
1	5	1	0	0	170	50
1	6	1	0	0	140	50
2	1	1	1	0	60	60
2	2	1	0	0	70	60
2	3	1	0	0	50	60
2	4	1	0	0	50	60
2	5	1	0	0	60	60
2	6	1	0	0	50	60
3	1	0	0	0	0	0
3	2	0	0	0	0	0
3	3	0	0	0	0	0
3	4	0	0	0	0	0
3	5	0	0	0	0	0
3	6	0	0	0	0	0

For all the periods the reserves provided are above the reserve requirements. Yes, we always should enforce ramp-feasibility of reserve requirements.