3 Playing with Numbers

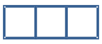
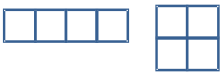
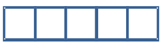
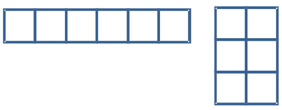
Factors and Multiples

Active Maths 1

To understand the concept of factors and multiples.

Exploring the Concept

1. Copy the table

Number of squares	Sketch of rectangle formed	Dimensions of each rectangle
2		1 × 2
3		1 × 3
4		1 × 4, 2 × 2
5		1 × 5
6		1 × 6, 2 × 3
⋮		
20		

2. Use square tiles to complete the table.

Drawing Conclusions

1. For the numbers 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20 more than one rectangle can be formed?

2. For the numbers 2, 3, 5, 7, 11, 13, 17, 19 only one rectangle can be formed?

3. For the numbers in which only one rectangle is formed, what do you notice about the dimensions of the rectangle?

When two or more numbers are multiplied, each number of the product is called a factor.

$1 \times 7 = 7$ — The factors of 7 are 1 and 7.

$1 \times 6 = 6$ and $2 \times 3 = 6$ — The factors of 6 are 1, 6, 2 and 3.

and the product is a multiple of each of its factors.

Do You Remember ?

- 1 is neither prime nor composite.

Facts about factors and multiples

(1) $1 \times 2 = 2$, $1 \times 3 = 3$, $1 \times 5 = 5$

So, 1 is a factor of every number.

(2) $2 \times 1 = 2$, $3 \times 1 = 3$, $4 \times 1 = 4$

and it is true for all numbers. So, it can be said that every number is a factor of itself.

(3) $1 \times 8 = 8$, $2 \times 4 = 8$, $4 \times 2 = 8$, $8 \times 1 = 8$

Clearly, $1 < 8$, $2 < 8$, $4 < 8$, and $8 = 8$. We can say that every factor of the number is less than or equal to the given number.

(4) Factors of 4 are 1, 2 and 4

Factors of 8 are 1, 2, 4 and 8

Factors of 16 are 1, 2, 4, 8 and 16

Factors of 64 are 1, 2, 4, 8, 16, 32 and 64

The number of factors are 3, 4, 5 and 7 respectively, i.e., the numbers of factors are countable. Thus, the number of factors of a given number are always finite, i.e., they can be counted.

(5) $7 \times 1 = 7$, $7 \times 2 = 14$, $7 \times 3 = 21$, $7 \times 4 = 28$, $7 \times 5 = 35$

$7 = 7$, $14 > 7$, $21 > 7$, $28 > 7$, $35 > 7$, Thus 7, 14, 21, 28, 35 are multiples of 7 and all these are either equal to or greater than 7. Thus, every multiple is equal to or greater than the given number.

(6) Multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, ... The number of multiples do not end. So, it can be said that the number of multiples of a given number is infinite.

(7) 2 is a multiple of 2 and 32 is a multiple of 32. Thus, every number is a multiple of itself.

Building Concepts 1

Find two numbers whose difference is 2 and product is 24.

Explanation

Here we have to find two factors of 24 whose difference is 2

And the product is 24.

All factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

Clearly, 4 and 6 are two factors of 24 such that 6 – 4 = 2 and

6 × 4 = 24.

Hence required numbers are 6 and 4.

Numerical Ability 1

How many multiples of 17 are in between 35 and 100?

Solution

There are only 3 multiples of 17 that are in between 35 and 100. They are 51, 68 and 85.

17 × 3 = 51

17 × 4 = 68

17 × 5 = 85

Check your Concepts 1

1. Write all the factors of 36, 48 and 68.
2. Write first 6 multiples of 7 and 13.

Perfect number

A number is called a **perfect number** if the sum of all its factors is equal to twice the number.

Factors of 6 are 1, 2, 3 and 6

Now, the sum of the factors of 6

= 1 + 2 + 3 + 6 = 12 = 2 times 6

Factors of 28 are 1, 2, 4, 7, 14 and 28

Now, the sum of the factors of 28

= 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 times 28.

The numbers like 6 and 28 are called perfect numbers.

Prime and composite numbers

Active Maths 2

To understand the concept of prime and composite numbers

Exploring the Concept

1. Copy the following table.

Numbers	Factors	Number of factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5		
6		
7		
8		
9		
10		
11		
12		

2 is the smallest prime number which is even. Every prime number except 2 is odd.

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2. Complete the table.

Drawing Conclusions

1. How many numbers do you find having only one factor ? Can you find some more numbers which have only one factor ?
2. How many numbers do you find having exactly 2 factors ? Can you find some more numbers which have only two factors ?
3. List 5 more numbers having more than two factors.

A whole number that has exactly two unique factors, 1 and the number itself, is a **prime number**. A number greater than 1 with more than two factors is a **composite number**.

1 is neither prime nor composite number.

Numerical Ability 2

Tell whether each number is prime, composite, or neither.

78, 91, and 71

Solution

Factors of 78 are 1, 2, 3, 6, 13, 26, 39 and 78.

Since, 78 has more than two factors, it is a composite number.

Factors of 91 are 1, 7, 13 and 91.

Since, 91 has more than two factors, it is a composite number.

Factors of 71 are 1 and 71.

Since, 71 has exactly two factors, it is a prime number.

Sieve of Eratosthenes

Eratosthenes was a Greek mathematician in the third century B.C. He gave a method to find the prime numbers from 1 to 100, without actually checking the number of factors.

List all numbers from 1 to 100, as shown below.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Step 1 : Cross out 1 because it is not a prime number.

Step 2 : Encircle 2, cross out all the multiples of 2, other than 2 itself, i.e. 4, 6, 8 and so on.

Step 3 : You will find that the next uncrossed number is 3. Encircle 3 and cross out all the multiples of 3, other than 3 itself.

Step 4 : The next uncrossed number is 5. Encircle 5 and cross out all the multiples of 5 other than 5 itself.

Step 5 : Continue this process till all the numbers in the list are either encircled or crossed out.

All the encircled numbers are prime numbers. All the crossed out numbers, other than 1 are composite numbers.

This method is called the Sieve of Eratosthenes.

To determine whether a given number is prime or not

Case 1. When the number lies between 100 and 200.

To check whether a number between 100 and 200 is prime or not, test its divisibility by a prime number less then 15, i.e., 2, 3, 5, 7, 11 and 13. If it is divisible by any of them, then it is not a prime number, otherwise it is a prime number.

Building Concepts 2

Is 161 a prime number ?

Explanation

Since 13 × 13 = 169 > 161. Therefore, on testing divisibility by each of the prime numbers 3, 5, 7, 11 we find that it is divisible by 7. Hence, 161 is not a prime number.

Case 2. When the number lies between 100 and 400.

To check whether a number between 100 and 400 is prime or not, test its divisibility by a prime number less then 20, i.e., 2, 3, 5, 7, 11, 13, 17 and 19. If it is divisible by any of them, then it is not a prime number, otherwise it is a prime number.

Building Concepts 3

Is 353 a prime number ?

Explanation

Since 19 × 19 = 361 > 353, we test divisibility by 2, 3, 5, 7, 11, 13, 17. We find that it is not divisible by any one of them. Hence, 353 is a prime number.

Twin Primes

A twin prime is a prime number that differs from another prime number by two. Some examples of twin prime pairs are (3, 5), (5, 7), (11, 13), (17, 19), (29, 31) and (41, 43). Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin.

Co-prime numbers

Two numbers are said to be co-prime or relatively prime if they have only 1 as a common factor. E.g. 2 and 3 are co-prime.

> Any two prime numbers are always co primes, e.g., 3 and 5, or 11 and 13, But two co prime need not be both prime numbers.
>
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The two numbers in the pair of co-prime numbers need not be both prime. They both can be composite, or prime, or one composite and the other prime.

Building Concepts 4

Are 35 and 39 co-prime numbers ?

Explanation

35 = 7 × 5 × 1

39 = 3 × 13 × 1

Though both 35 and 39 are composite numbers, the only factor common between them is 1. Therefore, 35 and 39 are co-prime numbers.

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Check your Answers 1

1. Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36,
   Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
   Factors of 68 are 1, 2, 4, 17, 34, 68
2. Multiples of 7 are 7, 14, 21, 28, 35, 42
   Multiples of 13 are 13, 26, 39, 52, 65, 78

A set of three consecutive prime numbers differing by 2, is called a prime triplet. The only prime triplet known so far is (3, 5, 7).

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Check your Concepts 2

1. Find the largest prime number that you need to test as a divisor to determine whether each of the following numbers is prime or not.
   (i) 109 (ii) 239 (iii) 397 (iv) 317
   (v) 361 (vi) 263
2. List all the prime numbers less than 25.
3. Give two examples each of co-prime numbers satisfying the condition that numbers of the pair should be
   (i) both prime (ii) both composite
   (iii) one prime and other composite.
4. Which of the following pair of numbers are co-prime numbers?
   (i) 88 and 187 (ii) 675 and 392

Divisibility Rules

Divisibility by 2

A number is divisible by two, if it has any of the digits 0, 2, 4, 6 or 8 in its ones place.
E.g. 24, 632, 478, 500 etc. are divisible by two whereas 301, 783 etc. are not divisible by two.

Divisibility by 3

A number is divisible by three, if the sum of its digits is a multiple of three. E.g 27, 66 are divisible by three because the sum of their digits 2 +7 = 9, 6 + 6 =12 are multiples of three, whereas 31, 88 are not divisible by three because their sums 3 +1 = 4, 8 + 8 =16 are not a multiples of three.

Divisibility by 4

A number with 3 or more digits is divisible by 4, if the number formed by its last two digits (i.e. ones and tens) is divisible by 4. E.g. 284 is divisible by 4 because 84 is divisible by 4 whereas 315 is not divisible by 4 because 15 is not divisible by 4.

Divisibility by 5

A number which has either 0 or 5 in its ones place is divisible by 5. E.g. 15, 40 are divisible by 5 whereas 12, 78 are not divisible by 5.

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Divisibility by 6

A number is divisible by 6, if it is divisible by 2 and 3 both. E.g. 72 is divisible by 6 because it is divisible by 2 and 3 both whereas 81 is not divisible by 6 because it is divisible by 3 but not by 2.

Divisibility by 8

A number with 4 or more digits is divisible by 8, if the number formed by its last three digits (i.e. ones, tens and hundreds) is divisible by 8. E.g.74512 is divisible by 8 because 512 is divisible by 8.

Divisibility by 9

A number is divisible by 9, if the sum of its digits is a multiple of 9. E.g. 756 is divisible by 9 because 7 + 5 + 6 = 18, is divisible by 9.

Divisibility by 10

A number is divisible by 10, if it has 0 in the ones place. E.g. 20, 10 , 900 etc. are divisible by 10.

Divisibility by 11

A number is divisible by 11, if the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number is either 0 or divisible by 11.

E.g. 121, 1331 are divisible by 11 whereas 3456788 is not. The following table will make it more clear to you.

Number	Sum of the digits (at odd places from right)	Sum of the digits (at even places from right)	Difference	Divisible by 11
121	1 + 1 = 2	2	2 – 2 = 0	yes
1331	1 + 3 = 4	3 + 1 = 4	4 – 4 = 0	yes
3456788	8 + 7 + 5 + 3 = 23	8 + 6 + 4 = 18	23 – 18 = 5	no

Divisibility by 25

A number is divisible by 25, if the number formed by the digits at the tens and the unit place is divisible by 25.

E.g. 8750, 23275, 8926825 are divisible by 25 because the number formed by tens and unit place in these numbers viz., 50, 75, 25 respectively are divisible by 25.

Some more divisibility rules

(1) If a number is divisible by another number then it is divisible by each of the factors of that number.
E.g. 48 is divisible by 12 and the factors of 12 are 1, 2, 3, 4, 6, 12. Clearly, each of the factors of 12 divide 48 exactly.

(2) If a number is divisible by two co-prime numbers then it is divisible by their product also.
E.g. 60 is divisible by 3 and 5 which are co-primes. 60 is also divisible by $3 \times 5 = 15$.

(3) If two given numbers are divisible by a number, then their sum is also divisible by that number.
E.g. The number 27 and 33 both are divisible by 3, then their sum $27 + 33 = 60$ is also divisible by 3.

(4) If two given numbers are divisible by a number, then their difference is also divisible by that number.
E.g. The number 16 and 20 are divisible by 4. Then, their difference $20 - 16 = 4$ is also divisible by 4.

Be Alert !

✸ We cannot divide any number by 0, or number divided by 0 is not defined. For e.g., $\frac{7}{0}$ = not defined

Building Concepts 5

Using divisibility test, determine which of the following numbers are divisible by 2, by 3, by 5, by 9 and by 11?

(i) 472 (ii) 369 (iii) 3040 (iv) 8265 (v) 230514
(vi) 406747

Explanation

We know that a number is divisible by 2 if its unit's digit is 0, 2, 4, 6, or 8.
∴ Numbers divisible by 2 are: 472, 3040, 230514.
A number is divisible by 3 if the sum of its digits is a multiple of 3.
In 472, sum of the digits is $4 + 7 + 2 = 13$ which is not divisible by 3.
In 369, sum of the digits is $3 + 6 + 9 = 18$ which is divisible by 3.
In 3040, sum of the digits is $3 + 4 = 7$ which is not divisible by 3.
In 8265, sum of the digits is $8 + 2 + 6 + 5 = 21$ which is divisible by 3.
In 230514, sum of the digits is 15 which is divisible by 3.
In 406747, sum of the digits is 28 which is not divisible by 3.
∴ Numbers divisible by 3 are: 369, 8265, 230514.
A number is divisible by 5 if the unit's digit is '0' or '5'.
∴ Numbers divisible by 5 are: 3040, 8265.
The sum of the digits in (ii) is 18 which is completely divisible by 9.
∴ The number divisible by 9 is 369.
In 406747 the sum of digits at odd places i.e. $4 + 6 + 4 = 14$ and sum of the digits at even places
i.e. $0 + 7 + 7 = 14$ and difference is $14 - 14 = 0$
∴ 406747 is divisible by 11.

Numerical Ability 3

Find out whether 896 is divisible by 7 or not.

Solution

In the number 896, last digit is 6.

Twice of 6 is $6 \times 2 = 12$

Number formed by other digits = 89

Difference = 89 – 12

= 77, which is divisible by 7.

Hence, 896 is divisible by 7.

Check your Answers 2

1. (i) 7 (ii) 13 (iii) 19 (iv) 17 (v) 19 (vi) 13
2. All prime numbers less than 25 are 2, 3, 5, 7, 11, 13, 17, 19, 23
3. (i) (2, 3) (ii) (4, 9) (iii) (3, 8)
4. (ii) 675 and 392 are co-prime numbers.

Check your Concepts 3

1. Using divisibility tests, determine which of the following numbers are divisible by 2; by 3; by 4; by 5; by 6; by 8; by 9; by 10 ; by 11 (say, yes or no):

Number	Divisible by								
	2	3	4	5	6	8	9	10	11
168	......	......	......	......	......	......	......	......	......
748	......	......	......	......	......	......	......	......	......
275	......	......	......	......	......	......	......	......	......
5500	......	......	......	......	......	......	......	......	......
1035	......	......	......	......	......	......	......	......	......
2158	......	......	......	......	......	......	......	......	......
8370	......	......	......	......	......	......	......	......	......
43974	......	......	......	......	......	......	......	......	......
9075	......	......	......	......	......	......	......	......	......
5736	......	......	......	......	......	......	......	......	......
63921	......	......	......	......	......	......	......	......	......
871245	......	......	......	......	......	......	......	......	......
183692	......	......	......	......	......	......	......	......	......
7039452	......	......	......	......	......	......	......	......	......
2152	......	......	......	......	......	......	......	......	......

2. Think 2 more examples each of the special rules of divisibility mentioned above.

Prime factorisation

When a number is expressed as a product of its factors, we say that the number has been factorised. E.g. we can write 72 as product of 12 and 6 or as a product of 8 and 9 or as a product of 2, 4 and 9, in each of these cases, we will say that the number 72 has been factorised into two or three factors, as the case may be.

Most uniquely we can express 72 as $72 = 2 \times 2 \times 2 \times 3 \times 3$.

In this factorization of 72, all factors are prime numbers. It is representing the prime factorisation of 72.

Such a factorisation of a number in which the number is expressed as a product of primes is called prime factorisation of the number.

Note: When writing the prime factorization, it is customary to write the prime factors in ascending order, that is from least to greatest.

There can be many factorization of a number n but there can be only one prime factorization.

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Building Concepts 6

Express 108 as a product of prime factors and draw a factor tree.

Explanation

(i) Divide 108 starting from the smallest prime number which will divide it exactly.

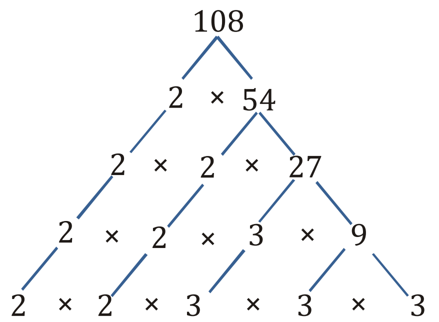
(ii) Continue dividing the quotient until that number is no more divisible by the smallest prime number.

(iii) Then try to divide the remaining quotient by the next higher prime number that will divide this quotient.

2	108
2	54
3	27
3	9
3	3
	1

$\therefore 108 = 2 \times 2 \times 3 \times 3 \times 3$

(iv) Carry on this process until the last quotient is also a prime number. Factor tree for 108 is



Numerical Ability 4

Write the greatest 4 – digit number and express it as a product of prime factors.

Solution

The greatest 4 – digit number = 9999. On factorising 9999, we have.

3	9999
3	3333
11	1111
101	101
	1

Thus, the prime factorisation of 9999 = 3 × 3 × 11 × 101. 101 is a prime number as it is not divisible by 2, 3, 5, 7 (we need not to check by 11 since $11^2 > 101$).

Be Alert !

✸ Prime factorization of 54 is 2 × 3 × 3 × 3 not 9 × 6.

Common factors and common multiples

The factor of a number can divide the number and the multiple of a number is divisible by the number. E.g. 2 being a factor of 12 can divide 12 and 16. As 12 and 16 are multiples of 4, is divisible by 4.

Building Concepts 7

Find the common factors of 54 and 72.

Explanation

The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54.

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72.

The common factors are 1, 2, 3, 6, 9 and 18.

> In prime factorization, 1 will not come as product as it is not a prime number.
>
> SPOT LIGHT

Numerical Ability 5

List three common multiples of 8 and 12.

Solution

Multiples of 8 are 8, 16, 24 , 32, 40, 48 , 56, 64, 72 , 80, ...

Multiples of 12 are 12, 24 , 36, 48 , 60, 72 , 84, ...

Common multiples of 8 and 12 are 24, 48 and 72.

Active Maths 3

To go deep into the concept of common factors and common multiples

Exploring the Concept

Complete the following tables:

1.

Numbers	Factors	Common factors
4 and 18		
4 and 15		
4, 12 and 16		

2.

Numbers	Multiples	Common multiples
4 and 6		
3, 5 and 6		

Drawing Conclusions

1. Is it possible for two numbers to always have a common factor?

2. What is that number which is always one of the common factors of given numbers?

3. The possible number of common factors of the given numbers is ______ (finite/infinite).

4. The possible numbers of common multiples of the given numbers is ________(finite/infinite).

Highest common factor (HCF)

The highest common factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also known as **Greatest Common Divisor** (GCD).

You can find HCF of given numbers in three ways as follows:

(a) Listing factors

(b) Prime factorisation method

(c) Continued division method

The largest number which exactly divides the given two or more numbers is called the Highest Common Factor (HCF) of these numbers.

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Numerical Ability 6

Find the HCF of 18, 27 and 45 by listing factors.

Solution

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The factors of 27 are 1, 3, 9 and 27.

The factors of 45 are 1, 3, 5, 9, 15 and 45.

The common factors are 1, 3, and 9.

The highest common factor (HCF) is 9.

> The HCF of given numbers is smaller than or equal to the smallest number.
>
> **SPOT LIGHT**

Numerical Ability 7

Find the HCF of 98 and 112 using the prime factorisation method.

Solution

98 = 2 × 7 × 7

112 = 2 × 2 × 2 × 2 × 7

The highest common factor = 2 × 7

So, HCF = 2 × 7 = 14

$$\begin{array}{r|l} 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 112 \\ \hline 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

HCF by continued division method

This method of division was invented by Greek mathematician "Euclid". Divide the larger number by the smaller and then divide the previous divisor by the remainder until the remainder is zero. The last divisor is the HCF of the numbers.

Numerical Ability 8

Find the HCF of 255 and 357 by continued division method.

Solution

```
255 ) 357 (
     -255
      102 ) 255 (
           -204
HCF →        51 ) 102 ( 2
                 -102
                    0
```

Since 51 is the last divisor, so it is the HCF of 255 and 357.

Numerical Ability 9

Find the HCF of 2261, 3059 and 3325.

Solution

Step - I

```
2261)3059(1
    -2261
      798)2261(2
         -1596
           665)798(1
               665
               133)665(5
                  -665
                     0
```

Step - 2

```
133)3325(25
    -266
    133)665(5
       -665
          0
```

$\therefore$ HCF of 2261 and 3059 = 133

Therefore, HCF of 2261, 3059 and 3325 is **133.**

Be Alert !

✸ HCF of 5000 and 5001 is 1 not 0.

HCF of two co prime numbers will always be 1. HCF of two or more prime numbers is 1.

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Problems on HCF

Building Concepts 8

Find the greatest number that will divide 79, 117, and 59 leaving the remainders, 7, 9 and 11 respectively.

Explanation

As the word greatest is used, we have to find the highest common factor that divides 79 leaving a remainder 7, 117 leaving a remainder 9, and 59 leaving a remainder 11. First subtract the respective remainders from the corresponding numbers and then find the HCF. Hence the numbers which are divisible by the highest common factor are:

79 – 7 = 72, 117 – 9 = 108 and 59 – 11 = 48. The HCF of 72, 108 and 48 = 2 × 2 × 3 = 12.

Hence, 12 is the greatest number that will divide 79, 117 and 59 leaving remainders 7, 9 and 11 respectively.

Building Concepts 9

The length, breadth and height of a room are 8 m 25 cm, 6m 75 cm and 4 m 50 cm respectively. Determine the longest tape which can measure the three dimensions of the room exactly.

Explanation

Length = 8 m 25 cm = (8 × 100 + 25) cm = 825 cm

Breadth = 6 m 75 cm = (6 × 100 + 75) cm = 675 cm

Height = 4 m 50 cm = (4 × 100 + 50) cm = 450 cm

675)825(1
−675
150)675(4
−600
75)150(2
−150
0

75)450(6
−450
0

∴ HCF of 825, 675 and 450 = 75

Hence, the required largest tape = 75 cm.

Building Concepts 10

A rectangular courtyard is 20 m 16 cm long and 15 m 60 cm broad. It is to be paved with square stones of the same size. Find the least possible number of such stones.

Explanation

Length = 20 m 16 cm = 2016 cm

Breadth = 15 m 60 cm = 1560 cm

1560)2016(1
−1560
456)1560(3
−1368
192)456(2
−384
72)192(2
−144
48)72(1
−48
24)48(2
−48
0

HCF of 1560 and 2016 = 24

∴ Side of each square stone required = 24 cm

∴ The least number of stones = $\frac{1560}{24} \times \frac{2016}{24} = 65 \times 84 = 5460$

∴ The number of the square stones required = 5460.

Note: HCF is mentioned as the largest, greatest, highest or maximum number or value, remember that it is factor and hence it is smaller than all the numbers or equal to the smallest numbers.

Least Common Multiple (LCM)

The least common multiple of two or more numbers is the least of their common multiples.
You can find LCM of given numbers in three ways as follows:
(a) Listing multiples
(b) Prime factorisation method.
(c) Common division.

Numerical Ability 10

Find the LCM of 8 and 12 by listing multiples.

Solution

Multiples of 8: 8, 16, **24**, 32, 40, **48**, 56, ...
Multiples of 12: 12, **24**, 36, **48**, 60, ...
Common multiples of 8 and 12 are: 24, 48, 72, ...
The Least Common Multiple (LCM) is 24.

Numerical Ability 11

Find the LCM of 84 and 96 using the prime factorisation method.

Solution

$$\begin{array}{c|c} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$84 = 2 \times 2 \times 3 \times 7$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

The common factors of 84 and 96 are 2, 2, and 3. Hence, the HCF is $2 \times 2 \times 3 = 12$. But in this case, we are not finding the HCF, we are finding the LCM.

The LCM of co-prime numbers is equal to the product of co- prime.

SPOT LIGHT

The common factors are $2 \times 2 \times 3$. The factors which are not common to both the numbers are 7, 2, 2, 2.

LCM is the product of the common factors and the factors which are not common.

That is, $\underbrace{2 \times 2 \times 3}_{\text{common factors}} \times \underbrace{2 \times 2 \times 2 \times 7}_{\text{other factors}} = 672$

This means that 672 is a multiple of 84 and 96 and is the smallest multiple common to both these numbers. So, 672 is the LCM of 84 and 96.

LCM by common division method

Step 1: Write the given numbers in a row separated by commas.

Step 2 : Divide these numbers by the least prime number which divides at least one of the given numbers.

Step 3 : Write the quotients and the numbers that are not divisible by the prime number in the second row. Then repeat Steps 2 and 3 with the rows and continue till the numbers in a row are all 1.

Step 4 : The LCM is found out by multiplying all the prime divisors and quotients other than 1.

Numerical Ability 12

Find the LCM of 88 and 64.

Solution

2	88, 64
2	44, 32
2	22, 16
2	11, 8
2	11, 4
2	11, 2
11	11, 1
	1, 1

The LCM is the product of all the prime divisors and the quotients. Common divisors

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Quotient = 11.

The LCM of 88 and 64 is

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 = 704$.

Problems on LCM

Building Concepts 11

Find the smallest number which when divided by 25, 40 and 60 leaves remainder 7 in each case.

Explanation

2	25, 40, 60
2	25, 20, 30
5	25, 10, 15
	5, 2, 3

$\therefore$ LCM of 25, 40, 60 = $2 \times 2 \times 5 \times 5 \times 2 \times 3 = 600$

$\therefore$ The required smallest number = $600 + 7 = 607$.

HCF is a factor of LCM. In other words, LCM is a multiple of HCF.

SPOT LIGHT

Building Concepts 12

A boy saves Rs. 4.65 daily. Find the least number of days in which he will be able to save an exact number of rupees.

Explanation

₹ 4.65 = 465 paise

The number of paise which is an exact number of 100. By the condition of the question, the paise in the number of rupees saved by the boy will be a multiple of 465.

∴ LCM of 100 and 465 = 5 × 20 × 93 = 9300

5	100, 465
	20, 93

∴ The boy saves 9300 paise.

∴ The required number of days = 9300 ÷ 465 = 20.

The LCM of given numbers is greater than equal to the largest of the numbers.

SPOT LIGHT

Numerical Ability 13

The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 AM after what time will they change again simultaneously?

Solution

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

∴ LCM = 2 × 2 × 2 × 2 × 3 × 3 × 3

= 432

$$\begin{array}{r} 60 \overline{)\ 432\ } 7 \\ -420 \\ \hline 12 \end{array}$$

432 seconds = 7 minutes 12 seconds. Therefore,

the traffic lights will change simultaneously after 7 minutes 12 seconds.

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Numerical Ability 14

Find the smallest 3-digit number which is exactly divisible by 4, 5 and 6.

Solution

The LCM of 4, 5 and 6 is divisible by 4, 5 & 6.

LCM = 2 × 2 × 5 × 3 = 60

But we need the smallest 3-digit number which is a multiple of 60 as 4, 5 and 6 divides exactly only multiple of 60.

$$\begin{array}{r|l} 2 & 4,\ 5,\ 6 \\ \hline 2 & 2,\ 5,\ 3 \\ \hline 3 & 1,\ 5,\ 3 \\ \hline 5 & 1,\ 5,\ 1 \\ \hline & 1,\ 1,\ 1 \end{array}$$

These are 120, 180, 240, ...

The smallest 3-digit number which is exactly divisible by 4, 5 and 6 is 120.

Do You Remember ?

✸ HCF is always less than LCM of two given numbers.

Relationship between HCF and LCM

Active Maths 4

To derive the relation between HCF and LCM of two numbers

Exploring the Concept

1. Complete the following table

Numbers	HCF	LCM	HCF × LCM	Product of the numbers
6, 10				
25, 80				
12, 30				

Drawing Conclusions

What do you observe?

1. If a and b are two numbers, then

   $a \times b = \text{HCF} \times \text{LCM}$

2. $a = \dfrac{\text{HCF} \times \text{LCM}}{b}; \ b = \dfrac{\text{HCF} \times \text{LCM}}{a}$

3. $\text{HCF} = \dfrac{a \times b}{\text{LCM}}; \ \text{LCM} = \dfrac{a \times b}{\text{HCF}}$

Note: If two numbers a and b are there and b is multiple of a then

HCF (a, b) = a

LCM (a, b) = b

Numerical Ability 15

The HCF and LCM of two numbers are 13 and 1989 respectively. If one of the numbers is 117, determine the other.

Solution

Product of two numbers = Their HCF × LCM

$\therefore$ Other number = $\dfrac{\text{HCF} \times \text{LCM}}{\text{First Number}}$

$= \dfrac{13 \times 1989}{117}$

$= 221$

LCM of numbers is exactly divisible by HCF.

SPOT LIGHT

Building Concepts 13

Can two numbers have 14 as their HCF and 204 as their LCM? Give reasons in support of your answer.

Explanation

LCM of two numbers is exactly divisible by their HCF.
Since 204 is not divisible by 14, so the HCF and LCM of two numbers cannot be 14 and 204 respectively.

$$\begin{array}{r|l} 14 & 204 \;\; | \; 14 \\ & -14 \\ \hline & \;\; 64 \\ & -56 \\ \hline & \;\;\;\; 8 \end{array}$$

Check your Answers 3

1.

Number	Divisible by								
	2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**11**
168	Yes	Yes	Yes	No	Yes	Yes	No	No	No
748	Yes	No	Yes	No	No	No	No	No	Yes
275	No	No	No	Yes	No	No	No	No	Yes
5500	Yes	No	Yes	Yes	No	No	No	Yes	Yes
1035	No	Yes	No	Yes	No	No	Yes	No	No
2158	Yes	No	No	No	No	No	No	No	No
8370	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
43974	Yes	Yes	No	No	Yes	No	Yes	No	No
9075	No	Yes	No	Yes	No	No	No	No	Yes
5736	Yes	Yes	Yes	No	Yes	Yes	No	No	No
63921	No	Yes	No	No	No	No	No	No	Yes
871245	No	Yes	No	Yes	No	No	Yes	No	No
183692	Yes	No	Yes	No	No	No	No	No	No
7039452	Yes	Yes	Yes	No	Yes	No	No	No	No
2152	Yes	No	Yes	No	No	Yes	No	No	No

2. 72, 105

Memory Map

Playing with Numbers

Factors and Multiples
- When two or more numbers are multiplied, each number of the product is called a factor.
- The product is a multiple of each of its factors.

Twin Primes
A twin prime is a pair of prime numbers that differs from each other by 2.

Co-prime numbers
Two numbers are said to be co-prime or relatively prime if they have only 1 as a common factor.

Perfect Numbers
A number is called a perfect number if the sum of all its factors is equal to twice the number.

Divisibility Rules
Divisible by 2, 3, 4, 5, 6, 8, 9, 10, 11, 25

Prime and composite numbers
A whole number that has exactly two unique factors, 1 and the number itself, is a prime number. A number greater than 1 with more than two factors is a composite number.

★ 1 is neither prime nor composite number.

Prime factorisation
When a number is expressed as a product of its factors, we say that the number has been factorised.

Sieve of Eratosthenes

Highest common factor (HCF)
- Listing factors
- By Prime factorisation method
- By Continued division method

Least Common Multiple (LCM)
- Listing Multiples
- Prime factorisation method
- Common division

Relationship between HCF and LCM
If a and b are two numbers then,

$a \times b = HCF \times LCM$