6 Integers

The need of negative numbers

As we know that when a smaller whole number is subtracted from larger whole number we get a whole number but what about 4 – 8, 3 – 9, 8 – 10 etc...?

Clearly there are no whole numbers to represent them. So, there is a need to extend our whole number system to represent the above differences.

Corresponding to natural number 1, 2, 3, 4, 5, 6, ..., we introduce new numbers denoted by –1, –2, –3, –4, –5, –6...., respectively such that 1 + (–1) =0, 2 + (–2) = 0, 3 + (–3) = 0, and so on.

The oppositeness of two quantities may be indicated by representing one as a positive and the other as a negative number.

We say that –1 and 1 are the opposites of each other;

–2 and 2 are the opposites of each other;

 –3 and 3 are the opposites of each other, and so on.

Do You Remember ?

★ Integers were introduced by Arbermouth Holst in 1563.

Integers

Numbers greater than 0 are called positive numbers. Extending the number line to the left of 0 allows us to picture negative numbers those are less than 0.

When a single + sign or no sign is in front of a number, the number is a positive number.

When a single – sign is in front of a number, the number is a negative number.

–5 indicates "negative five". 5 and +5 indicates "positive five".

The number 0 is neither positive nor negative.

Number Line

Representing integers on number line

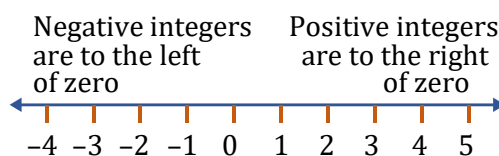Integers can be represented on a number line.

The number line shows that every integer has an opposite number except '0'.

The numbers +1, +2, +3 ..... are positive numbers, denoted by +Z.

The numbers –1, –2, –3 ..... are negative numbers, denoted by –Z.

The positive and negative integers together with 0 are integers, denoted by Z or I.

thus Z = {.... –3, –2, –1, 0, 1, 2, 3,...}

Negative integers are to the left of zero   Positive integers are to the right of zero
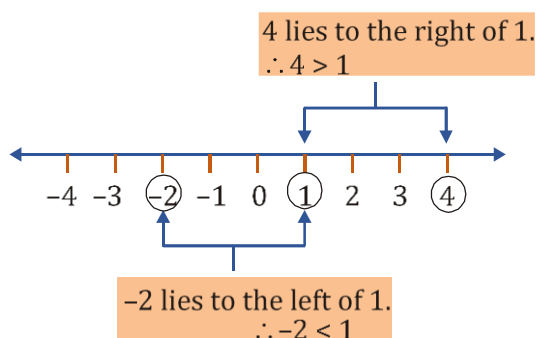
–4 –3 –2 –1 0 1 2 3 4 5

Do You Remember ?

✦ The symbol for integer is Z and Z stands for Zahlen which is German word.

Comparing the values of two integers

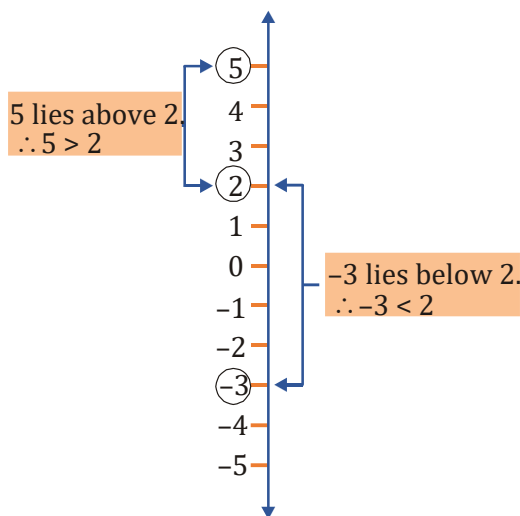Number line can be used to compare the values of two integers.

Horizontal number line

(i) On a horizontal number line, an integer is greater than the integer on its left.

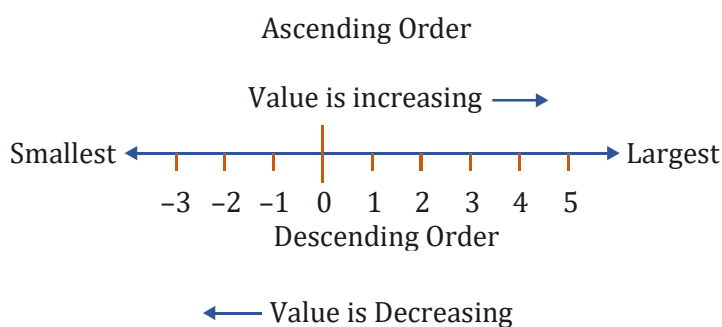(ii) On a horizontal number line, an integer is less than the integer on its right.

4 lies to the right of 1.
∴ 4 > 1

–4 –3 –2 –1 0 1 2 3 4

–2 lies to the left of 1.
∴ –2 < 1

The smallest positive integers is 1.

SPOT LIGHT

Vertical number line

(i) On a vertical number line, an integer is greater than the integer below it.

(ii) On a vertical number line, an integer is less than the integer above it.

```
          5
5 lies above 2.  4
∴ 5 > 2       3
          2
          1
          0        −3 lies below 2.
         −1        ∴ −3 < 2
         −2
         −3
         −4
         −5
```

Arranging integers in order

(i) Number lines can be used to arrange the order of integers in increasing or decreasing order.

(ii) The value of integers on a horizontal number line increases from left to right and decreases from right to left.

Ascending Order

Value is increasing ⟶

Smallest ⟵   ⟶ Largest

−3 −2 −1 0 1 2 3 4 5

Descending Order

⟵ Value is Decreasing

The greatest negative integers is −1.

SPOT LIGHT

Do You Remember ?

★ Negative numbers were finally accepted into the number line in the nineteenth century.
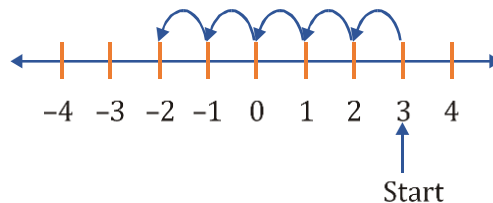
Building Concepts **1**

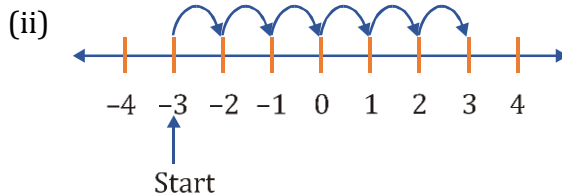**Use a number line to answer the following questions.**

(i) Which number shall we reach if we move 5 numbers to the left of 3?

(ii) Which number shall we reach if we move 6 numbers to the right of −3?

Explanation

(i) 3 – (+5) = – 2

Start

Moving 5 numbers to the left of 3, we reach the point –2.

(ii)

Start

Moving 6 numbers to the right of –3, we reach the point 3.

Building Concepts 2

Fill in the blanks by the appropriate symbol '>' or '<' in each of the following cases.

(i) 0......3 (ii) –7......0 (iii) 7......–5   (vi) –3......–8

Explanation

(i)   0......3

Since 0 is less than 3 and on the left hand side of 3.

0 < 3.

(ii)   –7......0

–7 is on the left side of 0 so it is less than 0.

–7 < 0.

(iii)   7......–5.

–5 is less than 7 and 7 is on the right hand side of –5.

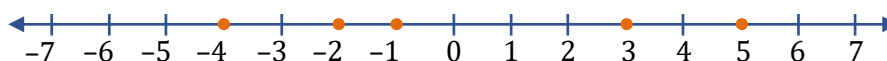7 > –5.

(vi)   –3......–8

–8 is on the left side of –3.

–3 > –8.

Numerical Ability 1

Represent the integers –4, –1, 3, 5, –2 on the number line.

Solution

Draw a number line and show the points corresponding to the integers –4, –1, 3, 5, and –2 by the dots.

Quick Tips

★ Two integers are opposite, if they are on the same distance away from zero, but on opposite sides of the number line.

★ There is no such thing as the lowest or largest integer.

1 is the lowest positive integer, and –1 is the largest negative integer.

Check your Concepts — 1

Which integer in each of the following pairs is to the right of the other on the number line?
(i) 1, 7 (ii) –2, –5   (iii) 0, –3   (iv) –5, 8

Writing positive and negative integers to represent word descriptions

A positive or negative number is used to denote

I.   An increase or decrease in value

For e.g.,

(i)   Rs. 70 withdrawn is denoted by –Rs. 70.

(ii) Rs. 70 deposited is denoted by +Rs. 70.

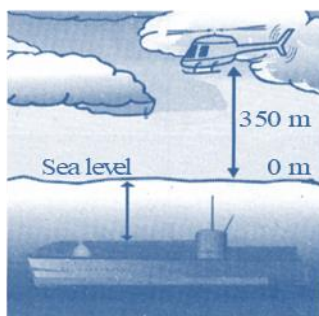**II. Values more than zero or less than zero**

For e.g.,

(i)   –18°C denotes a temperature that is 18°C below 0°C.

(ii) +18°C denotes a temperature that is 18°C above 0°C.

III. A positive direction or a negative direction (opposite direction)

For e.g.,

(i)   5 m denotes a direction 5 m to the right.

(ii) –5 m denotes a direction 5 m to the left.

IV. Position above or below sea level

For e.g.,

(i)   Sea level is taken as 0 m.

(ii) The helicopter flies 350 m above sea level or +350 m.

(iii) The submarine lies 150 m below sea level or –150 m.

Building Concepts 3

Write the numbers in the following situations with appropriate sign.

(i) 100 m below sea level

(ii) A gain of Rs. 600

Explanation

(i) 100 m below sea level.

Since, it is below sea level, the sign will be negative.

Therefore, it can be written as –100 m.

(ii) A gain of Rs. 600.

A gain indicates that the sign will be positive.

Therefore, it can be written as + Rs.600.

Numerical Ability 2

Write the opposite of the following:

(i)   Withdrawn of Rs. 1000   **(ii) 50 km North**

(iii)   5°C temperature falls   **(iv) Won by 2 seconds**

Solution

(i) Deposit of Rs. 1000   (ii) 50 km South

(iii) 5°C temperature increase   (iv) Lose by 2 seconds

Quick Tips

✸ Positive integers may be written with or without a sign.

Check your Answers 1

(i) 7   (ii) –2

(iii) 0   (iv) 8

Check your Concepts 2

The absolute value of a negative or a positive integer is the positive value of the integer as it represents the distance of the number from zero.

SPOT LIGHT

Use a positive or a negative number to denote each of the following.

(i) 18 m below sea level   (ii) Loss of Rs. 254

(iii) 30 km to the East

Be Alert !

★ Remember that positive integers represent gains, or deposits, and negative integers represent losses, or withdrawals.
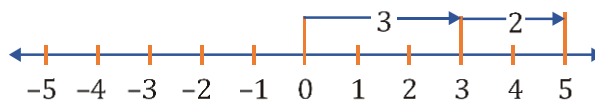
Operations on integers

Using number line-
Addition of integers

You know how to use the number line to add whole numbers. You can also use the number line in the same way to add positive and negative numbers.
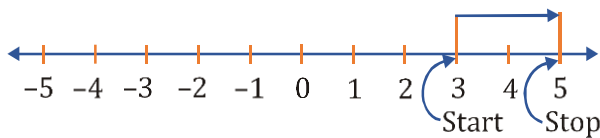
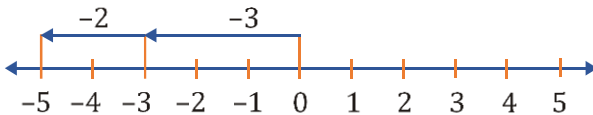(i) Adding two positive integers.

For e.g., add 3 and 2

3 + 2 = 5

First move 3 places to the right from the origin. Then move 2 units further to the right.
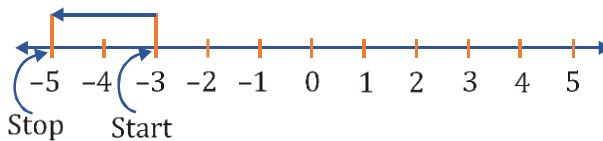
Start at 3 and move 2 units to the right.

(ii) Adding two negative integers

For e.g., add – 3 and – 2

> For every positive integer, there exists a negative integer at the same distance from zero in the opposite direction. These two integers are called the opposite of each other.
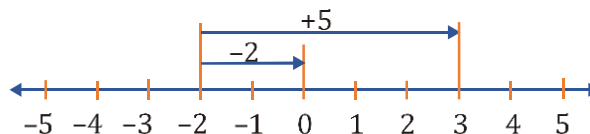>
> **SPOT LIGHT**

−3 + (−2) = − 5

First move 3 places to the left from the origin. Then move 2 units further to the left.

Start at −3 and move 2 units to the left.

(iii) Adding a positive integer and a negative integer

For e.g., add – 2 and 5

(−2) + 5 = 3

First move 2 units to the left from 0. Then move 5 units to the right from this point.

II. Using basic operation

Rule 1: To add two integers of like signs, find the sum of their absolute values and place the common sign before the sum.

E.g. –2 – 5 = –(2 + 5) = –7

Rule 2: To add two integers of unlike signs, find the difference of their absolute values and place the sign of the integer which has the larger absolute value before this difference.

E.g. (–2 + 5) = 3

E.g. 2 + (–5)= –3

Be Alert !

The operation of addition has closure, commutative, associative, additive inverse and successor properties.

SPOT LIGHT

General mistake by student

★ 100 – 150 = – 50, not 50

Building Concepts 4

Add the following :

(i) 2 + 3   (ii) 2 + (–3)   (iii) –2 + (–3)

Explanation

(i) 2 + 3 = 5

(ii) 2 + (–3) = 2 – 3 = –1

   (Find 3 – 2 and since 3 > 2 and it is a negative integer, the answer will also be negative.)

(iii) (–2) + (–3) = –2 – 3 = –5

   (Add the absolute values and place the common sign which is negative or minus sign in the answer.)

Numerical Ability 3

Add the following :

(i) 11 + (+9)   (ii) 121 + (+22)   (iii) 21 + (–11)   (iv) 22 + (–121)
(v) –13 + (+15)   (vi) (–26) + (+13)   (vii) (–33) + (–55)   (viii) (–81) + (–132)

Solution

(i)   11 + (+9) = 11 + 9 = 20

       (Add as both the numbers have the same sign.)

(ii)   121 + (+22) = 121 + 22 = 143

(iii)   21 + (–11) = 21 – 11 = 10 [∵ 21 > 11]0000

(iv)   22 + (–121) = –121 + 22

       (Subtract as both the numbers have different signs.)

       = –(121 – 22) [∵ 121 > 22] = –99

(v) $-13 + (+15) = 15 + (-13) = 15 - 13 = 2$ [$\because 15 > 13$]

(vi) $-26 + (+13) = -(26 - 13)$

(Subtract as both have opposite signs.) $= -13$ [$\because 26 > 13$]

(vii) $(-33) + (-55) = -(33 + 55)$

(Add as both have the same sign.) $= -88$

(viii) $(-81) + (-132)$

(Add as both have the same sign) $= -(81 + 132) = -213$

Subtraction of integers

If a and b are two integers then a-b is equal to a + (−b), i.e., to subtract b from a, change the sign of b and add to a.

Rule: (i) Change the sign of the subtrahend.

(ii) Add by the rules for adding integers.

In general, 'a–b' means the displacement from the point of b to the point of a. Eg.

(i) $(-)\dfrac{\begin{array}{r}8\\4\end{array}}{4}$ (ii) $(-)\dfrac{\begin{array}{r}-8\\-4\end{array}}{-4}$

Building Concepts **5**

Subtract :

(i) – 8 from 5   **(ii) 8 from 5**   **(iii) – 8 from – 5**   **(iv) 8 from – 5**

Explanation

(i) $(+)\dfrac{\begin{array}{r}5\\-8\end{array}}{13}$ or $5 - (-8) = 5 + 8 = 13$

Step 1. Change the sign of –8. It becomes +8.

Step 2. Add 8 to 5

You can show it on the number line as under
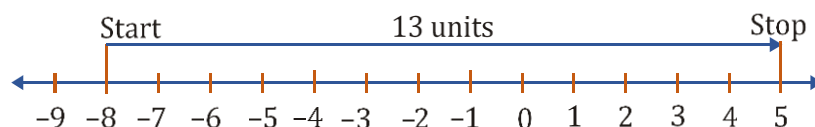
Start at –8 and find the displacement from –8 to 5. To arrive at 5, you have to move 13 units in the positive direction.
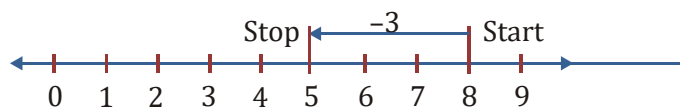
(ii) $(-)\dfrac{\begin{array}{c}5\\ +8\end{array}}{-3}$ or 5 – 8

5 + (–8) = –3

Step 1. Change the sign of 8. You get –8.

Step 2. Add 5 to –8

On the number line, start at 8 and move so as to arrive at 5. For this you have to move 3 units in the negative direction.

> The two integers which are opposite of each other are called additive inverse of each other as their sum is zero.
>
> **SPOT LIGHT**

(iii) $(+)\dfrac{\begin{array}{c}-5\\ -8\end{array}}{3}$ or   –5 – (–8) = –5 + (8) = 3

Step 1. Change the sign of –8. It becomes +8.

Step 2. Add 8 to –5

On the number line, start at –8 and move so as to arrive at –5. For this you have to move 3 units in the positive direction.

(iv) $(-)\dfrac{\begin{array}{c}-5\\ +8\end{array}}{-13}$ or –5 – (+8) = –5 + (–8) = –13

Step 1. Change the sign of 8. You get –8.

Step 2. Add –8 to –5

On the number line, start at 8 and move so as to arrive at –5. For this you have to move 13 units in the negative direction.
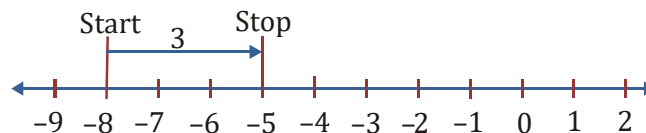

Do You Remember ?

★ a + b = b + a

∴   4 + 3 = 3 + 4

Rule of signs in addition and subtraction

If a is any number, then

+(+a) = +a

+(–a) = –a

–(+a) = –a

–(–a) = +a

The sign of the number inside the brackets remains unchanged if there is a positive sign before it. The sign of the number inside the bracket changes if there is a negative sign before it.

Building Concepts 6

Subtract the sum of 837 and –487 from the sum of –392 and 792.

Explanation

Sum of 837 and –487 = 837 + (–487) = 837 – 487 = 350

Sum of –392 and 792 = –392 + (792) = 400

Now 400 – 350 = 50

Numerical Ability 4

The sum of two integers is –449. If one of them is –336, determine the other.

Solution

Sum of two integers = –449

$\Rightarrow$ –336 + other integer = –449

other integer = sum –(–336) = –449 – (–336)

= –449 + 336 = –113

Numerical Ability 5

Find the value of –3 + 7 + (–9).

Solution

–3 + 7 + (–9) = –3 + 7 – 9 = 4 – 9 = –5

Numerical Ability 6

Find the value of –12 – [(–15) + (–3) – 8].

Solution

–12 – [(–15) + (–3) – 8] = –12 – [(–15) + (–3) + (–8)]

= –12 – [(–15) + (–11)] = –12 – [–26] = –12 + 26 = 14

Subtraction and division of integers does not follow associative property.

SPOT LIGHT

Check your Answers 2
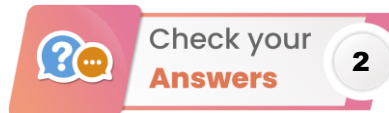
(i) –18 m (ii) Rs. –254 (iii) +30 km

Check your Concepts 3

1. Use the number line and add (–7) + (–3) + (–1).
2. Add without using number line 4004, –299.
3. Simplify : 742 – [–342 – 47 + 30].

If a, b and c are three integers then
$a \div b \neq b \div a$ and
$a \div (b \div c) \neq (a \div b) \div c$

SPOT LIGHT

Properties of Integers

Additive Inverse

The sum of an integer and its opposite is 0.

Thus, if a is an integer then $a + (–a) = 0$

a and –a are called opposites or negatives or additive inverses of each other.

Successor and predecessor of an integer

Let a be an integer then (a + 1) is called the successor of a and (a – 1) is called the predecessor of a. Eg.

The successor of –18 is –18 + 1 = –17 and the predecessor of –18 – 1 = –19.

Building Concepts 7

(i) Find the additive inverse of 70 and –1002.

(ii) Find the successor and predecessor of –39.

(iii) Find an integer a such that

(a) a + (–4) = 0 (b) a + 3 = 0

If a, b and c are three integers then
$a – b \neq b – a$ and
$a – (b – c) \neq (a – b) – c.$

SPOT LIGHT

Explanation

(i) The additive inverse of 70 is –70 and –1002 is 1002.

(ii) The successor of –39 is –39 +1 = –38.

The predecessor of –39 is –39 –1 = –40.
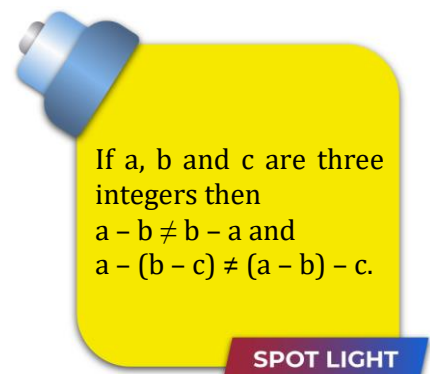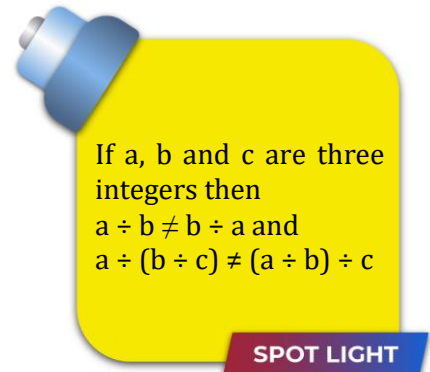
(iii) (a) a = 4 (b) a = (–3)

Numerical Ability 7

Find the sum of the successor and additive inverse of 452.

Solution

Successor of 452 is 452 + 1 = 453

Additive inverse of 452 = – 452

Sum of successor and additive inverse of 452 = 453 + (–452) = 453 – 452 = 1

★ $a \div b \neq b \div a$

∴ $4 \div 2 \neq 2 \div 4$

Word Problems

Building Concepts 8

Shyam has overdrawn his checking account by Rs.38. The bank debited him Rs.20 for an overdraft fee. Later, he deposited Rs.150. What is his current balance?

Explanation

Total amount deposited= Rs. 150

Amount overdrew by Shyam= Rs. 38

⇒   Debit amount = –38   [Debit is represented as negative integer]

Amount charged by bank= Rs. 20

⇒   Debit amount= –20

Total amount debited = (–38) + (–20) = –58

Current balance= Total deposit +Total Debit

⇒   150 + (–58) = 92 [Subtract and give the sign of greater number]

Hence, the current balance is Rs. 92.

Numerical Ability 8

Anna is a microbiology student. She was doing research on optimum temperature for the survival of different strains of bacteria. Studies showed that bacteria X need optimum temperature of –31°C while bacteria Y need optimum temperature of –56°C. What is the temperature difference?

Solution

Bacteria X need optimum temperature = –31°C

Bacteria Y need optimum temperature = –56°C

Temperature difference = (–31) – (–56)

⇒ –31 + 56 = 25   [Subtract and give the sign of greater number]

Hence, temperature difference is 25°C.

Numerical Ability 9

A submarine submerges at the rate of 5 m/min. If it descends from 20 m above the sea level, how long will it take to reach 250 m below sea level?

Solution

Initial position = 20 m   (above sea level)

Final position = –250 m   (below sea level)

Total depth it submerged = (–250 – 20) = –270 m

negative sign shows that the submarine travelled below sea level.

Thus, the submarine travelled 270 m below sea level.

Time taken to submerge 1 meter = $\dfrac{1}{5}$ minutes

Time taken to submerge 270 m = 270 × $\dfrac{1}{5}$ = 54 min

Time = $\dfrac{\text{distance}}{\text{speed}} = \dfrac{270}{5} = 54\,\text{min}$

Hence, the submarine will reach 250 m below sea level in 54 minutes.

Numerical Ability **10**

The diagram below shows a pendulum tied to a string.

(i) When the pendulum was released from the table, it dropped to a height of 80 cm below the table. It was then pulled 35 cm up. How far is the pendulum from the table now?

(ii) The temperature of a town is –14°C at night. During the day, the temperature increases by 7°C. What is the temperature of the town during the day?

Table

35 cm

80 cm
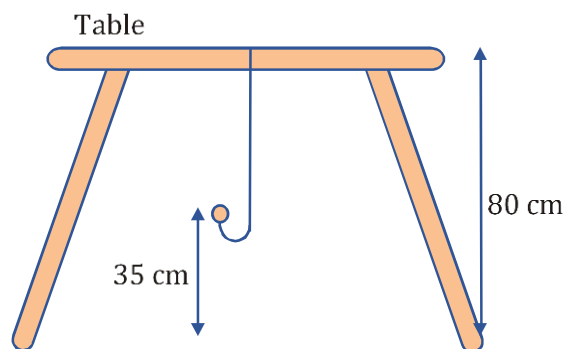
Solution

(i) The distance of the pendulum from the table
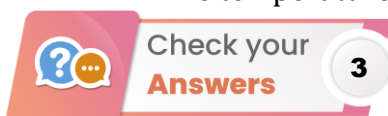
$\Rightarrow$ –80 + 35

$\Rightarrow$ –45 cm

$\therefore$ The pendulum is 45 cm below the table.

(ii) The question involves the sum of negative integer and a positive integer.

–14 °C + 7 °C = –7 °C

The temperature of the town during the day is –7 °C.

Check your Answers **3**

1.   –11   **2.** 3705   **3.** 1101

Memory Map

Integers

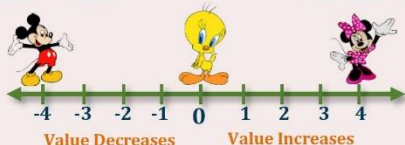Negative numbers
Z or I = {……,-4, -3, -2, -1, 0, 1, 2, 3, 4,……}

The number 0 is neither positive nor negative.

Number Line

NEGATIVE INTEGERS ARE TO THE LEFT OF ZERO　　ZERO IS THE CENTER　　POSITIVE INTEGERS ARE TO THE RIGHT OF ZERO

-4 -3 -2 -1 0 1 2 3 4

Value Decreases　　Value Increases

Comparing the values of two integers

To compare two integers, we use symbols greater than (>), less than (<) and 'equal to' (=) symbol.

Operations on integers

👉 Addition
👉 Subtraction
👉 Multiplication
👉 Division

Rule of signs in addition and subtraction

👉 +(+a) = +a,
👉 +(−a) = −a,
👉 −(+a) = −a,
👉 −(−a) = +a

Successor and predecessor of an integer

Let a be an integer, then (a + 1) is called the successor of a and (a − 1) is called the predecessor of a.

Properties of Integers

Integer Property	Addition	Multiplication	Subtraction	Division
Closure Property	$x + y \in Z$	$x \times y \in Z$	$x - y \in Z$	$x \div y \notin Z$
Commutative Property	$x + y = y + x$	$x \times y = y \times x$	$x - y \neq y - x$	$x \div y \neq y \div x$
Associative Property	$x + (y + z) = (x + y) + z$	$x \times (y \times z) = (x \times y) \times z$	$(x - y) - z \neq x - (y - z)$	$(x \div y) \div z \neq x \div (y \div z)$
Identity Property	$x + 0 = x = 0 + x$	$x \times 1 = x = 1 \times x$	$x - 0 = x \neq 0 - x$	$x \div 1 = x \neq 1 \div x$
Distributive Property	$x \times (y + z) = (x \times y) + (x \times z),$		$x \times (y - z) = (x \times y) - (x \times z)$	

Additive Inverse: If a is an integer, then a + (−a) = 0.