

# 12

## Ratio and Proportion



5 : 2

### Comparison

There are two ways of comparison

(i) By taking difference

(ii) By division

**By taking difference :** For example, Leena and Kavita got 45 marks and 90 marks respectively in their science examination. So, marks of Leena are 45 less than the marks of Kavita. This kind of comparison is called comparison by difference.

**By division :** In the above example, another way to make comparison is to say that Leena got  $\frac{45}{90}$

or  $\frac{1}{2}$  the marks that Kavita got or Kavita got  $\frac{90}{45} = 2$ , that is, twice the marks that Leena got. This

kind of comparison is called comparison by division.

A comparison by division is called ratio.



Do You

Remember ?

★ Ratio can be written as a fraction. Fractions may also be written as ratio.

E.g.  $\frac{p}{q}$  or  $p : q$

### Ratio

The relation between two quantities (both of the same kind and in the same unit) obtained on dividing one quantity by the other, is called the ratio. The ratio between two numbers or quantities is denoted by the colon ":".

Thus, the ratio between two quantities  $p$  and  $q = p : q$ . The numbers  $p$  and  $q$  are called the terms of the ratio  $p : q$ . The first term  $p$  is the antecedent and second term  $q$  is the consequent. Here,  $p$  and  $q$  are two non-zero numbers.

**Note :** The ratio  $p : q$  has no units. It is independent of the units of  $p$  and  $q$ .



### Quick Tips

- ★ The ratio  $a : b$  is defined only if  $a$  and  $b$  are non-zero numbers.
- ★ The ratio  $p : q$  is equal to  $1 : 1$  only when  $p = q$ .



### Be Alert !

- ★ The ratio  $p : q$  and  $q : p$  are not equal. They are equal only if  $p = q$ . So,  $\frac{p}{q} \neq \frac{q}{p}$ .



### Building Concepts

1

**Find the ratio of the following :**

- 36 minutes to 2 hours
- 42cm to 5metres
- 16gm to 2kg
- 34 days to 2 years

### Explanation

- Change both 36 minutes and 2 hours in same unit.

Now, 36 minutes = 36 minutes

2 hours =  $2 \times 60$  minutes = 120 minutes

$$\text{Ratio of 36 min. to 2 hours} = \frac{36 \text{ minutes}}{2 \text{ hours}} = \frac{36 \text{ min}}{120 \text{ min}} = \frac{36}{120} = \frac{36 \div 12}{120 \div 12} = \frac{3}{10} \text{ or } 3 : 10$$

- 5 metres =  $5 \times 100$  cm = 500 cm

$$\therefore \text{Ratio of 42 cm to 5 metres} = \frac{42}{5 \times 100} = \frac{42 \div 2}{500 \div 2} = \frac{21}{250} \text{ or } 21 : 250$$

- 16 gm to 2 kg

2 kg =  $2 \times 1000$  gm = 2000 gm

$$\text{The ratio of 16 gm to 2 kg} = \frac{16}{2000} = \frac{1}{125} \text{ or } 1 : 125$$

- 34 days to 2 years

2 years =  $2 \times 365$  days = 730 days

$$\text{The ratio of 34 days to 730 days} = \frac{34 \div 2}{730 \div 2} = \frac{17}{365} \text{ or } 17 : 365$$

**Simplest form of ratio**

A ratio is in its simplest form if the terms of the ratio have no common factors other than 1. This is also called “reduced form” or “lowest term” of a ratio.



Express the following ratios in its simplest form :

(i) 400 : 256

(ii) 194 : 2000

**Explanation**

(i) 400 : 256

The H.C.F of 400 and 256 is 16

$$\therefore 400 : 256 = \frac{400}{256} = \frac{400 \div 16}{256 \div 16} = \frac{25}{16} = 25 : 16$$

(ii) 194 : 2000

The H.C.F. of 194 and 2000 is 2

$$\therefore 194 : 2000 = \frac{194 \div 2}{2000 \div 2} = \frac{97}{1000} = 97 : 1000$$



There are 20 girls and 25 boys in a class.

(i) What is the ratio of number of girls to the number of boys ?

(ii) What is the ratio of number of girls to the total number of students in the class ?

**Solution**

Number of girls in a class = 20

Number of boys in a class = 25

(i) Ratio of number of girls to the number of boys = 20 : 25 = 4 : 5

(ii) Ratio of number of girls to the total number of students in the class = 20 : (20 + 25)  
= 20 : 45 = 4 : 9



Out of 2,000 students in a school, 750 opted for basketball, 800 opted for cricket and the remaining opted for table tennis. If a student can opt for only one game, find the ratio of:

(i) Number of students who opted for basketball to those who opted for table tennis.

(ii) Number of students who opted for cricket to those who opted for basketball.

(iii) Number of students who opted for basketball to the total number of students.

**Solution**

Total number of students = 2,000

Number of students who opted for basketball = 750

Number of students who opted for cricket = 800

Number of students who opted for table tennis =  $2,000 - (800 + 750) = 450$

- (i) Number of students who opted for basketball to the number of students who opted for table tennis =  $750 : 450 = 5 : 3$
- (ii) Number of students who opted for cricket to the number of students who opted for basketball =  $800 : 750 = 16 : 15$
- (iii) Number of students who opted for basketball to the total number of students =  $750 : 2000 = 3 : 8$



- Find the ratio of following and convert it into simplest form.
  - 40 minutes to 1.5 hours
  - 40 cm to 1.5 m
  - 75 paise to ₹ 1
  - 400 mL to 2 L
- Out of 40 students in a class, 9 like football, 15 like cricket and the remaining like tennis. Find the ratio of
  - Number of students liking cricket to number of students liking tennis.
  - Number of students liking cricket to the total number of students.

**Equivalent ratios**

If we multiply or divide both the numerator and the denominator by the same number, we get an equivalent ratio.

$$\text{E.g., } \frac{40}{100} = \frac{40 \div 2}{100 \div 2} = \frac{20}{50},$$

$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

**Comparison of ratios****Steps**

- Write the given ratios as fractions in the simplest form.
- Find the LCM of the denominators of the fractions.
- Convert them into like fractions with same denominators.
- Compare the numerators and arrange the fractions.
- Then respective ratio are also in the same order.





Compare 5 : 12 and 3 : 8.

**Explanation**

$$5 : 12 = \frac{5}{12}, 3 : 8 = \frac{3}{8}$$

LCM of 8, 12 = 24

$$5 : 12 = \frac{5}{12} \times \frac{2}{2} = \frac{10}{24}$$

$$3 : 8 = \frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$$

$$10 > 9 \Rightarrow \frac{10}{24} > \frac{9}{24} \Rightarrow 5 : 12 > 3 : 8$$



- ★ If two ratios are equivalent, their simplest forms are the same.
- ★ Equivalent ratio represent the same proportion or scaling factor.
- ★ Compare the fractions

$$\frac{a}{b} \text{ and } \frac{c}{d}$$

$$\frac{a}{b} \times \frac{c}{d}$$

$$ad \square bc$$

$$\text{If } ad > bc, \text{ then } \frac{a}{b} > \frac{c}{d}$$

$$ad < bc, \text{ then } \frac{a}{b} < \frac{c}{d}$$

$$ad = bc, \text{ then } \frac{a}{b} = \frac{c}{d}$$



The ratio of the number of girls to the number of boys in a school is 5 : 8. In another school the ratio of the number of girls to the number of boys is 7 : 10. Which school has a higher ratio of girls?

**Explanation**

The ratios of girls to boys in the two schools are 5 : 8 and 7 : 10. Since the number of girls form the numerator in both the cases, the school which has a greater ratio has a higher number of girls. We have two fractions  $\frac{5}{8}$  and  $\frac{7}{10}$ . We can compare these fractions by converting both the fractions into fractions with same denominator.

The LCM of 8 and 10 is 40.

$$\frac{5}{8} = \frac{5 \times 5}{8 \times 5} = \frac{25}{40}, \frac{7}{10} \times \frac{4}{4} = \frac{28}{40}$$

$$\frac{28}{40} > \frac{25}{40}$$

So, the second school with the ratio 7 : 10 has a higher ratio of girls.



**The number of stamps in the collections of Jaya, Soumya, and Mamta are in the ratio 3 : 4 : 5. If Soumya has a collection of 108 stamps, find the number of stamps that Jaya and Mamta each has.**

**Solution**

The number of stamps in the collection of Jaya, Soumya, and Mamta are in the ratio = 3 : 4 : 5

Let the number of stamps with Jaya be 3x.

Then Soumya will have 4x stamps and Mamta will have 5x stamps.

Given that Soumya's 4x = 108 stamps

$$x = \frac{108}{4} = 27 \text{ stamps}$$

Number of stamps with Jaya's = 3x = 3 × 27 = 81 stamps

Number of stamps with Mamta's = 5x = 5 × 27 = 135 stamps

∴ Jaya has 81 stamps and Mamta has 135 stamps.

**Dividing a number in the given ratio**

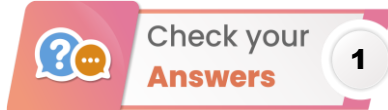
If the number p is to be divided in the ratio a : b,

then the first part =  $\left(\frac{a}{a+b}\right) \times p$  and the second part  $\left(\frac{b}{a+b}\right) \times p$

For example, let's divide 35 in the ratio 2 : 3.

$$\text{First part} = \left(\frac{2}{2+3}\right) \times 35 = \frac{2}{5} \times 35 = 14;$$

$$\text{second part} = \left(\frac{3}{2+3}\right) \times 35 = \frac{3}{5} \times 35 = 21$$



1. (i) 4 : 9 (ii) 4 : 15 (iii) 3 : 4 (iv) 1 : 5  
2. (i) 15 : 16 (ii) 3 : 8

### Proportion

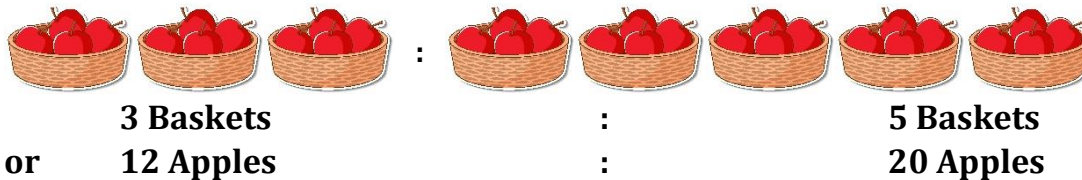
If two ratios are equal, we say they are in proportion. If four numbers  $a, b, c$  and  $d$  are such that the ratio of the first two is equal to the ratio of the last two, i.e.,  $a : b = c : d$ , then we say  $a, b, c$  and  $d$  are in proportion.

The symbol ' $::$ ' can also be used to denote equality of two ratios, i.e.,  $a : b :: c : d$ , read as 'a is to b as c is to d' also means  $a, b, c$  and  $d$  are in proportion. Here  $a$  and  $d$  are called extreme terms and  $b$  and  $c$  are called mean terms. The number  $d$  is also known as the fourth proportional to  $a, b$  and  $c$ .

For example, 3, 4, 18 and 24 are in proportion because  $3 : 4 = 18 : 24$ .

If  $a, b, c, d$  are in proportion, which implies,  $\frac{a}{b} = \frac{c}{d}$ , then

$a \times d = b \times c$ , i.e., Product of the extremes = Product of the means



So, we can write it as,  $3 : 5 :: 12 : 20$

### Continued proportion

Three numbers  $a, b, c$  are said to be in continued proportion if  $a, b, b$  and  $c$  are in proportion. i.e.,

$\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$ . If  $a, b, c$  are in continued proportion then  $b$  is known as the mean proportion of

$a$  and  $c$ ,  $c$  is known as the third proportion.



- ★ If  $a, b$  and  $c$  are in continued proportion then, ratio of first two and last two is same.  
i.e.,  $a : b = b : c$



**Determine if the following are in proportion or not?**

- (i) 4, 24, 5, 30 (ii) 10, 20, 30, 40

### Explanation

- (i) 4, 24, 5, 30

$$\text{Ratio of 4 to 24} = \frac{4}{24} = 1 : 6$$

$$\text{Ratio of 5 to 30} = \frac{5}{30} = 1 : 6$$

Since,  $4 : 24 = 5 : 30$

Therefore, 4, 24, 5, 30 are in proportions.

(ii) 10, 20, 30, 40

$$\text{Ratio of 10 to 20} = \frac{10}{20} = 1 : 2$$

$$\text{Ratio of 30 to 40} = \frac{30}{40} = 3 : 4$$

Since,  $10 : 20 \neq 30 : 40$ 

Therefore, 10, 20, 30, 40 are not in proportion.

**Find the value of x, If  $15 : 60 :: x : 20$ .****Solution**

$$15 : 60 :: x : 20 \quad (\text{Given})$$

$$\Rightarrow \frac{15}{60} = \frac{x}{20}$$

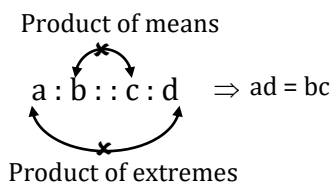
$$\Rightarrow \frac{1}{4} = \frac{x}{20}$$

$$\Rightarrow x = 5$$



★ For every proportion,

Product of means = product of extremes

**Unitary method**

Look at this problem.

If the cost of 3 pens is ₹ 12, what will be the cost of 8 pens ?

The cost of 3 pens is ₹ 12. So, we know that the cost of one pen will be lesser than ₹ 12. We can find the cost of one pen by dividing ₹ 12 by 3. The cost of one pen is ₹  $12 \div 3 = ₹ 4$ . If we have to find the cost of 8 pens, we know that it will be more than the cost of one pen, and further that it will be 8 times the cost of one pen. So, we multiply ₹ 4 by 8 and we get ₹ 32. So, the cost of 8 pens is ₹  $4 \times 8 = ₹ 32$ .

In the process of our calculation, we first find out the value of one unit (in this case the cost of one pen). So, this method of problem solving is called the unitary method.



Number of pens	Cost of pens	
$\Rightarrow$ 		i.e, Rs. 12
$\Rightarrow$ 		i.e, Rs. 4



**If 96 tiles required in filling 3 rooms, then, how many tiles will be required in filling 9 rooms ?**

**Explanation**

- $\therefore$  Number of tiles required in 3 rooms = 96 tiles
- $\therefore$  Number of tiles required in 1 room =  $\frac{96}{3} = 32$  tiles
- $\therefore$  Number of tiles required in 9 rooms =  $32 \times 9 = 288$  tiles

## Memory Map

### Comparison

There are two ways of comparison

- (i) By taking difference
- (ii) By division

### Ratio

The ratio between two quantities  $p$  and  $q$  is  $p : q$ .

#### Equivalent ratios :

If we multiply or divide both the numerator and the denominator by the same number, we get an equivalent ratio.

#### Simplest form of ratio :

A ratio is in its simplest form if the terms of the ratio have no common factors other than 1.

#### Comparison of ratios

Steps

- (i) Write the given ratios as fractions in the simplest form.
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- (iv) Compare the numerators and arrange the fractions.
- (v) Then respective ratio are also in the same order.

### Proportion



3 Baskets : 5 Baskets

or 12 Apples : 20 Apples

So, we can write it as,  $3 : 5 :: 12 : 20$

### Dividing a number in the given ratio

If the number  $p$  is to be divided in the

ratio  $a : b$ , then the first part =  $\left(\frac{a}{a+b}\right) \times p$   
and the second part  $\left(\frac{b}{a+b}\right)$

### Continued proportion

Three numbers  $a, b, c$  are said to be in continued proportion if  $a, b$  and  $c$  are in proportion. i.e.,  $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$  If  $a, b, c$  are in continued proportion then  $b$  is known as the mean proportion of  $a$  and  $c$ ,  $c$  is known as the third proportion.

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