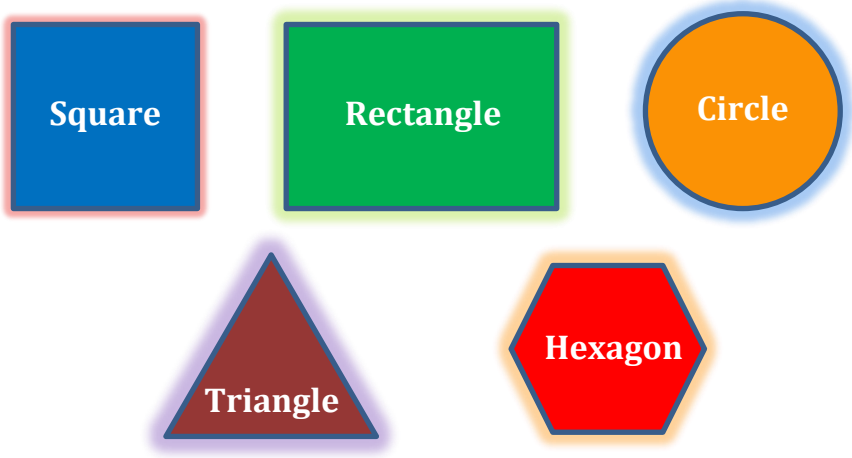
10 Mensuration



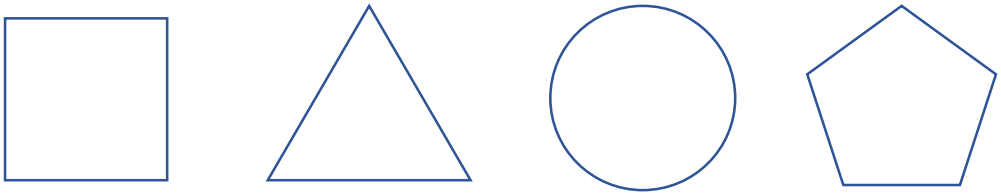
Mensuration

Mensuration is a branch of mathematics. It is a study of various geometrical shapes, their length, breadth, and area for two dimensional shapes and volumes for three dimensional shapes.

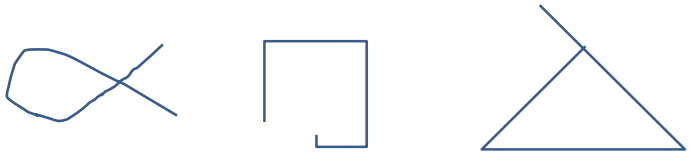
Mensuration deals with the measurement of length, area or volume of various geometrical shapes.

Relevance of mensuration: In real life situations, we may need to measure distances like if we have to run around a park or we have to deal with planar surface measurements like we have to plant grass on ground.

Closed figures : In a plane figures if initial and end points coincides with each other then those figures are called closed figures.



Open figures : In a plane figures if we have different initial and end points, then those figures are called open figures.



Polygon : A closed figure made up of only line segments is called a polygon.

(i) Regular polygon : A polygon having all sides and all internal angles equal is called a regular polygon.

Regular Polygon

(ii) Irregular polygon : A polygon which does not have all its sides equal and not all interior angles are equal in measure.

Building Concepts 1

How many meters will make 3.1 km.

Explanation

1 km = 1000 m

= 3.1 × 1000

= 3100 m

Building Concepts 2

Convert :

(i) 103 kℓ to ℓ **(ii) 1008 cm to m**

Explanation

(i) 1 kℓ = 1000 ℓ

= 103 × 1000 ℓ

= 103000 ℓ

(ii) 1 m = 100 cm

$\frac{1008}{100}$ = 10.08 m

Conversion of units

1 m = 100 cm

1 km = 1000 m

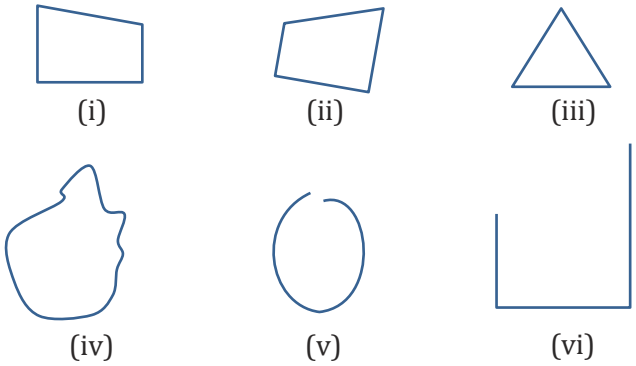
1kℓ = 1000 litres

1 ℓ = 1000 mℓ

SPOT LIGHT

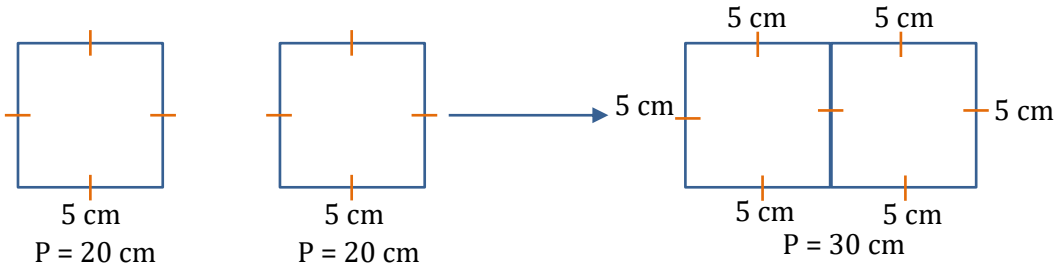
Perimeter of figures

If we have to build a wall around a field, we need to know the length of the wall to be built. This length of the wall along the boundary is called the perimeter of the field. It depends on the size and shape of the field.



In above figure, if a wall is built around in figure (i), (ii), (iii) and (iv) the boundaries will be closed. This length of the wall is called the perimeter. However, in figure (v) and (vi) the boundaries are not closed. The lengths of the walls in such cases are not called perimeter.

If two or more surfaces meet, then their perimeter reduces as

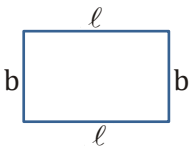


Quick Tips

✸ The perimeter of a closed figure is the length of the boundary of the figure.

Perimeter of a rectangle

The perimeter of a rectangle is the sum of all its sides. The opposite sides of a rectangle are equal. In figure, if the longer side, is ℓ units and the shorter side is b units.



Perimeter = length + breadth + length + breadth

= 2(length) + 2(breadth)

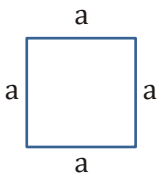
= 2(length + breadth)

= 2(ℓ + b) units

where ℓ = length, b = breadth

Perimeter of a square

A square is a special rectangle having all the four sides equal. In figure, if one side of a square measures 'a' unit, we can say that both the length and the breadth are 'a' unit each.



Perimeter of a square = 2 (ℓ + b)

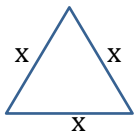
= 2 (a + a)

= 2 × 2a = 4a units

= 4 times the side $\Rightarrow$ So, perimeter of a square = 4 × side

The perimeter of a square is equal to four times the length of the side.

Perimeter of an equilateral triangle



A triangle with all sides are equal is called an equilateral triangle. In figure, if the length of one side of an equilateral triangle is 'x' units, its perimeter will be :

side + side + side = x + x + x = 3x units

Perimeter of a regular polygon

In general, if all the sides and angles of a polygon are equal, that is, if it is a regular polygon, its perimeter will be the product of the length of its side with the number of sides.

Let one side of a regular polygon be 'a' units.

Perimeter of a regular pentagon = 5a units

Perimeter of a regular hexagon = 6a units

Perimeter of a regular octagon = 8a units

If the sides of closed figure are not of same measure, then the figure is said to be irregular.

SPOT LIGHT

To find the cost of fencing

Cost of fencing = Perimeter × Cost per unit

Quick Tips

- Perimeter can be found of only closed figures.
- Perimeter of 'n' sided regular polygon of each side a = 'na' units

Be Alert !

- The dimensions of any geometrical figure must have the same unit while calculating perimeter.

Building Concepts 3

(i) The sides of a triangle are 3 cm, 4 cm and 5 cm long. Find its perimeter.

(ii) The length of the sides of a triangular field are 15 m, 20 m and 24 m.

Find the total distance travelled by the boy moving along its boundary, in making

(a) One complete round

(b) Seven complete rounds

Explanation

(i) Perimeter of triangle = 3 cm + 4 cm + 5 cm = 12 cm

(ii) Perimeter of triangular field = 15 m + 20 m + 24 m = 59 m

(a) Distance travelled by the boy in making one complete round = 59 m

(b) Distance travelled by the boy in 7 complete rounds = 59 × 7 = 413 m

Numerical Ability 1

A playground is rectangular in shape. If its length is 60 m and width is 45 m, find

(i) Its perimeter.

(ii) Total cost of fencing at the rate of ₹ 2.50 per metre.

Solution

(i) Perimeter of rectangular playground

= 2 (length + width)

= 2 (60 + 45) m

= 2 (105) m = 210 m

(ii) Cost of fencing of 1 metre = ₹ 2.50

Cost of fencing of 210 metres = ₹ 210 × 2.50 = ₹ 525

Check your Concepts 1

Aditya runs around a square park of side 50 m three times. Shweta runs twice around a rectangular park of length 100 m and breadth 60 m. Who covers more distance?

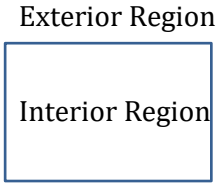
Be Alert !

- Don't forget to write the answer of all mensuration problem with proper units.
  Units of Area = sq. units
  Units of perimeter = units

Area of figures

The amount of surface of the plane covered by a closed figure is called its area.

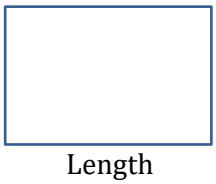
For every closed figure, there are two regions: Exterior region and interior region.



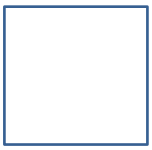
The term 'area' refers to the measure of the total interior region.

Formulas to find areas of certain standard figures are given below.

(i) Area of a rectangle = Length × Breadth



(ii) Area of a square = Side × Side



(iii) Length of a rectangle = $\frac{\text{Area}}{\text{Breadth}}$

(iv) Breadth of a rectangle = $\frac{\text{Area}}{\text{Length}}$

Quick Tips

- If area of a rectangle and square are same then perimeter of square will always be less than that of the rectangle.
- If perimeter of rectangle and square are same then area of square will always be more than that of rectangle.

Numerical Ability 2

A table-cloth measures 3 m by 2 m 50 cm. Another table-cloth measures 5 m by 1 m 50 cm. Which rectangular table-cloth costs more if they are made of the same cloth?

Solution

Length of first table-cloth = 3 m

Breadth of first table-cloth = 2 m 50 cm = 2.5 m

Area of first table-cloth = 3 m × 2.5 m = 7.5 sq. m

Similarly, length of second table-cloth = 5 m

Breadth of second table-cloth = 1 m 50 cm = 1.5 m

Area of second table-cloth = 5 m × 1.5 m = 7.5 sq. m

Since, the two table-cloths have equal area, their costs are same.

Numerical Ability 3

A floor is 6 m long and 5 m wide. A square carpet of side 4 m is laid on the floor. Find the area of the floor that is not carpeted.

Solution

Area of the floor = Length × Breadth

= 6 m × 5 m = 30 sq. m

Area of the square carpet = Side × Side

= 4 m × 4 m = 16 sq. m

Area of the floor that is not carpeted = 30 – 16 = 14 sq. m

Numerical Ability 4

Carpeting of a room measuring 8 m by 6 m is to be done. What length of the carpet is just enough if the roll of the carpet is 3 m wide?

Solution

Length of the room = 8 m

Breadth of the room = 6 m

Area to be carpeted = 8 m × 6 m = 48 sq. m

Let carpet of length ℓ be sufficient to cover the floor.

Now, area of the carpet = area of the floor

ℓ × 3 = 48 sq. m

ℓ = 16 m

Hence, length of the carpet is 16m.

SPOT LIGHT

For a same given area of rectangle and square, a square has the smallest perimeter.

Numerical Ability 5

The cost of cultivating a rectangular field at the rate of ₹ 3 per square metre is ₹ 1,728. If the breadth of the field is 18 m, find the cost of fencing the field at ₹ 8.5 per metre.

Solution

Area of the rectangular field = $\frac{\text{Total cost}}{\text{Cost per square metre}} = \frac{1728}{3} = 576$ sq.m

Now,

Length × Breadth = 576 sq. m

$\Rightarrow$ Length × 18 = 576 sq. m

$\therefore$ Length of the field = $\frac{576}{18}$ = 32 m

Perimeter of the field = 2(ℓ + b) = 2(32 + 18) = 100 m

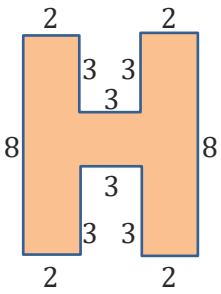
Cost of fencing the field = ₹ 8.5 × 100 = ₹ 850

SPOT LIGHT

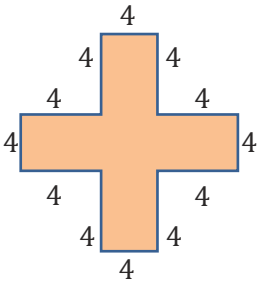
For a same given perimeter of rectangle and square, a square has the largest area.

Numerical Ability 6

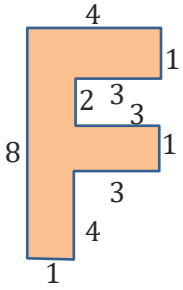
Split the following shapes into rectangles and squares and find their areas. (The measures are given in centimetres).



(i)



(ii)



(iii)

Solution

(i) Area of fig. I (rectangle) = 8 cm × 2 cm = 16 sq. cm

Area of fig. II (rectangle) = 8 cm × 2 cm = 16 sq. cm

Area of fig. III (rectangle) = 3 cm × 2 cm = 6 sq. cm

So, Area of complete figure = (16 + 16 + 6) sq. cm = 38 sq. cm

(ii) Area of fig. I (square) = 4 cm × 4 cm = 16 sq. cm

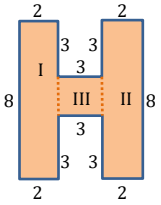
So, Area of 5 such figures = 5 cm × 16 cm = 80 sq. cm

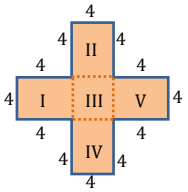
(iii) Area of fig. I (rectangle) = 8 cm × 1 cm = 8 sq. cm

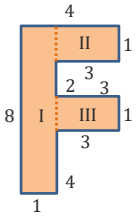
Area of fig. II (rectangle) = 3 cm × 1 cm = 3 sq. cm

Area of fig. III (rectangle) = 3 cm × 1 cm = 3 sq. cm

So, Area of complete figure = (8 + 3 + 3) sq. cm = 14 sq. cm







Area of Irregular figures/shapes

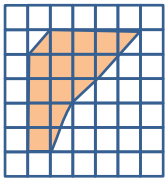
To find area of irregular shapes, it can be divided into multiple familiar shapes, such as triangles, squares and rectangles. Then, we can get the total area by adding the area of those smaller shapes.

Steps to find area of an irregular shape:

- The area of full circle is taken as 1 sq. unit.
- Ignore portion of the area that are less than half a square.
- If more than half of square is in a region, just count it as one square.
- If exactly half the square is counted, take its area as $\frac{1}{2}$ sq. unit.

Numerical Ability 7

Find the area of the shaded figure?



Solution

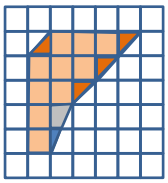
Rules for finding the area by counting the squares

Area of complete squares – 1 sq. unit each

Area of two half squares – 1 sq. unit

Area of more than half squares –1 sq. unit each

Ignore the less than half-filled squares.



Completely filled squares	10
Half filled squares	4
Less than half filled squares	1
More than half filled squares	1

$\therefore$ Total Area = Number of complete squares + $\frac{1}{2}$ (Number of half squares) + Number of more than half-filled squares

Therefore, area of the shaded figure = $10 + \frac{1}{2} \times 4 + 1 = 13$ sq. cm.

Check your Answers 1

Shweta covers more distance.

Memory Map

Mensuration deals with the measurement of length, area or volume of various geometrical shapes.

Closed figures

Open figures

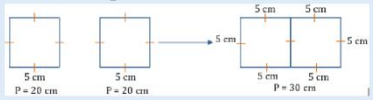
Polygon

A closed figure made up of only line segments is called a polygon.

Regular polygon

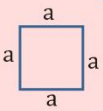
Perimeter of figures

Sum of length of all sides



Perimeter of a square

Perimeter of a square = 4 × side



Perimeter of a rectangle

Perimeter = 2(length + breadth)
where ℓ = length, b = breadth

Perimeter of an equilateral triangle

Perimeter = side + side + side
= x + x + x = 3x units

Perimeter of a regular polygon

- Perimeter of a regular pentagon = 5a units
- Perimeter of a regular hexagon = 6a units
- Perimeter of a regular octagon = 8a units

To find the cost of fencing

- Cost of fencing = Perimeter × Cost per unit

Area of figures

Exterior Region

Interior Region

Rectangle

- Area of a rectangle = Length × Breadth
- Length of a rectangle = $\frac{\text{Area}}{\text{Breadth}}$
- Breadth of a rectangle = $\frac{\text{Area}}{\text{Length}}$

Square

- Area of a square = side × side
= side^2