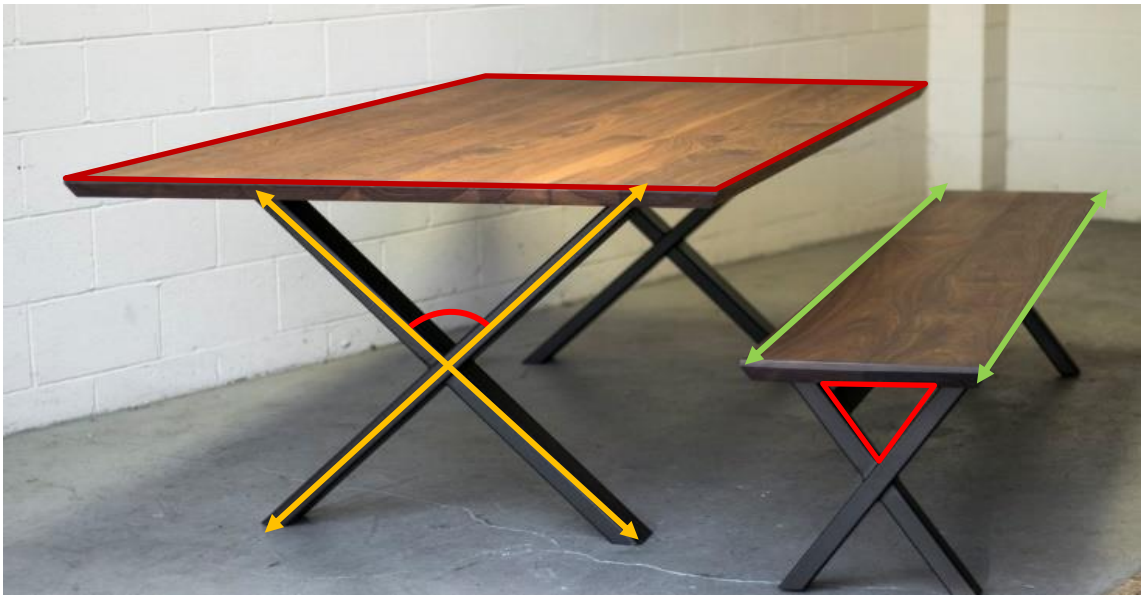
4 Basic Geometrical Ideas



Basic Terms

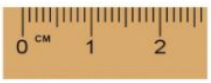
Point

A point is an exact location in space. It has no length and no width. We represent a point by a dot (.) and label it with a capital letter. Some representations of a point in everyday life are shown below.

A distant star in the sky

the tip of your pencil

Vertex of a square



Zero mark of your ruler

Line

A line is a straight path of points that extends on and on in both the directions without ever ending. It has one dimension. The length of a line cannot be measured.

To show that a line extends endlessly in both the directions, we use arrowheads at both the ends.

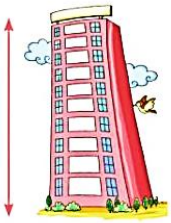
A line is named by using any two points on it. Thus, the line given alongside is named as line AB or line BA and represented as $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$. Through and two points, there is exactly one line.

A horizontal line goes straight across.

Horizontal line

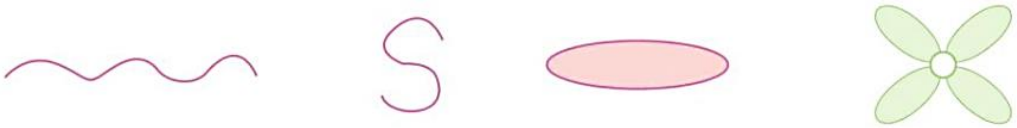
Horizon

A vertical line goes straight up and down.



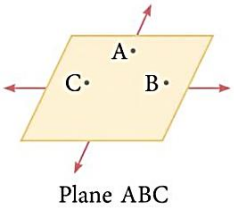
Vertical line

A line can also be curved. A curved line is called a curve.



Plane

A plane is a flat surface. A plane has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end. In mathematics, a plane goes on and on in all directions without end. We usually work with just a part of a plane. Points and lines lie on a plane. Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



Plane ABC

The given figure shows plane ABC. The order of the points does not matter.

Some representations of a plane surface from our everyday life are:

Surface of Blackboard

A wall of your room

Surface of a book

Screen of a flat TV

Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

Building Concepts 1

Answer the following questions in real life examples of Point, Line and Plane.

(i) A location of a place in the Map. **(ii) The tip of a needle.**

(iii) Lines of latitude and longitude. **(iv) The centre – line on a highway.**

(v) White – board.

Explanation

(i) Point (ii) Point (iii) Line (iv) Line

(v) Plane

Numerical Ability 1

Draw and label each of the following.

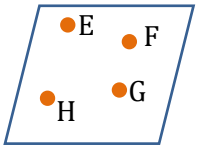
(i) $\overleftrightarrow{PQ}$ **(ii) Points S and T** **(iii) Plane EFGH**

Solution

(i)

(ii)

(iii)



Building Concepts 2

(i) Give two other names for $\overleftrightarrow{PQ}$ and for plane R.

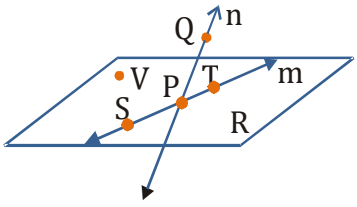
(ii) Name three points that are collinear. Name four points that are coplanar.

Explanation

(i) Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line n. Other names for plane R are plane SVT and plane PTV.

(ii) Points S, P and T lie on the same line, so they are collinear.

Points S, P, T and V lie in the same plane, so they coplanar.



Do You Remember ?

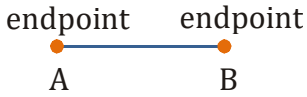
✸ Line segment is a part of line.

Line Segment and ray (defined terms)

In geometry, terms that can be described using known words such as point or line are called defined terms.

Line Segment

The line segment AB or segment AB, (written as $\overline{AB}$) consists of the endpoints A and B and all points on $\overline{AB}$ that are between A and B. Note that $\overline{AB}$ can also be named $\overline{BA}$.



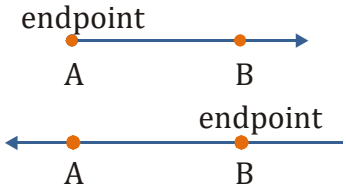
There are unlimited number of points on a line.

SPOT LIGHT

Ray

The ray AB (written as $\overrightarrow{AB}$) consists of the endpoint A and all points on $\overrightarrow{AB}$ that lie on the same side of A as B.

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.

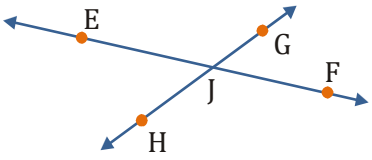


If point C lies on $\overrightarrow{AB}$ between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.

Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear.

Lines, segments, and rays are coplanar if they lie in the same plane.

Building Concepts 3



(i) Give another name of $\overline{GH}$.

(ii) Name all rays with end point J. Which of these rays are opposite rays?

Explanation

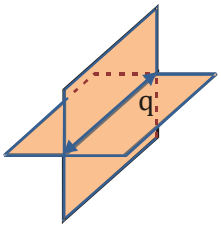
(i) Another name for $\overline{GH}$ is $\overline{HG}$.

(ii) The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$ and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

Intersecting lines

When two or more lines intersect at a common point they are known as intersecting lines. The point at which they cross each other is known as point of intersection.

(i) The intersection of two different lines is a point.



(ii) The intersection of two different planes is a line

Note: There are unlimited number of lines through a point. Three or more lines in a plane are said to be concurrent if all of them pass through the same point and this point is called point of concurrence of the given lines.

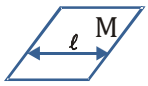
Building Concepts 4

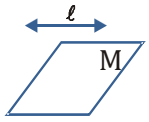
(i) Sketch a plane and a line that is in the plane.

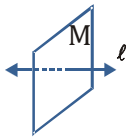
(ii) Sketch a plane and a line that does not intersect the plane.

(iii) Sketch a plane and a line that intersects the plane at a point.

Explanation

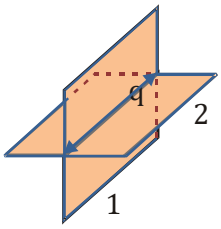
(i) 

(ii) 

(iii) 

Numerical Ability 2

Sketch two planes that intersect in a line.



Solution

Step 1: Draw a vertical plane. Shade the plane.

Step 2: Draw a second plane that is horizontal. Shade this plane with a different colour. Use dashed lines to show where one plane is hidden.

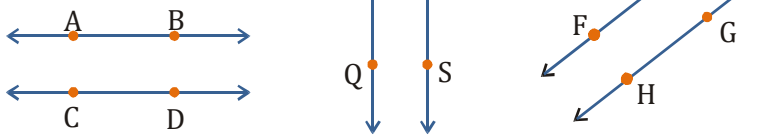
Step 3: Draw the line of intersection.

Parallel lines

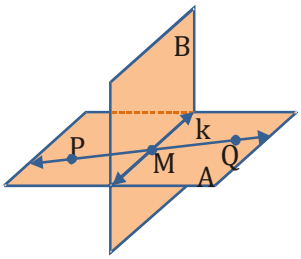
Two lines in the same plane either meet or do not meet. Lines that do not meet are parallel lines. AB and CD are horizontal parallel lines. PQ and RS are vertical parallel lines.

The perpendicular distance between two parallel lines remains the same throughout.

SPOT LIGHT



Note: The rails of a railway line, opposite edges of a ruler and the opposite sides of a rectangle are examples of parallel lines.



Be Alert !

- If two lines are not parallel, then they are intersecting.

Check your Concepts 1

1. Sketch two different lines that intersect a plane at the same point.

2. Use the diagram at the right.

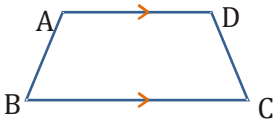
(a) Name the intersection of $\overleftrightarrow{PQ}$ and line k.

(b) Name the intersection of plane A and plane B.

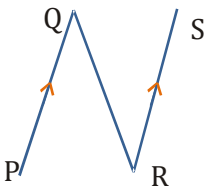
(c) Name the intersection of line k and plane A.

3. Name the parallel line segment in each of the following figures.

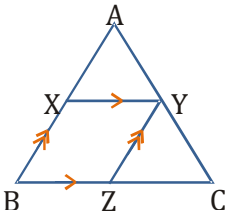
(a)



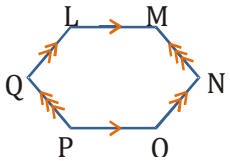
(b)



(c)



(d)



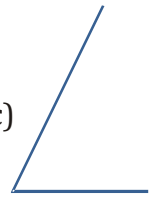
Plane figure, Interior and Exterior Figure

In geometry, any collection of points is called a figure. If all the points in a figure are in one plane, the figure is a plane figure.

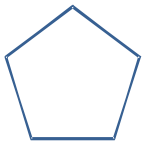
So we can say that a line segment is plane figure.

Open and closed figures

Open figures do not begin and end at the same points.

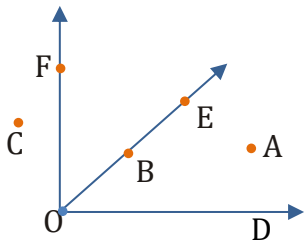
(a)  (b)  (c) 

Closed figures begin and end at the same points.

(a)  (b) 

Building Concepts 5

In the given diagram, name the point (s)



(i) In the interior of ∠DOE

(ii) In the exterior of ∠EOF

(iii) On ∠EOF

Explanation

(i) A

(ii) C, A, D

(iii) B, E, O, F

A closed figure that does not intersect itself is called a simple closed figure.

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Numerical Ability 3

Which of the following figure is simple closed figure?

(i) 

(ii) 

(iii) 

(iv) 

Solution

Option (iii) is simple closed figure as a closed figure that does not intersect itself is called simple closed figure.

Active Maths 1

Identify the open and close figures.

Exploring the concept: Make two points, P and Q, on your paper. Start at P and draw a path to Q ; then draw a different path from Q back to P.

(a) 

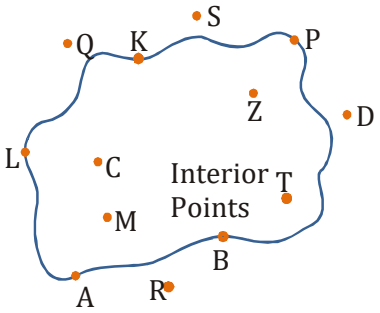
(b) 

Drawing conclusion: You end at the same point P after drawing the figure. Such figures are called closed figures.

Interior and exterior of a figure

Your house has a boundary wall. The boundary wall separate your house from the main road and adjoining houses. There can be three cases - you are inside your house, you are just at the gate of the house or you are on the road, i.e. outside the house.

Similarly, if we have a closed figure, there are three cases :



(i) Interior (inside) to the figure.

(ii) 'On' (boundary) the figure.

(iii) Exterior (outside) of the figure.

Points (C, M, T, Z) lie in the interior of figure.

Points (K, L, A, B, P) lie on the boundary.

Points (Q, R, D, S) lie in exterior of the figure.

Do You Remember ?

✸ Triangle is the smallest polygon with minimum sides.

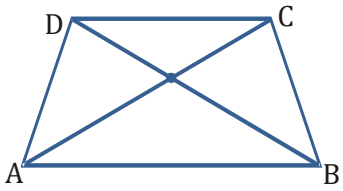
Polygons and Angles

A simple closed figure formed of three or more line segments is called a polygon.

Sides, vertices and diagonals of a polygon

Sides

The line segments forming a polygon are called its sides. ABCD is a polygon in which AB, BC, CD and DA are its four sides.



Vertices

The meeting point of a pair of sides of a polygon is called its vertex. Thus, A, B, C, D are the four vertices of given polygon ABCD.

Adjacent sides

Any two sides of a polygon having a common end point are called its adjacent sides.

Thus (AB, BC), (BC, CD), (CD, DA) and (DA, AB) are four pairs of adjacent sides in the given polygon ABCD.

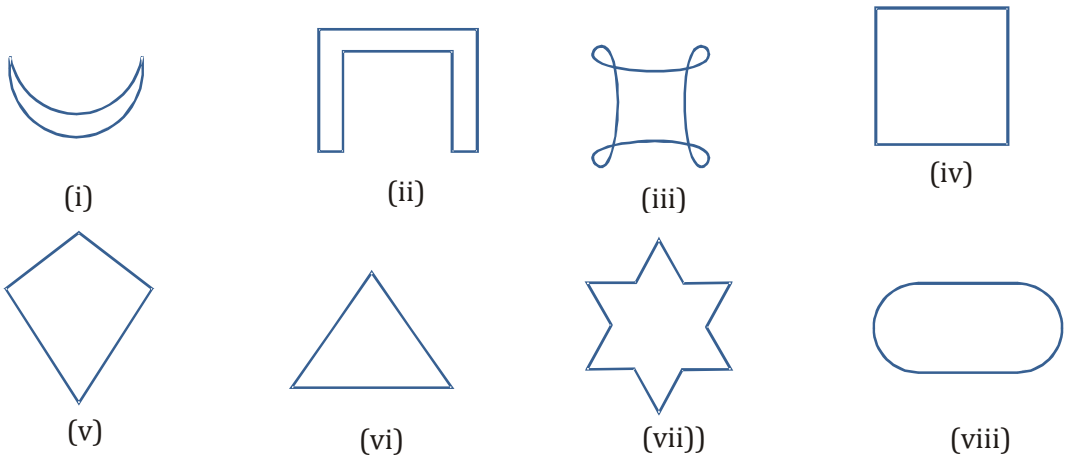
Here (A, B), (B, C), (C, D) and (D, A) are the pairs of adjacent vertices.

Diagonals : A line segment joining two non-adjacent vertices of a polygon is called its diagonal.

Thus, AC and BD are the diagonals of the given polygon ABCD.

Numerical Ability 4

Which of the following are simple closed figures and which are polygons ?



Solution

Figures (i), (ii), (iv), (v), (vi), (vii), (viii) are simple closed figures because they do not intersect itself. Figure (ii), (iv), (v), (vi), (vii) are polygons because they are made of three or more line segments.

Do You Remember ?

- 90° is the measurement of right angle.

Angles

An angle is made up of two rays that have the same end point. The end point at which the two rays meet is called the vertex of the angle. Each of the rays that form the angle are called the arms of the angle.

The sign of angle is represented by ∠. Here, angle is ∠BAC.

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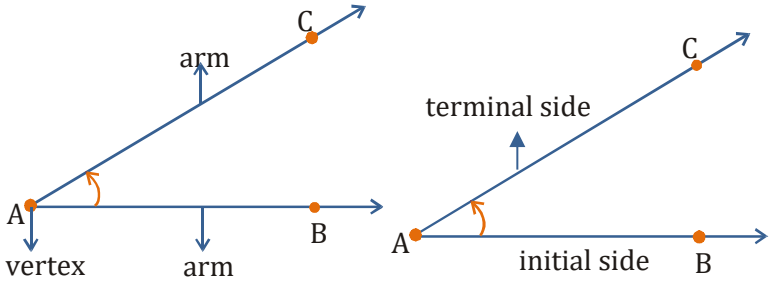
The size of an angle depends on the rotation up to the terminal side. The amount of this rotation is called the measure of the angle.

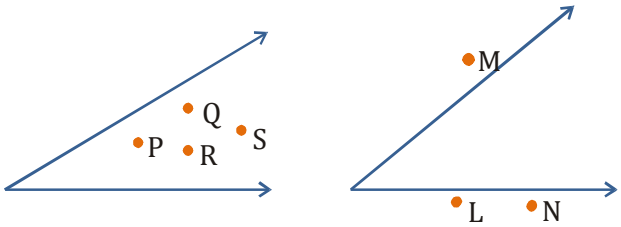


The ray which represents the starting position is called the initial side of the angle and the ray which indicates the stopping position is called the terminal side of the angle.

Interior and exterior of an angle

Like any plane figure, an angle divides the plane in which it lies into two parts. One part is called the inside region or the interior of an angle. The other part is called the outside region or the exterior of an angle.

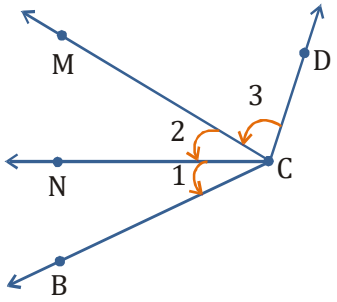
The points P, Q, R, S lie in the interior of an angle. The points L, M, N lie in the exterior of an angle.



(i) (ii)

Building Concepts 6

Name the angles ∠1, ∠2, ∠3 and ∠1 + ∠2 in the following figure.



Explanation

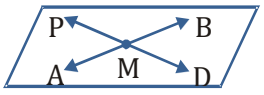
∠1 = ∠NCB ∠2 = ∠MCN

∠3 = ∠DCM ∠1 + ∠2 = ∠MCB

Note: A quarter turn of a ray $\overrightarrow{OA}$ about "O" describe an angle called a right angle. The measure of right angle is 90°.

Check your Answers 1

1.



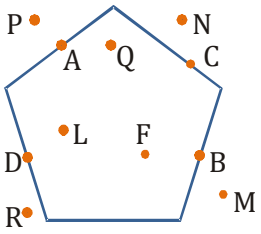
2. (a) M (b) line k (c) line k

3. (a) AD||BC (b) PQ||RS

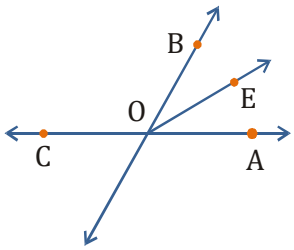
(c) XY||BC and YZ||BA (d) LM||PO, LQ||NO, MN||QP

Check your Concepts 2

1. Name the points which are inside the figure, on the figure and outside the figure.



2. Which of the following rays are the arms of ∠BOA ?



(a) $\overrightarrow{OB}$, $\overrightarrow{OE}$ (b) $\overrightarrow{OE}$, $\overrightarrow{OA}$ (c) $\overrightarrow{OB}$, $\overrightarrow{OA}$ (d) $\overrightarrow{OC}$, $\overrightarrow{OA}$

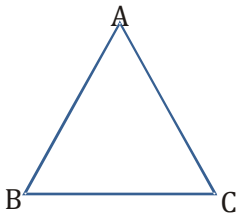
SPOT LIGHT

The centroid divides the median in the ratio of 2 : 1

Triangle

A triangle is a simple closed figure made of three-line segments.

Triangle ABC is denoted by the symbol ΔABC.



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The sum of the angles of a triangle is 180°, or 2 right angles.

ΔABC has :

(i) Three sides, namely AB, BC and CA ;

(ii) Three angles, namely ∠BAC, ∠ABC and ∠BCA to be denoted by ∠A, ∠B and ∠C respectively.

The three sides and three angles of a triangle are together called the six parts or six elements of the triangle.

In ΔABC, the points A, B and C are called its vertices.

Clearly, A is the vertex opposite to the side BC.

Similarly, B is the vertex opposite to the side CA.

And, C is the vertex opposite to the side AB.

Do You Remember ?

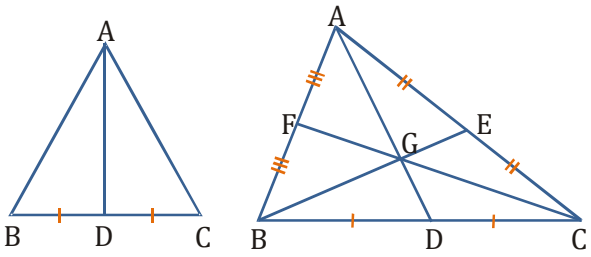
✸ In equilateral triangle median and altitude are same line.

Median of a triangle

A line segment joining a vertex to the midpoint of the side opposite to the vertex is called a median of a triangle.

Here in ΔABC, D, E, F are the mid points of the sides BC, AC and AB respectively. The medians are AD, BE and CF.

The point of intersection of three medians is called centroid (G).

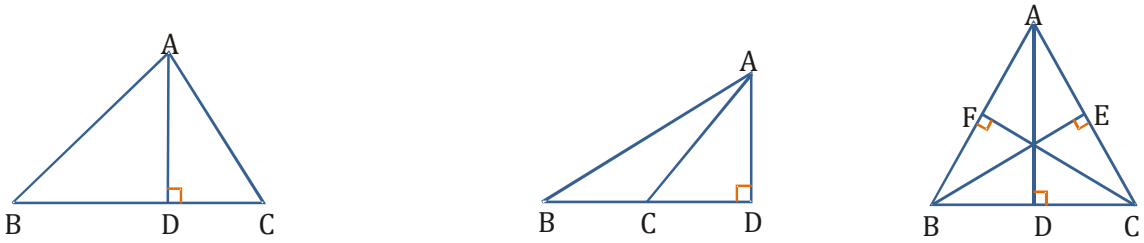


Altitude of a triangle

An altitude of a triangle is the perpendicular drawn from a vertex to the opposite side. If we take BC as the base, then AD is called the height of the triangle.

Every triangle has three altitudes, one from each vertex.

The point of intersection of altitude is called orthocentre.



Note: Two triangle are said to be congruent if every angle of one is equal to the corresponding angle of the other and every side of one is equal to the corresponding side of the other.

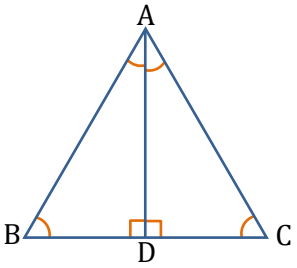
Numerical Ability 5

(i) Identify all the triangles in figure.

(ii) How many angles are there in the figure? Write their names.

(iii) Write the names of six line segments.

(iv) Which two triangles have ∠B as common in the figure? Name them.



Solution

(i) ΔABD, ΔADC, ΔABC

(ii) There are 8 angles in the figure $\angle ABD$, $\angle BAD$, $\angle ADB$, $\angle ADC$, $\angle DAC$, $\angle ACD$, $\angle BAC$, $\angle BDC$

(iii) AB, BD, DC, BC, AC, AD

(iv) ΔABD and ΔABC have $\angle B$ as common.

Check your Answers 2

1. (a) The points inside the figure are Q, L, F.

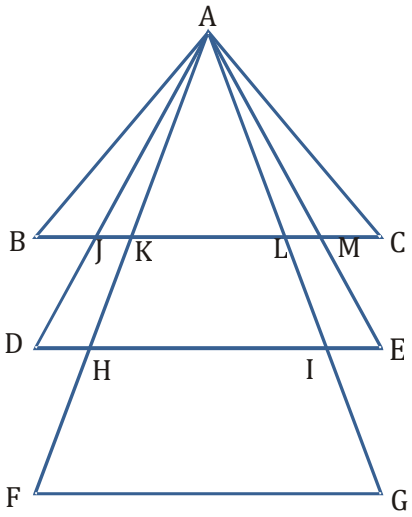
(b) The points outside the figure are N, P, M, R.

(c) The points lie on the figure are A, C, B, D.

2. (c) $\overrightarrow{OB}$, $\overrightarrow{OA}$

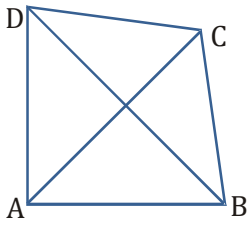
Check your Concepts 3

1. Identify as many triangles as you can in the Christmas tree.



Quadrilaterals

A quadrilateral is a four sided polygon. It has 4 sides and 4 angles.



(i) The four points A, B, C and D are called the vertices.

(ii) The four line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ are called the sides.

(iii) It has 4 angles namely $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle CDA$.

(iv) The line segments $\overline{AC}$ and $\overline{BD}$ are called the diagonals.

(v) The pair of vertices such as A, C and B, D are called the opposite vertices.

(vi) Two sides such as AB and BC which have one common vertex B are called its adjacent sides and such other are BC, CD, CD, DA and DA, AB.

(vii) Sides having no common vertex are called opposite sides. Eg. AB and CD ; AD and BC.

(viii) Angles such as $\angle A$ and $\angle B$ having one common side AB are called adjacent angles.

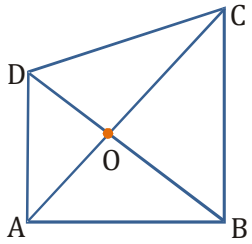
(ix) $\angle A$ and $\angle C$ or $\angle B$ and $\angle D$ having no common side are called opposite angles.

(x) The quadrilateral is named in the cyclic manner in which the vertices are named, i.e. we name it as ABCD and not as ACBD.

(xi) The sum of the angle of a quadrilateral is 360°

Building Concepts 7

Take a point O inside a given quadrilateral ABCD. Join the point O to the vertices A, B, C and D. Into which figures the quadrilateral will be divided?

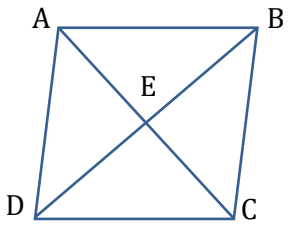


Explanation

The quadrilateral is divided into 4 triangles that are ΔBOC, ΔCOD, ΔDOA, ΔAOB.

Numerical Ability 6

Study the given figure and answer the following.



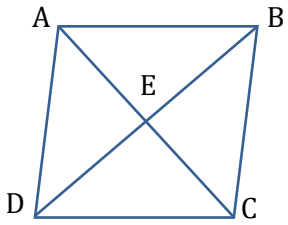
(i) Name the angle EDC in three different ways.

(ii) Name the vertex of angle AEB?

(iii) How many angles are formed at the vertex D?

(iv) How many triangles can you locate?

Solution



(i) ∠EDC can be named as ∠CDE, ∠BDC and ∠CDB.

(ii) E is the vertex of ∠AEB

(iii) Three angles are formed at vertex D i.e. ∠ADB, ∠BDC and ∠ADC.

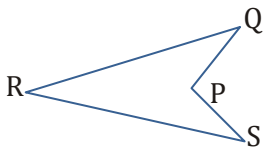
(iv) Eight triangles i.e.; ΔDEC, ΔAEB, ΔAED, ΔBEC, ΔADC, ΔABC, ΔBAD and ΔBCD

Diameter is the longest chord of the circle.

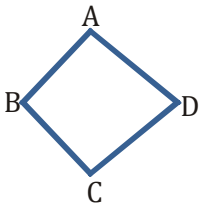
SPOT LIGHT

Note:

- A quadrilateral in which the measure of one angle is more than 180° is called a concave quadrilateral.



- A quadrilateral in which the measure of each angle is less than 180° is called a convex quadrilateral



Circles

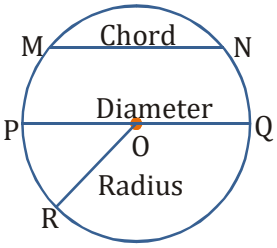
A circle is a simple closed curve all of whose points are at the same distance from a given point O in the same plane. The given point O is called the centre of the circle.

Part of a circle

Radius : A line segment joining the centre of a circle to any point on the circle is called a radius of that circle.

Chord : A line segment joining any two points on a circle is called a chord of that circle.

Diameter : A chord that passes through the centre of a circle is called a diameter of that circle. The diameter is twice the radius.



In figure, MN, PQ and OR are chord, diameter and radius respectively. All the diameters of a circle meet at the centre of the circle.

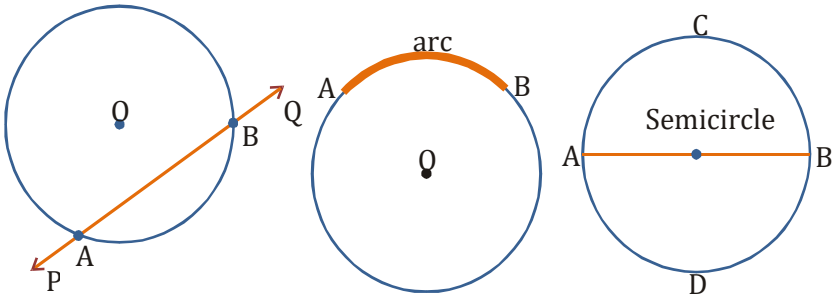
Circumference : The perimeter of a circle is called its circumference. In other words, the length of the boundary of the interior of a circle is its circumference.

Secant : A line which intersects or meets the circle at two distinct points is called a secant.

$\overleftrightarrow{PQ}$ is a secant.

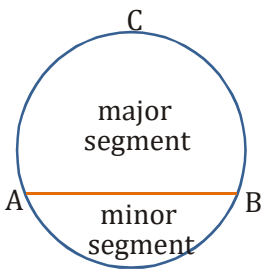
Arc : A part (continuous) of a circle is called an arc.

Semi circle : A diameter divides a circle into two equal parts which are called semi circles.

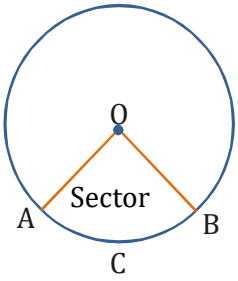


Segment : A chord AB of a circle divides the area enclosed by it into two parts which are called segments.

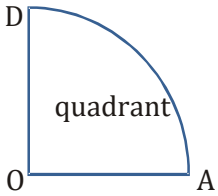
The smaller part is called a minor segment and the larger part is called a major segment.



Sector and quadrant : The part of a circle enclosed by any two radii of the circle is called a sector of the circle. In this figure, OACB is a sector.



If two radii are at right angles to each other, the sector is called a quadrant. A quadrant is ¼ the of a circle.

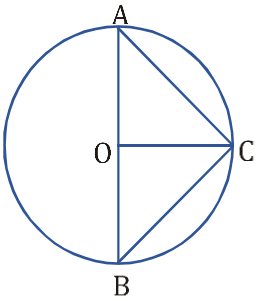


Check your Answers 3

1. 22 triangles

Building Concepts 8

Use circle to name following figures.



(i) Three radii

(ii) Three chords

(iii) A diameter

(iv) A triangle that has the centre of the circle as a vertex.

Explanation

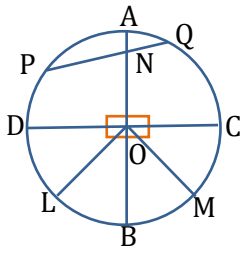
(i) The three radii are OA, OB and OC.

(ii) The three chords are AC, BC and AB.

(iii) The diameter is AB.

(iv) The triangle that has centre as a vertex are ΔAOC and ΔBOC.

Numerical Ability 7



Two or more circles drawn with the same centre are called concentric circles

Here, r ≠ R but centre (O) is same

SPOT LIGHT

(i) COA, AOD, DOB, BOC are four _____ of the circle.
(ii) PAQNP is a minor _____ of the circle.
(iii) PBQNP is a _____ segment of a circle.

Solution

(i) COA, AOD, DOB, BOC four quadrants of the circle.
(ii) PAQNP is a minor segment of the circle.
(iii) PBQNP is a major segment of a circle.