

NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 3.1

1. Write all the factors of the following numbers:

- (a) 24 (b) 15
(c) 21 (d) 27
(e) 12 (f) 20
(g) 18 (h) 23
(i) 36

Sol. (a) 24

$24 = 1 \times 24, 24 = 2 \times 12, 24 = 3 \times 8,$
 $24 = 4 \times 6, 24 = 6 \times 4$
 \therefore Factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24

(b) 15

$15 = 1 \times 15, 15 = 3 \times 5, 15 = 5 \times 3$
 \therefore Factors of 15 are 1, 3, 5, and 15

(c) 21

$21 = 1 \times 21, 21 = 3 \times 7, 21 = 7 \times 3$
 \therefore Factors of 21 are 1, 3, 7, and 21

(d) 27

$27 = 1 \times 27, 27 = 3 \times 9, 27 = 9 \times 3$
 \therefore Factors of 27 are 1, 3, 9, and 27

(e) 12

$12 = 1 \times 12, 12 = 2 \times 6,$
 $12 = 3 \times 4, 12 = 4 \times 3$
 \therefore Factors of 12 are 1, 2, 3, 4, 6, and 12

(f) 20

$20 = 1 \times 20, 20 = 2 \times 10,$
 $20 = 4 \times 5, 20 = 5 \times 4$
 \therefore Factors of 20 are 1, 2, 4, 5, 10, and 20

(g) 18

$18 = 1 \times 18, 18 = 2 \times 9,$
 $18 = 3 \times 6, 18 = 6 \times 3$
 \therefore Factors of 18 are 1, 2, 3, 6, 9, and 18

(h) 23

$23 = 1 \times 23, 23 = 23 \times 1$
 \therefore Factors of 23 are 1 and 23

(i) 36

$36 = 1 \times 36, 36 = 2 \times 18,$
 $36 = 3 \times 12, 36 = 4 \times 9,$
 $36 = 6 \times 6$
 \therefore Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36

2. Write first five multiples of:

- (a) 5 (b) 8
(c) 9

Sol. (a) $5 \times 1 = 5, 5 \times 2 = 10,$

$5 \times 3 = 15, 5 \times 4 = 20, 5 \times 5 = 25$

\therefore The required multiples are 5, 10, 15, 20, and 25.

(b) $8 \times 1 = 8, 8 \times 2 = 16,$

$8 \times 3 = 24, 8 \times 4 = 32, 8 \times 5 = 40$

\therefore The required multiples are 8, 16, 24, 32, and 40.

(c) $9 \times 1 = 9, 9 \times 2 = 18, 9 \times 3 = 27,$

$9 \times 4 = 36, 9 \times 5 = 45$

\therefore The required multiples are 9, 18, 27, 36, and 45.

3. Match the items in column 1 with the items in column 2.

Column - 1		Column - 2	
(i)	35	(a)	Multiple of 8
(ii)	15	(b)	Multiple of 7
(iii)	16	(c)	Multiple of 70
(iv)	20	(d)	Factors of 30
(v)	25	(e)	Factors of 50
-		(f)	Factors of 20

Sol.

Column - 1		Column - 2	
(i)	35	(b)	Multiple of 7
(ii)	15	(d)	Factors of 30
(iii)	16	(a)	Multiple of 8
(iv)	20	(f)	Factors of 20
(v)	25	(e)	Factors of 50

4. Find all the multiples of 9 up to 100.**Sol.** $9 \times 1 = 9$, $9 \times 2 = 18$, $9 \times 3 = 27$,

$9 \times 4 = 36$,

$9 \times 5 = 45$

$9 \times 6 = 54$, $9 \times 7 = 63$, $9 \times 8 = 72$

$9 \times 9 = 81$, $9 \times 10 = 90$,

$9 \times 11 = 99$

Therefore, the multiples of 9 up to 100 are 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, and 99

EXERCISE : 3.2**1.** What is the sum of any two

(a) Odd numbers? (b) Even numbers?

Sol. (a) The sum of two odd numbers is even.

e.g., $1 + 3 = 4$

$13 + 19 = 32$

(b) The sum of two even numbers is even.

e.g., $2 + 4 = 6$

$10 + 18 = 28$

2. State whether the following statements are True or False:

(a) The sum of three odd numbers is even.

(b) The sum of two odd numbers and one even number is even.

(c) The product of three odd numbers is odd.

(d) If an even number is divided by 2, the quotient is always odd.

(e) All prime numbers are odd.

(f) Prime numbers do not have any factors.

(g) Sum of two prime numbers is always even.

(h) 2 is the only even prime number.

(i) All even numbers are composite numbers.

(j) The product of two even numbers is always even.

Sol. (a) False $3 + 5 + 7 = 15$, i.e., odd(b) True $3 + 5 + 6 = 14$, i.e., even(c) True $3 \times 5 \times 7 = 105$, i.e., odd(d) False $4 \div 2 = 2$, i.e., even

(e) False 2 is a prime number and it is also even

(f) False 1 and the number itself are factors of the prime number

(g) False $2 + 3 = 5$, i.e., odd

(h) True

(i) False 2 is a prime number

(j) True $2 \times 4 = 8$, i.e., even**3.** The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers up to 100.**Sol.** 17, 71

37, 73

79, 97

4. Write down separately the prime and composite numbers less than 20.**Sol.** Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.

Composite numbers less than 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18.

5. What is the greatest prime number between 1 and 10?**Sol.** Prime numbers between 1 and 10 are 2, 3, 5, and 7. Among these numbers, '7' is the greatest.

6. Express the following as the sum of two odd primes.

- (a) 44 (b) 36
(c) 24 (d) 18

Sol. (a) $44 = 37 + 7$ (b) $36 = 31 + 5$
(c) $24 = 19 + 5$ (d) $18 = 11 + 7$

7. Give three pairs of prime numbers whose difference is 2.

[Remark: Two prime numbers whose difference is 2 are called twin primes].

Sol. 3, 5
41, 43
71, 73

8. Which of the following numbers are prime?

- (a) 23 (b) 51
(c) 37 (d) 26

Sol. (a) $23 = 1 \times 23$
 $23 = 23 \times 1$
23 has only two factors, 1 and 23.
Therefore, it is a prime number.

(b) $51 = 1 \times 51$
 $51 = 3 \times 17$
51 has four factors, 1, 3, 17, 51. Therefore, it is not a prime number. It is a composite number.

(c) $37 = 1 \times 37$
 $37 = 37 \times 1$
It has only two factors, 1 and 37.
Therefore, it is a prime number.

(d) $26 = 1 \times 26$
 $26 = 2 \times 13$
26 has four factors (1, 2, 13, 26).
Therefore, it is not a prime number. It is a composite number.

9. Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

Sol. Between 89 and 97, both of which are prime numbers, there are 7 composite numbers. They are
90, 91, 92, 93, 94, 95, 96

Factors of 90 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

Factors of 91 are 1, 7, 13, 91

Factors of 92 are 1, 2, 4, 23, 46, 92

Factors of 93 are 1, 3, 31, 93

Factors of 94 are 1, 2, 47, 94

Factors of 95 are 1, 5, 19, 95

Factors of 96 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

10. Express each of the following numbers as the sum of three odd primes:

- (a) 21 (b) 31
(c) 53 (d) 61

Sol. (a) $21 = 3 + 7 + 11$ (b) $31 = 5 + 7 + 19$
(c) $53 = 3 + 19 + 31$ (d) $61 = 11 + 19 + 31$

11. Write five pairs of prime numbers less than 20 whose sum is divisible by 5.
(Hint : $3+7 = 10$)

Sol. $2 + 3 = 5$
 $2 + 13 = 15$
 $3 + 17 = 20$
 $7 + 13 = 20$
 $19 + 11 = 30$

12. Fill in the blanks:

- (a) A number which has only two factors is called a _____.
(b) A number which has more than two factors is called a _____.
(c) 1 is neither _____ nor _____.
(d) The smallest prime number is _____.
(e) The smallest composite number is _____.
(f) The smallest even number is _____.

- Sol.** (a) Prime number
 (b) Composite number
 (c) Prime number, composite number
 (d) 2
 (e) 4
 (f) 2

EXERCISE : 3.3

1. Using divisibility tests, determine which of the following numbers are divisible by 2; by 3; by 4; by 5; by 6; by 8; by 9; by 10; by 11 (say, yes or no):

	2	3	4	5	6	8	9	10	11
128	Yes	No	Yes	No	No	Yes	No	No	No
990
1586
275
6686
639210
429714
2856
3060
406839

Sol.

	2	3	4	5	6	8	9	10	11
990	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes
1586	Yes	No	No	No	No	No	No	No	No
275	No	No	No	Yes	No	No	No	No	Yes
6686	Yes	No	No	No	No	No	No	No	No
639210	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes
429714	Yes	Yes	No	No	Yes	No	Yes	No	No
2856	Yes	Yes	Yes	No	Yes	Yes	No	No	No
3060	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
406839	No	Yes	No	No	No	No	No	No	No

2. Using divisibility tests, determine which of the following numbers are divisible by 4; by 8
- (a) 572 (b) 726352
 (c) 5500 (d) 6000
 (e) 12159

- (f) 14560 (g) 21084
 (h) 31795072 (i) 1700
 (j) 2150

Sol. (a) 572

The last two digits are 72. Since 72 is divisible by 4, the given number is divisible by 4.

The last three digits are 572. Since 572 is not divisible by 8, the given number is also not divisible by 8.

(b) 726352

The last two digits are 52. As 52 is divisible by 4, the given number is also divisible by 4. The last three digits are 352. Since 352 is divisible by 8, the given number is also divisible by 8.

(c) 5500

Since last two digits are 00, it is divisible by 4. The last 3 digits are 500. Since 500 is not divisible by 8, the given number is also not divisible by 8.

(d) 6000

Since the last 2 digits are 00, the given number is divisible by 4.

Since the last 3 digits are 000, the given number is divisible by 8.

(e) 12159

The last 2 digits are 59. Since 59 is not divisible by 4, the given number is also not divisible by 4. The last 3 digits are 159. Since 159 is not divisible by 8, the given number is not divisible by 8.

(f) 14560

The last two digits are 60. Since 60 is divisible by 4, the given number is divisible by 4. The last 3 digits are 560. Since 560 is divisible by 8, the given number is divisible by 8.

(g) 21084

The last two digits are 84. Since 84 is divisible by 4, the given number is divisible by 4.

The last three digits are 084. Since 084 is not divisible by 8, the given number is not divisible by 8.

(h) 31795072

The last two digits are 72. Since 72 is divisible by 4, the given number is divisible by 4. The last three digits are 072. Since 072 is divisible by 8, the given number is divisible by 8.

(i) 1700

The last two digits are 00. Since 00 is divisible by 4, the given number is divisible by 4.

The last three digits are 700. Since 700 is not divisible by 8, the given number is not divisible by 8.

(j) 2150

The last two digits are 50. Since 50 is not divisible by 4, the given number is not divisible by 4.

The last three digits are 150. Since 150 is not divisible by 8, the given number is not divisible by 8.

3. Using divisibility tests, determine which of following numbers are divisible by 6:

(a) 297144

(b) 1258

(c) 4335

(d) 61233

(e) 901352

(f) 438750

(g) 1790184

(h) 12583

(i) 639210

(j) 17852

Sol. (a) 297144

Since the last digit of the number is 4, it is divisible by 2.

On adding all the digits of the number, the sum obtained is 27. Since 27 is divisible by 3, the given number is also divisible by 3.

As the number is divisible by both 2 and 3, it is divisible by 6.

(b) 1258

Since the last digit of the number is 8, it is divisible by 2. On adding all the digits of the number, the sum obtained is 16. Since 16 is not divisible by 3, the given number is also not divisible by 3. As the number is not divisible by both 2 and 3, it is not divisible by 6.

(c) 4335

The last digit of the number is 5, which is not divisible by 2. Therefore, the given number is also not divisible by 2. On adding all the digits of the number, the sum obtained is 15. Since 15 is divisible by 3, the given number is also divisible by 3.

As the number is not divisible by both 2 and 3, it is not divisible by 6.

(d) 61233

The last digit of the number is 3, which is not divisible by 2. Therefore, the given number is also not divisible by 2.

On adding all the digits of the number, the sum obtained is 15. Since 15 is divisible by 3, the given number is also divisible by 3.

As the number is not divisible by both 2 and 3, it is not divisible by 6.

(e) 901352

Since the last digit of the number is 2, it is divisible by 2.

On adding all the digits of the number, the sum obtained is 20. Since 20 is not divisible by 3, the given number is also not divisible by 3. As the number is not divisible by both 2 and 3, it is not divisible by 6.

(f) 438750

Since the last digit of the number is 0, it is divisible by 2.

On adding all the digits of the number, the sum obtained is 27. Since 27 is divisible by 3, the given number is also divisible by 3. As the number is divisible by both 2 and 3, it is divisible by 6.

(g) 1790184

Since the last digit of the number is 4, it is divisible by 2.

On adding all the digits of the number, the sum obtained is 30. Since 30 is divisible by 3, the given number is also divisible by 3.

As the number is divisible by both 2 and 3, it is divisible by 6.

(h) 12583

Since the last digit of the number is 3, it is not divisible by 2.

On adding all the digits of the number, the sum obtained is 19. Since 19 is not divisible by 3, the given number is also not divisible by 3.

As the number is not divisible by both 2 and 3, it is not divisible by 6.

(i) 639210

Since the last digit of the number is 0, it is divisible by 2.

On adding all the digits of the number, the sum obtained is 21. Since 21 is divisible by 3, the given number is also divisible by 3.

As the number is divisible by both 2 and 3, it is divisible by 6.

(j) 17852

Since the last digit of the number is 2, it is divisible by 2.

On adding all the digits of the number, the sum obtained is 23. Since 23 is not divisible by 3, the given number is also not divisible by 3.

As the number is not divisible by both 2 and 3, it is not divisible by 6.

4. Using divisibility tests, determine which of the following numbers are divisible by 11:

(a) 5445

(b) 10824

(c) 7138965

(d) 70169308

(e) 10000001

(f) 901153

Sol. (a) 5445

Sum of the digits at odd places = $5 + 4 = 9$

Sum of the digits at even places = $4 + 5 = 9$

Difference = $9 - 9 = 0$

As the difference between the sum of the digits at odd places and the sum of the digits at even places is 0, therefore, 5445 is divisible by 11.

- (b) 10824
Sum of the digits at odd places = $4 + 8 + 1 = 13$
Sum of the digits at even places = $2 + 0 = 2$
Difference = $13 - 2 = 11$
The difference between the sum of the digits at odd places and the sum of the digits at even places is 11, which is divisible by 11. Therefore, 10824 is divisible by 11.
- (c) 7138965
Sum of the digits at odd places = $5 + 9 + 3 + 7 = 24$
Sum of the digits at even places = $6 + 8 + 1 = 15$
Difference = $24 - 15 = 9$
The difference between the sum of the digits at odd places and the sum of digits at even places is 9, which is not divisible by 11. Therefore, 7138965 is not divisible by 11.
- (d) 70169308
Sum of the digits at odd places = $8 + 3 + 6 + 0 = 17$
Sum of the digits at even places = $0 + 9 + 1 + 7 = 17$
Difference = $17 - 17 = 0$
As the difference between the sum of the digits at odd places and the sum of the digits at even places is 0, therefore, 70169308 is divisible by 11.
- (e) 10000001
Sum of the digits at odd places = 1
Sum of the digits at even places = 1
Difference = $1 - 1 = 0$
As the difference between the sum of the digits at odd places and the sum of the digits at even places is 0, therefore, 10000001 is divisible by 11.
- (f) 901153
Sum of the digits at odd places = $3 + 1 + 0 = 4$
Sum of the digits at even places

$$= 5 + 1 + 9 = 15$$

$$\text{Difference} = 15 - 4 = 11$$

The difference between the sum of the digits at odd places and the sum of the digits at even places is 11, which is divisible by 11. Therefore, 901153 is divisible by 11.

5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3:

(a) 6724 (b) 4765 2

Sol. (a) 6724

$$\text{Sum of the remaining digits} = 19$$

To make the number divisible by 3, the sum of its digits should be divisible by 3.

The smallest multiple of 3 which comes after 19 is 21.

$$\text{Therefore, smallest digit} = 21 - 19 = 2$$

$$\text{Now, } 2 + 3 + 3 = 8$$

$$\text{However, } 2 + 3 + 3 + 3 = 11$$

If we put 8, then the sum of the digits will be 27 and as 27 is divisible by 3, the number will also be divisible by 3.

Therefore, the largest digit is 8.

- (b) 4765 2

$$\text{Sum of the remaining digits} = 24$$

To make the number divisible by 3, the sum of its digits should be divisible by 3.

As 24 is already divisible by 3, the smallest digit that can be placed here is 0.

$$\text{Now, } 0 + 3 = 3$$

$$3 + 3 = 6$$

$$3 + 3 + 3 = 9$$

$$\text{However, } 3 + 3 + 3 + 3 = 12$$

If we put 9, then the sum of the digits will be 33 and as 33 is divisible by 3, the number will also be divisible by 3.

Therefore, the largest digit is 9.

6. Write a digit in the blank space of each of the following numbers so that the number formed is divisible by 11:

(a) 92 ___ 389 (b) 8 ___ 9484

Sol. (a) 92_389

Let a be placed in the blank.

$$\text{Sum of the digits at odd places} = 9 + 3 + 2 = 14$$

$$\text{Sum of the digits at even places} = 8 + a + 9 = 17 + a$$

$$\text{Difference} = 17 + a - 14 = 3 + a$$

For a number to be divisible by 11, this difference should be zero or a multiple of 11.

$$\text{If } 3 + a = 0, \text{ then}$$

$$a = -3$$

However, it cannot be negative.

The closest multiple of 11, which is near to 3, has to be taken. It is 11 itself.

$$3 + a = 11$$

$$a = 8$$

Therefore, the required digit is 8.

(b) 8_9484

Let a be placed in the blank.

$$\text{Sum of the digits at odd places} = 4 + 4 + a = 8 + a$$

$$\text{Sum of the digits at even places} = 8 + 9 + 8 = 25$$

$$\text{Difference} = 25 - (8 + a) = 17 - a$$

For a number to be divisible by 11, this difference should be zero or a multiple of 11.

$$\text{If } 17 - a = 0, \text{ then}$$

$$a = 17$$

This is not possible.

A multiple of 11 has to be taken. Taking 11, we obtain

$$17 - a = 11$$

$$a = 6$$

Therefore, the required digit is 6.

EXERCISE : 3.4

1. Find the common factors of:

(a) 20 and 28 (b) 15 and 25

(c) 35 and 50 (d) 56 and 120

Sol. (a) Factors of 20 = 1, 2, 4, 5, 10, 20

$$\text{Factors of 28} = 1, 2, 4, 7, 14, 28$$

$$\text{Common factors} = 1, 2, 4$$

(b) Factors of 15 = 1, 3, 5, 15

$$\text{Factors of 25} = 1, 5, 25$$

$$\text{Common factors} = 1, 5$$

(c) Factors of 35 = 1, 5, 7, 35

$$\text{Factors of 50} = 1, 2, 5, 10, 25, 50$$

$$\text{Common factors} = 1, 5$$

(d) Factors of 56 = 1, 2, 4, 7, 8, 14, 28, 56

$$\text{Factors of 120} = 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120$$

$$\text{Common factors} = 1, 2, 4, 8$$

2. Find the common factors of:

(a) 4, 8 and 12 (b) 5, 15 and 25

Sol. (a) 4, 8, 12

$$\text{Factors of 4} = 1, 2, 4$$

$$\text{Factors of 8} = 1, 2, 4, 8$$

$$\text{Factors of 12} = 1, 2, 3, 4, 6, 12$$

$$\text{Common factors} = 1, 2, 4$$

(b) 5, 15, and 25

$$\text{Factors of 5} = 1, 5$$

$$\text{Factors of 15} = 1, 3, 5, 15$$

$$\text{Factors of 25} = 1, 5, 25$$

$$\text{Common factors} = 1, 5$$

3. Find first three common multiples of:

- (a) 6 and 8 (b) 12 and 18

Sol. (a) 6 and 8

Multiples of 6 = 6, 12, 18, 24, 30

Multiples of 8 = 8, 16, 24, 32

3 common multiples = 24, 48, 72

(b) 12 and 18

Multiples of 12 = 12, 24, 36, 48

Multiples of 18 = 18, 36, 54, 72

3 common multiples = 36, 72, 108

4. Write all the numbers less than 100 which are common multiples of 3 and 4.

Sol. Multiples of 3 = 3, 6, 9, 12, 15...

Multiples of 4 = 4, 8, 12, 16, 20...

Common multiples = 12, 24, 36, 48, 60, 72, 84, 96

5. Which of the following numbers are co-prime?

- (a) 18 and 35 (b) 15 and 37

- (c) 30 and 415 (d) 17 and 68

- (e) 216 and 215 (f) 81 and 16

Sol. (a) Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 35 = 1, 5, 7, 35

Common factor = 1

Therefore, the given two numbers are co-prime.

(b) Factors of 15 = 1, 3, 5, 15

Factors of 37 = 1, 37

Common factors = 1

Therefore, the given two numbers are co-prime.

(c) Factors of 30 = 1, 2, 3, 5, 6, 10, 15, 30

Factors of 415 = 1, 5, 83, 415

Common factors = 1, 5

As these numbers have a common factor other than 1, the given two numbers are not co-prime.

(d) Factors of 17 = 1, 17

Factors of 68 = 1, 2, 4, 17, 34, 68

Common factors = 1, 17

As these numbers have a common factor other than 1, the given two numbers are not co-prime.

(e) 216 and 215

Factors of 216 = 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 108, 216

Factors of 215 = 1, 5, 43, 215

Common factors = 1

Therefore, the given two numbers are co-prime.

(f) 81 and 16

Factors of 81 = 1, 3, 9, 27, 81

Factors of 16 = 1, 2, 4, 8, 16

Common factors = 1

Therefore, the given two numbers are co-prime

6. A number is divisible by both 5 and 12. By which other number will that number be always divisible?

Sol. Factors of 5 = 1, 5

Factors of 12 = 1, 2, 3, 4, 6, 12

As the common factor of these numbers is 1, the given two numbers are co-prime and the number will also be divisible by their product, i.e. 60, and the factors of 60, i.e., 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.

7. A number is divisible by 12. By what other numbers will that number be divisible?

Sol. Since the number is divisible by 12, it will also be divisible by its factors i.e., 1, 2, 3, 4, 6, 12. Clearly, 1, 2, 3, 4, and 6 are numbers other than 12 by which this number is also divisible.

EXERCISE : 3.5

1. Which of the following statements are true ?

- (a) If a number is divisible by 3, it must be divisible by 9.
- (b) If a number is divisible by 9, it must be divisible by 3.
- (c) A number is divisible by 18, if it is divisible by both 3 and 6.
- (d) If a number is divisible by 9 and 10 both, then it must be divisible by 90.
- (e) If two numbers are co-primes, at least one of them must be prime.
- (f) All numbers which are divisible by 4 must also be divisible by 8.
- (g) All numbers which are divisible by 8 must also be divisible by 4.
- (h) If a number exactly divides two numbers separately, it must exactly divide their sum.
- (i) If a number exactly divides the sum of two numbers, it must exactly divide the two numbers separately.

Sol. (a) False

6 is divisible by 3, but not by 9.

(b) True, as $9 = 3 \times 3$

Therefore, if a number is divisible by 9, then it will also be divisible by 3.

(c) False

30 is divisible by 3 and 6 both, but it is not divisible by 18.

(d) True, as $9 \times 10 = 90$

Therefore, if a number is divisible by 9 and 10 both, then it will also be divisible by 90.

(e) False

15 and 32 are co-primes and also composite.

(f) False

12 is divisible by 4, but not by 8.

(g) True, as $8 = 2 \times 4$

Therefore, if a number is divisible by 8, then it will also be divisible by 2 and 4.

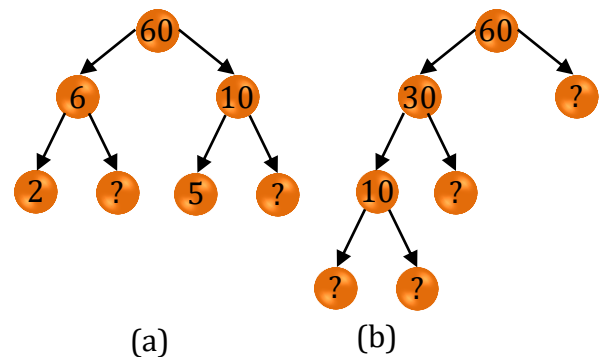
(h) True

2 divides 4 and 8 as well as 12. ($4 + 8 = 12$)

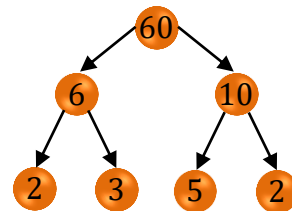
(i) False

2 divides 12 but does not divide 7 and 5.

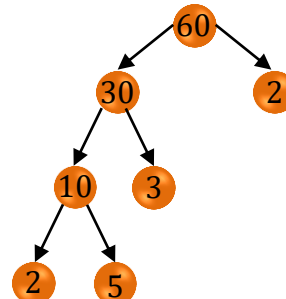
2. Here are two different factor trees for 60. Write the missing numbers.



Sol. (a) As $6 = 2 \times 3$ and $10 = 5 \times 2$



(b) As $60 = 30 \times 2$, $30 = 10 \times 3$, and $10 = 5 \times 2$

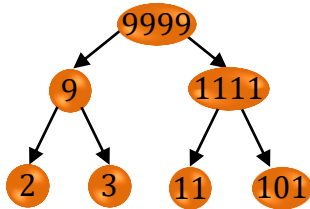


3. Which factors are not included in the prime factorization of a composite number?

Sol. 1 and the number itself

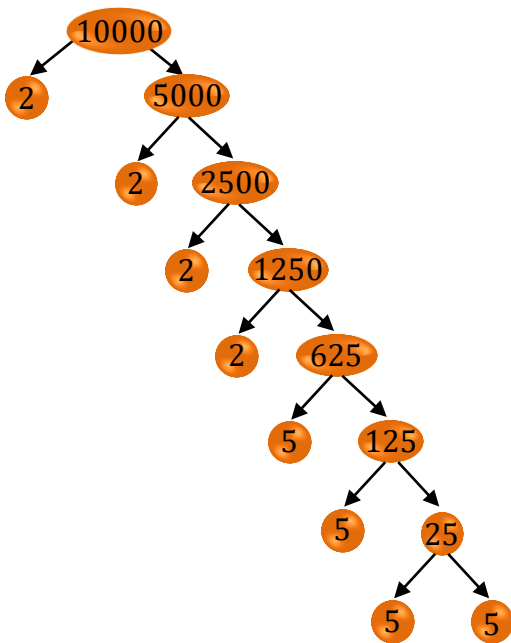
4. Write the greatest 4-digit number and express it in terms of its prime factors.

Sol. Greatest four-digit number = 9999
 $9999 = 3 \times 3 \times 11 \times 101$



5. Write the smallest 5-digit number and express it in the form of its prime factors.

Sol. Smallest five-digit number = 10,000
 $10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$



6. Find all prime factors of 1729 and arrange them in ascending order. Now state the relation, if any; between two consecutive prime factors.

Sol.

7	1729
13	247
19	19
	1

$$1729 = 7 \times 13 \times 19$$

$$13 - 7 = 6, 19 - 13 = 6$$

The difference of two consecutive prime factors is 6.

7. The product of three consecutive numbers is always divisible by 6. Verify this statement with the help of some examples.

Sol. $2 \times 3 \times 4 = 24$, which is divisible by 6

$$9 \times 10 \times 11 = 990, \text{ which is divisible by } 6$$

$$20 \times 21 \times 22 = 9240, \text{ which is divisible by } 6$$

8. The sum of two consecutive odd numbers is divisible by 4. Verify this statement with the help of some examples.

Sol. $3 + 5 = 8$, which is divisible by 4

$$15 + 17 = 32, \text{ which is divisible by } 4$$

$$19 + 21 = 40, \text{ which is divisible by } 4$$

9. In which of the following expressions, prime factorization has been done?

(a) $24 = 2 \times 3 \times 4$ (b) $56 = 7 \times 2 \times 2 \times 2$

(c) $70 = 2 \times 5 \times 7$ (d) $54 = 2 \times 3 \times 9$

Sol. (a) $24 = 2 \times 3 \times 4$

Since 4 is composite, prime factorisation has not been done.

(b) $56 = 7 \times 2 \times 2 \times 2$

Since all the factors are prime, prime factorisation has been done.

(c) $70 = 2 \times 5 \times 7$

Since all the factors are prime, prime factorisation has been done.

(d) $54 = 2 \times 3 \times 9$

Since 9 is composite, prime factorisation has not been done.

10. Determine if 25110 is divisible by 45.

[Hint : 5 and 9 are co-prime numbers. Test the divisibility of the number by 5 and 9].

Sol. $45 = 5 \times 9$

Factors of 5 = 1, 5

Factors of 9 = 1, 3, 9

Therefore, 5 and 9 are co-prime numbers. Since the last digit of 25110 is 0, it is divisible by 5.

Sum of the digits of 25110 = $2 + 5 + 1 + 1 + 0 = 9$

As the sum of the digits of 25110 is divisible by 9, therefore, 25110 is divisible by 9.

Since the number is divisible by 5 and 9 both, it is divisible by 45.

11. 18 is divisible by both 2 and 3. It is also divisible by $2 \times 3 = 6$. Similarly, a number is divisible by both 4 and 6. Can we say that the number must also be divisible by $4 \times 6 = 24$? If not, give an example to justify our answer:

Sol. No. It is not necessary because 12 and 36 are divisible by 4 and 6 both but are not divisible by 24.

12. I am the smallest number, having four different prime factors. Can you find me?

Sol. Since it is the smallest number of such type, it will be the product of 4 smallest prime numbers.

$$2 \times 3 \times 5 \times 7 = 210$$

EXERCISE : 3.6

1. Find the HCF of the following numbers:

(a) 18, 48

(b) 30, 42

(c) 18, 60

(d) 27, 63

(e) 36, 84

(f) 34, 102

(g) 70, 105, 175

(h) 91, 112, 49

(i) 18, 54, 81

(j) 12, 45, 75

Sol. (a) 18, 48

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 2 & 3 \\ \hline & 1 \end{array}$$

$$18 = 2 \times 3 \times 3$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{HCF} = 2 \times 3 = 6$$

(b) 30, 42

$$\begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$30 = 2 \times 3 \times 5$$

$$42 = 2 \times 3 \times 7$$

$$\text{HCF} = 2 \times 3 = 6$$

(c) 18, 60

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$18 = 2 \times 3 \times 3$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$\text{HCF} = 2 \times 3 = 6$$

(d) 27, 63

$$\begin{array}{r|l} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$27 = 3 \times 3 \times 3$$

$$63 = 3 \times 3 \times 7$$

$$\text{HCF} = 3 \times 3 = 9$$

(e) 36, 84

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$84 = 2 \times 2 \times 3 \times 7$$

$$\text{HCF} = 2 \times 2 \times 3 = 12$$

(f) 34, 102

$$\begin{array}{r|l} 2 & 34 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 102 \\ \hline 3 & 51 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$$34 = 2 \times 17$$

$$102 = 2 \times 3 \times 17$$

$$\text{HCF} = 2 \times 17 = 34$$

(g) 70, 105, 175

$$\begin{array}{r|l} 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 5 & 175 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$70 = 2 \times 5 \times 7$$

$$105 = 3 \times 5 \times 7$$

$$175 = 5 \times 5 \times 7$$

$$\text{HCF} = 5 \times 7 = 35$$

(h) 91, 112, 49

$$\begin{array}{r|l} 2 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 112 \\ \hline 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$91 = 7 \times 13$$

$$112 = 2 \times 2 \times 2 \times 2 \times 7$$

$$49 = 7 \times 7$$

$$\text{HCF} = 7$$

(i) 18, 54, 81

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$18 = 2 \times 3 \times 3$$

$$54 = 2 \times 3 \times 3 \times 3$$

$$81 = 3 \times 3 \times 3 \times 3$$

$$\text{HCF} = 3 \times 3 = 9$$

(j) 12, 45, 75

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$12 = 2 \times 2 \times 3$$

$$45 = 3 \times 3 \times 5$$

$$75 = 3 \times 5 \times 5$$

$$\text{HCF} = 3$$

2. What is the HCF of two consecutive?

(a) Numbers?

(b) Even numbers? (c) Odd numbers?

Sol. (a) 1 e.g., HCF of 2 and 3 is 1.

(b) 2 e.g., HCF of 2 and 4 is 2.

(c) 1 e.g., HCF of 3 and 5 is 1.

3. HCF of co-prime numbers 4 and 15 was found as follows by factorization:

$4 = 2 \times 2$ and $15 = 3 \times 5$ since there is no common prime factors, so HCF of 4 and 15 is 1. Is the answer correct? If not, what is the correct HCF?

Sol. No. The answer is not correct. 1 is the correct HCF.

EXERCISE : 3.7

1. Renu purchases two bags of fertilizer of weight 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertilizer exact number of times.

Sol. Weight of the two bags = 75 kg and 69 kg
Maximum weight = HCF (75, 69)

$$\begin{array}{r|l} 3 & 75 \\ \hline 4 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 69 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$$75 = 3 \times 5 \times 5$$

$$69 = 3 \times 23$$

$$\text{HCF} = 3$$

Hence, the maximum value of weight, which can measure the weight of the fertilizer exact number of times, is 3 kg.

2. Three boys step off together from the same spot. Their steps measure 63cm, 70cm and 77cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?

Sol. Step measure of 1st Boy = 63 cm
Step measure of 2nd Boy = 70 cm
Step measure of 3rd Boy = 77 cm
LCM of 63, 70, 77

$$\begin{array}{r|l} 2 & 63, 70, 77 \\ \hline 3 & 63, 35, 77 \\ \hline 3 & 21, 35, 77 \\ \hline 5 & 7, 35, 77 \\ \hline 7 & 7, 7, 77 \\ \hline 11 & 1, 1, 11 \\ \hline & 1, 1, 1 \end{array}$$

$$\text{LCM} = 2 \times 3 \times 3 \times 5 \times 7 \times 11 = 6930$$

Hence, the minimum distance each should cover so that all can cover the distance in complete steps is 6930 cm.

3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Sol. Length = 825 cm = $3 \times 5 \times 5 \times 11$
Breadth = 675 cm = $3 \times 3 \times 3 \times 5 \times 5$
Height = 450 cm = $2 \times 3 \times 3 \times 5 \times 5$
Longest tape = HCF of 825, 675, and 450
 $= 3 \times 5 \times 5 = 75$ cm
Therefore, the longest tape is 75 cm.

4. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.

Sol. Smallest number = LCM of 6, 8, 12

$$\begin{array}{r|l} 2 & 6, 8, 12 \\ \hline 2 & 3, 4, 6 \\ \hline 2 & 3, 2, 3 \\ \hline 2 & 3, 1, 3 \\ \hline & 1, 1, 1 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 2 \times 3 = 24$$

We have to find the smallest 3-digit multiple of 24.

It can be seen that $24 \times 4 = 96$ and $24 \times 5 = 120$.

Hence, the smallest 3-digit number which is exactly divisible by 6, 8, and 12 is 120.

5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.

Sol. LCM of 8, 10, and 12

$$\begin{array}{r|l} 2 & 8, 10, 12 \\ \hline 2 & 4, 5, 6 \\ \hline 2 & 2, 5, 3 \\ \hline 3 & 1, 5, 3 \\ \hline 5 & 1, 5, 1 \\ \hline & 1, 1, 1 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 = 120$$

We have to find the greatest 3-digit multiple of 120.

It can be seen that $120 \times 8 = 960$ and $120 \times 9 = 1080$.

Hence, the greatest 3-digit number exactly divisible by 8, 10, and 12 is 960.

6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?

Sol. Time period after which these lights will change = LCM of 48, 72, 108

$$\begin{array}{r|l} 2 & 48, 72, 108 \\ \hline 2 & 24, 36, 54 \\ \hline 2 & 12, 18, 27 \\ \hline 2 & 6, 9, 27 \\ \hline 2 & 3, 9, 27 \\ \hline 3 & 1, 3, 9 \\ \hline 3 & 1, 1, 3 \\ \hline & 1, 1, 1 \end{array}$$

LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$ They will change together after every 432 seconds i.e., 7 min 12 seconds.

Hence, they will change simultaneously at 7:07:12 a.m.

7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of the three containers exact number of times.

Sol. Maximum capacity of the required tanker = HCF of 403, 434, 465

$$403 = 13 \times 31$$

$$434 = 2 \times 7 \times 31$$

$$465 = 3 \times 5 \times 31$$

$$\text{HCF} = 31$$

\therefore A container of capacity 31ℓ can measure the diesel of 3 containers exact number of times

8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.

Sol. LCM of 6, 15, 18

$$\begin{array}{r|l} 2 & 6, 15, 18 \\ \hline 2 & 3, 15, 9 \\ \hline 2 & 1, 5, 3 \\ \hline 5 & 1, 5, 1 \\ \hline & 1, 1, 1 \end{array}$$

$$\text{LCM} = 2 \times 3 \times 3 \times 5 = 90$$

$$\text{Required number} = 90 + 5 = 95$$

9. Find the smallest 4-digit number which is divisible by 18, 24 and 32.

Sol. LCM of 18, 24, and 32

$$\begin{array}{r|l} 2 & 18, 24, 32 \\ \hline 2 & 9, 12, 16 \\ \hline 2 & 9, 6, 8 \\ \hline 2 & 9, 3, 4 \\ \hline 2 & 9, 3, 2 \\ \hline 3 & 9, 3, 1 \\ \hline 3 & 3, 1, 1 \\ \hline & 1, 1, 1 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$$

We have to find the smallest 4-digit multiple of 288.

It can be observed that $288 \times 3 = 864$ and $288 \times 4 = 1152$.

Therefore, the smallest 4-digit number which is divisible by 18, 24, and 32 is 1152.

10. Find the LCM of the following numbers:

(a) 9 and 4

(b) 12 and 5

(c) 6 and 5

(d) 15 and 4

Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?

Sol. (a)
$$\begin{array}{r|l} 2 & 9, 4 \\ \hline 2 & 9, 2 \\ \hline 3 & 9, 1 \\ \hline 3 & 3, 1 \\ \hline & 1, 1 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 3 \times 3 = 36$$

$$\begin{array}{r|l}
 2 & 12, 5 \\
 \hline
 2 & 6, 5 \\
 \hline
 3 & 3, 5 \\
 \hline
 5 & 1, 5 \\
 \hline
 & 1, 1
 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 3 \times 5 = 60$$

$$\begin{array}{r|l}
 2 & 6, 5 \\
 \hline
 3 & 3, 5 \\
 \hline
 5 & 1, 5 \\
 \hline
 & 1, 1
 \end{array}$$

$$\text{LCM} = 2 \times 3 \times 5 = 30$$

$$\begin{array}{r|l}
 2 & 15, 4 \\
 \hline
 2 & 15, 2 \\
 \hline
 3 & 15, 1 \\
 \hline
 5 & 5, 1 \\
 \hline
 & 1
 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 3 \times 5 = 60$$

Yes, it can be observed that in each case, the LCM of the given numbers is the product of these numbers. When two numbers are co-prime, their LCM is the product of those numbers. Also, in each case, LCM is a multiple of 3.

- 11.** Find the LCM of the following numbers in which one number is the factor of the other.

(a) 5, 20 (b) 6, 18

(c) 12, 48 (d) 9, 45

What do you observe in the results obtained?

Sol. (a) 5, 20

$$\begin{array}{r|l}
 2 & 5, 20 \\
 \hline
 2 & 5, 10 \\
 \hline
 5 & 5, 5 \\
 \hline
 & 1, 1
 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 5 = 20$$

(b) 6, 18

$$\begin{array}{r|l}
 2 & 6, 18 \\
 \hline
 3 & 3, 9 \\
 \hline
 3 & 1, 3 \\
 \hline
 & 1, 1
 \end{array}$$

$$\text{LCM} = 2 \times 3 \times 3 = 18$$

(c) 12, 48

$$\begin{array}{r|l}
 2 & 12, 48 \\
 \hline
 2 & 6, 24 \\
 \hline
 2 & 3, 12 \\
 \hline
 2 & 3, 6 \\
 \hline
 3 & 3, 3 \\
 \hline
 & 1, 1
 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 = 48$$

(d) 9, 45

$$\begin{array}{r|l}
 3 & 9, 45 \\
 \hline
 3 & 3, 15 \\
 \hline
 5 & 1, 5 \\
 \hline
 & 1, 1
 \end{array}$$

$$\text{LCM} = 3 \times 3 \times 5 = 45$$

Yes, it can be observed that in each case, the LCM of the given numbers is the larger number. When one number is a factor of the other number, their LCM will be the larger number.