11 Algebra

Variables

The height of growing child changes with time

Let us fill the empty boxes

(i) $\square$ + 2 = 8 **(ii) 9 – $\square$ = 4** **(iii) 9 + 7 = $\square$**

Here the empty box $\square$ stands for an unknown number. Let us write these problems a little differently.

(i) Cost of a chocolate + ₹ 2 is equal to ₹ 8.

(ii) Sapna has 9 fruits in a bag. Some fruits are taken by Reema from her bag and 4 fruits are left.

(iii) Anshu has 9 strawberries. His cousin has given him 7 more. How many strawberries does Anshu have now?

In all above examples, we have to find answers, which are not known. They are referred to as unknowns. We can put any symbol or picture of these unknowns in the place of the box.

Generally, for convenience, letters of the English alphabets such as a, b, c, ... x, y, z, etc., are used instead of the box.

So, these three examples can also be written as

(i) $x + 2 = 8$ (ii) $9 - x = 4$ (iii) $9 + 7 = x$

After solving we find that the values for x are 6, 5 and 16 respectively. When we use letters to denote numbers, we call them literal numbers. Literal numbers are generally referred to as variables as their values vary and are not fixed.

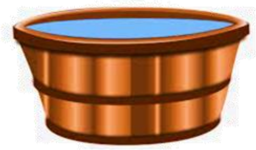
Variables are used with plus or minus sign to indicate the addition or subtraction of unspecified numbers.

> The word algebra is derived from the title of the book 'Al-jabar w'al almugabalah'.
>
> SPOT LIGHT

Constants

The quantities with fixed numerical values are called constants.

E.g., –3, 2, 217, 25, etc, …… are constants or numerals.



Capacity of a given container is fixed

Building Concepts 1

(i) Use the symbols of algebra to write an expression which indicates the sum of any two numbers.

(ii) Use the symbols of algebra to write an expression which indicates that a number is being added to itself.

(iii) How do the expression x + y and x + x differ ?

(iv) Indicate that d is to be subtracted from c.

Explanation

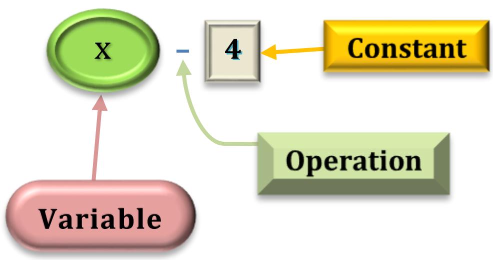
(i) Since we are not dealing with specific numbers, we may indicate this sum by writing ____ + ____, where the blanks represent any two numbers. However, instead of using blank spaces, we can let x represents one number and y represents the other number. Then, the sum may be indicated as x + y. The expression x + y says, "any number x plus any number y".

(ii) If we represent this unspecified number by x, then the expression x + x indicates that a number is being added to itself.

(iii) Both expressions indicate sums. However, the expression x + y uses two different letters to indicate that the numbers are not necessarily the same, although they may be the same. Whereas the expression x + x uses the same letter to indicate that the first addend is the same number as the second addend.

(iv) Since d represents the number, which is to be subtracted, it must appear after the minus sign. Thus, c – d indicates that d is to be subtracted from c.

Framing of an algebraic expression



An expression made up of variables (or literals) and constants (or numerals) connected by signs of arithmetic operations is called an algebraic expression.

E.g., 2y + 10, 5z, 9a + b, 3x etc. are algebraic expressions.

Addition of variables

The sum of x and 10 is (x + 10). Just as the sum of 9 and 10 is written as 9 + 10. Addition could also be understood by using the following statements.

(i) 8 more than y is written as y + 8.

(ii) x added to 4 is x + 4.

(iii) Increase a by 10 is a + 10.

(iv) A number which is 17 more than that of x is x + 17.

Subtraction of variables

If we are asked to subtract 9 from 13, we write it as 13 – 9. Similarly, if we are asked to subtract 3 from y, we write it as y – 3 and "x subtracted from 13" is written as 13 – x. Subtraction could also be understood from the following statements.

(i) Decrease x by 20 is x – 20.

(ii) b diminished from 15 is 15 – b.

(iii) 7 less than z means z – 7.

(iv) Subtract 14 from y is y – 14.

Be Alert !

- 7 less than x

  (7 – x) ✗ (x – 7) ✓

- 5 subtracted from x:

  (x – 5) ✓ (5 – x) ✗

Building Concepts 2

Express each of the following as an algebraic expression :

(i) 9 added to the sum of x and 9.

(ii) Decrease the sum of a and b by c.

(iii) y less than the sum of x and 11.

Explanation

(i) The sum of x and 9 is (x + 9). 9 is added to (x + 9), then the expression becomes as (x + 9) + 9.

(ii) The sum of a and b is a + b. Sum (a + b) is decreased by c, then it becomes (a + b) – c.

(iii) The sum of x and 11 is (x + 11). y less than x + 11 is (x + 11) – y.

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Multiplication of variables

Multiplication is the process of repeated addition.

E.g. $7 + 7 + 7 = 3 \times 7 = 21$

$x + x + x + x = 4 \times x = 4x$

The product of two variables x and y is written as xy.

Division of variables

In algebra, division of two variables a and b i.e. $a \div b$ is written as $\frac{a}{b}$, i.e. a by b.

Building Concepts 3

Express each of the following as an algebraic expression:

(i) x times y.

(ii) Three times of a number subtracted from 14.

(iii) 9 multiplied by the sum of c and d.

(iv) Quotient of x by 7.

(v) Quotient of y by 19 added to z.

(vi) Quotient of a by b added to the product of m and n.

Explanation

(i) x times y is written as xy.

(ii) Suppose the number is x. Three times of x is 3x. 3x is subtracted from 14, i.e. $14 - 3x$.

(iii) The sum of c and d is $(c + d)$. 9 multiplied by $(c + d)$ is $9(c + d)$.

(iv) Quotient of x by 7 means x is divided by 7, i.e. written as $x \div 7$ or $\frac{x}{7}$

(v) Quotient of y by 19 is written as $\frac{y}{19}$. $\frac{y}{19}$ is added to z which is written as $\frac{y}{19} + z$.

(vi) Quotient of a by b is written as $\frac{a}{b}$. This $\frac{a}{b}$ is added to the product of m and n that is written as $\frac{a}{b} + mn$.

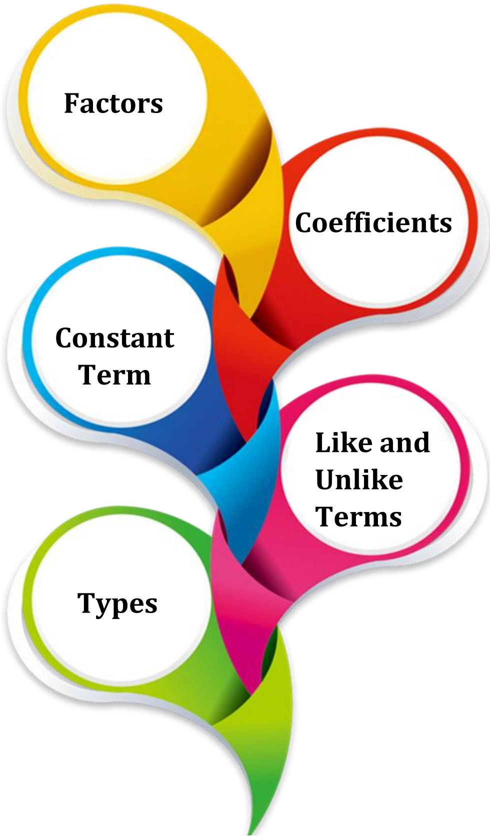
Check your Concepts 1

Express each of the following as an algebraic expression

1. The sum of x, y and z is decreased by y.

2. Write the predecessor of a number x.

Algebraic expressions basic terms and types



A combination of constants and variables connected by any one or more of the symbols +, –, × and ÷ is called an algebraic expression. The several parts of the expression, separated by the sign + or – are called the terms of the expression. Thus,

(i) The expression $5x - 9y + 1$ has three terms, namely $5x$, $-9y$ and 1.

(ii) The expression $6xyz - 6x^3y + 3y^2 + 4x^3$ has four terms, namely $6xyz$, $-6x^3y$, $3y^2$ and $4x^3$.

Some terms related to algebraic expressions

Factors : When two or more numbers and literals are multiplied then each one of them is called a factor of the product.

The constant factor is called a numerical factor, while a literal one is known as a literal factor.

For example,

(i) In $7x^2y$, we have 7 as the numerical factor, whereas x^2 and y are the literal factors.

(ii) In $-5y^3xz$, the numerical factor is -5, whereas y^3, x and z are the literal factors.

Coefficient : In a product of numbers and literals, any of the factors is called the coefficient of the product of other factors. For example,

(i) In $7x^2y$, the coefficient of x^2 is $7y$ and the coefficient of y is $7x^2$.

(ii) In $-xy^2z$, the coefficient of $-x$ is y^2z and the coefficient of x is $-y^2z$.

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Quick Tips

✸ If letters are used to denote numbers, they are called literals.

Building Concepts 4

Write down the coefficient of

(i) x^2 in $6x^3y$ (ii) -3 in $-6xy^3$

Explanation

(i) $6x^3y$ can be written as $= 6x \times x^2 \times y = 6xy \times x^2$

$\therefore$ The coefficient of x^2 is $6xy$.

(ii) $-6xy^3$ can be written as $= -3 \times 2 \times xy^3 = -3 \times 2xy^3$

$\therefore$ The coefficient of -3 is $2xy^3$.

Constant terms : A term of the expression having no literal factor is called a constant term.

E.g., In expression $x^2 + y^2 - \frac{7}{8}$, the constant term is $-\frac{7}{8}$.

Like terms : The terms having the same literal factors are called like or similar terms.

E.g., 5x, 9x etc.

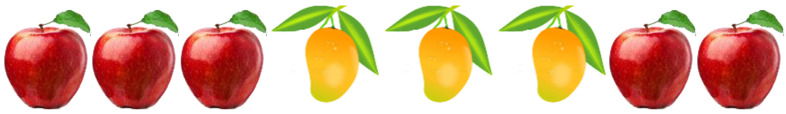
Unlike terms : The terms not having the same literal factors are called unlike or dissimilar terms.

E.g., $9x^2$, 5x etc.

In the expression, $6x^2y + 5xy^2 - 8xy - 7yx^2$, we have $6x^2y$ and $-7yx^2$ as like terms, whereas $5xy^2$ and $-8xy$ are unlike terms.

Simplify: 3a + 3b + 2a

If we think of "a" being Apples and "b" being Mangoes, then we have the following situation



We can see that by combining the like objects, the above can be simplified to be 5 Apples and 3 Mangoes, which in Algebra is :

$3a + 3b + 2a = 5a + 3b$

Various types of algebraic expressions are as follows

Monomial : An expression which contains only one term is known as a monomial. Thus, $4x$, $5x^3y$, -1, 100 etc. are all monomials.

Binomial : An expression containing two terms is called a binomial. Thus, $5x - y^3$, $7x + 1$, $z^3 + y$ etc. are all binomials.

Trinomial : An expression containing three terms is called a trinomial. Thus, $5x + y^2 + z$, $x^3 - y^2z + 1$ etc. are trinomials.

Quadrinomial : An expression containing four terms is called a quadrinomial.

Thus, $a^2 + b^2 + c^2 - abc$, $x^3 + y^3 + z^3 + 3xyz$ etc. are all quadrinomials.

Polynomials : A mathematical expression of one or more algebraic terms each of which consist of a constant multiplied by one or more variable raised to a non-negative integral power. Thus, $ax^2 + bx + c$ is polynomial.

Quick Tips

- "Mono" means 'one or single'
- "Bi" means 'two'
- "Tri" means 'three'
- "Quad" means 'four'
- "Poly" means 'many' and "Nomial" means 'terms'

Building Concepts 5

Identify monomials, binomials and trinomials in the following

(i) $-x^2yz$ **(ii)** $7x^3y^3z^3 - 2z^2 + 3xy$

(iii) $4y^2 - 5y$ **(iv)** $7x - 5x$

Explanation

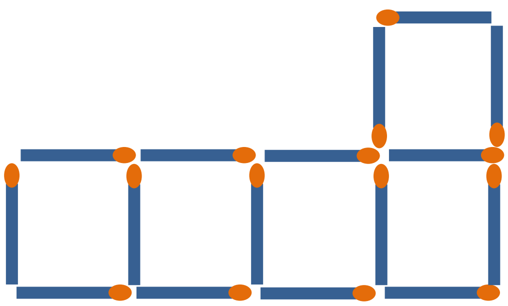
(i) The expression $-x^2yz$ has only one term. It is called monomial.

(ii) The expression $7x^3y^3z^3 - 2z^2 + 3xy$ has three terms. It is called trinomial.

(iii) The expression $4y^2 - 5y$ has two terms, so it is called binomial.

(iv) The expression $7x - 5x = 2x$ has only one term. It is called monomial.

Algebra as generalisation



The most fruitful use of algebra is in generalisation. When we say that the perimeter of a rectangle is equal to $2(\ell + b)$ or the area of a rectangular surface is $\ell \times b$; where ℓ stands for the length of the rectangle and b for its breadth, we have a general formula which can be used to determine the perimeter and area of any rectangular surface. This generalisation is true for any rectangle irrespective of the numerical values of length and breadth.

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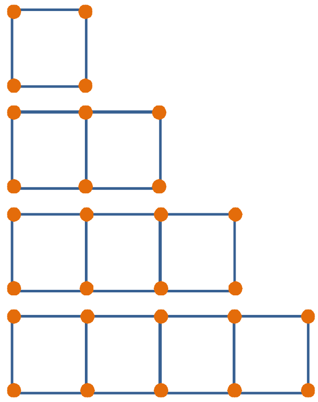
Look at the number series : 1, 5, 9, 13, 17, 21

(i) $1 = 4 \times 1 - 3$ (ii) $5 = 4 \times 2 - 3$ (iii) $9 = 4 \times 3 - 3$

(iv) $13 = 4 \times 4 - 3$ (v) $17 = 4 \times 5 - 3$ (vi) $21 = 4 \times 6 - 3$



A careful observation will give us the general formula for the terms as $4x - 3$. Once you get this general formula, we can find any term in the number series.

E.g., 7th term $= 4 \times 7 - 3 = 28 - 3 = 25$

10th term $= 4 \times 10 - 3 = 40 - 3 = 37$

100th term $= 4 \times 100 - 3 = 400 - 3 = 397$

Look at the patterns of squares made with the matchsticks given.

Number of squares	1	2	3	4	5	n
Number of matchsticks	4	7	10	13	16	3n+1

If there are n squares, then the number of matchsticks will be $3n + 1$. This means that n can take any value and accordingly the number of matchsticks will be $3n + 1$. This is a generalised statement.

Check your Answers 1

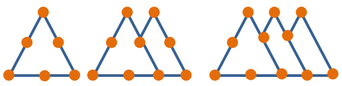
1. $(x + y + z) - y$ **2.** $(x - 1)$

SPOT LIGHT

A variable is always written after the constant in multiplication of a variable with a constant.

Check your Concepts 2

Observe the following pattern of overlapping triangles made with matchsticks and complete the table. Also, write the general rule that gives the number of matchsticks.



Number of triangles	1	2	3	4	5	10	....	n
Number of matchsticks	6	10	14	....	....	....	....	....

Algebraic equations

An equation is a mathematical statement equating two quantities. The expression on either side of the equal sign (=) are called members of the equation.

E.g., $2x + 9 = 11$, $5x - 3 = 7$, $2y + 9 = 17$

In an equation, the terms on the left-hand side (L.H.S.) of the equal sign are equal to the terms on the right-hand side (R.H.S.) of the equal sign.

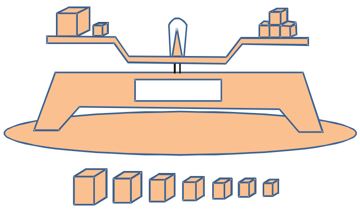
An equation is like a weighing balance having equal weights on each pan of the balance.

$7x - 12 = 3x + 40$.

L.H.S. = R.H.S.

Solving one equation using the principle of balance

To solve an equation is to determine the value(s) of the variable (or unknown) that will make the equation true.



The value(s) of the unknown that balances an equation is called the root(s) of the equation or solution of the equation. If the root is substituted for the unknown quantity in the equation and the equation balances, then the root is said to satisfy the equation. Once this is done, the answer (root) is said to have been verified.

(i) Using addition : If the same or equal quantity is added to both sides of an equation, the equation remains true.

(ii) Using subtraction : If the same number or equal quantity is subtracted on both sides of an equation, the equation remains true.

(iii) Using multiplication : If the same number or equal quantity is multiplied on both sides of an equation, the equation remains true.

(iv) Using division : If the same number or quantity divides both sides of an equation (except by zero), the equation holds true.

Quick Tips

✸ An equation is satisfied only for a definite value of the variable.

✸ There is difference between algebraic expression and algebraic equation.
E.g., algebraic expression: $5x + 7$, algebraic equation: $5x + 7 = 14$

Building Concepts 6

Solve :

(i) $x - 9 = 8$ **(ii)** $x + 9 = 8$ **(iii)** $\frac{x}{3} = 4$ **(iv)** $2x = 8$

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Explanation

(i) $x - 9 = 8$

$x - 9 + 9 = 8 + 9$ (9 added to both sides)

$x = 17$

(ii) $x + 9 = 8$

$x + 9 - 9 = 8 - 9$ (9 subtracted to both sides)

$x = -1$

(iii) $\frac{x}{3} = 4$

$\frac{x}{3} \times 3 = 4 \times 3$ (3 is multiplied to both sides)

$x = 12$

(iv) $2x = 8$

$\frac{2x}{2} = \frac{8}{2}$ (Both sides are divided by 2)

$x = 4$

Numerical Ability 1

Solve :

(i) $3x - 4 = 5$ **(ii) $\frac{2x}{3} = 12$** **(iii) $\frac{3x}{2} - 5 = 4$**

Solution

(i) $3x - 4 = 5$

$3x - 4 + 4 = 5 + 4$ [4 is added to both sides]

$3x = 9$

$\frac{3x}{3} = \frac{9}{3}$ [Both sides are divided by 3]

$x = 3$

(ii) $\frac{2x}{3} = 12$

$\frac{2x}{3} \times 3 = 12 \times 3$ [Both sides are multiplied by 3]

$2x = 36$

$\frac{2x}{2} = \frac{36}{2}$ [Both sides are divided by 2]

$x = 18$

In an equation, the value of the quantity which is not known is referred to as the unknown number or the unknown value.

SPOT LIGHT

(iii) $\frac{3x}{2} - 5 = 4$

$\frac{3x}{2} - 5 + 5 = 4 + 5$ [Both sides are added by 5]

$\frac{3x}{2} = 9$

$\frac{3x}{2} \times 2 = 9 \times 2$ [Both sides are multiplied by 2]

$3x = 18$

$\frac{3x}{3} = \frac{18}{3}$ [Both sides are divided by 3]

$x = 6$

Be Alert !

✸ Don't forget to check the answer by putting obtained value of variable in equations.

Check your Answers 2

Number of triangles	4	5	10	....	n
Number of matchsticks	18	22	42	....	4n+2

Check your Concepts 3

Pick the solution from the values given in the bracket next to the equation. Show that the other values do not satisfy the equation.

(i) $\frac{q}{2} = 7$; (7, 2, 10, 14)

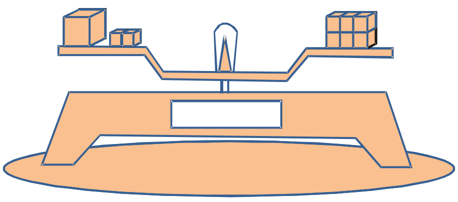
(ii) $3x + 16 = 4$; (3, – 3, 4, – 4)

Systematic method

A much better method of solving an equation is the systematic method as the trial-and-error method could take a lot of time.

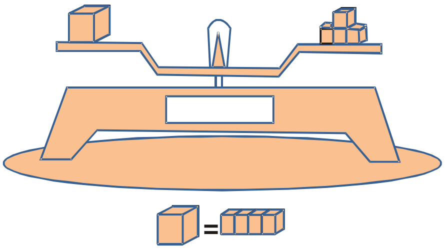
Let us consider the example of the balance.

One big cube and two small cubes are balanced by 6 small cubes.



Look at the second balance figure. How many small cubes will balance the big cube?

If we remove two small cubes from the left-hand side of the first balance, a big cube will be left there. To balance this, we have to remove two small cubes from the right-hand side. Then there will be four small cubes balancing the big cube.



Numerical Ability 2

If 5 is subtracted from three times a number, the result is 16. Find the number.

Solution

Let the required number be x.

According to the sum,

$3x - 5 = 16$

$3x = 16 + 5$

$3x = 21 \Rightarrow x = \dfrac{21}{3} = 7$

Hence, the required number is 7.

Numerical Ability 3

A number is increased by 26 and the new number obtained is divided by 3. If the resulting number is 18. Find the original number.

Solution

Let the original number = x

According to the sum,

$(x + 26) \div 3 = 18$

$\dfrac{x+26}{3} = 18 \Rightarrow x + 26 = 18 \times 3 \Rightarrow x + 26 = 54 \Rightarrow x = 54 - 26 = 28$

Hence, the original number is 28.

Numerical Ability 4

When three consecutive natural numbers are added, the sum is 66, find the numbers.

Solution

Let first natural number be x

Then second number = x + 1, and third number = x + 2

According to the sum, x + x + 1 + x + 2 = 66

3x + 3 = 66

$3x = 66 - 3 = 63 \Rightarrow 3x = 63 \Rightarrow x = \frac{63}{3} = 21$

$\therefore$ First natural number = 21

Second number = 21 + 1 = 22 and third number = 22 + 1 = 23

Hence, numbers are 21, 22 and 23.

Check your Answers 3

(i) q = 14 (ii) x = – 4

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Memory Map

1. **Algebra**

2. **Variable**

A **variable's value** can change over time; it is not fixed. A square's length can be any number, it's a variable.

3. **Constant**

A quantity having a fixed numerical value is called a constant.

4. **Coefficient**

Any factor of a term of an algebraic expression is called the coefficient of the remaining factor of the term.

5. **Solution of an equation**

The value of a variable in an equation that satisfies the equation is known as the solution.

6. **Polynomial**

A polynomial is defined as an expression of one or more than one algebraic terms.
For example: $6x^4 + 2x^3 + 3$

7. **Degree of a polynomial**

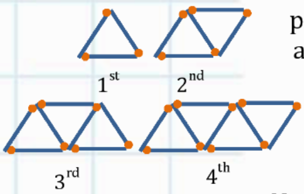
The degree of a polynomial is the highest power of the variable in a polynomial expression.
Degree of $(x^5 + x^3 + x^2 + x^1 + x^0) = 5$

8. **Framing of an algebraic expression**

An expression made up of variables (or literals) and constants (or numerals) connected by signs of arithmetic operations is called an algebraic expression.

9. **Algebra as generalisation**

Matchstick patterns are used for producing letters and other forms.



$\therefore$ Number of matchsticks used in n^{th} pattern = $2n + 1$

10. **Addition and subtraction of algebraic expressions**

For addition and subtraction of two or more algebraic expressions, we first collect like terms and then find the sum and difference of coefficients of these terms.

11. **Algebraic equations**

An equation is a mathematical statement equating two quantities. An equation has two sides, LHS and RHS, with the equal (=) sign in the middle.
e.g., $3x + 9 = 15$

12. **Solving one equation using the principle of balance**

To solve an equation is to determine the value(s) of the variable (or unknown) that will make the equation true.

13. **Systematic method**

A much better method of solving an equation is the systematic method as the trial and error method could take a lot of time.
One big cube and two small cubes are balanced by 6 small cubes.