Comp Econ Homework 8

Date 1st April

Due Date: 7th April

Place all of the following in a single Jupyter notebook with the usual naming convention `firstname_lastname.ipynb`. Remember that you can use LaTeX expressions in the notebook. When you've finished your notebook, remember to restart the kernel (see the "Kernel" menu) and run again, just to check that everything runs with a fresh start. Also, go to the "File" menu and try to download your notebook as a PDF. If it doesn't work, some part of your notebook is broken. (Probably a LaTeX expression.) This makes it harder for me to print out and assess your work. Please try to fix it.

Exercise 1. Let S be a discrete set. Show that if $\mathscr{P}(S)$ is compact as a subset of $(\ell_1(S), \|\cdot\|)$, then S must be finite.

Exercise 2. Consider the determinisitic Markov chain $X_{t+1} = X_t + 1$ on $\mathbb{Z}$. The corresponding stochastic kernel is $p(x, y) = \mathbb{1}\{x = y - 1\}$. Show that no stationary distribution exists.

The next exercise relates to (s, S) inventory dynamics. I'll use the symbols (q, Q) because the symbol S is already taken. Throughout, q and Q are integers with $0 \leq q \leq Q$.

Consider a single firm with time t inventory X_t. At the start of time t, if $X_t \leq q$, then the firm orders $Q - X_t$. Otherwise it orders nothing. At the end of t demand D_{t+1} is observed, and the firm meets this demand up to (but not exceeding) its current stock level. Any remaining inventory is carried over to the next period. This gives the next period inventory X_{t+1}.

Demand $\{D_t\}$ is assumed to be IID with

$$\mathbb{P}\{D_t = d\} = (1/2)^{d+1} \quad \text{for} \quad d = 0, 1, \ldots$$

The associated stochastic kernel p is defined (at least implicitly) by the expression

$$p(x, y) = \mathbb{P}\{X_{t+1} = y \mid X_t = x\}$$

Exercise 3. Argue that this stochastic kernel is globally stable for every q and Q. (It's fine if you can come up with a reasonable probabilistic argument that doesn't necessarily involve a lot of calculations.)

Exercise 4. Let ψ^* be the stationary distribution corresponding to given (q, Q). Using `quantecon`'s `MarkovChain` class, compute the stationary distribution when $q = 2$ and $Q = 5$. You should get

$$\psi_2^* = (0.0625, 0.0625, 0.125, 0.25, 0.25, 0.25).$$

(By now hopefully you know that you should write a function that does most of the work.)

Exercise 5. Now compute the stationary distribution iteratively, using the ideas from the lecture. Terminate when successive iterates are closer together than some tolerance in the sense of ℓ_1 distance. Compare your result with the result from the previous exercise.

Exercise 6. Finally, set $Q = 20$ and plot the stationary distribution for $q = 2, 5, 10, 15$. Give some interpretation for what you see.