HW6

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1.A square matrix M is said to be positive definite if for any vector x, $x^{\intercal}\ Mx > 0$. Consider $x^{\intercal}A^{\intercal}Ax$. Since A has linearly independent columns Ax is invertible and also $x^{\intercal}A^{\intercal} = Ax$. Furthermore, $A^{\intercal}A$ is $k \times k$. Also, since $x^{\intercal}A^{\intercal} = Ax$, we have that $x^{\intercal}A^{\intercal}Ax = \|Ax\| = \|\mathrm{x}^{\intercal}A^{\intercal}\| \geq 0$.But the norm is only when x is the zero vector. Thus we may conclude that $A^{\intercal}A$ is positive definite. All the eigenvalues of a positive definite matrix are positive(strictly), so the determinant is non-zero, and so finally we conclude that the matrix is invertible.

2.Let $P = X(X'X)^{-1}X'$ and suppose k=n. Then, both X and X' are invertivle and $P = X(X'X)^{-1}X' = X'^{-1}X'X(X'X)^{-1}X'XX^{-1} = X'^{-1}(X'X)(X'X)^{-1}(X'X)X^{-1} = X'^{-1}(X'X)X^{-1} = X'^{-1}X'XX^{-1} = I$. Geometrically, when we take projection we are trying to find a $b \in R^k$ such that the the distance to Y is minimized. Y however already in the span onto which are projecting, so the projection matrix is then clearly the identity.

3. First we note that if S=1, the vector 1 is an orthogonal basis. It also obvious that $\|1_{R^N}\|^2 = N$. Thus $1_{R^N}/N$ is an orthonormal basis. Thus the projection of Y onto the this orthonormal basis is given by $\frac{\langle\ Y, 1_{R^N}\rangle\ \vec{1}}{N} = \frac{\sum_{k=1}^{N} y_k \vec{1}}{N} = \bar{y}\vec{1}$.

4. Since Py is a projection onto S, $\hat{u}\ \perp\ Py$ implies that $\hat{u} \perp S$. Then $\langle\ \hat{u}, s\rangle = 0$, where s $\in S$(since Py is linear combination of vectors in S). Clearly the vector of all ones is in S, since it is the linear combination of itself and weights of 0 for all the other basis vectors. But then the inner product of $\hat{u}$ with the vector of ones is the sum of the entries of $\hat{u}$ and must be 0.

5. Suppose $x \in S \cap S^{\perp}$. Then since $x \in S$ and $x \in S^{\perp}$,$\langle\ x, x\rangle = 0$. But this implies that $\|x\|^2 = 0$, which means x can only be the 0 vector.