AMS 595 Abby Bindelglass
Project 2: How Long is the Coast of Britain 10/5/25

In this assignment, the goal was to estimate the circumference of a Mandelbrot Set fractal with function approximation. The arc length of a visible part of the Mandelbrot Set's upper boundary was estimated by sampling the boundary, fitting a polynomial to the curved part, and integrating the fitted curve.

I used the function fractal(c) to iterate from z=0, $z \to z^2 + c$. The point was considered outside if $|z|>2$ and up to 100 iterations were tested if not. The function returned either the iteration where the point escaped, or 0 if there was no escape in the 100 iterations.

For x in [-2,1], 1000 equally spaced samples were generated. For each x, an indicator along the vertical line was defined: +1 if the fractal escaped, or -1 if there was no escape within 100 iterations. The boundary was bracketed with s=0 as the start, inside, and e=1.5 as the end, outside. The bisection was applied until the interval width was below the tolerance, tol=10^{-6}, with a 60-step safety cap. The (x,y) pairs coming from this approximated the upper boundary.

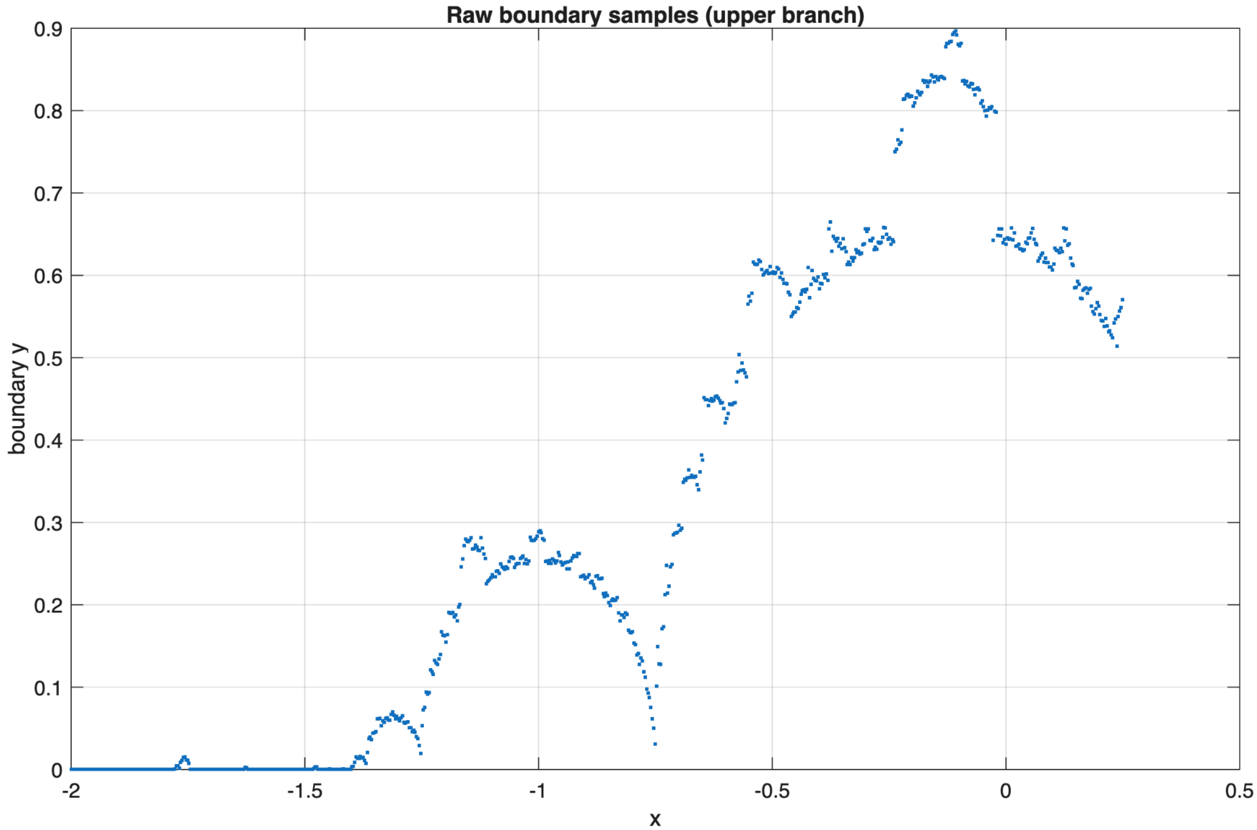
I plotted the raw boundary in order to determine what bounds to use for the fit. The left and right ends of the plot looked flat, so those portions were not included in the x range, in order to improve the fit. I used the bounds [-1.34,0.25]. Then, I fitted a 15-degree polynomial, f(x), to the trimmed points using polyfit, and visualized polyval(p,x) and the data on the same plot to verify the quality of the fit.

The boundary length over the chosen fit interval, [s,e], is $L = \int_s^e \sqrt{1 + (f'(x))^2 dx}$. I used polyder(p) to form f'(x), and evaluated the integral with MATLAB's integral function, with the bounds being restricted to the fitted data's span.

Using 1000 samples and trims [-1.34,0.25], the 15-degree polynomial seemed to closely follow the plot of the curved boundary. The estimated arc length over [s,e]=[-1.33934,0.249249] was around 2.42118429. These were the printed s and e bounds, because the uniform x-grid nodes do not land exactly on the chosen trim values.

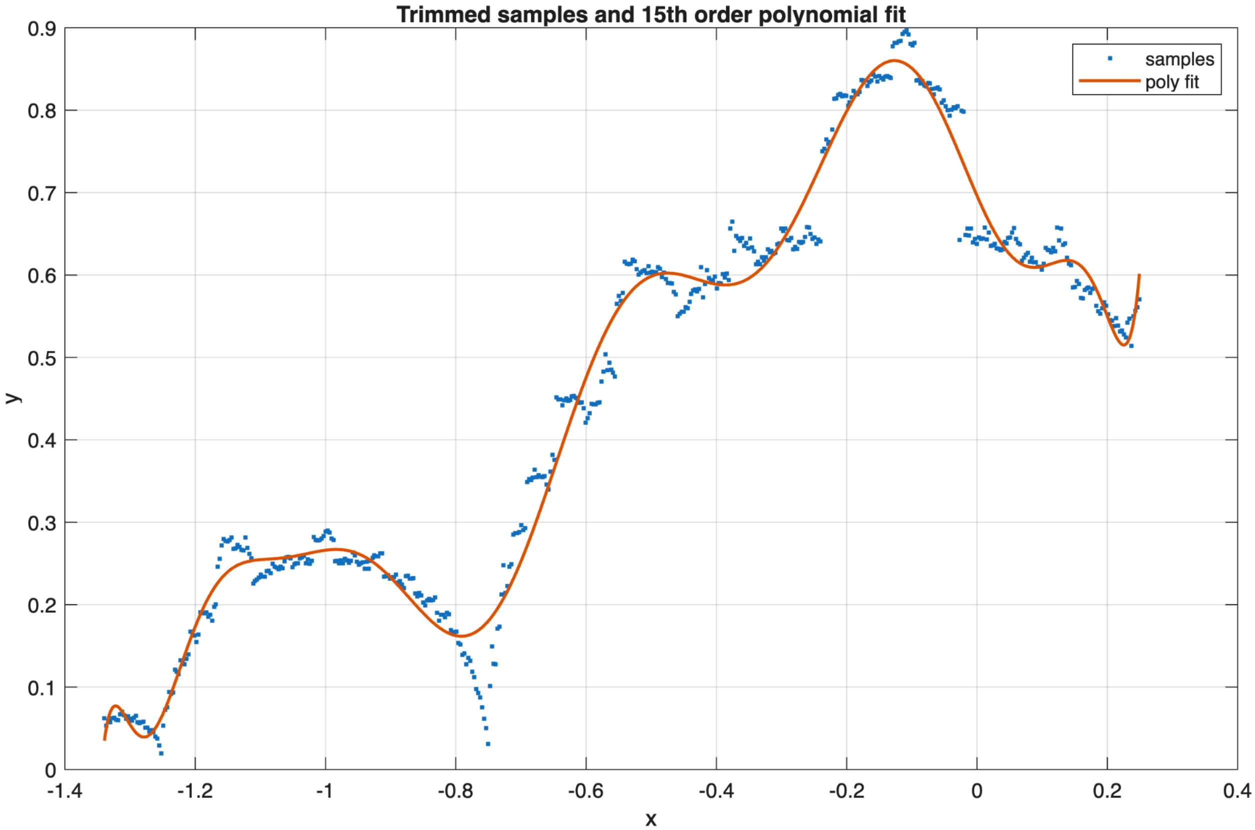
The length of the top edge of the Mandelbrot boundary was estimated on the curved part. The flat ends were trimmed for a better fit with the degree-15 polynomial. The resulting length depends on the trim and number of x-samples used. This exhibits the "coastline paradox" idea that more detail can change the measured length. With more samples, the estimate would likely be more accurate.

Figure 1



Caption: Scatter plot of (x,y) boundary points found by bisection for x range [-2,1]. The boundary goes flat to the left and right of the fractal, so these tails are excluded from the fitting process.

Figure 2



Caption: Shows the data restricted to the range [-1.34,0.25] with the fitted 15-degree polynomial overlaid. The fitted curve is used to compute the arc length.