EIE4003
Final Project Presentation

Massive MIMO Performance With Imperfect Channel Reciprocity and Channel Estimation Error

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Content

1. Introduction / Background

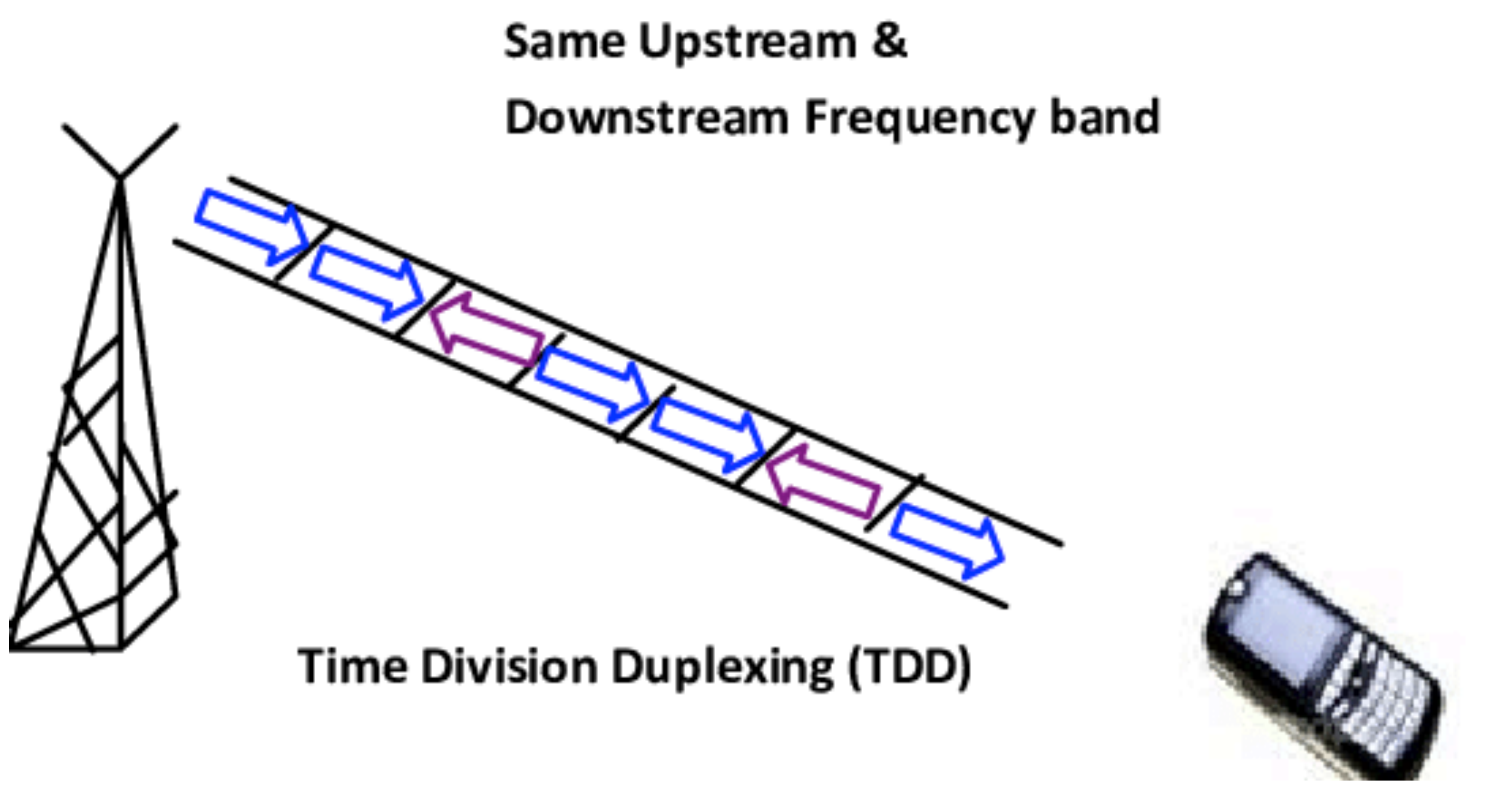
1. Introduction/Background

- *Massive MIMO Performance With Imperfect Channel Reciprocity and Channel Estimation Error*
- **Core concepts:**
  1. TDD and Channel Reciprocity
  2. Channel Estimation
  3. Massive MIMO

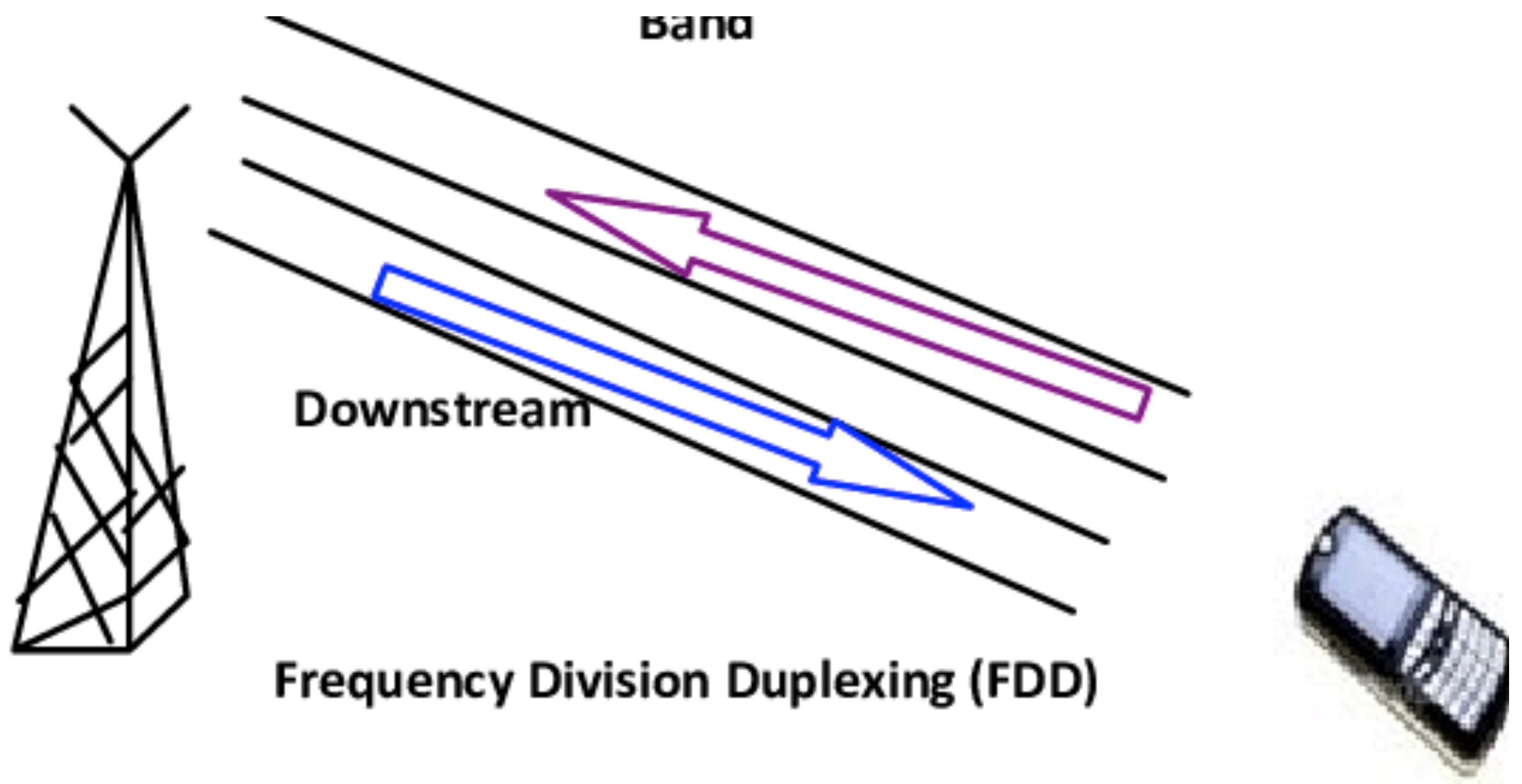
1. Introduction/Background

1. TDD and Channel Reciprocity

- TDD - Time Division Duplexing



- FDD - Frequency Division Duplexing



1. Introduction/Background

1. TDD and Channel Reciprocity

- Channel Reciprocity $H_r = H_t^T$:
  - TDD: time difference < channel coherence time ✓ smaller pilot overhead
  - FDD: freq. difference < channel coherence bandwidth
- Imperfect Channel Reciprocity —> RF chain mismatch

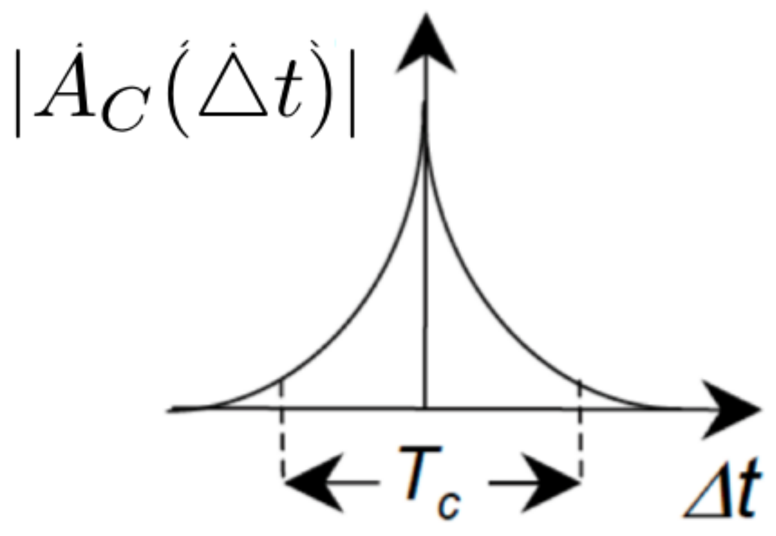


Fig.1 Channel Coherence Time

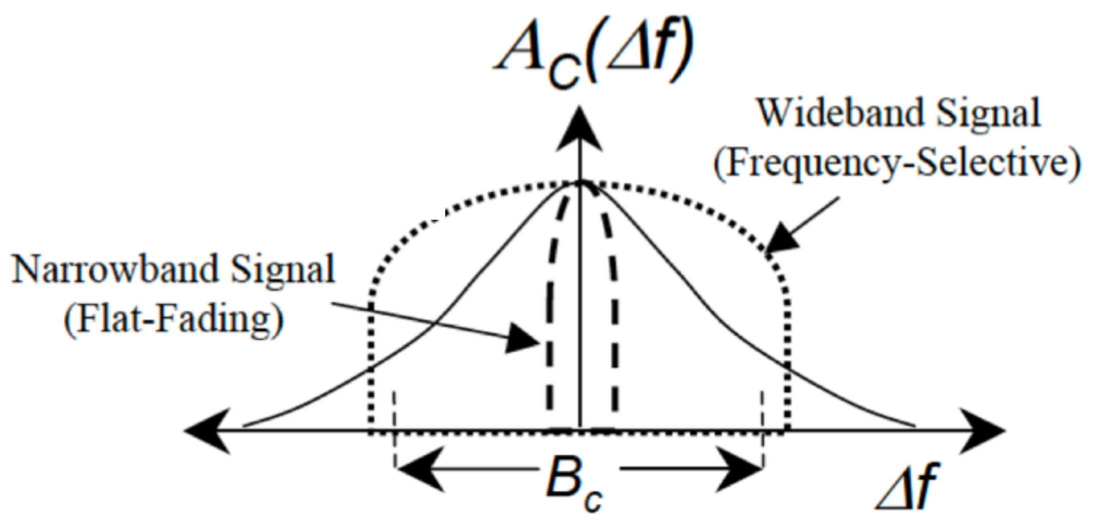


Fig.2 Channel Coherence Bandwidth

1. Introduction/Background

2. Channel Estimation

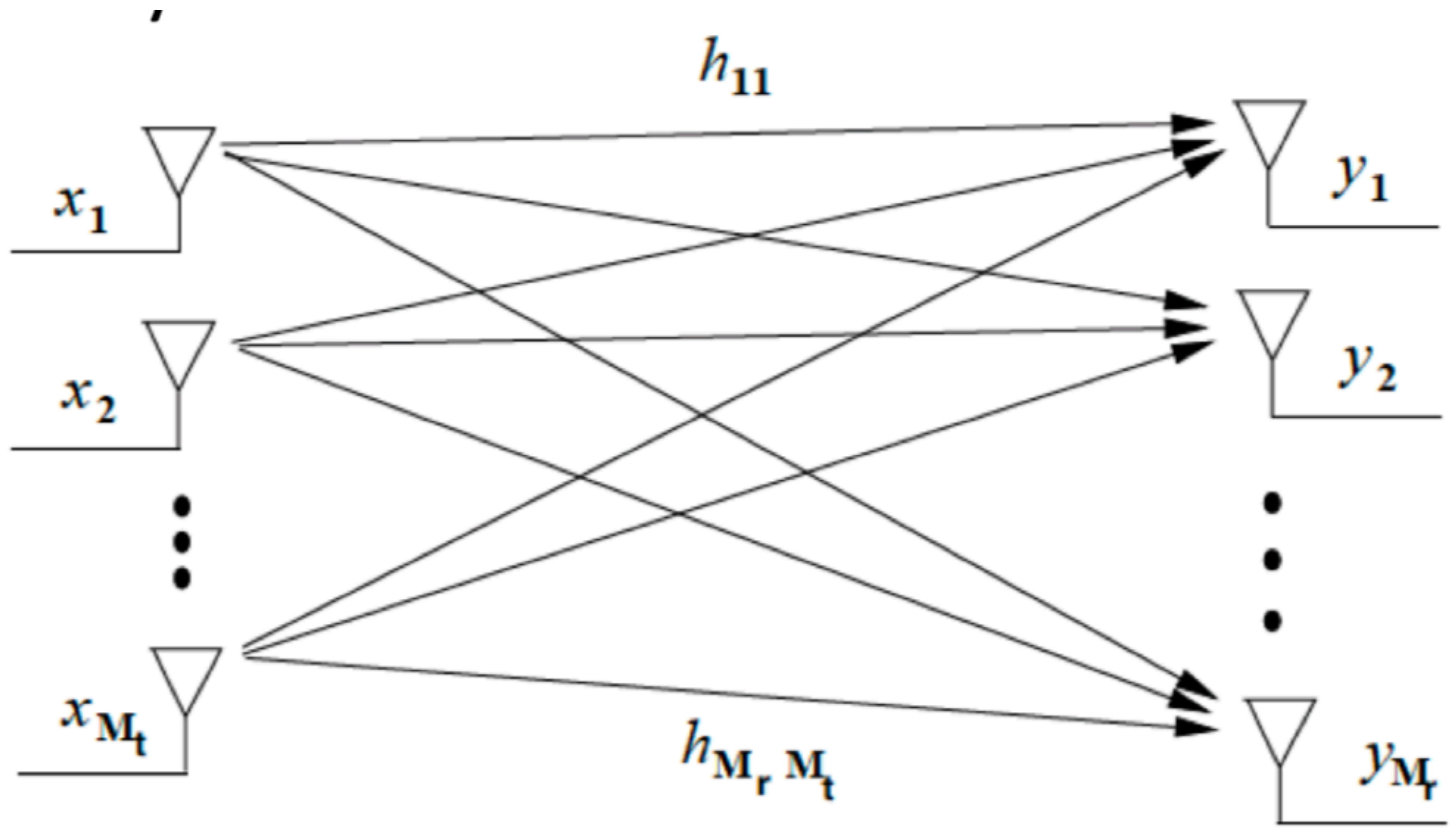
- Utilize Channel Reciprocity $H_u = H_d^T$
- But with Estimation Error

$$\text{Base Station} \quad \underset{H_u = H_d^T}{\overset{H_d}{\rightleftarrows}} \quad \text{User Equipment}$$

1. Introduction/Background

3. Massive MIMO

- Natural extend of MIMO
- $M \to \infty, N \to \infty$
- Millimeter Wave (mmWave)
- Ultra-dense Network
- Channel Hardening -> linear precoding



1. Introduction/Background

- **Core concepts:**

1. TDD and Channel Reciprocity — — — Imperfect Channel Reciprocity
2. Channel Estimation — — — Channel Estimation Error
3. Massive MIMO — — — Asymptotic SINR Analysis

2. Theory Section

A. System Model
B. SINR for MRT and ZF
C. Asymptotic SINR Analysis

2A. Theory - System Model

- Imperfect Channel Reciprocity
- Channel Estimation Error

    - **Basic Setting:**
        - 1 BS with M antennas
        - K user with single antenna
        - M >> k
        - Antenna coupling negligible
        - Within channel coherence time

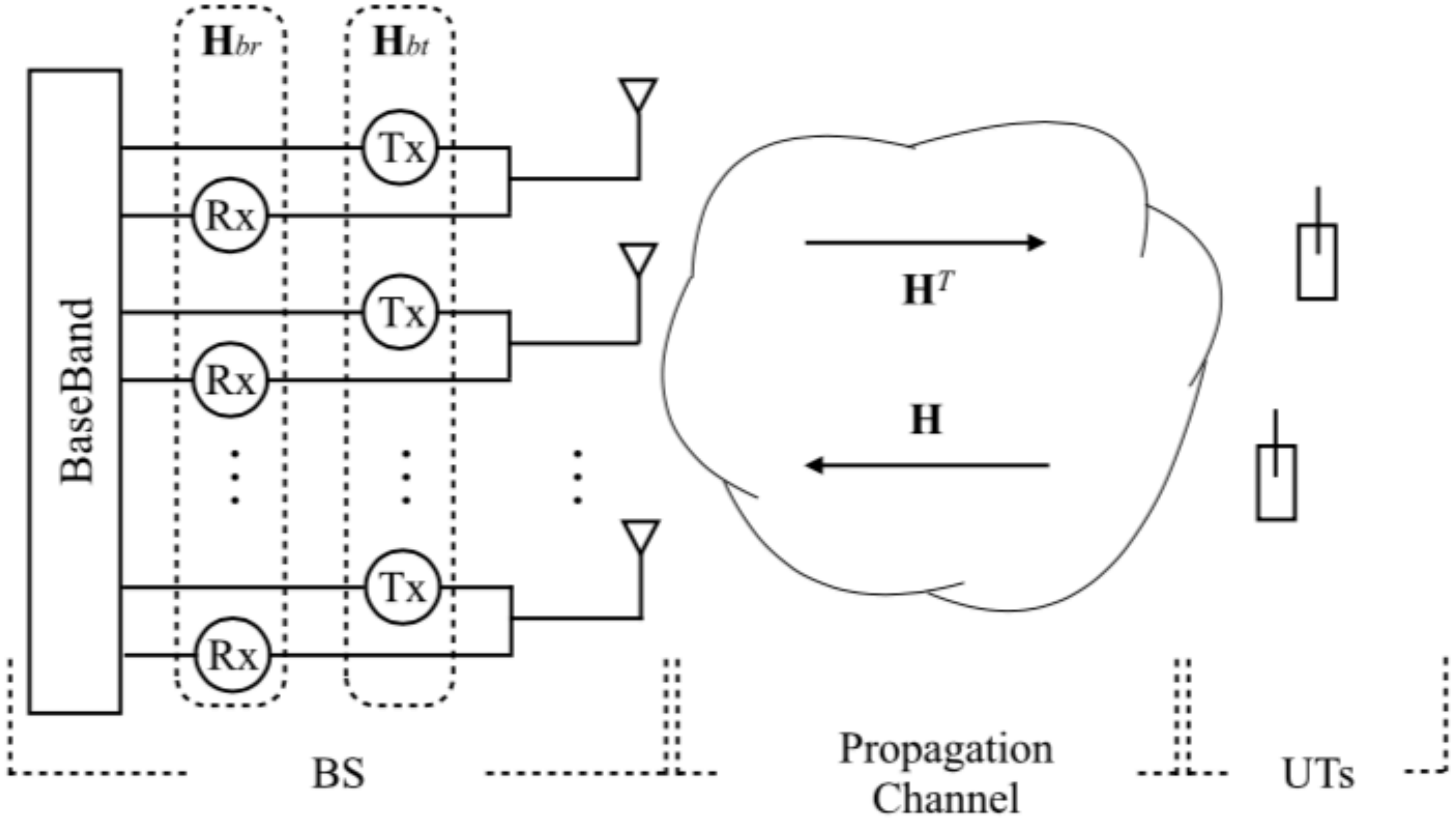


Fig. 1. A massive MU-MIMO TDD system.

2A. Theory - System Model

- **1. Imperfect Channel Reciprocity (e.g. in RX)**
  - Effective response of RF: $H_{br} = diag(h_{br,1}, \cdots, h_{br,i}, \cdots, h_{br,M})$, $\mathbf{H} \in \mathbb{C}^{M\times K}$
  - Amplitude/Phase: $h_{br,i} = A_{br,i} \exp\left(j\varphi_{br,i}\right)$
  - Independent Variable: (truncated gaussian distribution)

$$
\begin{aligned}
A_{bt,i} &\sim \mathcal{N}_{\mathrm{T}}(\alpha_{bt,0}, \sigma_{bt}^2),\ A_{bt,i} \in [a_t, b_t],\\
\varphi_{bt,i} &\sim \mathcal{N}_{\mathrm{T}}(\theta_{bt,0}, \sigma_{\varphi_t}^2),\ \varphi_{bt,i} \in [\theta_{t,1}, \theta_{t,2}],\\
A_{br,i} &\sim \mathcal{N}_{\mathrm{T}}(\alpha_{br,0}, \sigma_{br}^2),\ A_{br,i} \in [a_r, b_r],\\
\varphi_{br,i} &\sim \mathcal{N}_{\mathrm{T}}(\theta_{br,0}, \sigma_{\varphi_r}^2),\ \varphi_{br,i} \in [\theta_{r,1}, \theta_{r,2}],
\end{aligned}
$$

2A. Theory - System Model

- **1. Imperfect Channel Reciprocity (e.g. in RX)**

  - RF Mismatch: $$\mathbf{E} \triangleq \mathbf{H}_{bt}\mathbf{H}_{br}^{-1} = \mathrm{diag}\left(\frac{h_{bt,1}}{h_{br,1}}, \cdots, \frac{h_{bt,i}}{h_{br,i}}, \cdots, \frac{h_{bt,M}}{h_{br,M}}\right)$$

  - RF Mismatch is a multiplicative error, which will be shown later.

2A. Theory - System Model

- **2. Channel Estimation Error**
  - Additive Random Error:
    - $\hat{\mathbf{H}}_u = \sqrt{1-\tau^2}\mathbf{H}_{br}\mathbf{H} + \tau\mathbf{V}$

$\hat{\mathbf{H}}_u$: estimation of uplink channel

$\mathbf{H_u}$: actual uplink channel response

$\mathbf{H}$: propagation channel

$\mathbf{V}$: channel estimation error

  - Estimation Variance Parameter $\tau \in [0,1]$:
    - $\tau = 0$, perfect estimation
    - $\tau = 1$, only estimation error, uncorrelated with real channel response

2A. Theory - System Model

- **Entire Channel Model:**
  - Estimated downlink by Channel Reciprocity: $\hat{\mathbf{H}}_d = \hat{\mathbf{H}}_u^T = \sqrt{1-\tau^2}\mathbf{H}^T\mathbf{H}_{br} + \tau\mathbf{V}^T$
  - Actual downlink channel response: $\mathbf{H}_d = \mathbf{H}^T\mathbf{H}_{bt}$
  - Comparison: $\mathbf{H}_d = \dfrac{1}{\sqrt{1-\tau^2}}\left(\hat{\mathbf{H}}_d - \tau\mathbf{V}^T\right) \cdot \underbrace{\mathbf{H}_{br}^{-1}\mathbf{H}_{bt}}_{\text{reciprocity errors}}$
- **Receive Signal:**

$$\mathbf{y} = \sqrt{\rho_d}\lambda\mathbf{H}_d\mathbf{W}\mathbf{s} + \mathbf{n} = \sqrt{\rho_d}\lambda\mathbf{H}^T\mathbf{H}_{bt}\mathbf{W}\mathbf{s} + \mathbf{n}$$

2B. Theory - SINR for MRT and ZF

- **MRT** (maximum ratio transmission):
  - a match filter approach to maximize SNR:

$$\mathbf{W}_{\mathrm{mrt}} = \hat{\mathbf{H}}_d^H = \sqrt{1-\tau^2}\mathbf{H}_{br}^* \mathbf{H}^* + \tau \mathbf{V}^*$$

- **ZF** (zero-forcing):
  - Cancel the interference:

$$\mathbf{W}_{\mathrm{zf}} = \hat{\mathbf{H}}_d^H \left(\hat{\mathbf{H}}_d \hat{\mathbf{H}}_d^H \right)^{-1}$$

2B. Theory - SINR for MRT and ZF

- **SINR** (Signal to Interference and Noise Ratio):

$$y_k = \underbrace{\sqrt{\rho_d}\lambda \mathbf{h}_k^T \mathbf{H}_{bt} \mathbf{w}_k s_k}_{\text{Desired Signal}} + \underbrace{\sqrt{\rho_d}\lambda \sum_{i=1, i\neq k}^{K} \mathbf{h}_k^T \mathbf{H}_{bt} \mathbf{w}_i s_i}_{\text{Inter-user Interference}} + \underbrace{n_k}_{\text{Noise}} .$$

- And it can be approximated by:

$$\mathrm{SINR}_k = \mathbb{E}\left\{\frac{P_s}{P_I + \sigma_k^2}\right\} \approx \mathbb{E}\left\{P_s\right\} \mathbb{E}\left\{\frac{1}{P_I + \sigma_k^2}\right\}$$

2C. Theory - Asymptotic SINR Analysis

- **Asymptotic SINR:** (Channel Reciprocity Error Only)

$$\lim_{\substack{M \to \infty \\ K \gg 1}} \mathrm{SINR}_{k,\mathrm{mrt}} = \boxed{\frac{M}{K}} A_I \qquad \lim_{\substack{M \to \infty \\ M \gg K \gg 1,}} \mathrm{SINR}_{k,\mathrm{zf}} = \boxed{\frac{M}{K}} \left(\frac{1}{A_I^{-1} - 1} \right)$$

  - **Upper Bound** due to inter-user interference (ISI): MRT = ZF
  - **Ceiling Effect** due to the reciprocity error (A_I)

2C. Theory - Asymptotic SINR Analysis

- **Asymptotic SINR:** (Compound Effect of two Errors)

$$\lim_{\substack{M \to \infty, \\ K \gg 1}} \mathrm{SINR}_{k,\mathrm{mrt}} = \boxed{\frac{M}{K}} \left(\frac{\rho_d \tilde{B}_I}{\rho_d + A_t^{-1}} \right) \qquad \lim_{\substack{M \to \infty \\ K \gg 1}} \mathrm{SINR}_{k,\mathrm{zf}} = \boxed{\frac{M-K}{K}} \left(\frac{\rho_d \tilde{B}_I}{\rho_d \left(1 - \tilde{B}_I\right) + A_t^{-1}} \right)$$

- **Upper Bound** due to inter-user interference (ISI): <u>MRT > ZF</u>
- **Ceiling Effect** due to the compound error: $\tilde{B}_I \approx \left(1 - \tau^2\right) A_I$
  - Note that also affected by <u>transmit power</u>

3. Simulation Result

A. Channel Reciprocity Error
B. Compound Effect
C. MRT ZF Comparison

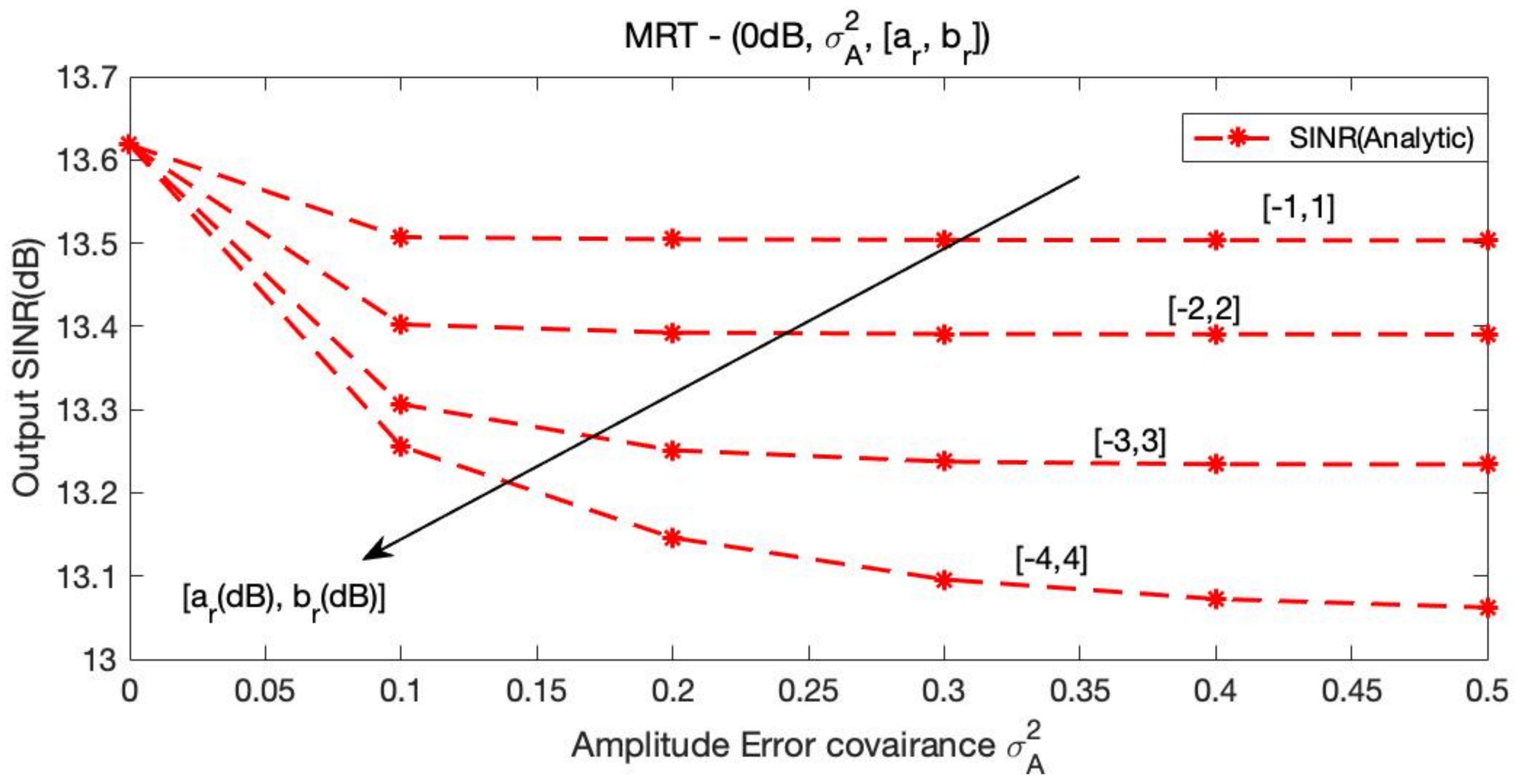
3A. Result - Channel Reciprocity Error

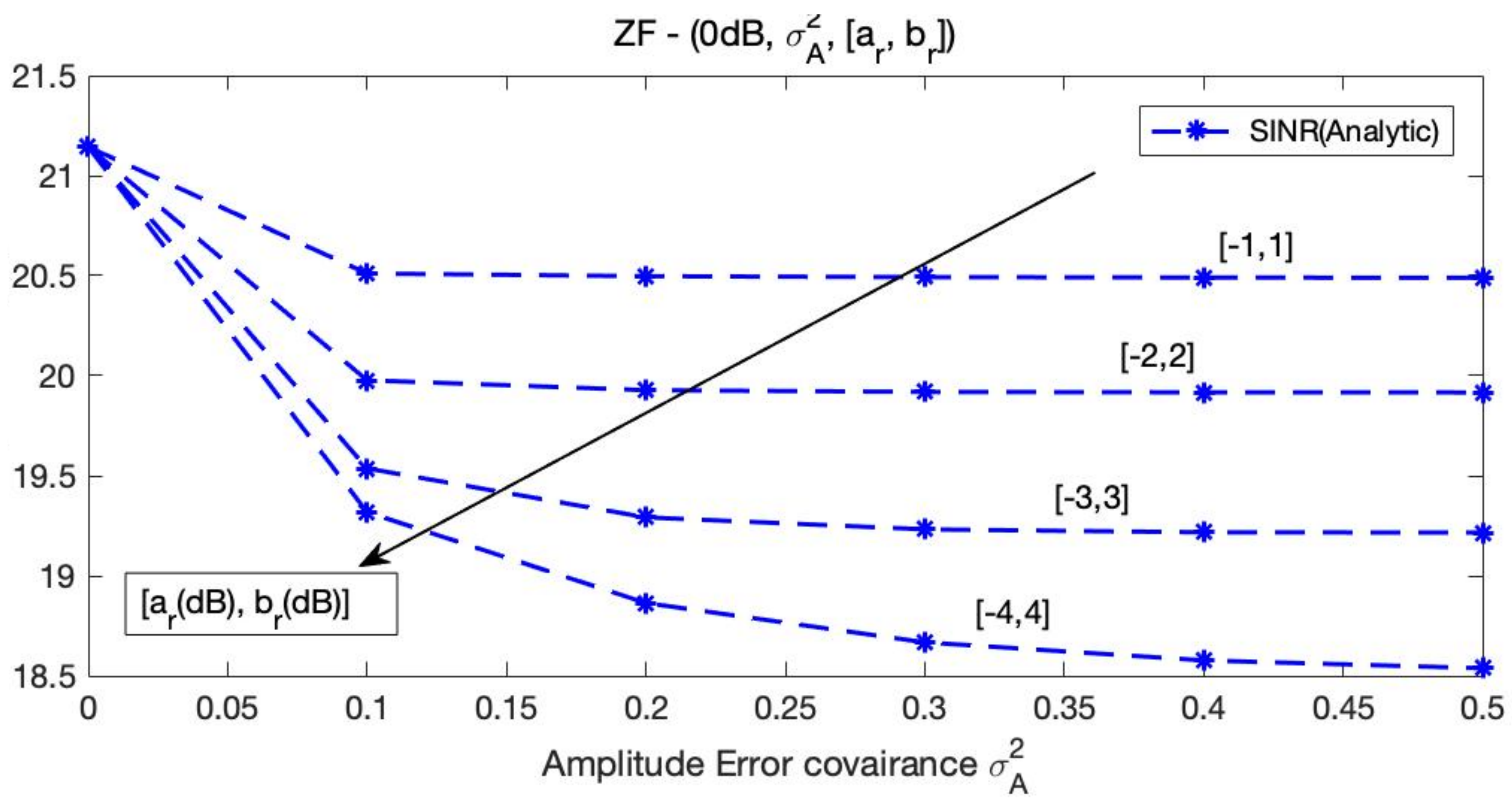
- **Cases:**
    1. Fix phase, amplitude error only
    2. Fix amplitude, phase error only
    3. Both phase and amplitude error
- **Scenarios** in each case**:**
    1. Change truncated range and variance
    2. Compare TX and RX
    3. Compare MRT and ZF

3A. Result - Channel Reciprocity Error

- **Cases 1: Fix phase, amplitude error only**

    1. SINR degrades with the increase of variance and truncate range.

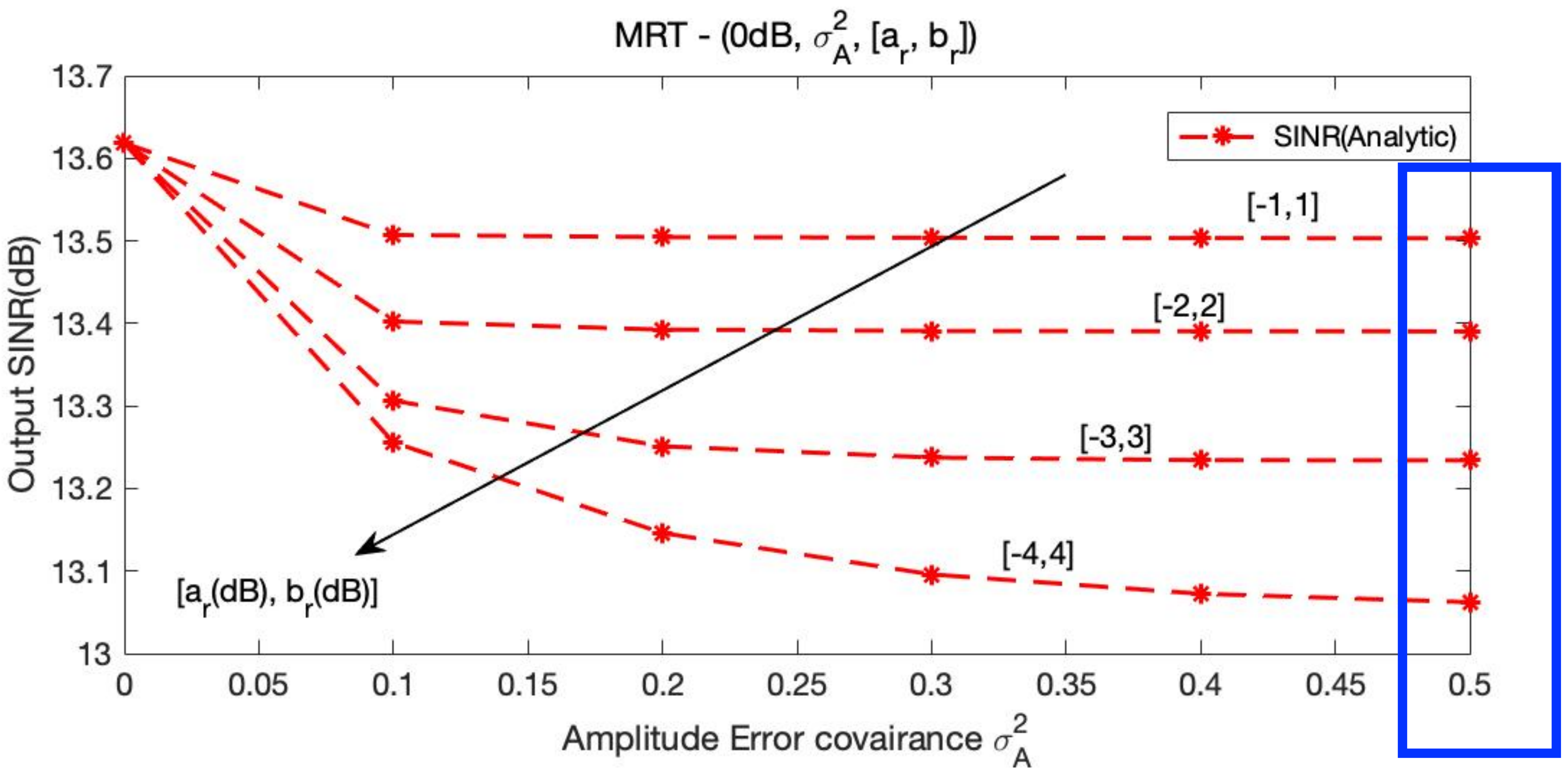


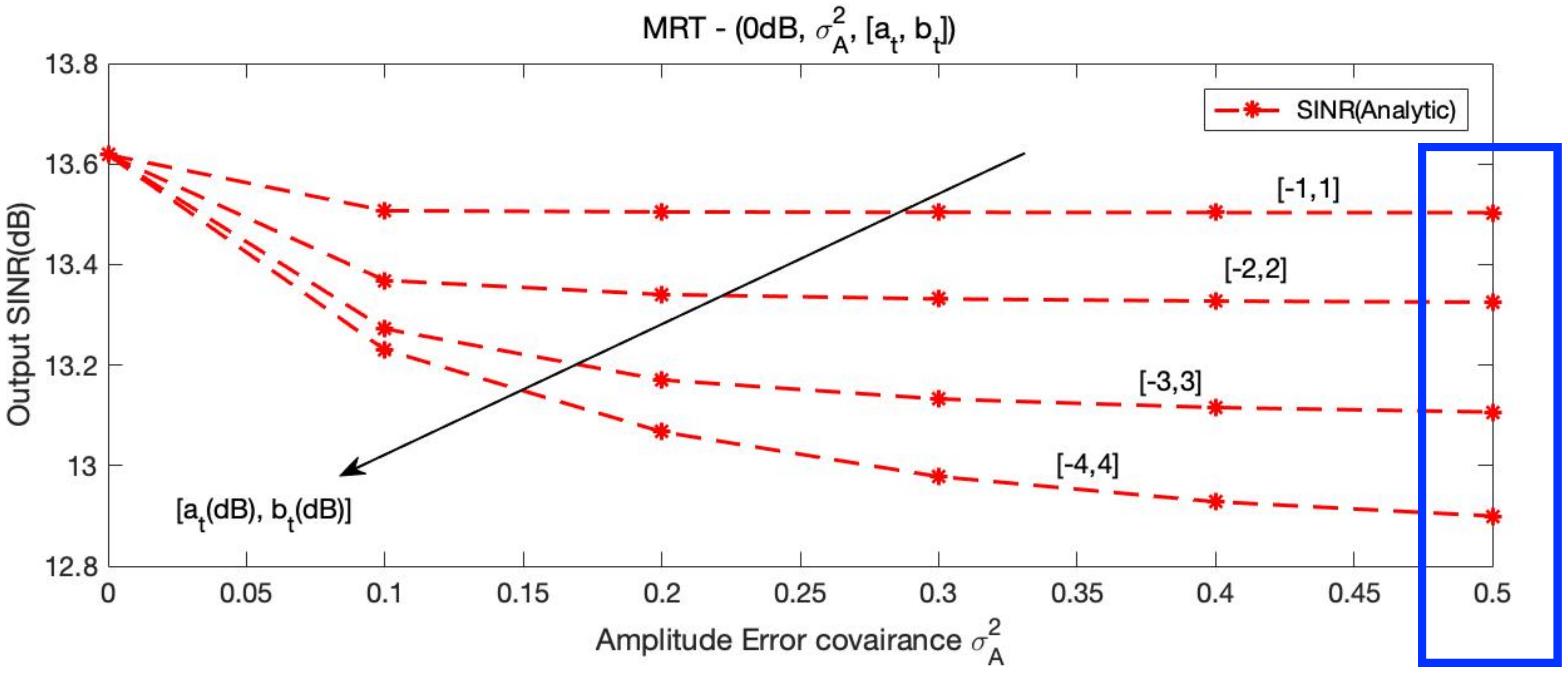


3A. Result - Channel Reciprocity Error

- **Cases 1: Fix phase, amplitude error only**

  2. TX and RX brings slightly different impact to SINR

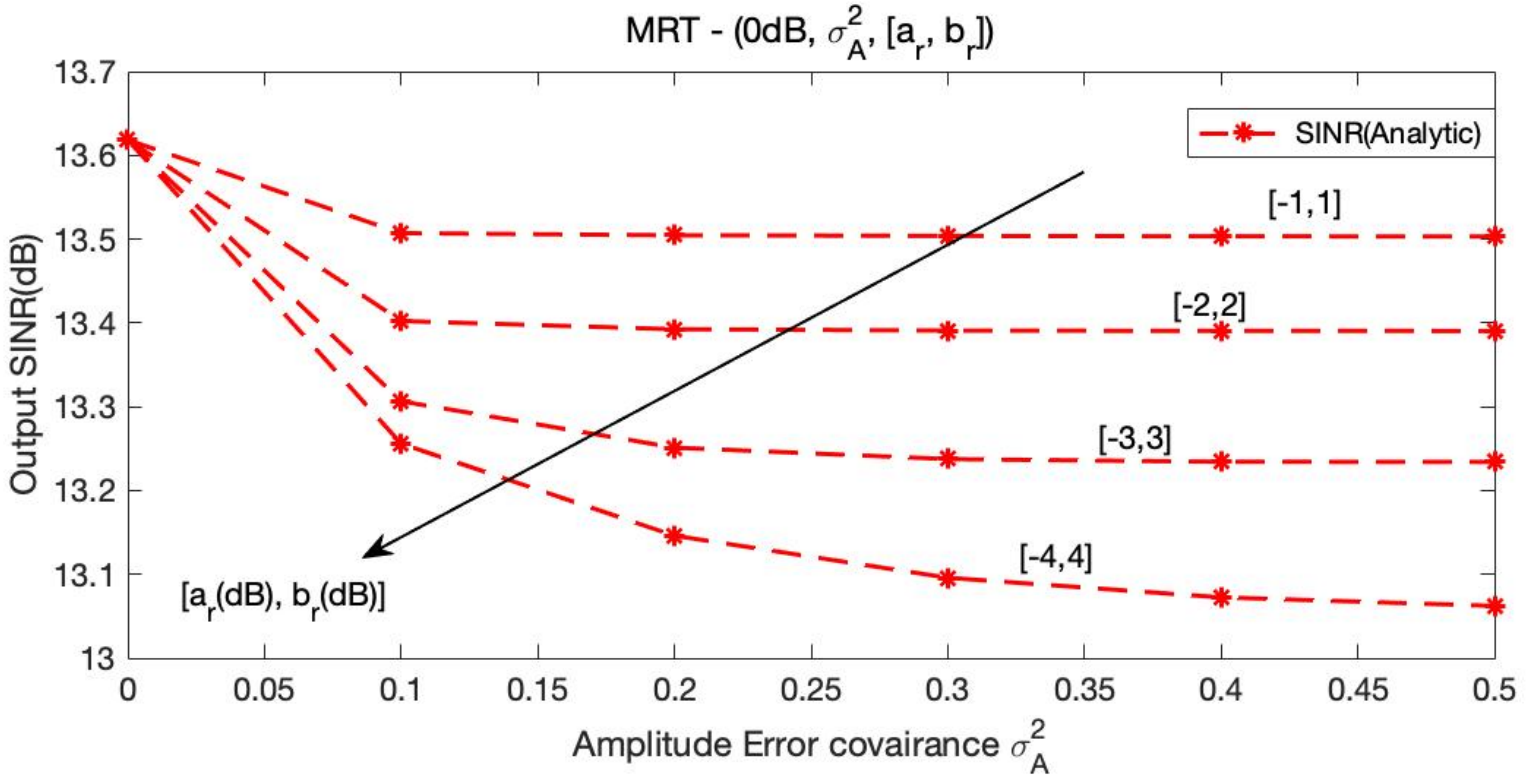


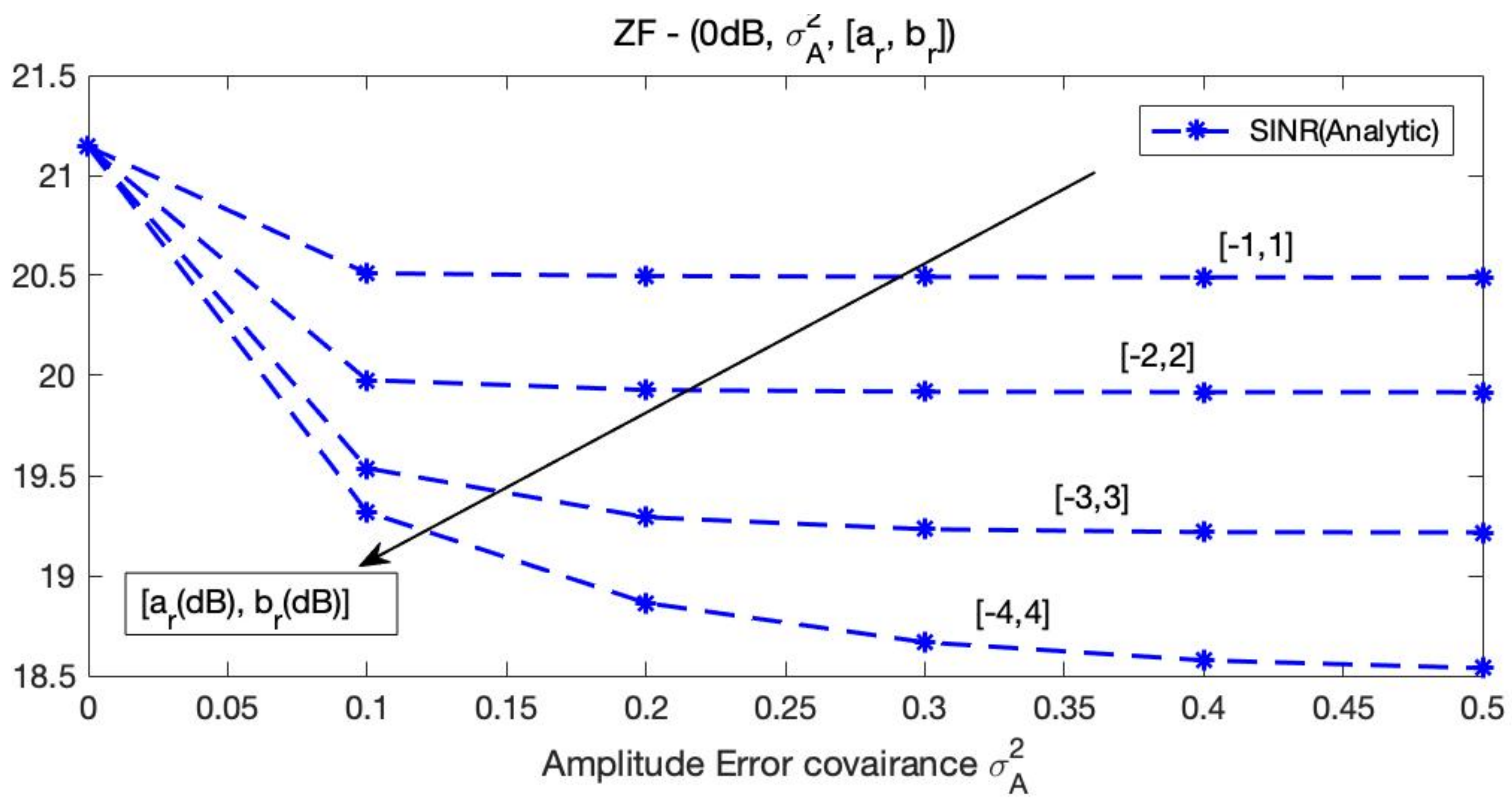


3A. Result - Channel Reciprocity Error

- **Cases 1: Fix phase, amplitude error only**

3. ZF is more sensitive to error than MRT





13.62 - 13.03 = 0.59dB << 21.2 - 18.5 = 2.7dB

3A. Result - Channel Reciprocity Error

- **Cases 2: Fix amplitude, phase error only**

1. Similar to case 1
2. Phase has more impact than amplitude
3. Error mean has less impact than truncate range on SINR

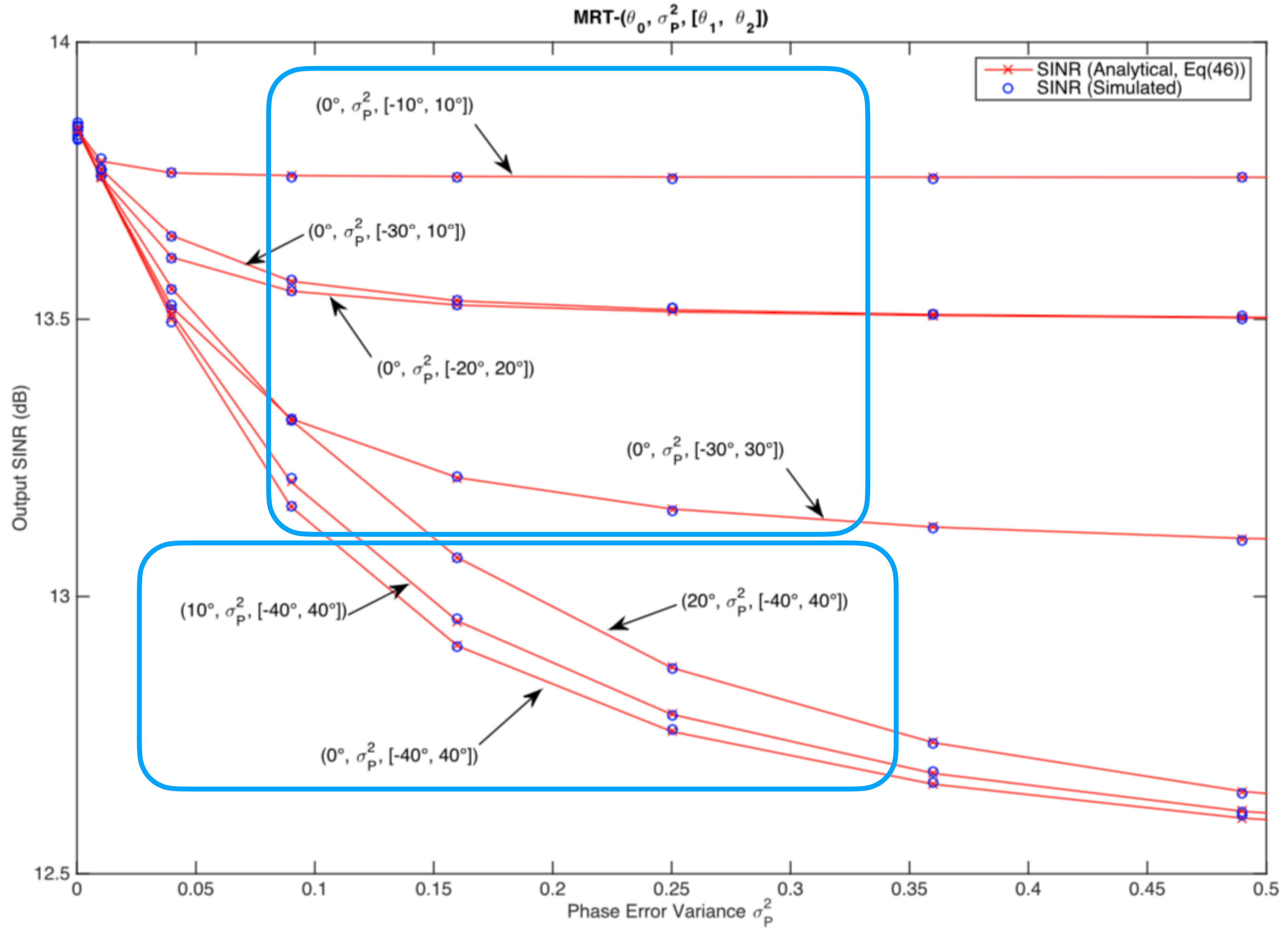
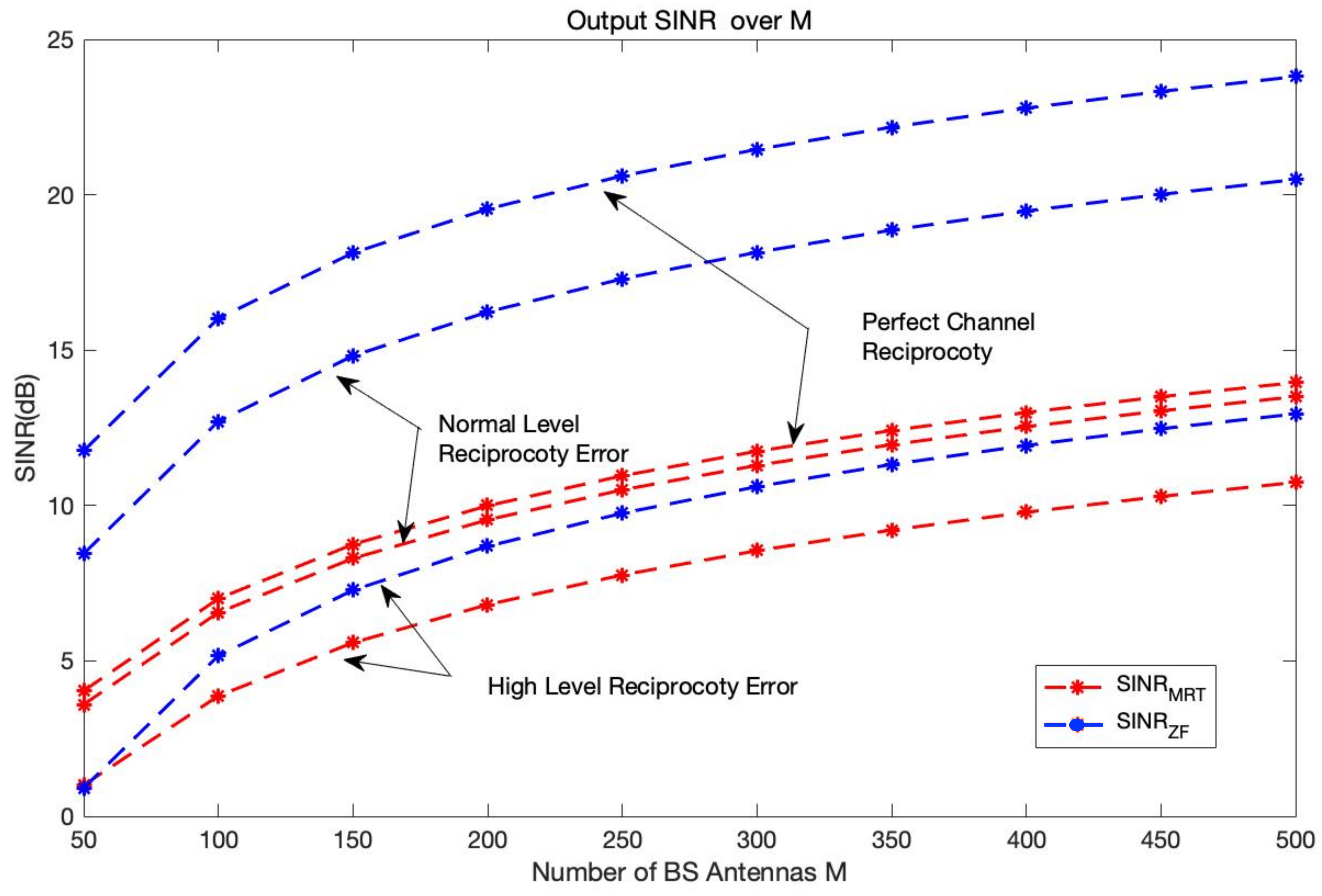
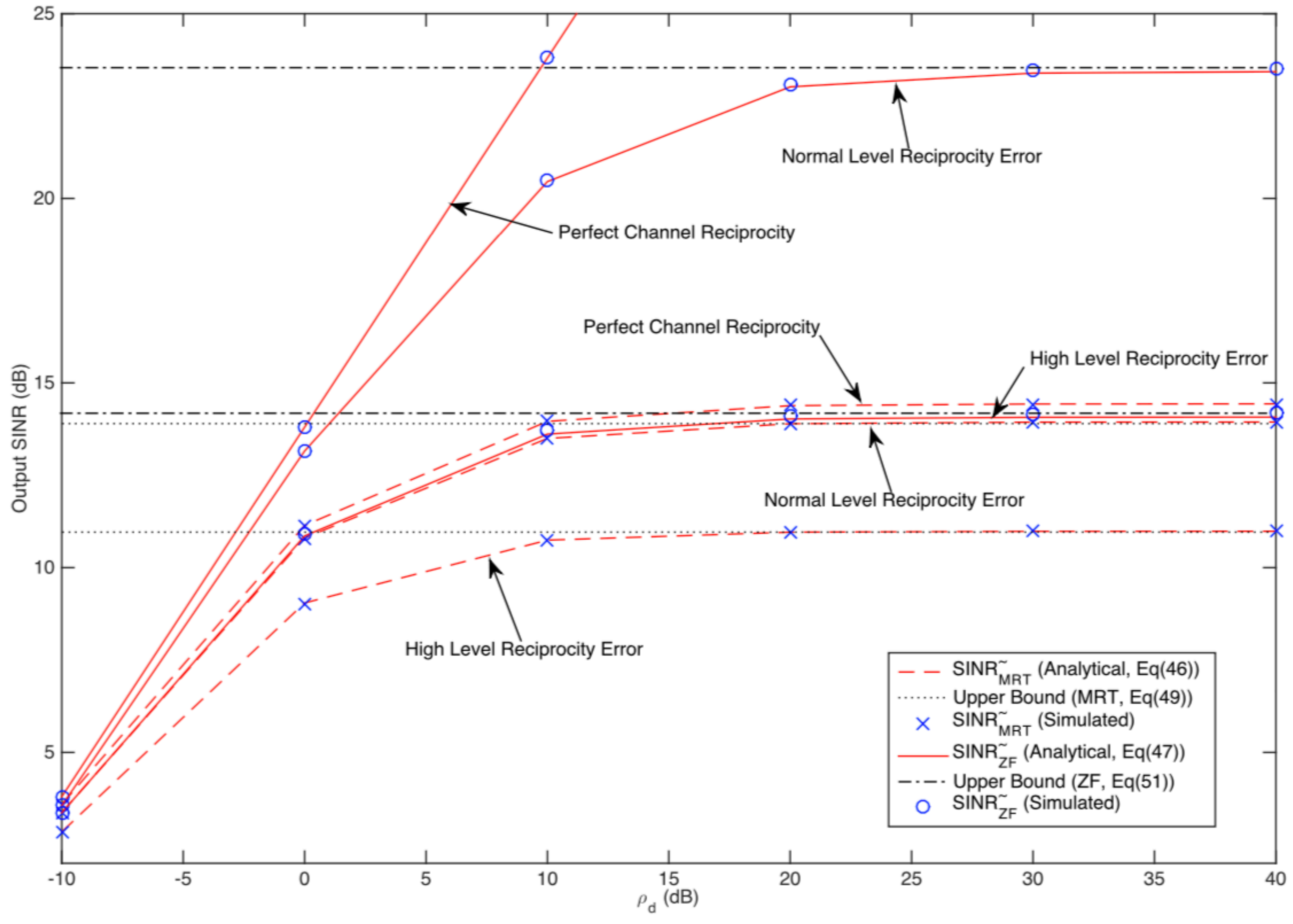


Fig. 4. Output SINR with MRT precoding in the presence of fixed amplitude errors and different combinations of phase errors.

3A. Result - Channel Reciprocity Error

- **Cases 3: both phase/amp error**
  1. SINR increases with the number of antennas in BS
  2. Ceiling Effect (previous introduced): $$\lim_{\substack{M \to \infty \\ K \gg 1}} \mathrm{SINR}_{k,\mathrm{mrt}} = \frac{M}{K} A_I$$
  3. ZF outperforms MRT

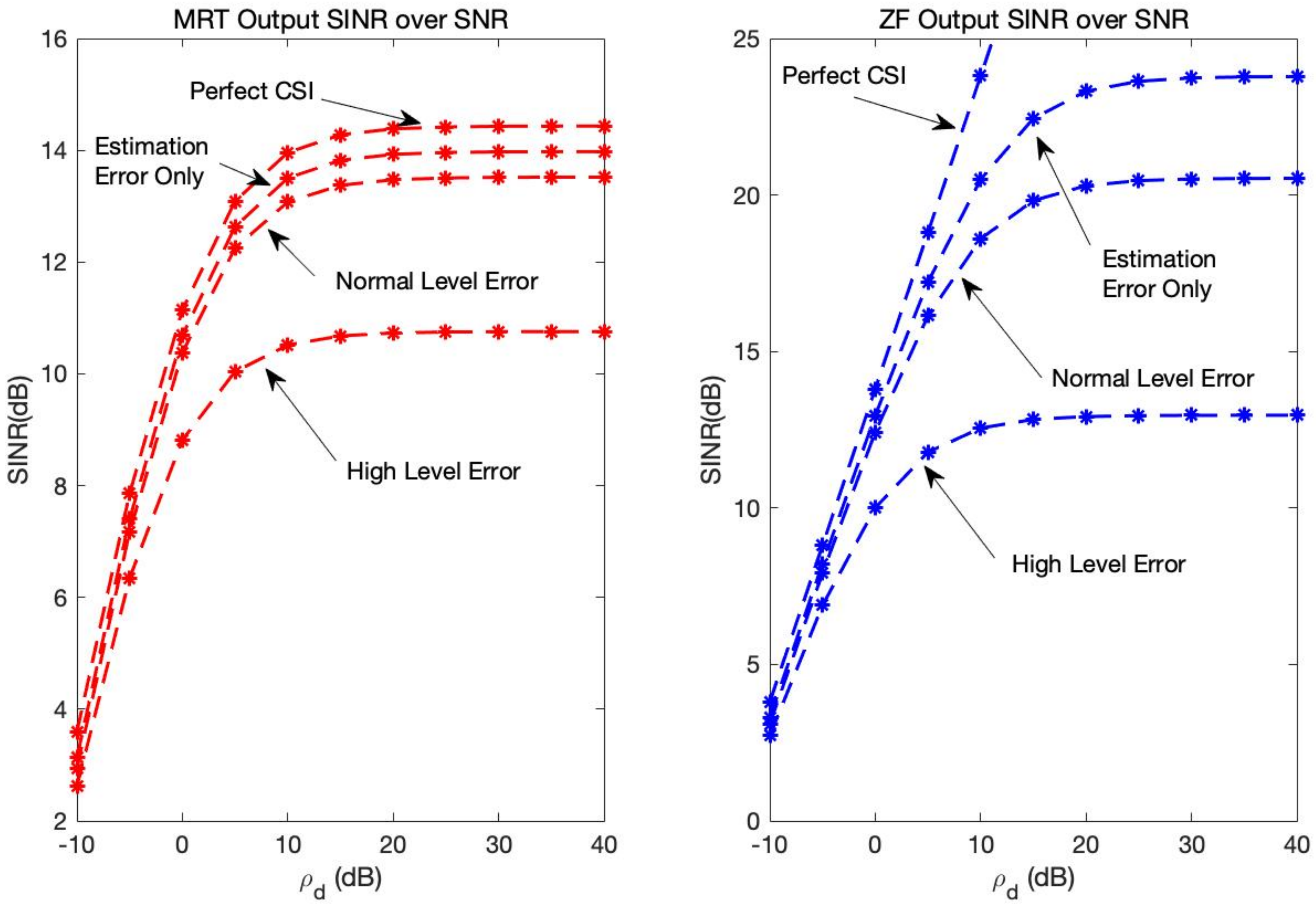




3B. Result - Compound Effect

- **Compound Effect:** channel reciprocity error + estimation error

1. Degradation dur to error
2. Again, ZF outperforms MRT

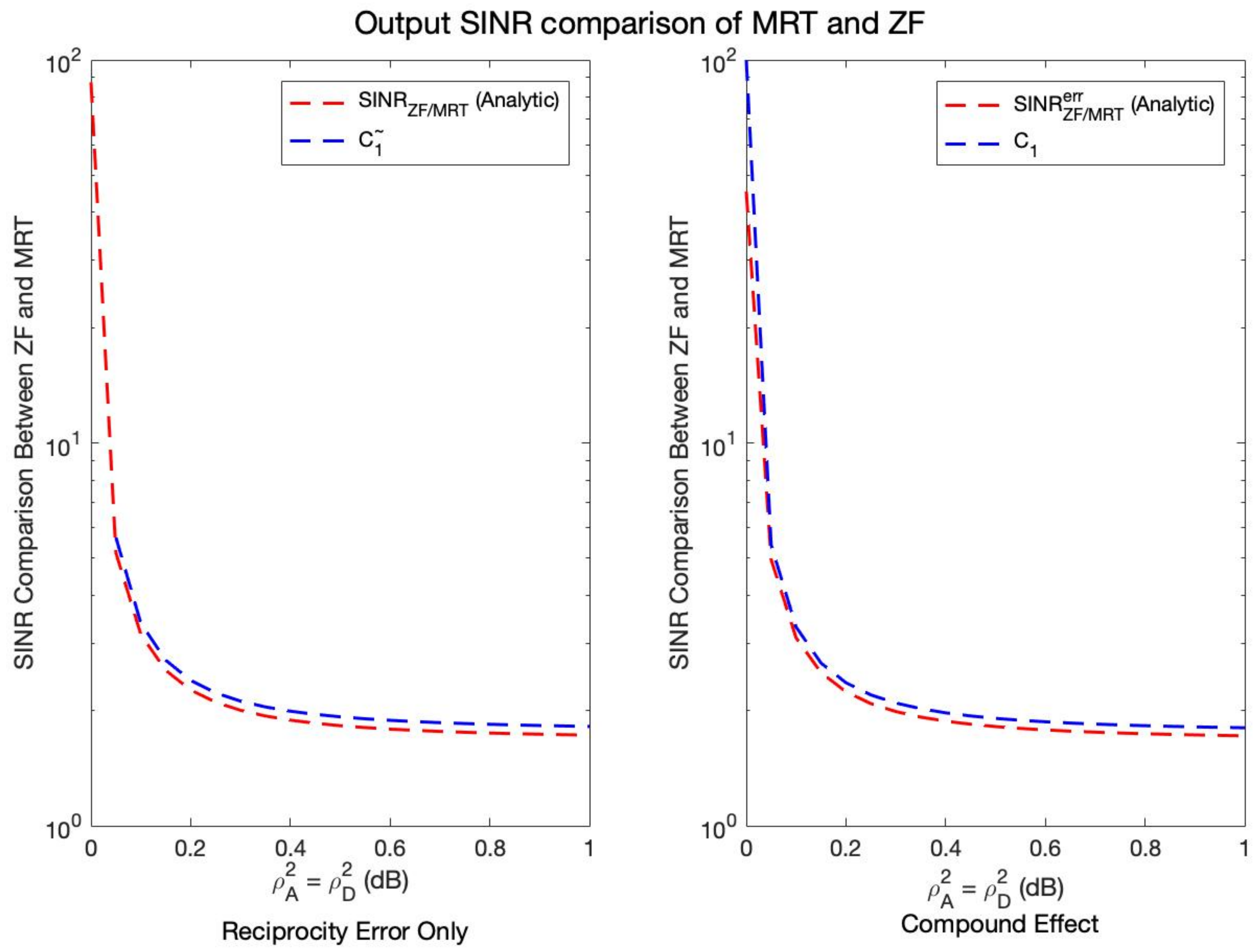


3C. Result - MRT ZF Comparison

- **MRT ZF Comparison:**

ZF generally outperforms MRT in both cases

$$
\begin{cases}
\tilde{C}_I \triangleq \lim\limits_{\substack{\rho_d(1-A_I)\gg 1\\ M\to\infty, M\gg K\gg 1}} \dfrac{\mathrm{SI\tilde{N}R}_{k,\mathrm{Zf}}}{\mathrm{SINR}_{k,\mathrm{mrt}}} = \dfrac{1}{1-A_I} > 1 \\[2ex]
C_I \triangleq \lim\limits_{\substack{M\to\infty,\\ M\gg K\gg 1}} \dfrac{\mathrm{SINR}_{k,\mathrm{zf}}}{\mathrm{SINR}_{k,\mathrm{mrt}}} = \dfrac{1}{1-\tilde{B}_I} \geq 1
\end{cases}
$$



4. Conclusion/Discussion

4. Conclusion

- In conclusion, we **model** the system with
  - Imperfect Channel Reciprocity -> ceiling effect
  - Channel Estimation Error
- And analyze the output **SINR** in
  - Normal MIMO
  - Massive MIMO (asymptotic)
- Finally indicate **factors** that may affect output SINR, and **compare** different precoding scheme for choosing precoding scheme
  -> ZF outperforms MRT, but MRT is more robust to error

4. Discussion

- **Further research:**
  - Consider other linear precoding scheme such as MMSE, or non-linear precoding scheme such as dirty paper coding
  - Consider computational complexity and energy efficiency
  - Possibly extend to FDD
  - $M \approx k$ in ultra dense network?

5. Reference List

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