

Laplace Transform Table

Properties and Rules

Function

Transform

Already know.

$f(t)$	$F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$	(Definition)
$a f(t) + b g(t)$	$a F(s) + b G(s)$	(Linearity)
$e^{zt} f(t)$	$F(s - z)$	(s-shift)

Will learn in a future session.

$f'(t)$	$sF(s) - f(0^-)$	
$f''(t)$	$s^2F(s) - sf(0^-) - f'(0^-)$	
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0^-) - \dots - f^{(n-1)}(0^-)$	
$tf(t)$	$-F'(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	
$u(t - a)f(t - a)$	$e^{-as}F(s)$	(t-translation)
$u(t - a)f(t)$	$e^{-as}\mathcal{L}(f(t + a))$	(t-translation)
$(f * g)(t) = \int_{0^-}^{t^+} f(t - \tau)g(\tau) d\tau$	$F(s)G(s)$	
$\int_{0^-}^{t^+} f(\tau) d\tau$	$\frac{F(s)}{s}$	(integration rule)

Interesting, but not included in this course.

$\frac{f(t)}{t}$	$\int_s^{\infty} F(\sigma) d\sigma$	
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(The function table is on the next page.)

Function Table

<u>Function</u>	<u>Transform</u>	<u>Region of convergence</u>
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Already know.

1	$1/s$	$\text{Re}(s) > 0$
e^{at}	$1/(s - a)$	$\text{Re}(s) > a$
t	$1/s^2$	$\text{Re}(s) > 0$
t^n	$n!/s^{n+1}$	$\text{Re}(s) > 0$
$\cos(\omega t)$	$s/(s^2 + \omega^2)$	$\text{Re}(s) > 0$
$\sin(\omega t)$	$\omega/(s^2 + \omega^2)$	$\text{Re}(s) > 0$
$e^{zt} \cos(\omega t)$	$(s - z)/((s - z)^2 + \omega^2)$	$\text{Re}(s) > \text{Re}(z)$
$e^{zt} \sin(\omega t)$	$\omega/((s - z)^2 + \omega^2)$	$\text{Re}(s) > \text{Re}(z)$
$\delta(t)$	1	all s
$\delta(t - a)$	e^{-as}	all s
$\cosh(kt) = \frac{e^{kt} + e^{-kt}}{2}$	$s/(s^2 - k^2)$	$\text{Re}(s) > k$
$\sinh(kt) = \frac{e^{kt} - e^{-kt}}{2}$	$k/(s^2 - k^2)$	$\text{Re}(s) > k$

Will learn in this session.

$\frac{1}{2\omega^3}(\sin(\omega t) - \omega t \cos(\omega t))$	$\frac{1}{(s^2 + \omega^2)^2}$	$\text{Re}(s) > 0$
$\frac{t}{2\omega} \sin(\omega t)$	$\frac{s}{(s^2 + \omega^2)^2}$	$\text{Re}(s) > 0$
$\frac{1}{2\omega}(\sin(\omega t) + \omega t \cos(\omega t))$	$\frac{s^2}{(s^2 + \omega^2)^2}$	$\text{Re}(s) > 0$

Will learn in a future session.

$u(t - a)$	e^{-as}/s	$\text{Re}(s) > 0$
$t^n e^{at}$	$n!/(s - a)^{n+1}$	$\text{Re}(s) > a$

Interesting, but not included in this course.

$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$\text{Re}(s) > 0$
t^a	$\frac{\Gamma(a + 1)}{s^{a+1}}$	$\text{Re}(s) > 0$

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