

## Problem I

### Expected Value of a Permutation

You have an array of  $N$  integers  $A = [A_1, A_2, \dots, A_N]$ . Summing all integers in  $A$  is boring, so you decided to take it to the next level. You have a permutation  $P$  of 1 to  $N$  generated randomly. Each permutation from 1 to  $N$  has an equal probability to be chosen as  $P$ .

You also want to define arrays  $X_0, X_1, X_2, \dots, X_N$  and an integer  $Y$  as follows:

- $X_0 = A$
- $X_i$  for  $1 \leq i \leq N$  is defined as  $X_{i-1}$  but all integers whose indices are multiples of  $i$  are changed to 0.
- $Y = \text{sum}(X_1) + \text{sum}(X_2) + \dots + \text{sum}(X_N)$ , where  $\text{sum}(X_i)$  is the sum of all integers in the array  $X_i$ .

For example, if  $A = [4, 1, 2, 3, 4]$  and  $P = [3, 2, 4, 1, 5]$ , then:

- $X_0 = [4, 1, 2, 3, 4]$
- $X_1 = [4, 1, 0, 3, 4] \leftarrow P_1 = 3$ , so, the  $3^{\text{rd}}$  element of  $X_1$  is changed to 0.
- $X_2 = [4, 0, 0, 0, 4] \leftarrow P_2 = 2$ , so, the  $2^{\text{nd}}$  and  $4^{\text{th}}$  elements of  $X_2$  are changed to 0.
- $X_3 = [4, 0, 0, 0, 4] \leftarrow P_3 = 4$ , so, the  $4^{\text{th}}$  element of  $X_3$  is changed to 0.
- $X_4 = [0, 0, 0, 0, 0] \leftarrow P_4 = 1$ , so, all elements of  $X_4$  are changed to 0.
- $X_5 = [0, 0, 0, 0, 0] \leftarrow P_5 = 5$ , so, the  $5^{\text{th}}$  element of  $X_5$  is changed to 0.

Therefore,  $Y = 12 + 8 + 8 + 0 + 0 = 28$  in this case.

Since  $P$  is generated randomly, you are wondering the expected value of  $Y$ . Let  $\frac{C}{D}$  be the expected value of  $Y$  where  $C$  and  $D$  are relatively prime non-negative integers. Print the value of  $(C \times D^{-1}) \bmod 1000000007$ . In other words, you must print the value of the unique integer  $K$  ( $0 \leq K < 1000000007$ ) satisfying  $C \equiv DK \pmod{1000000007}$ .

#### Input

Input begins with an integer  $N$  ( $1 \leq N \leq 100000$ ) representing the number of integers in  $A$ . The second line contains  $N$  integers:  $A_i$  ( $0 \leq A_i \leq 10^9$ ) representing the array  $A$ .

#### Output

Output in a line the expected value of  $Y$  using the format specified in the problem description.

**Sample Input**

```
5
4 1 2 3 4
```

**Sample Output**

```
500000020
```

*Explanation for the sample input/output*

There are  $5! = 120$  possible permutations for the value of  $P$ .

- When the value of  $P = [3, 2, 4, 1, 5]$ , the value of  $Y = 28$  as described in the problem statement above.
- When the value of  $P = [2, 1, 3, 4, 5]$ , the value of  $Y = 10$ .
- ...

The sum of  $Y$  for all possible values of  $P$  is 1980. Therefore, the expected value of  $Y$  is  $\frac{1980}{120} = \frac{33}{2}$ . Since  $33 \equiv 2 \times 500000020 \pmod{1000000007}$ , you must print 500000020 for this sample case.