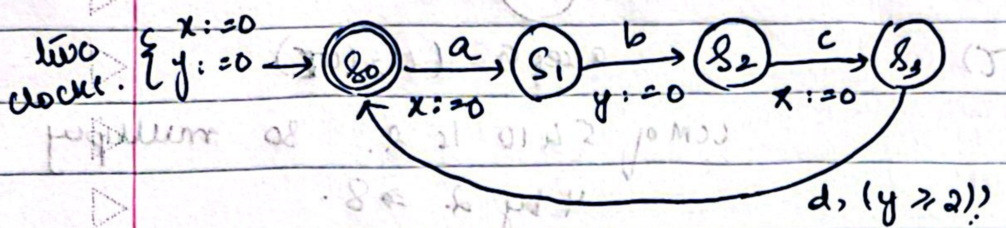
WEEK 7-2:

Example timed Word:



$(a, 2) \rightarrow (b, 2.7) \rightarrow (c, 2.8) \rightarrow (d, 5) \rightarrow \ldots$

a happens at time 2.

Timed Run:

when I traverse the edge a, x gets set to 0 & y remains as it is

$\langle s_0, [0, 0] \rangle \xrightarrow[2]{a} \langle s_1, [0, 2] \rangle \xrightarrow[2.7]{b} \langle s_2, [0.7, 0] \rangle \xrightarrow[2.8]{c}$

($[0,0]$: x, y; $[2,2]$; $[0.7, 2.7]$; bc $2.7 - 2 = 0.7$)

$\langle s_3, [0, 0.1] \rangle \xrightarrow[5]{d} \langle s_0, [2.2, 2.3] \rangle \longrightarrow \ldots$

($[0.8, 0.1]$; $[2.2, 2.3]$) – 2.2 bec b/w 5 & 2.8

Note: $y \geq 2$ so we could transition back to s_0

The untime of $L(AB) = (abcd)^\omega$

- $\langle s, [x_1, x_2] \rangle \rightarrow$ Instantaneous (ID) of Time Automaton.

  finite state; values of clocks.

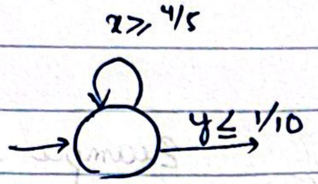
Issue: uncountable # of clock values, real #'s

<u>Dealing with timed Automaton A:</u>

1) All constants are integers WLOG (without loss of generality)

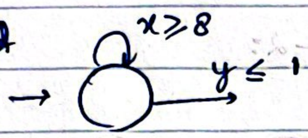
Multiply by LCM of all denominators of rational constants

• step 1 is finding 1D



Accepts (σ, τ)

step 2



accepts $(\sigma, 10\tau)$

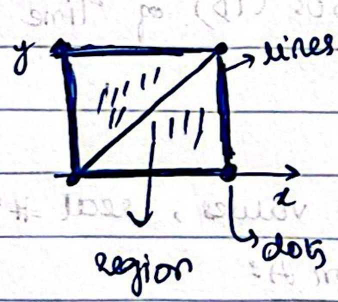
LCM of 5 & 10 is 2. so multiply 4 by 2. $\Rightarrow 8$.

Step 3 : combining ① & ②

Example : • Two clocks , • Two constraints : $x = 1$? , $y = 1$?

outgoing from current state could have different destinations

Which one is active first Depends on if : $x = y$ or $x < y$ or $y < x$



These are called clock regions
↳ am I on the dots or on the lines, on the regions.

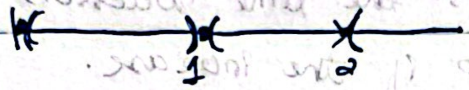
Step 4 : If the largest constant that clock x is compared to, c_x, then values of x bigger than c_x dont matter

largest constant clock x is compared to

Example: clock x, $c_x = 2$.

regions are $x = 0$, $0 < x < 1$, $x = 1$, $1 < x < 2$, $x = 2$, $x > 2$.

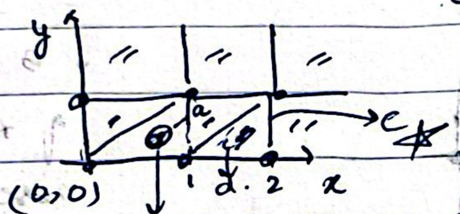
→ clock regions.



Ex 2: 2 clocks., x, y $C_x = 2$, $c_y = 1$

clock regions:

- 6 corner points
- 8 open regions
- 14 line segments



here both clocks are going to point a, so this point is a time successor of this region.

Time Successor Of A Region:

- forward analysis we have successors.
- backward " " " predecessors.

→ Clock region α'' is a time successor α if for each $v \in \alpha$ there exists A $t \in \mathbb{R}_+$ such that $v + t \in \alpha''$

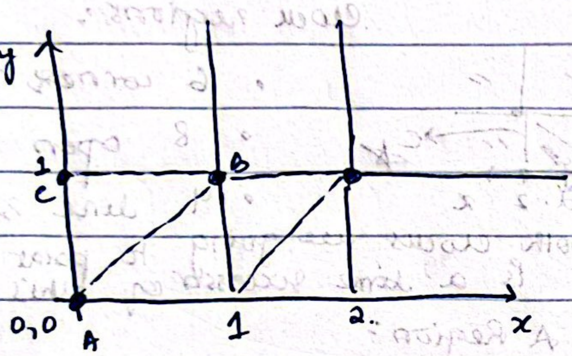
✱ segment c is time successor of all points in region d.

- basically where can I go to if I just let time flow.

Example:

$v = (2/3, 1/4) \xrightarrow{\text{transforms under } v+5} v+5 = (2/3 + 5,\ 1/4 + 5)$

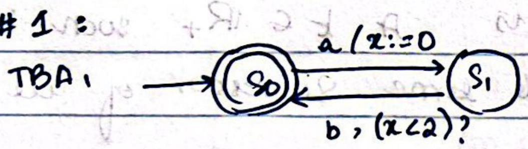
In English/words: the time successors are all regions the clocks can be in if the increase.

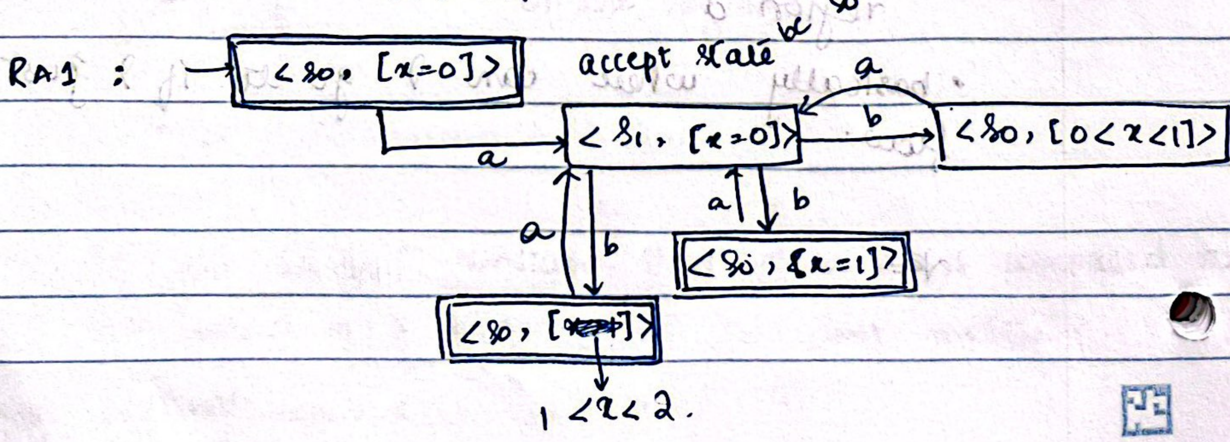


- B is a time successor of A if clock x was reset from B, C would be result which itself is a clock region.

Region Automaton:

Ex # 1:





Region Automaton:

Start state is $\langle s_0, [v_0] \rangle$ — set of all clocks set to 0.

(start state of figure) / "locations"

Arbitrary state $\langle$ discrete state, clock regions $\rangle$

An Edge

$\langle \langle \text{start} \rangle, \langle \text{destination} \rangle, \text{Event} \rangle$

eg $\langle \langle s, \alpha \rangle, \langle s', \alpha' \rangle, a \rangle$

I am here — I go here — when a happens.

Run of Timed Automata, A:

$r: \langle s_0, v_0 \rangle \xrightarrow[\tau_1]{\sigma_1} \langle s_1, v_1 \rangle \xrightarrow[\tau_2]{\sigma_2} \cdots$ (time)

Projection: $[r]: \langle s_0, [v_0] \rangle \xrightarrow{\sigma_1} \langle s_1, [v_1] \rangle \xrightarrow{\sigma_2} \cdots$

Valid Runs R' on RA: $r': \langle s_0, \alpha_0 \rangle \xrightarrow{\sigma_1} \langle s_1, \alpha_1 \rangle \xrightarrow{\sigma_2} \cdots$