

Task 1

2.1(a) :

To list the elements of the sample space for the set of integers between 1 and 50 that are divisible by 8, we need to find all integers within this range that are multiples of 8.

The multiples of 8 between 1 and 50 are:

- 8 (8×1)
- 16 (8×2)
- 24 (8×3)
- 32 (8×4)
- 40 (8×5)
- 48 (8×6)

$$S = \{8, 16, 24, 32, 40, 48\}$$

2.1(b) :

quadratic equation: $x^2 + 4x - 5 = 0$ for x .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $x^2 + 4x - 5 = 0$, we have:

- $a = 1$
- $b = 4$
- $c = -5$

Substituting these values into the quadratic formula, we get:

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-5)}}{2 \cdot 1}$$

First, calculate the discriminant:

$$4^2 - 4 \cdot 1 \cdot (-5) = 16 + 20 = 36$$

$$x = \frac{-4 \pm \sqrt{36}}{2}$$

$$x = \frac{-4 \pm 6}{2}$$

We have two possible solutions:

$$x = \frac{-4 + 6}{2} = 1$$

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$$x = 2 - 4 - 6 = 2 - 10 = -5$$

$$S = \{1, -5\}$$

2.3

To determine which events are equal to the event $A = \{1, 3\}$, we need to analyze each of the given events B, C, and D and compare them with A.

Event A

$$A = \{1, 3\}$$

Event B

$$B = \{x \mid x \text{ is a number on a die}\}$$

The numbers on a standard six-sided die are $\{1, 2, 3, 4, 5, 6\}$.

Event C

$$C = \{x \mid x^2 - 4x + 3 = 0\}$$

solve the quadratic equation $x^2 - 4x + 3 = 0$.

Using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1$, $b = -4$, and $c = 3$:

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$x = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$x = \frac{4 \pm 2}{2}$$

$$x = \frac{4 + 2}{2} \text{ or } x = \frac{4 - 2}{2}$$

$$\text{So, } x = 3 \text{ or } x = 1$$

$$\text{Thus, } C = \{1, 3\}$$

Event D

$$D = \{x \mid x \text{ is the number of heads when six coins are tossed}\}$$

The possible number of heads when tossing six coins ranges from 0 to 6.

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$$D = \{0, 1, 2, 3, 4, 5, 6\}$$

Comparison

- **Event A:** $\{1, 3\}$
- **Event B** $\{1, 2, 3, 4, 5, 6\}$
- **Event C:** $\{1, 3\}$
- **Event D:** $\{0, 1, 2, 3, 4, 5, 6\}$

Therefore, the events that are equal to $A = \{1, 3\}$ are:

- $C = \{1, 3\}$

2.14:

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, A = \{0, 2, 4, 6, 8\}, B = \{1, 3, 5, 7, 9\}, C = \{2, 3, 4, 5\}, D = \{1, 6, 7\}$$

$$1 - A \cup C = \{0, 2, 3, 4, 5, 6, 8\}$$

$$2 - A \cap B = \emptyset$$

$$3 - C' = S - C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 4, 5\} = \{0, 1, 6, 7, 8, 9\}$$

4-at first:

$$C' \cap D = \{0, 1, 6, 7, 8, 9\} \cap \{1, 6, 7\} = \{1, 6, 7\},$$

then:

$$(C' \cap D) \cup B = \{1, 6, 7\} \cup \{1, 3, 5, 7, 9\} = \{1, 3, 5, 6, 7, 9\}$$

5-first: $S \cap C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{2, 3, 4, 5\} = \{2, 3, 4, 5\}$, then :

$$(S \cap C)' = S - (S \cap C) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 4, 5\} = \{0, 1, 6, 7, 8, 9\}$$

6- at first:

$$D' = S - D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 6, 7\} = \{0, 2, 3, 4, 5, 8, 9\}$$

Then, find $A \cap C \cap D'$:

$$A \cap C \cap D' = \{2, 4\} \cap \{0, 2, 3, 4, 5, 8, 9\} = \{2, 4\}$$

Summary:

$$A \cup C = \{0, 2, 3, 4, 5, 6, 8\}$$

1. $A \cap B = \emptyset$
2. Complement of $C = \{0, 1, 6, 7, 8, 9\}$
3. $(C' \cap D) \cup B = \{1, 3, 5, 6, 7, 9\}$
4. Complement of $(S \cap C) = \{0, 1, 6, 7, 8, 9\}$
5. $A \cap C \cap D' = \{2, 4\}$

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2.63 :

1. Probability that PC is in the Bedroom

- $$P(\text{Bedroom}) = P(\text{Adult Bedroom}) + P(\text{Child Bedroom}) + P(\text{Other Bedroom})$$
$$P(\text{Bedroom}) = 0.03 + 0.15 + 0.14 = 0.32$$

2. Probability that PC is not in the Bedroom

$$P(\text{Not Bedroom}) = 1 - P(\text{Bedroom})$$
$$P(\text{Not Bedroom}) = 1 - 0.32 = 0.68$$

3. To determine in which room we would expect to find the PC, we look at the probabilities of the PC being in each room and choose the room with the highest probability.

From the given probabilities:

- Office or Den: 0.4
- Other Rooms: 0.28

Since the probability of the PC being in the office or den (0.4) is higher than the probability of it being in other rooms (0.28), we would expect to find the PC in the office or den.

2.58:

1. Probability of Getting a Total of 8

When two dice are tossed, the sample space consists of $6 \times 6 = 36$ possible outcomes, since each die has 6 faces.

The outcomes that result in a total of 8 are:

- (2, 6)
- (3, 5)
- (4, 4)
- (5, 3)
- (6, 2)

There are 5 such outcomes.

$$P(\text{Total of 8}) = 5/36$$

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2. Probability of Getting at Most a Total of 5

The outcomes that result in a total of 5 or less are:

- (1, 1)
- (1, 2)
- (1, 3)
- (1, 4)
- (1, 5)
- (2, 1)
- (2, 2)
- (2, 3)
- (2, 4)
- (3, 1)
- (3, 2)
- (3, 3)
- (4, 1)
- (4, 2)
- (5, 1)

There are 15 such outcomes.

$$P(\text{At most a total of 5}) = 15/36 = 5/12$$

2.76:

Nonsmokers: H=21 (Hypernation), NH=48 (None Hypernation)

- **Moderate Smokers:** H=36, NH=26
- **Heavy Smokers:** H=30, NH=19

1. Probability of Experiencing Hypernation Given That the Person Is a Heavy Smoker:

$$-P(\text{Hypernation} | \text{Heavy Smoker}) = p(\text{Hypernation} \& \text{heavy smoker}) / p(\text{heavy smoker})$$

-The total number of heavy smokers is $30 + 19 = 49$.

-The number of heavy smokers experiencing hypernation = 30.

$$-P(\text{Hypernation} | \text{Heavy Smoker}) = 30/49$$

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2. Probability of Being a Nonsmoker Given That the Person Is Experiencing No Hypernatation:

find $P(\text{Nonsmoker} | \text{No Hypernatation}) = \frac{p(\text{NOsmoker} \& \text{No Hypernatation})}{p(\text{No Hypernatation})}$

- The total number of individuals experiencing no hypernatation is $48 + 26 + 19 = 93$.
- The number of nonsmokers experiencing no hypernatation is 48.

$$P(\text{Nonsmoker} | \text{No Hypernatation}) = 48/93$$

2.110:

1. Probability That Exactly 2 of the Next 3 Patients Survive

- The probability of a patient surviving the operation is $p = 0.8$
- The probability of a patient not surviving the operation is $q = 1 - p = 0.2$

$$P(X=k) = \binom{n}{k} p^k q^{n-k}$$

For $n=3$ and $k=2$:

$$P(X=2) = \binom{3}{2} (0.8)^2 (0.2)^{3-2}$$

First, calculate the binomial coefficient $\binom{3}{2}$:

$$\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3 \times 2 \times 1}{2 \times 1} = 3$$

Now, substitute into the formula:

$$P(X=2) = 3 \times (0.8)^2 \times (0.2)^1$$

$$P(X=2) = 3 \times 0.64 \times 0.2 = 0.384$$

2. Probability That All of the Next 3 Patients Survive

We want to find the probability of all 3 successes (survivals) in 3 trials (operations).

For $n=3$ and $k=3$:

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$$P(X=3) = \binom{3}{3} (0.8)^3 (0.2)^{3-3}$$

The binomial coefficient $\binom{3}{3}$ is 1:

$$\binom{3}{3} = 1$$

$$P(X=3) = 1 \times (0.8)^3 \times (0.2)^0$$

$$P(X=3) = (0.8)^3 \times 1 = 0.512$$