Inferential statistics

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Section (2)

Chapter two

Sampling methods

*Methods of sample size distribution (allocation) in stratified sample:

1. The Best Method of Distribution (Optimal Allocation).

2. Nayman Allocation Method.

3. Proportional Method.

✓ The sampling design aims to find estimators with small variances at the lowest possible cost.

✓ After determining the sample size, there are ways to divide the sample size (n) into samples from different strata, which is affected by three factors:

1. (N_r) The size of each stratum, i.e. the number of observations in each class.

2. $(\sigma^2{}_r)$ The variance of each stratum, i.e. the variation in each class.

3. (C_r) The cost of each stratum, i.e. the total cost in each class.

Example (1):

Suppose we have the following data:

	First Stratum	second Stratum	third Stratum	N
(N_r)	155	62	93	310
(C_r)	9	9	16	
(σ_r)	5	15	10	

Required:

1. By using the Best Method of Distribution (Optimal Allocation), find the total sample size and distribute the sample size among the different strata.

2. By using Nayman Allocation Method, find the total sample size and distribute the sample size among the different strata.

3. By using Proportional Method, find the total sample size and distribute the sample size among the different strata, assume that the variances are equal (10) for all classes.

assume the limits of error estimation equals 2.

Solution

1. By using the Best Method of Distribution (Optimal Allocation), find the total sample size and distribute the sample size among the different strata.

***Steps of best sample size distribution (n):**

B=2, D= $\frac{B^2}{4} = \frac{4}{4} = 1$

B represents the bounds (limits) of error estimation.

1) Finding the weights *(W_r)* within each Stratum:

$$W_r = \frac{N_r \times \sigma_2}{\sqrt{C_r}} \div \left(\frac{N_1 \times \sigma_1}{\sqrt{C_1}} + \frac{N_2 \times \sigma_2}{\sqrt{C_2}} + \frac{N_3 \times \sigma_3}{\sqrt{C_3}}\right)$$

* $W_1 = \frac{155\times5}{\sqrt{9}} \div \left(\frac{155\times5}{\sqrt{9}} + \frac{62\times15}{\sqrt{9}} + \frac{93\times10}{\sqrt{16}}\right) = \mathbf{0.323}$

* $W_2 = \frac{62\times15}{\sqrt{9}} \div \left(\frac{155\times5}{\sqrt{9}} + \frac{62\times15}{\sqrt{9}} + \frac{93\times10}{\sqrt{16}}\right) = \mathbf{0.387}$

* $W_3 = \frac{93\times10}{\sqrt{16}} \div \left(\frac{155\times5}{\sqrt{9}} + \frac{62\times15}{\sqrt{9}} + \frac{93\times10}{\sqrt{16}}\right) = \mathbf{0.29}$

$$W_r = W_1 + W_2 + W_3 = 1$$

2)substitution of *(W_r)* in the following formula to determine the sample size *(n)*, which is:

$$n = \sum_{r=1}^{l} \frac{{N_r}^2 6^2 r}{W_r} \div [(N^2 D) + (\sum N_r {\sigma_r}^2)]$$

***Numerator value:**

$$= \sum_{r=1}^{l} \frac{{N_r}^2 6^2 r}{W_r}$$

$$= \frac{(155)^2\times(5)^2}{0.323} + \frac{(62)^2\times(15)^2}{0.387} + \frac{(93)^2\times(10)^2}{0.29} = 7076817.6$$

*Denominator value:

=$[(N^2D) + (\sum N_r\,\sigma_r{}^2)]$

=[((310)2 × (1)) + ($(155)^2 \times (5)^2 + (62)^2 \times (15)^2 + (93)^2 \times (10)^2$)]

=**123225**

(n) = 7076817.6 ÷ 123225 =57.43 → ≅ 58

3)Distribute the sample size (n = 58) among the different strata:

✓ $(n_1) = W_1 \times n = 0.323 \times 57.43 = \mathbf{18.55} \rightarrow \cong \mathbf{19}$

✓ $(n_2) = W_1 \times n = 0.387 \times 57.43 = \mathbf{22.23} \rightarrow \cong \mathbf{22}$

✓ $(n_3) = W_1 \times n = 0.29 \times 57.43 = \mathbf{16.65} \rightarrow \cong \mathbf{17}$

✓ $(n) = (n_1) + (n_2) + (n_2) = 58$

2. By using Nayman Allocation Method, find the total sample size and distribute the sample size among the different strata.

♣ Steps :

1) Finding the weights (W_r) within each Stratum:

$$W_r = \frac{N_{r\times}\sigma_r}{\sum N_{r\times}\sigma_r}$$

$$W_1 = \frac{N_{r\times}\sigma_r}{\sum N_{r\times}\sigma_r} = \frac{155\times5}{(155\times5)+(62\times15)+(93\times10)} = 0.294$$

$$W_2 = \frac{N_{r\times}\sigma_r}{\sum N_{r\times}\sigma_r} = \frac{62\times15}{(155\times5)+(62\times15)+(93\times10)} = 0.353$$

$$W_3 = \frac{N_{r\times}\sigma_r}{\sum N_{r\times}\sigma_r} = \frac{93\times10}{(155\times5)+(62\times15)+(93\times10)} = 0.353$$

$$W_r = W_1 + W_2 + W_3 = 1$$

2) Substitution of (Wr) in the following formula to determine the sample size (n), which is:

$$n = \sum_{r=1}^{l} \frac{N_r{}^2 6^2 r}{W_r} \div [(N^2 D) + (\sum N_r\, \sigma_r{}^2)]$$

***Numerator value:**

$$= \sum_{r=1}^{l} \frac{N_r{}^2 6^2 r}{W_r}$$

$$= \frac{(155)^2 \times (5)^2}{0.3} + \frac{(62)^2 \times (15)^2}{0.35} + \frac{(93)^2 \times (10)^2}{0.35} = \mathbf{6944369}$$

***Denominator value:**

$$= [(N^2 D) + (\sum N_r\, \sigma_r{}^2)]$$

$$= [((310)2 \times (1)) + ((155)^2 \times (5)^2 + (62)^2 \times (15)^2 + (93)^2 \times (10)^2)]$$

=123225

(n) = 6944369 ÷ 123225 =56.355 → ≅ 57

3)Distribute the sample size (n = 57) among the different strata:

✓ $(n_1) = W_1 \times n = 0.294 \times 57 = 16.758 \rightarrow \cong \mathbf{17}$

✓ $(n_2) = W_1 \times n = 0.353 \times 57 = \mathbf{20}.121 \rightarrow \cong \mathbf{20}$

✓ $(n_3) = W_1 \times n = 0.353 \times 57 = 20.121 \rightarrow \cong 20$

✓ $(n) = (n_1) + (n_2) + (n_2) = 57$

3. By using Proportional Method, find the total sample size and distribute the classes sample size among the different strata, assume that the variances are equal (10) for all.

♣ Steps

A. Finding the weights (W_r) within each Stratum:

$$W_r = \frac{N_r}{N}$$

$$W_1 = \frac{N_1}{N} = \frac{155}{310} = 0.5$$

$$W_2 = \frac{N_2}{N} = \frac{62}{310} = 0.2$$

$$W_3 = \frac{N_3}{N} = \frac{93}{310} = 0.3$$

2) Substitution of (Wr) in the following formula to determine the sample size (n), which is:

$$n = \sum_{r=1}^{l} \frac{N_r{}^2 6^2 r}{W_r} \div [(N^2 D) + (\sum N_r \sigma_r{}^2)]$$

***Numerator value:**

$$= \sum_{r=1}^{l} \frac{N_r{}^2 6^2 r}{W_r}$$

$$= \frac{(155)^2 \times (10)^2}{0.5} + \frac{(62)^2 \times (10)^2}{0.2} + \frac{(93)^2 \times (10)^2}{0.3} = \mathbf{9610000}$$

*Denominator value:

$=[(N^2D) + (\sum N_r\, \sigma_r{}^2)]$

$=[((310)2 \times (1)) + ((155)^2 \times (10)^2 + (62)^2 \times (10)^2 + (93)^2 \times (10)^2)]$

=127100

(n) = 9610000 ÷ **127100**=*75.6097→ ≅ 76*

3)Distribute the sample size (n = 76) among the different strata:

✓ $(n_1) = W_1 \times n = 0.5 \times 75.6 = 37.8 \rightarrow \cong 38$

✓ $(n_2) = W_1 \times n = 0.2 \times 75.6 = 15.12 \rightarrow \cong 15$

✓ $(n_3) = W_1 \times n = 0.3 \times 75.6 = 22.68 \rightarrow \cong 23$

✓ $(n) = (n_1) + (n_2) + (n_2) = 76$

Example (2):

Suppose we have the following data

	First Stratum	second Stratum	third Stratum	N
(N_r)	250	150	50	450
(C_r)	2	4	9	
$(\sigma^2{}_r)$	100	225	625	

Required:

1. By using the Best Method of Distribution (Optimal Allocation), find the total sample size and distribute the sample size among the different strata.

2. By using Nayman Allocation Method, find the total sample size and distribute the sample size among the different strata.

3. By using Proportional Method, find the total sample size and distribute the sample size among the different strata, assume that the variances are equal (10) for all classes.

assume the limits of error estimation equals 2.

Solution