Time Series Modeling

CCTS 40500

Autoregressive Integrated Moving Average Model

- **AR**: *Autoregression*. A model that uses the dependent relationship between an observation and some number of lagged observations
- **I**: *Integrated*. The use of differencing of raw observations (e.g. subtracting an observation from an observation at the previous time step) in order to make the time series stationary.
- **MA**: *Moving Average*. A model that uses the dependency between an observation and a residual error from a moving average model applied to lagged observations.

Autoregressive Integrated Moving Average Model

- **p**: The number of lag observations included in the model, also called the lag order.
- **d**: The number of times that the raw observations are differenced, also called the degree of differencing.
- **q**: The size of the moving average window, also called the order of moving average.

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Autocorrelation

&

Partial Autocorrelation

$$R(\tau) = \frac{1}{\sigma^2}\mathbf{E}[(X_t - \mu)(X_{t+\tau} - \mu)]$$

Moving Average Model

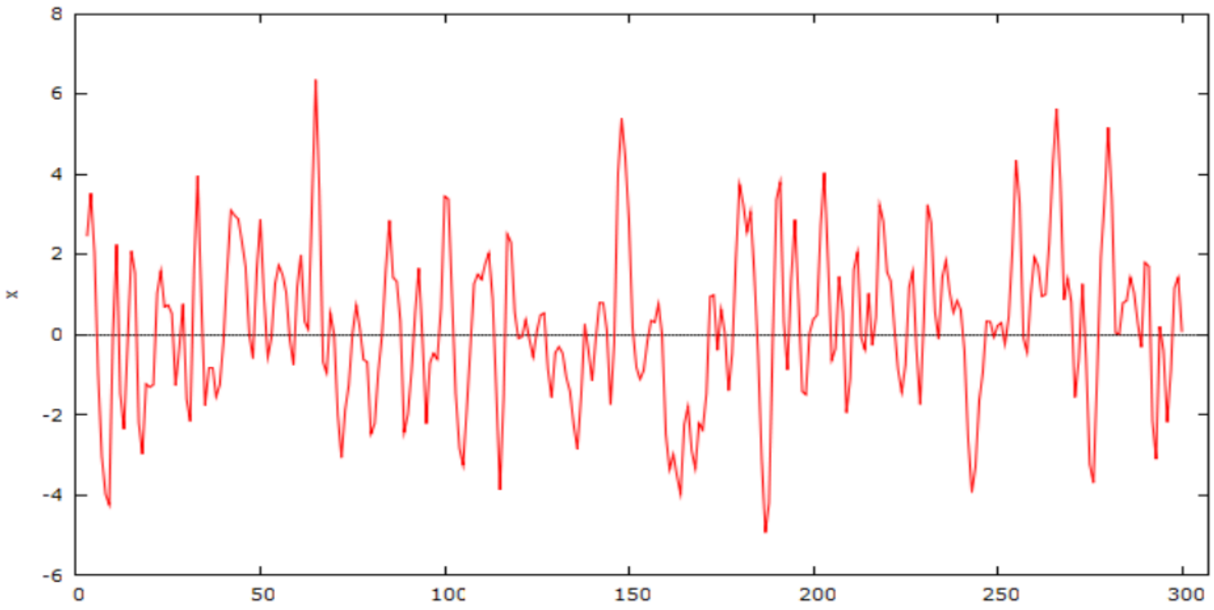

Figure : A simulated time series

Our time series seems the realization of a stationary process with zero mean, thus we can look at sample autocorrelation and partial autocorrelation function to establish the orders p and q of the ARMA model.

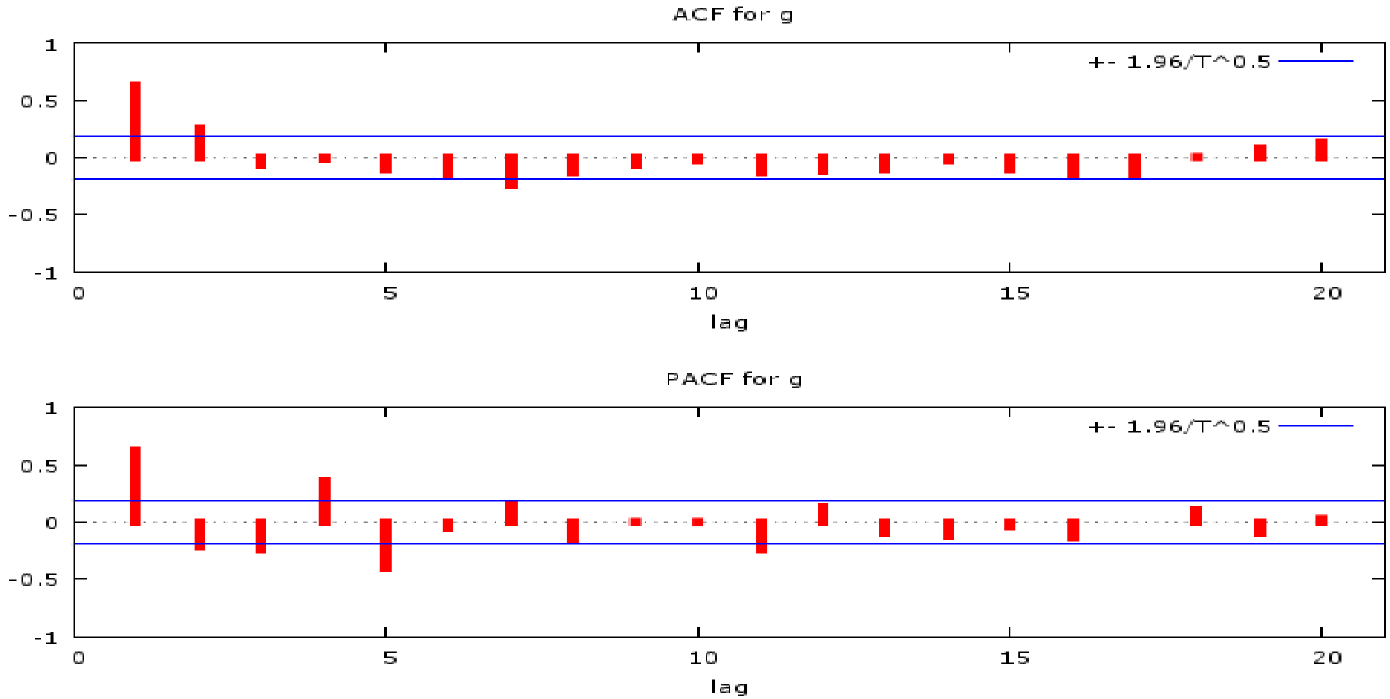

Figure : Sample autocorrelation and sample partial autocorrelation

Autoregressive Integrated Moving Average Model

ACF cuts off after lag 2

$$X_t = u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2},$$
$$u_t \sim WN(0, \sigma^2)$$

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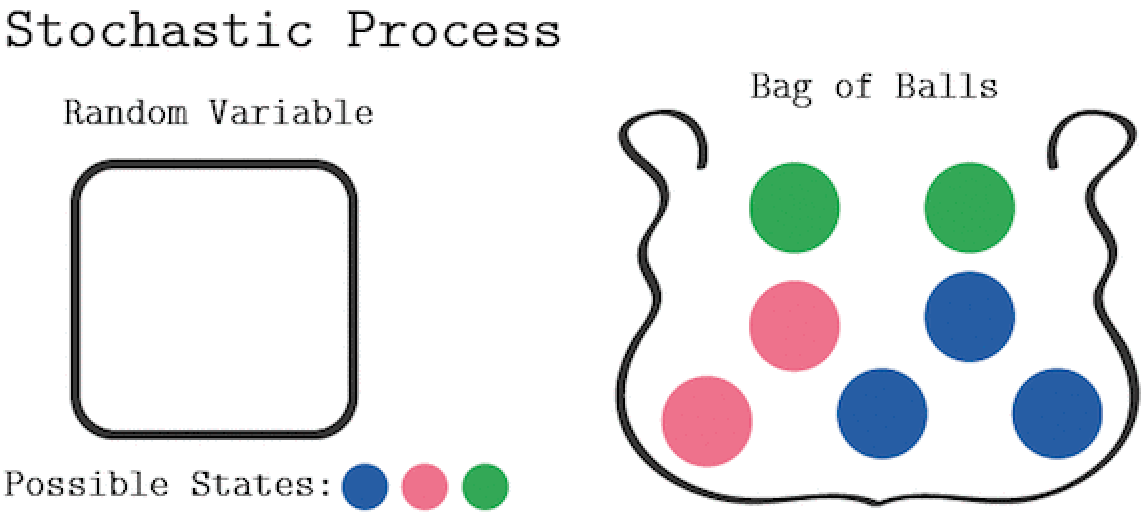
$$\text{If } d = 0 : y_t = Y_t$$

$$\text{If } d = 1 : y_t = Y_t - Y_{t-1}$$

$$\text{If } d = 2 : y_t = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2}) = Y_t - 2Y_{t-1} + Y_{t-2}$$

$$\boxed{\hat{y}_t = \mu + \sum_i \phi_i^d y_{t-i} - \sum_j \theta_j^q e_{t-j}}$$

Markov Chains



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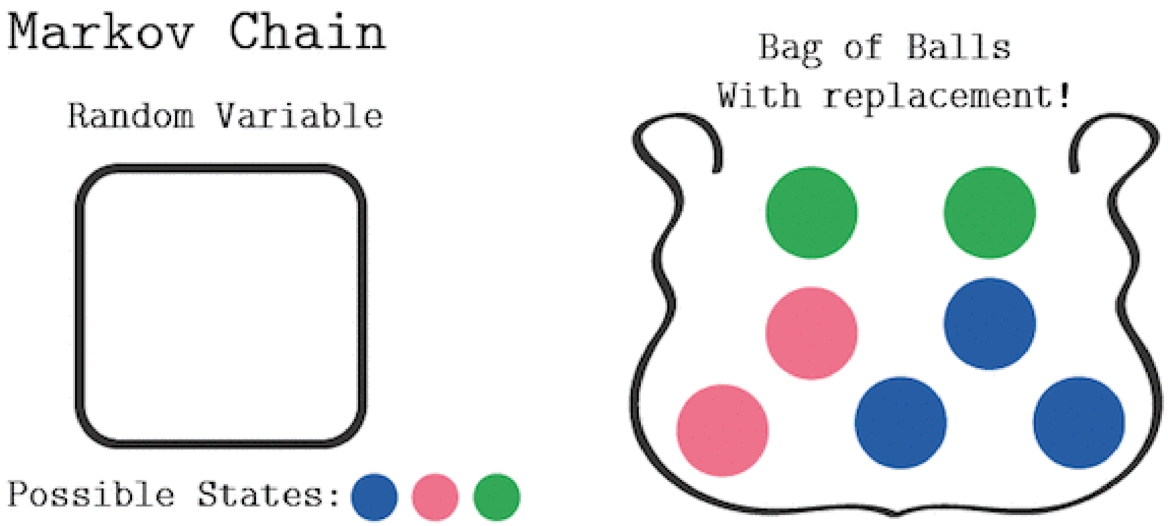

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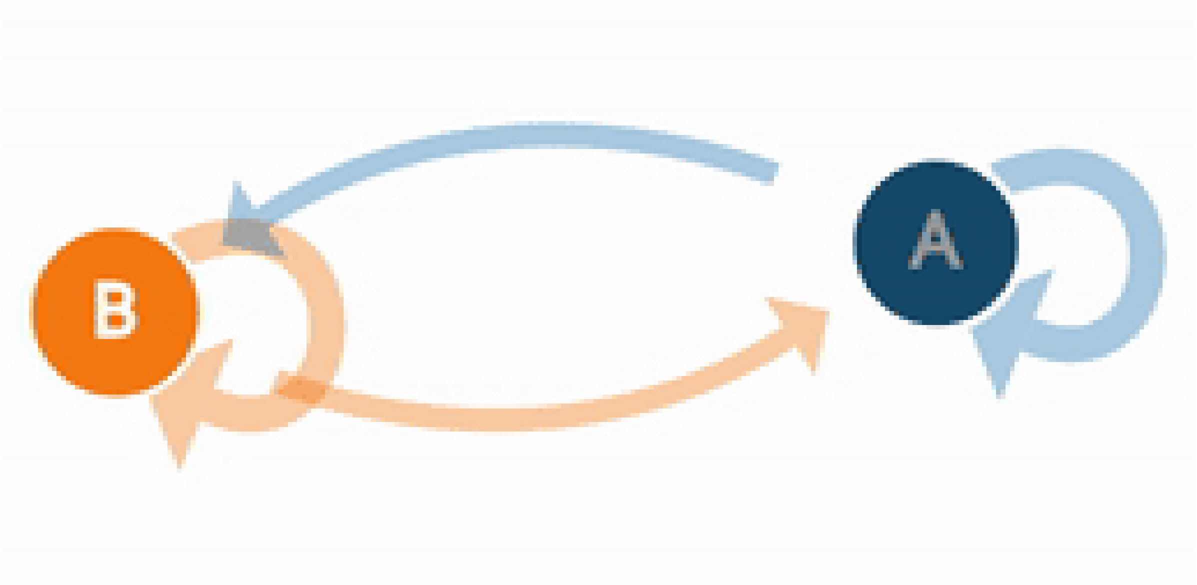

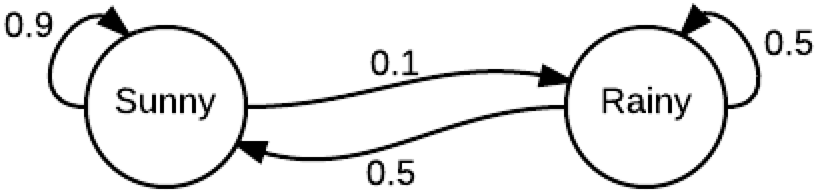

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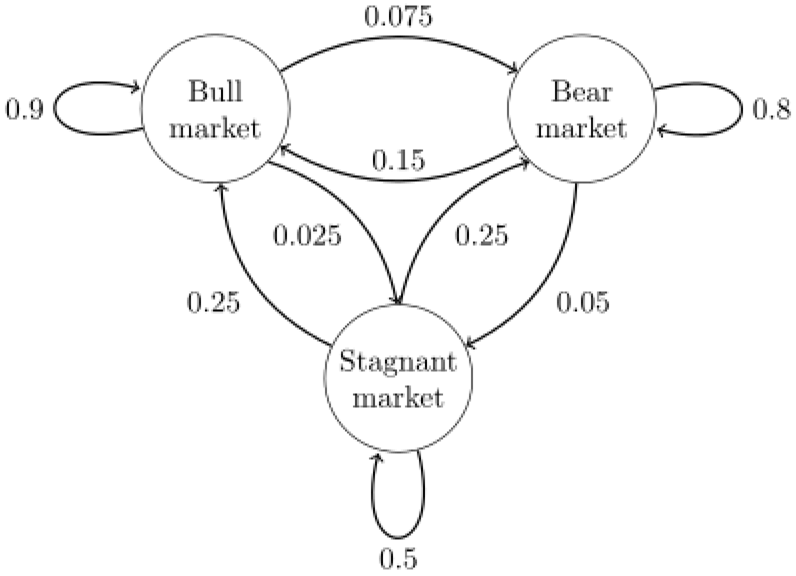

$$P = \begin{bmatrix} 0.9 & 0.075 & 0.025 \\ 0.15 & 0.8 & 0.05 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}.$$