Newcomb's speed of light

Week3-ex3, problem statement

R-template `ex_speed_of_light.Rmd`.

Data file `ex_speedOfLight.dat`.

(Here we redo the analysis from page 66 in BDA3.)

Simon Newcomb conducted experiments on speed of light in 1882. He measured the time required for light to travel a certain distance and here we will analyze a data recorded as deviations from 24,800 nanoseconds. The model used in BDA3 is %

$$\begin{aligned} y_i &\sim N(\mu, \sigma^2) \\ p(\mu, \sigma^2) &\propto \sigma^{-2}. \end{aligned}$$

% where y_i is the i'th measurement, μ is the mean of the measurement and σ^2 the variance of the measurements. Notice that this prior is improper ("uninformative"). This corresponds to widely used uniform prior for μ in the range $(-\infty, \infty)$, and uniform prior for $\log(\sigma)$ (BDA3 pp. 66, 52, and 21). Both priors are improper and cannot be found from Stan. You can use instead %

$$\begin{aligned} p(\mu) &\sim N(0, (10^3)^2) \\ p(\sigma^2) &\sim \text{Inv-}\chi^2(\nu = 4, s^2 = 1) \end{aligned} \tag{1}$$

In this exercise your tasks are the following:

1. Write a Stan model for the above model and sample from the posterior of the parameters. Report the posterior mean, variance and 95% central credible interval for μ and σ^2.
2. Additionally draw samples from the posterior predictive distribution of hypothetical new measurement $p(\tilde{y}|y)$. Calculate the mean, variance and 95% quantile of the posterior predictive distribution.
3. How does the posterior predictive distribution differ from the posterior of μ and Why?
4. Which parts of the model could be interpreted to correspond to aleatory and epistemic uncertainty? Discuss whether this distinction is useful here.
5. Instead of Inverse-χ^2 distribution the variance parameter prior has traditionally been defined using Gamma distribution for the precision parameter $\tau = 1/\sigma^2$. By using the results in Appendix A of BDA3 derive the analytic form of a Gamma prior for the precision corresponding to the prior (1). This should be of the form Gamma(α, β), where α and β are functions of ν and s^2.

Note! Many common distributions have multiple parameterizations, for which reason you need to be careful when interpreting others' works. The variance/precision parameter and their priors are notorious for this. The reason is mainly historical since different parameterizations correspond to different analytical solutions.

Grading

Total 10 points. 2 points from correct answer for each of the above steps.