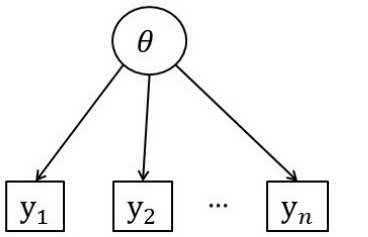
Directed acyclic graphs

Week4-ex1, problem statement

a)-c) 1) Write down the joint probability distribution of all the parameteters and observation variables y in the directed acyclic graphs (DAG) shown in Figure 1, and 2) write down a Stan pseudo code that tells how a model corresponding to that DAG would be written. You can assume that all variables get values in real numbers. An example of a model answer is provided on the right



Joint distribution

$$p(y_1, \ldots y_n, \theta) = p(\theta) \prod_{i=1}^{n} p(y_i|\theta)$$

Stan pseudo-code:

```
data{
   int<lower=0> n;
   real y[n];
}
parameters{
   real theta;
}
model{
   theta ~ p();
   for( i in 1 : n ) {
      y[i] ~ p(theta);
   }
}
```

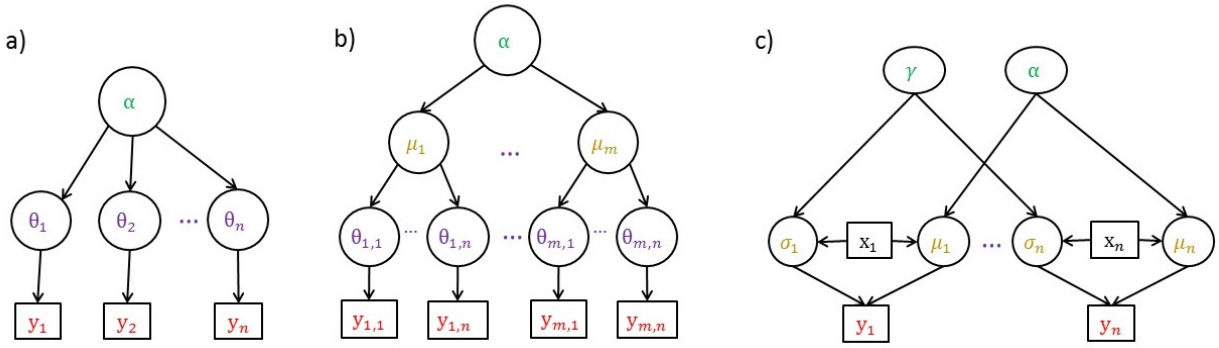


Figure 1: The DAGs for which the joint distribution and pseudo code have to be defined. Note variables denoted by x should be treated as covariates.

d) Draw a Directed acyclic graph (DAG) and write a Stan pseudo code of the following model

$$\begin{aligned}
y_{i,j} &\sim N(\mu_j, \sigma_j^2), i = 1, \ldots, n, j = 1, \ldots, J \\
\mu_j &\sim N(\mu_0, \phi) \\
\mu_0 &\sim N(0, 10^6) \\
\phi &\sim \text{Inv-}\chi^2(\nu_1, s_1^2) \\
\sigma_j^2 &\sim \text{Inv-}\chi^2(\nu_2, s_2^2)
\end{aligned}$$

See the previous problem for an example on the needed accuracy for the pseudo code.

Grading

Total 20 points. a)-c) Two points for correct joint density function and 3 points for correct pseudo-code. d) 3 points for correct DAG and 2 points for correct pseudo-code.