V = 8 1 if ball 1 is black

Yz = { 1 if ball z is black

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- 1. Consider a bag of 1 white and 1 black ball. Let the random variables be
 - Y_1 the outcome of drawing one ball from the bag
 - Y_2 the outcome of drawing second ball from the bag after replacing the first ball
- Are Y_1 and Y_2 independent?

• Are
$$Y_1$$
 and Y_2 exchangeable? $P(Y_1, Y_2 \mid \theta_1, \theta_2) = \Theta_1^{Y_1} \mid \Theta_2^{Y_2}$

· Y, and is are not independent

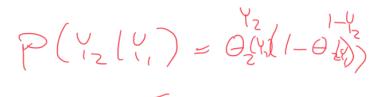
Example: exchangeability , y, and y, are exchangeable

2. Consider a bag of 1 white and 1 black ball. Let the random variables be

- Y_1 the outcome of drawing one ball from the bag
- Y_2 the outcome of drawing second ball from the bag without replacing the first ball
- Are Y_1 and Y_2 exchangeable?
- Are Y_1 and Y_2 independent?

- 3. Consider a bag of 10000 white and 10000 black balls. Let the random variables be
 - Y_1 the outcome of drawing one ball from the bag
 - Y_2 the outcome of drawing second ball from the bag without replacing the first ball
- Are Y_1 and Y_2 exchangeable?
- Are Y_1 and Y_2 independent? \bowtie
- Can we act as if Y_1 and Y_2 were independent?

$$T: G_{1} = \frac{1}{2}$$
 $\Theta_{2}(Y_{1}) = \frac{9999}{19999} \approx \frac{1}{2} i + \frac{1}{1} = 1$





- 4. Consider a bag of white and black balls (n is known but the proportion is unknown). Let the random variables be
 - Y_1 the outcome of drawing one ball from the bag
 - Y_2 the outcome of drawing second ball from the bag <u>after replacing the first</u> ball
- Are Y_1 and Y_2 exchangeable? $P(Y_1, Y_2, I_T) \stackrel{?}{=} P(Y_1, I_T) P(Y_2, I_T)$
- Are Y_1 and Y_2 independent?

 Can we act as if Y_1 and Y_2 were independent? $P(Y_1 | Y_2) = P(Y_1 | Y_2) = P(Y_1 | Y_2) = P(Y_2 | Y_2) = P(Y_1 | Y_2) = P(Y_2 | Y_2) = P(Y_1 | Y_2) = P(Y_1 | Y_2) = P(Y_2 | Y_2) = P(Y_1 | Y_2) = P(Y_1 | Y_2) = P(Y_2 | Y_2) = P(Y_1 | Y_2) = P(Y_2 | Y_2) = P(Y_1 | Y_2) = P(Y_$

$$T: \Theta_{1} = \frac{?}{?}$$

$$P(Y_{2}|Y, T) = \int P(Y_{2}|\theta, T) P(\theta|Y, T) d\theta \neq P(Y_{2}|T)$$

$$O_{1} = O_{2} = O$$

$$O_{1} = O_{2} = O$$

$$O_{1} = O_{2} = O$$
Bayesian Data Analysis, Jargo Vanhátaljo@helsinki, fi

- 5. Consider a bag of white and black balls (n is known but the proportion is unknown). Let the random variables be
 - Y_1 the outcome of drawing one ball from the bag
 - Y_2 the outcome of drawing second ball from the bag without replacing the first ball
- Are Y_1 and Y_2 exchangeable?
- Are Y_1 and Y_2 independent? \triangleright
- Can we act as if Y_1 and Y_2 were independent?

Yes, if n is very large (compare to case 3)

- 6. Consider a bag of white and black balls (<u>n is unknown</u> and the <u>proportion is unknown</u>). Let the random variables be
 - Y_1 the outcome of drawing one ball from the bag
 - Y_2 the outcome of drawing second ball from the bag <u>without replacing the</u> first ball
- Are Y_1 and Y_2 exchangeable?
- Are Y_1 and Y_2 independent? \bigcup_{δ}
- Can we act as if Y_1 and Y_2 were independent?

Depends on your prior assumptions on n.