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كلية الحاسبات و المعلومات

Scalar Quantization

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Quantization

Quantization:

- a process of representing a large – possibly infinite – set of values with a much smaller set.

Scalar quantization:

- a mapping of an input value ***x*** into a finite number of output values *(**Reconstruction values**)*

Quantization is one of the simplest and most general idea in **lossy compression.**

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Uniform Scalar Quantizer

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- A **uniform** scalar quantizer partitions the domain of input values into **equally spaced intervals.** Each Interval is defined by its **decision boundaries** (*AKA* **Range**)
- Each **interval** is represented by a distinct **codeword** (*AKA* **Q**).
- The output or **reconstruction value** (*AKA* $\mathbf{Q^{-1}}$) corresponding to each interval is taken to be the **midpoint** of the interval.
- The **length** of each interval is referred to as the ***step size***.

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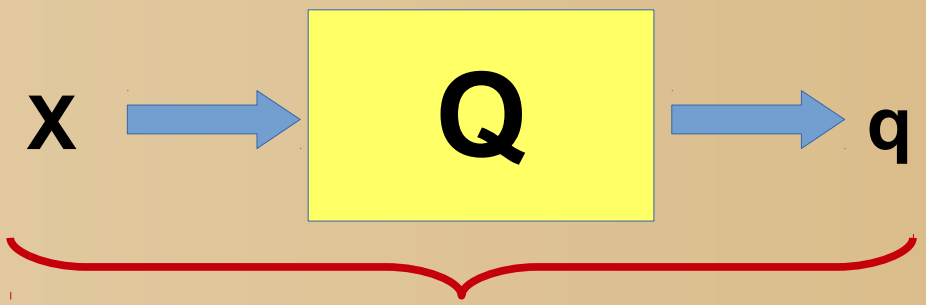
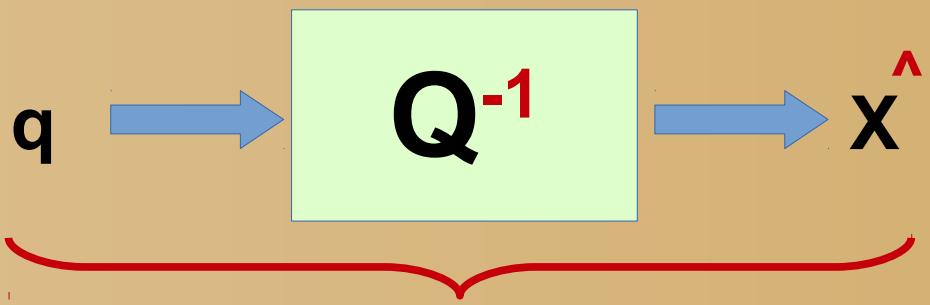
Quantization and De-Quantization



Quantization
(Encoding)



De-Quantization
(Decoding)

X: Input Value

q: Codeword for X
(Encoded value of X)

$\hat{X}$: Output Value
(Reconstructed Values of X)

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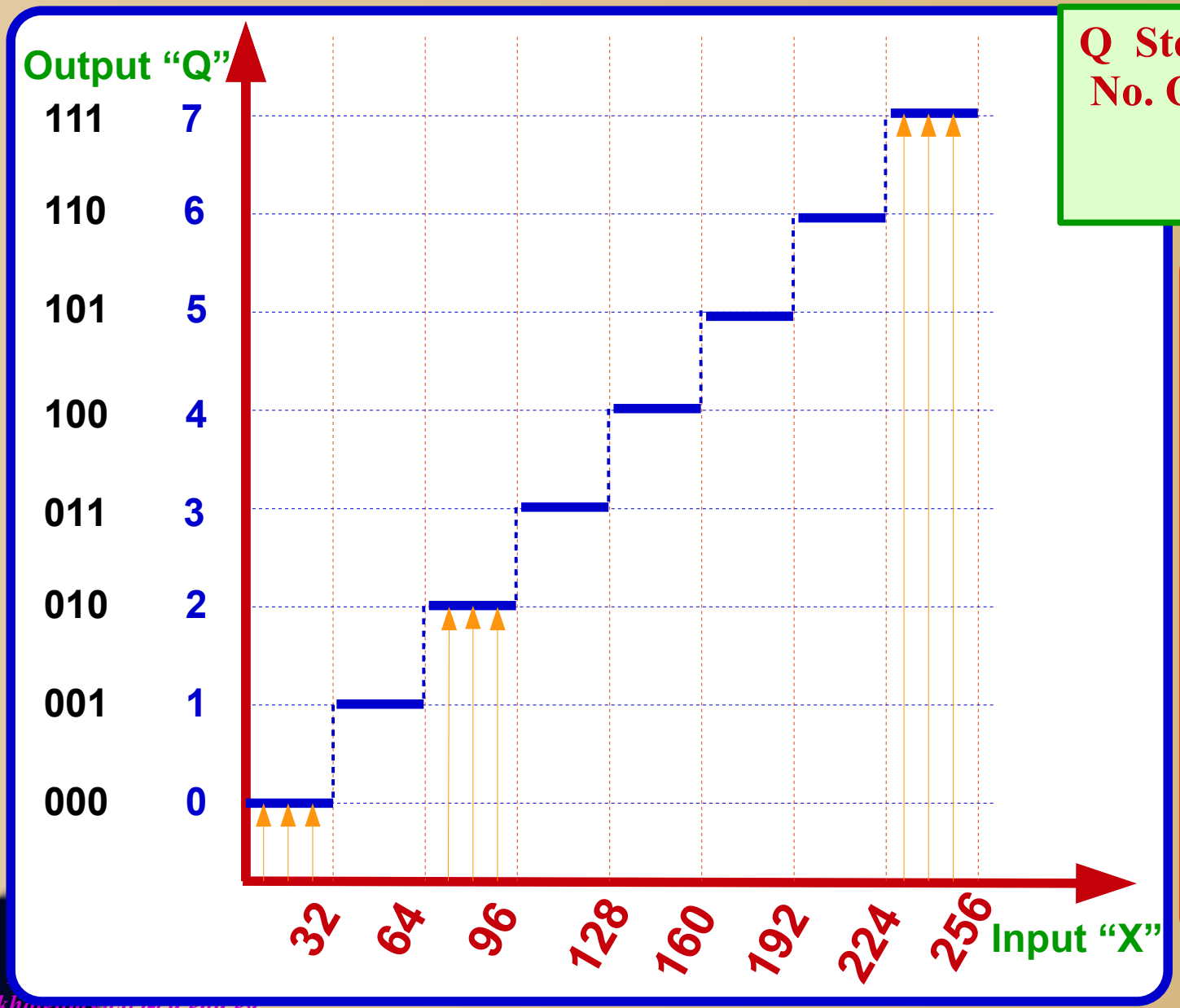
Scalar Quantization - Encoder (Input Output Mapping)



Q Step = 32, Full Scale =256

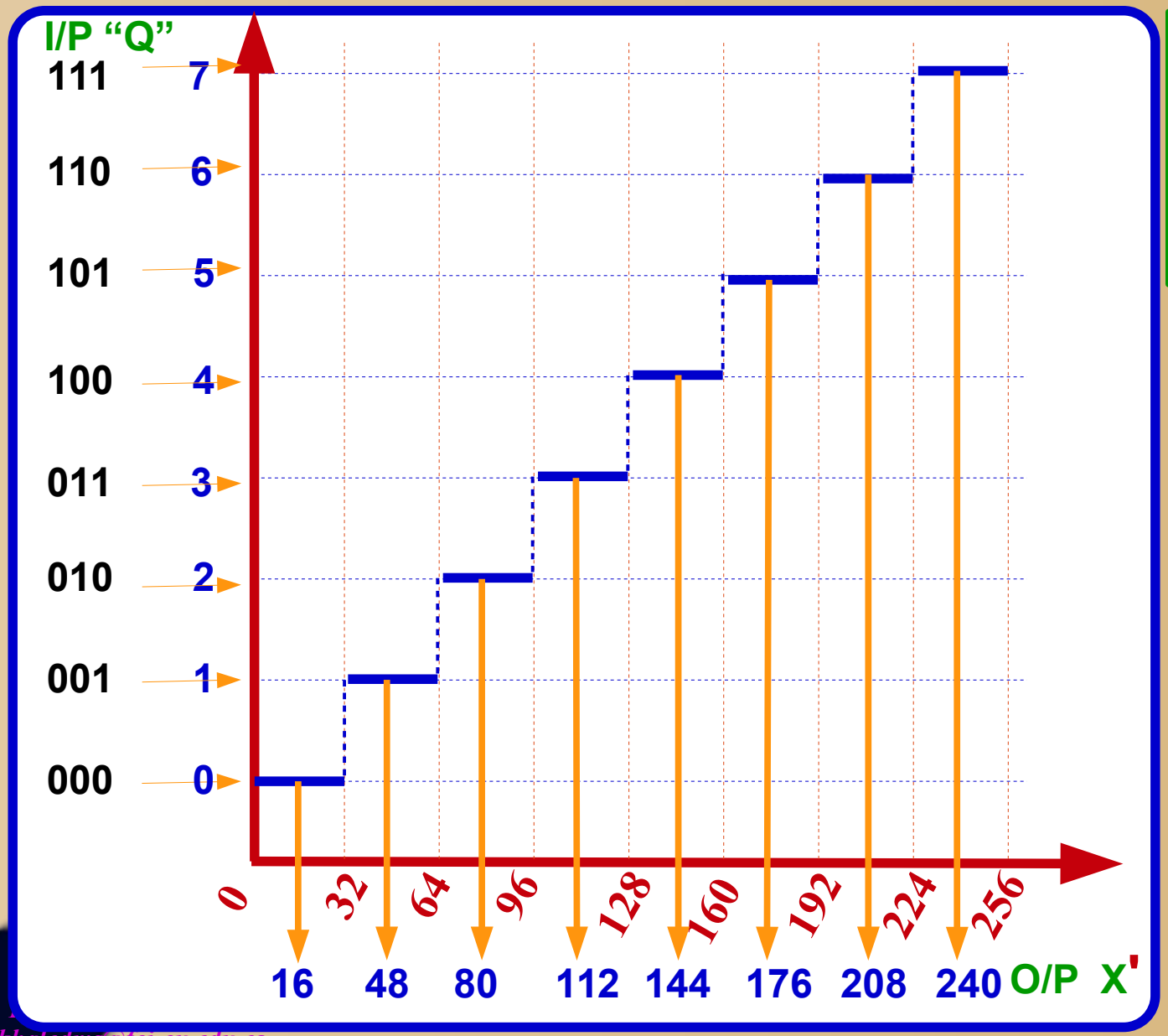
No. Of Steps

=Full Scale / Step

=256 / 32 = 8 Steps (3 bits)

Range	Q
0 → 31	0
32 → 63	1
64 → 95	2
96 → 127	3
128 → 159	4
160 → 191	5
192 → 223	6
224 → 225	7

Scalar Quantization - Decoder (Input Output Mapping)



Q Step = 32, Full Scale =256

No. Of Steps

=Full Scale / Step

=256 / 32 = 8 Steps (3 Bits)

Max Error = 1/2 Step = 16

Range	Q	Q^{-1}
0 → 31	0	→ 16
32 → 63	1	→ 48
64 → 95	2	→ 80
96 → 127	3	→ 112
128 → 159	4	→ 144
160 → 191	5	→ 176
192 → 223	6	→ 208
224 → 225	7	→ 240

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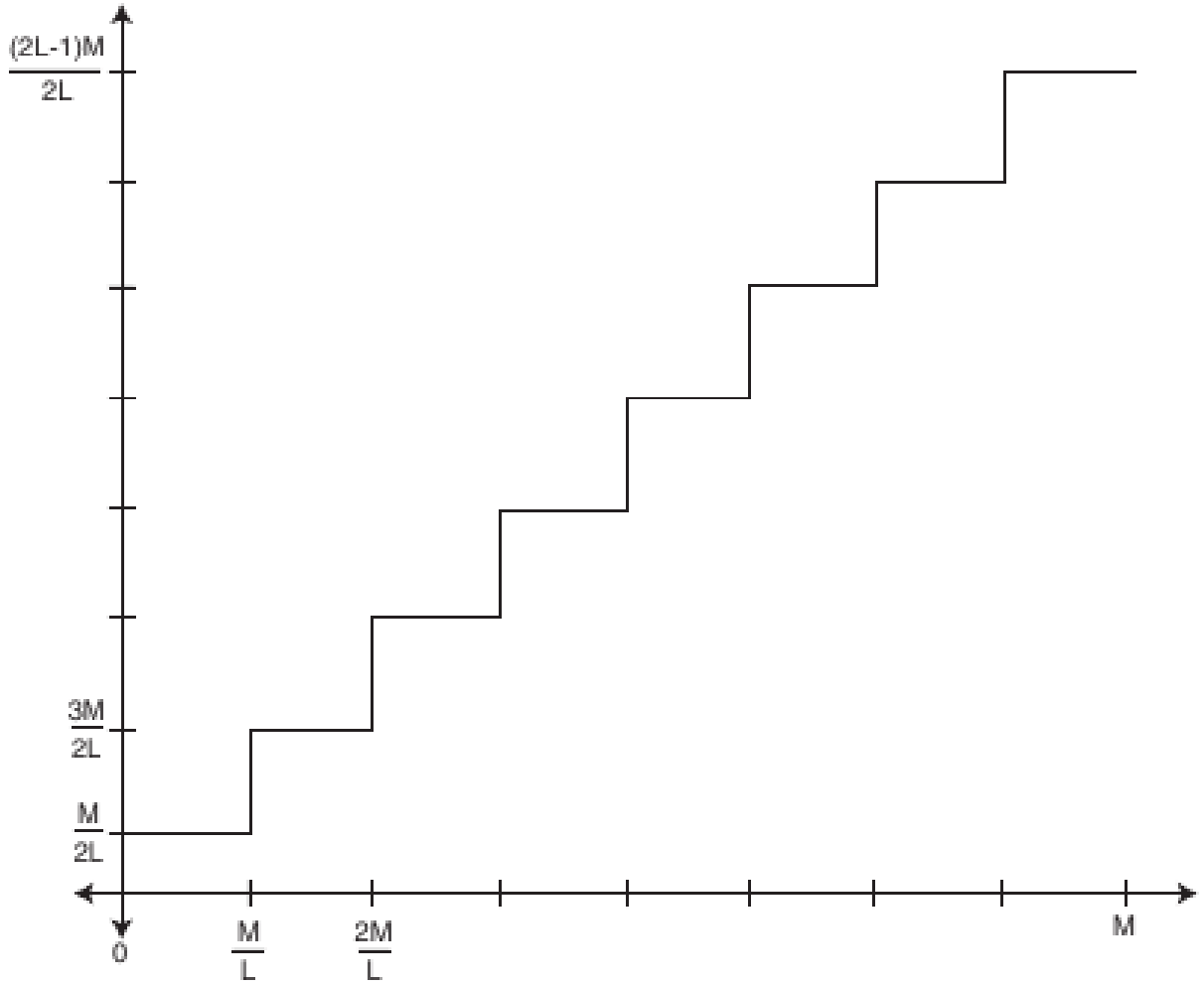
Design of Uniform Quantizer for a given number of steps



Full Scale:	**M**
Number of Steps:	**L**
Qunatization Step:	**M/L**
Number of bits:	**Log_2 L**
Max error =Half Step:	**M/2L**
First Q^{-1} :	**M/2L**

For n Bits Quantizer:

$L = 2^n$

For n=3, $L=2^3$ = 8 levels

Compression Ratio

Log_2 M : Log_2 L

For M=256 (2^8),

L =8 (2^3)

Ratio = 8:3

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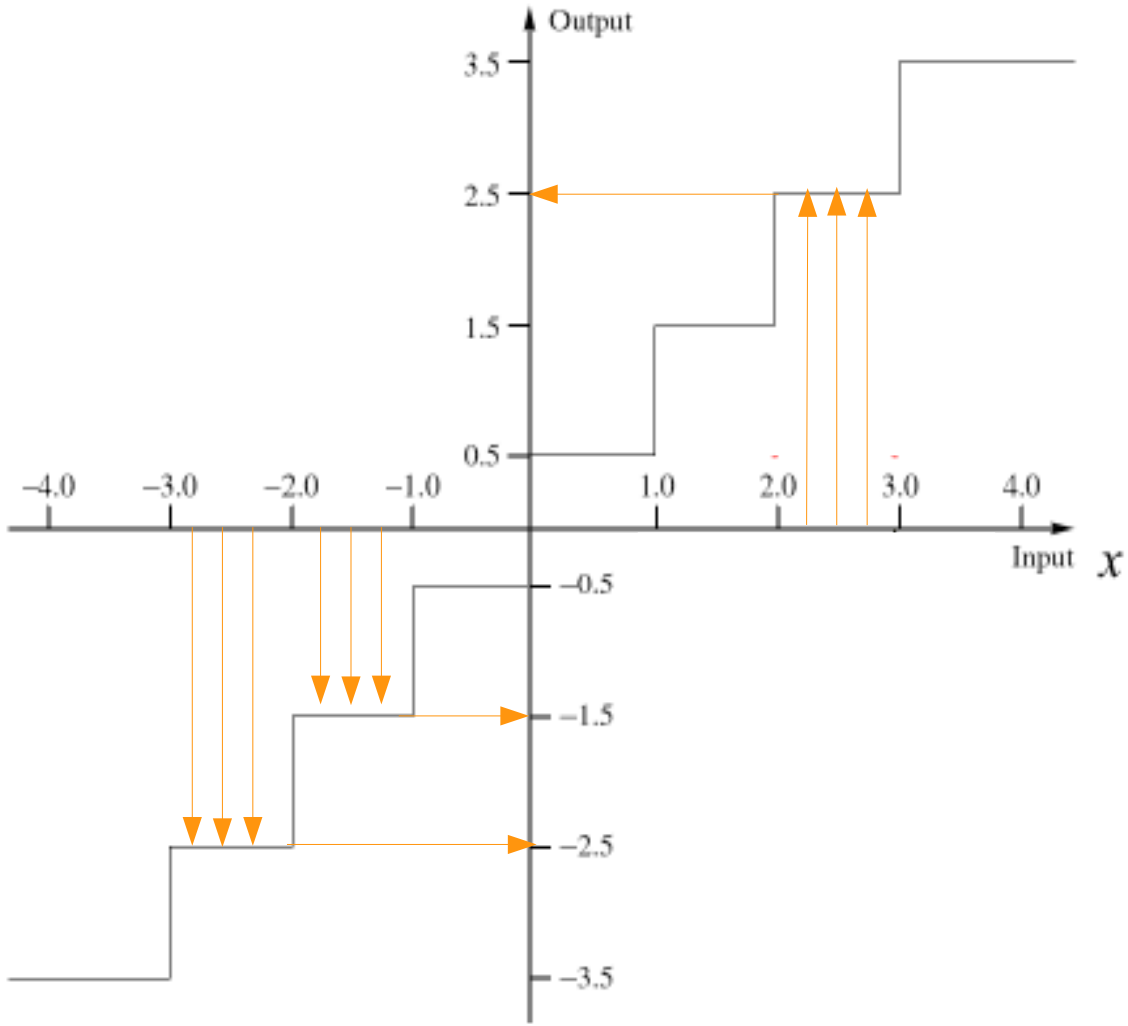
Scalar Quantizer with positive and negative input values



Q	Range	Q^{-1}
000	[-4....-3[	-3.5
001	[-3....-2[	-2.5
010	[-2...-1[	-1.5
011	[-1....0[	-0.5
100	[0...1[	0.5
101	[1....2[	1.5
110	[2....3[	2.5
111	[3....4[	3.5

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Types of Uniform Scalar Quantizers

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Two types of uniform scalar quantizers:

- **Midrise quantizers** have **even** number of output levels.
- **Midtread quantizers** have **odd** number of output levels, **including zero** as one of them

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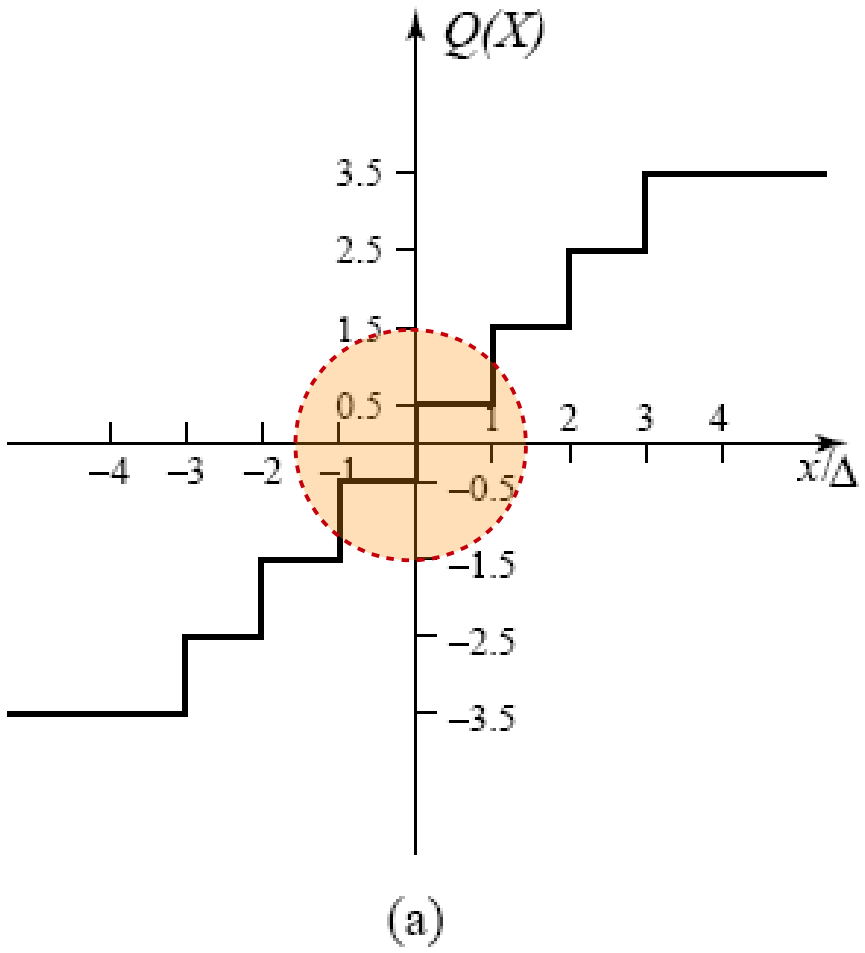
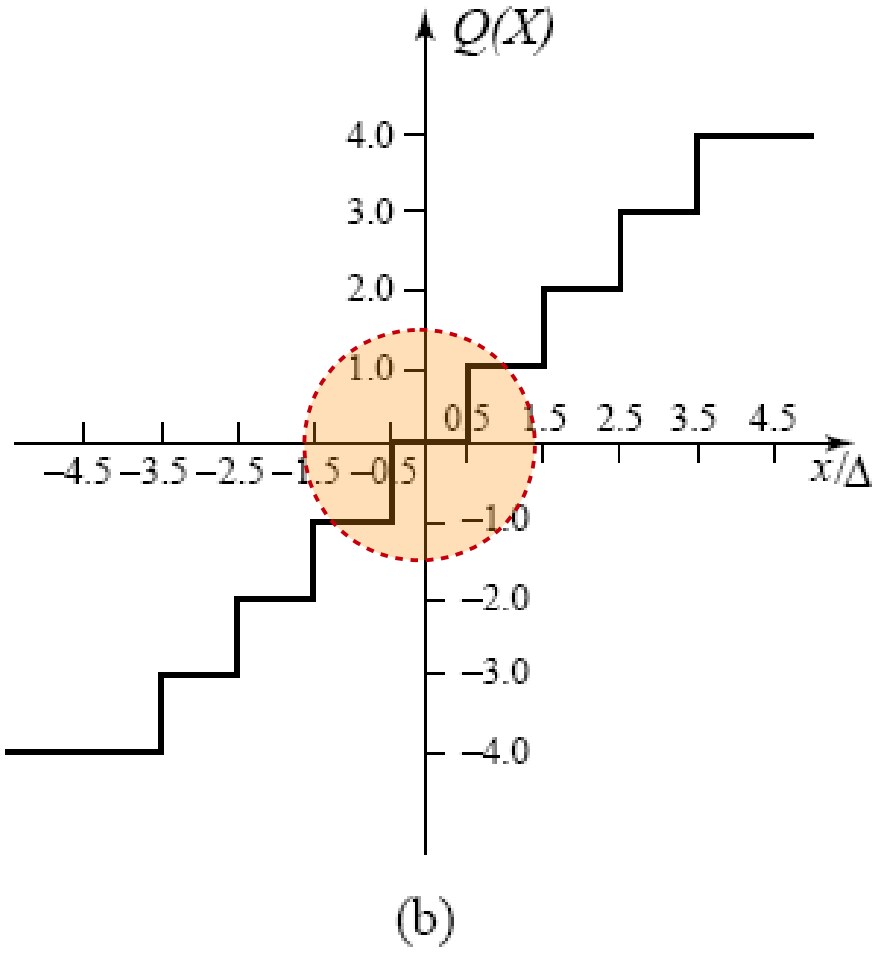
Types of Uniform Scalar Quantizers





Uniform Scalar Quantizers: (a) Midrise, (b) Midtread.

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Effect of Increasing number of bits (Number of Levels) On Quantization Error (Decompressed Image Quality)

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256 Levels (8 Bits) | 32 Levels (5 Bits) | 16 Levels (4 Bits)

8 Levels (3 Bits) | 4 Levels (2 Bits) | 2 Levels (1 Bits)

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Effect of Increasing number of bits (Number of Levels) On Quantization Error (Decompressed Image Quality)

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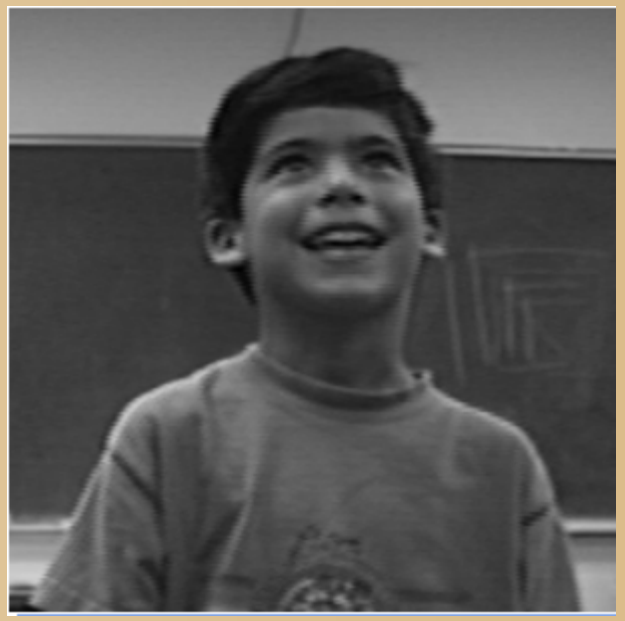
Original Image
8 bits / Pixel



3 bits / Pixel Image

2 bits / Pixel Image

1 bit / Pixel Image

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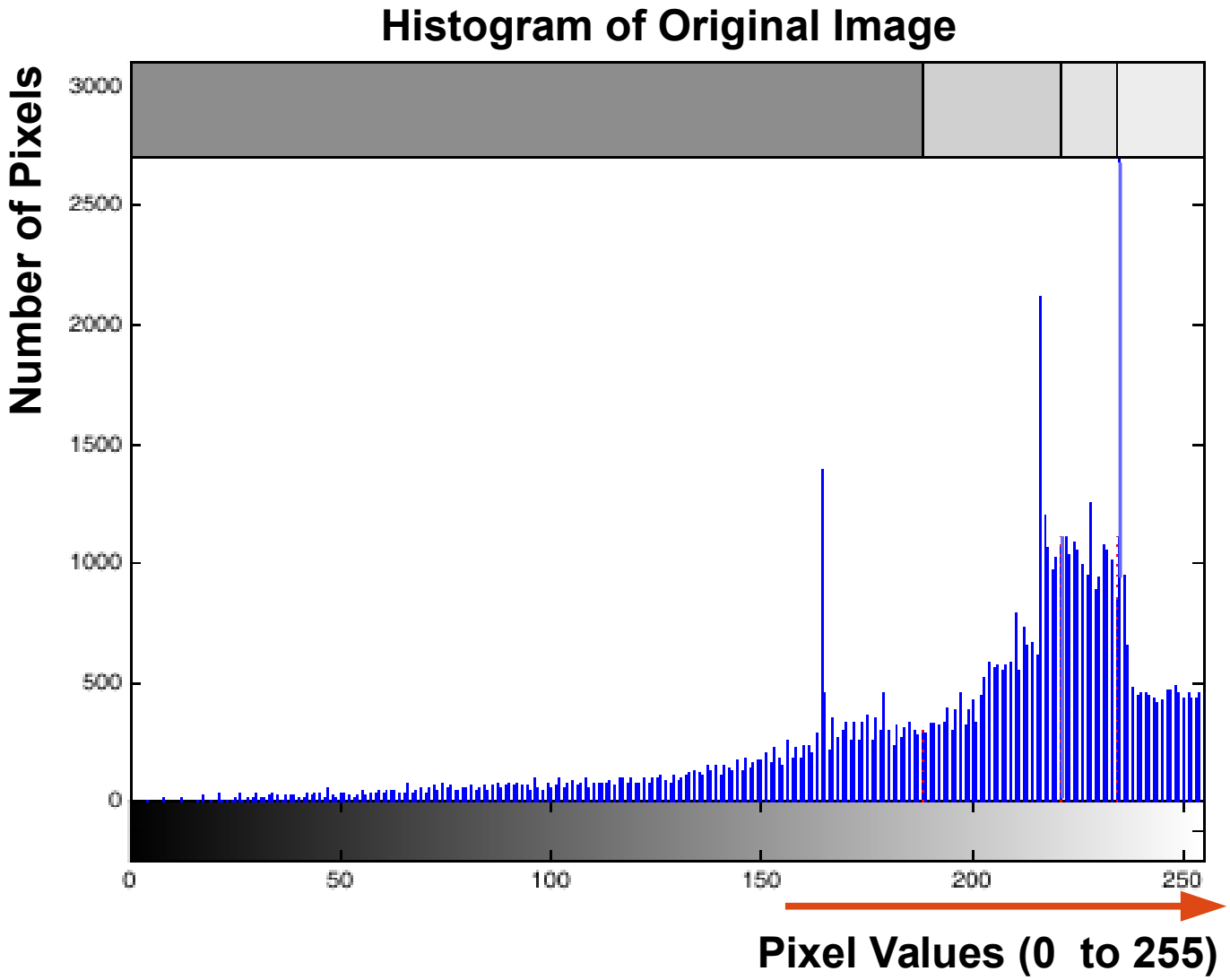
Is uniform Quantizer the Best ?



Original Image

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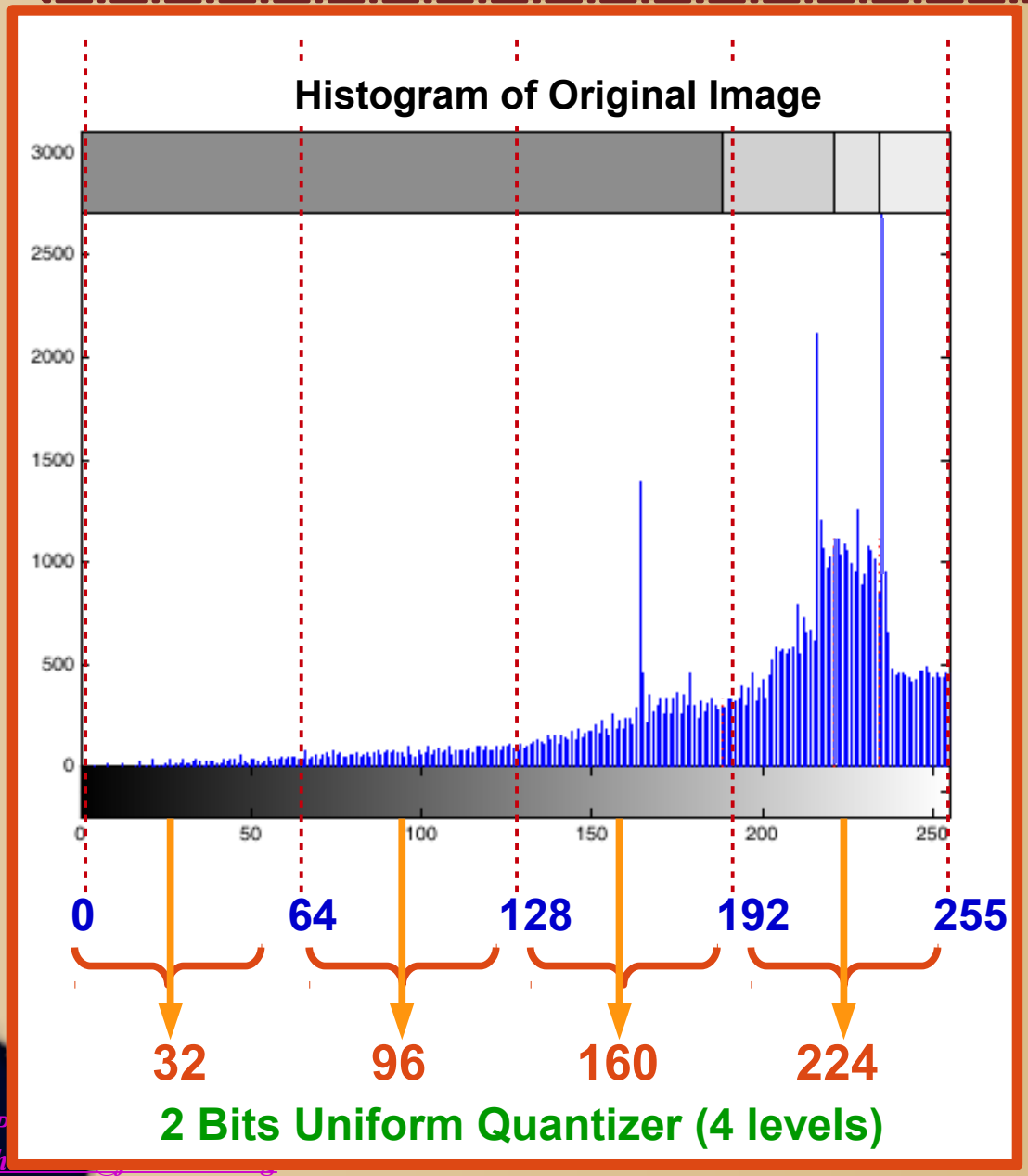
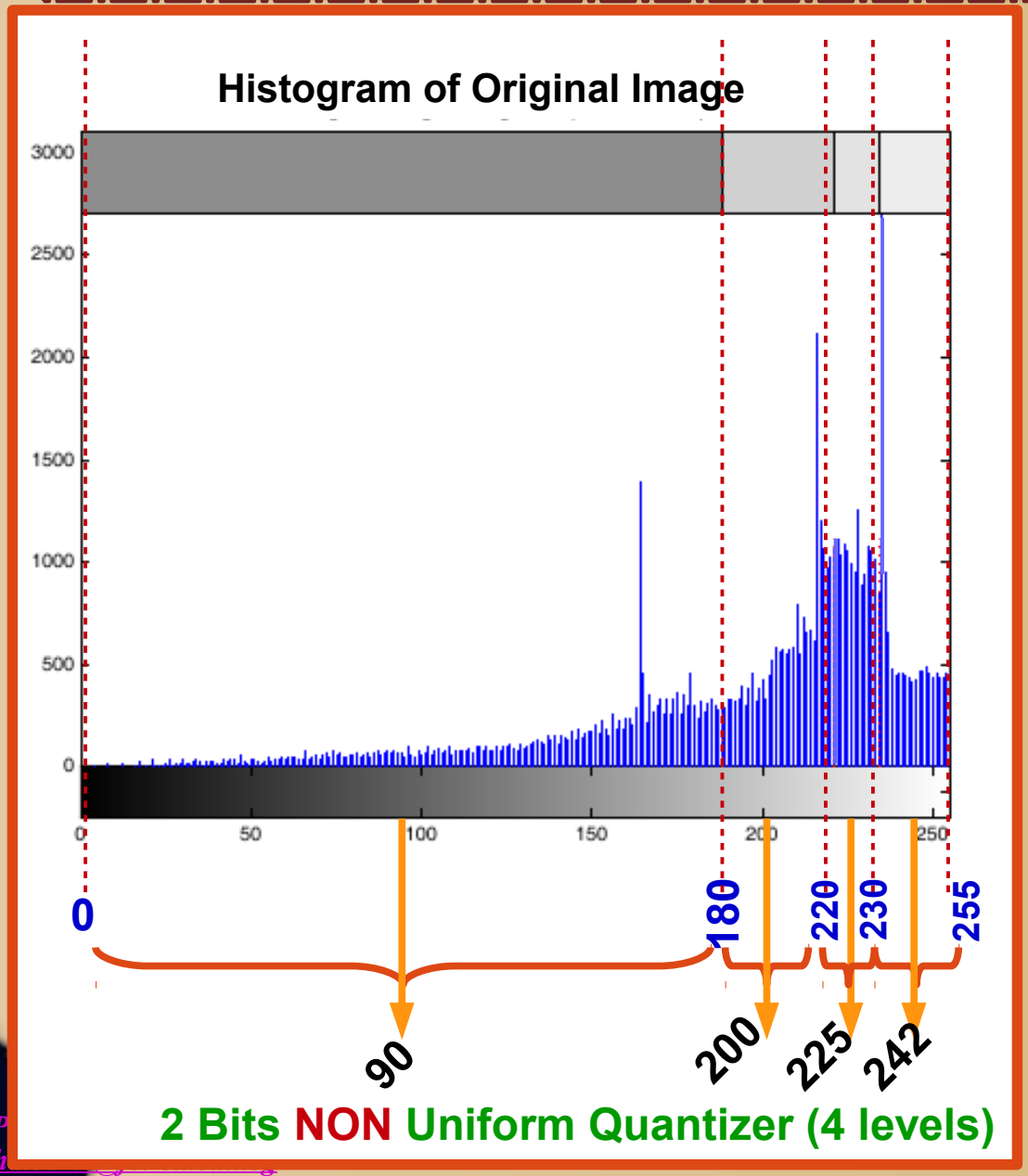
2 Bits Uniform Quantizer



Original Image (256 Gray Levels)

Quantized Image (4 Gray Levels)

2 Bits Non-Uniform Quantizer



Original Image (256 Gray Levels)

Quantized Image (4 Gray Levels)

Comparison Between Uniform and Non Uniform Scalar Quantizers

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Original Image
256 Levels (8 Bits)

2 Bits Compressed Image
Using Uniform Quantizer

2 Bits Compressed Image
Using Non Uniform Quantizer

Both Compressed images are of same size.
Compression ratio is 8:2 = 4:1 for both Images

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Example of Output Quality of Uniform and Non Uniform Quantizers

Example:

Compress the following Data using 2 bits uniform quantizer with step= 32, Full Scale=127

6, 15, 17, 60, 100, 90, 66, 59, 18, 3, 5, 16, 14, 67, 63, 2, 98, 92.

Calculate MSE (as Distortion Measure)

Range	Q	Q^{-1}
0... 31	0	16
32...63	1	48
64...95	2	80
96....127	3	112

Original	6	15	17	60	100	90	66	59	18	3	5	16	14	67	63	2	98	92
Q	0	0	0	1	3	2	2	1	0	0	0	0	0	2	1	0	3	2
Q^{-1}	16	16	16	48	112	80	80	48	16	16	16	16	16	80	48	16	112	80
Error	10	1	1	12	12	10	14	11	2	13	11	0	2	13	15	14	14	12
$Error^2$	100	1	1	144	144	100	196	121	4	169	121	0	4	169	225	196	196	144

Mean Square Error (MSE)= 1/18[100+1+1+144+144+100+196+....]= 2035 /18=113

Max Error = 15 (< ½ Step)

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Example of Output Quality of Uniform and Non Uniform Quantizers

Example:

Compress the following Data using the following 2 bits **Non** uniform quantizer

6, 15, 17, 60, 100, 90, 66, 59, 18, 3, 5, 16, 14, 67, 63, 2, 98, 92.

Calculate MSE (as Distortion Measure)

Range	Q	Q^{-1}
0...10	0	4
11....39	1	16
40...79	2	63
80...127	3	95

Original	6	15	17	60	100	90	66	59	18	3	5	16	14	67	63	2	98	92
Q	0	1	1	2	3	3	2	2	1	0	0	1	1	2	2	0	3	3
Q^{-1}	4	16	16	63	95	95	63	63	16	4	4	16	16	63	63	4	95	95
Error	2	1	1	3	5	5	3	4	2	1	1	0	2	4	0	2	3	3
$Error^2$	4	1	1	9	25	25	9	16	4	1	1	0	4	16	0	4	9	9

Mean Square Error (MSE)= 1/18[4+1+1+9+25+25+9+16+....] = 138 /18=7.66

Max Error = 5

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Design of Non Uniform Quantizer
(using LBG Algorithm with Splitting)

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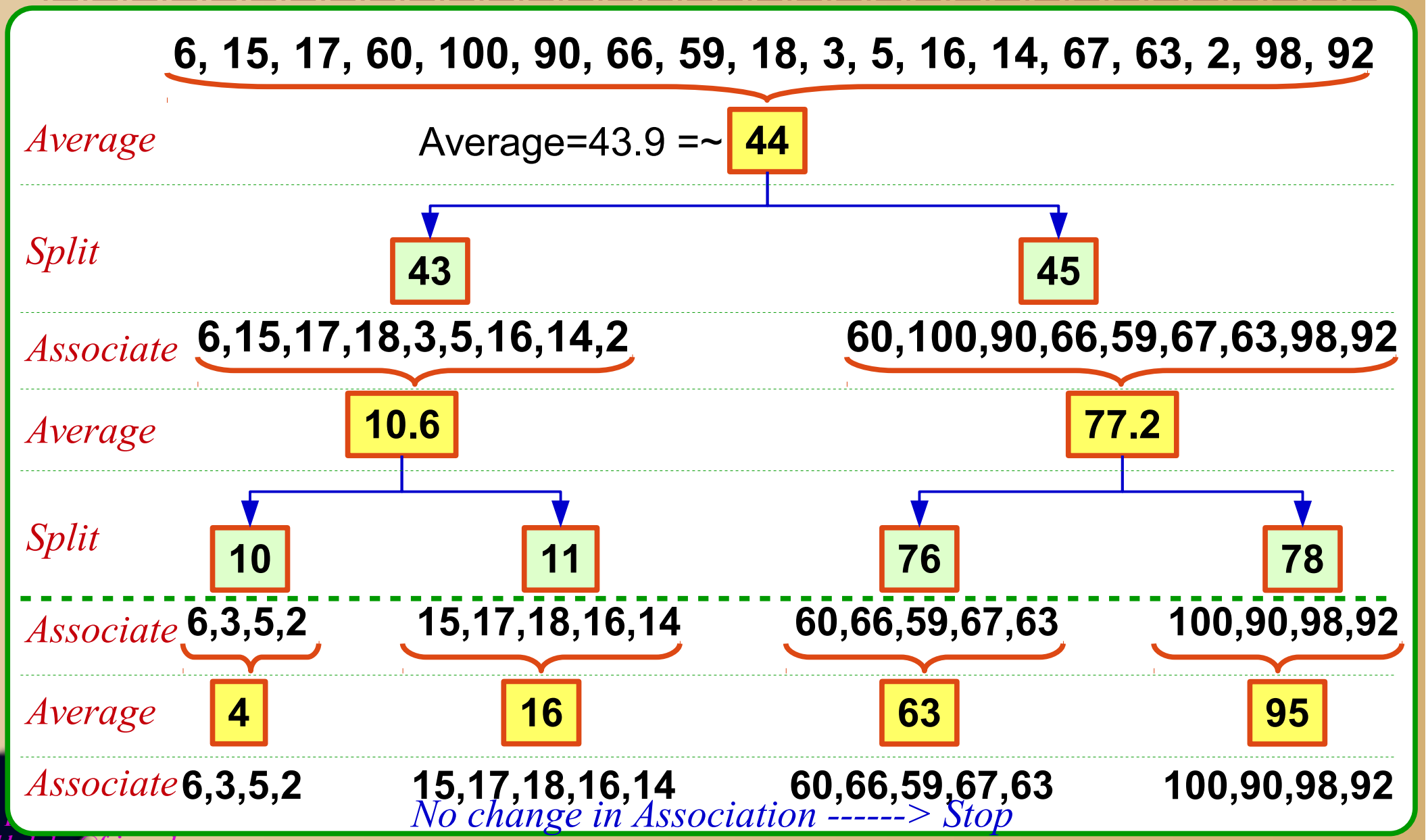

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(using LBG Algorithm with Splitting)

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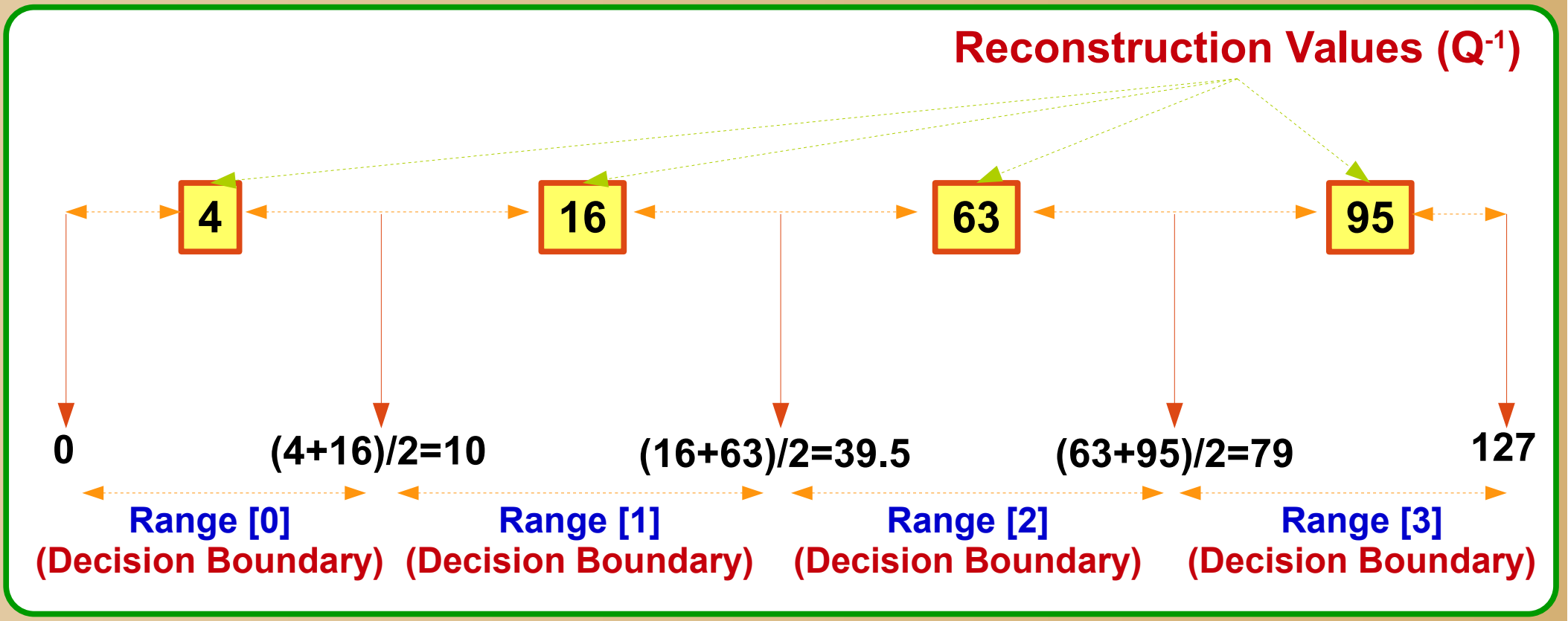

Range	Q	Q^{-1}
[0....10[	0	4
[10... 39.5[	1	16
[39.5....79[	2	63
[79.....127]	3	95

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