The Impact of Federal Reserve Interest Rates on S&P 500 Performance: A Statistical and Multivariate Analysis

Inflation as a Mediator Between Federal Funds Rate and S&P 500. Does inflation mediate the relationship between the federal funds rate and the S&P 500?

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Data Used: FredAPI

Series and Frames used |Data|Definition| |----|----------| |SP500|S&P 500 performance| | FEDFUNDS|Benchmark interest rate set by the Federal Reserve| |CPIAUCSL|Consumer price index for All Urban Consumers (Inflation Rates)|

Step 1: Imports and Setup

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import os
from fredapi import Fred
from functools import reduce
import plotly.io as pio
import plotly.express as px
pio.renderers.default = 'iframe'

color_pal = plt.rcParams["axes.prop_cycle"].by_key()["color"]


API_KEY = os.environ['FRED_API']

pd.set_option('display.max_columns',200)
pd.set_option('display.float_format','{:.2f}'.format)
plt.style.use('ggplot')
```

```
sns.set_context('notebook')
sns.set_style('darkgrid')

fred = Fred(api_key = API_KEY)

sp_search = fred.search('S&P',order_by='popularity')
sp_search.head()
```

```
                              id realtime_start realtime_end  \
series id
BAMLH0A0HYM2      BAMLH0A0HYM2     2024-12-16   2024-12-16
CSUSHPINSA          CSUSHPINSA     2024-12-16   2024-12-16
SP500                    SP500     2024-12-16   2024-12-16
BAMLH0A0HYM2EY  BAMLH0A0HYM2EY     2024-12-16   2024-12-16
BAMLC0A0CM          BAMLC0A0CM     2024-12-16   2024-12-16

                                                            title  \
series id
BAMLH0A0HYM2    ICE BofA US High Yield Index Option-Adjusted S...
CSUSHPINSA      S&P CoreLogic Case-Shiller U.S. National Home ...
SP500                                                     S&P 500
BAMLH0A0HYM2EY       ICE BofA US High Yield Index Effective Yield
BAMLC0A0CM      ICE BofA US Corporate Index Option-Adjusted Sp...

               observation_start observation_end     frequency  \
series id
BAMLH0A0HYM2          1996-12-31      2024-12-12  Daily, Close
CSUSHPINSA            1987-01-01      2024-09-01       Monthly
SP500                 2014-12-15      2024-12-13  Daily, Close
BAMLH0A0HYM2EY        1996-12-31      2024-12-12  Daily, Close
BAMLC0A0CM            1996-12-31      2024-12-12  Daily, Close

               frequency_short               units         units_short
\
series id

BAMLH0A0HYM2                 D             Percent                   %

CSUSHPINSA                   M  Index Jan 2000=100  Index Jan 2000=100

SP500                        D               Index               Index

BAMLH0A0HYM2EY               D             Percent                   %

BAMLC0A0CM                   D             Percent                   %


                    seasonal_adjustment seasonal_adjustment_short  \
series id
BAMLH0A0HYM2    Not Seasonally Adjusted                       NSA
CSUSHPINSA      Not Seasonally Adjusted                       NSA
```

```
SP500            Not Seasonally Adjusted                       NSA
BAMLH0A0HYM2EY  Not Seasonally Adjusted                       NSA
BAMLC0A0CM      Not Seasonally Adjusted                       NSA

                             last_updated popularity  \
series id
BAMLH0A0HYM2    2024-12-13 08:54:09-06:00         92
CSUSHPINSA      2024-11-26 08:12:02-06:00         88
SP500           2024-12-13 19:11:46-06:00         83
BAMLH0A0HYM2EY  2024-12-13 08:54:12-06:00         82
BAMLC0A0CM      2024-12-13 09:00:01-06:00         78

                                                            notes
series id
BAMLH0A0HYM2    The ICE BofA Option-Adjusted Spreads (OASs) ar...
CSUSHPINSA      For more information regarding the index, plea...
SP500           The observations for the S&P 500 represent the...
BAMLH0A0HYM2EY  This data represents the effective yield of th...
BAMLC0A0CM      The ICE BofA Option-Adjusted Spreads (OASs) ar...
```

```
sp500 = pd.DataFrame(fred.get_series('sp500',frequency='m'))
federal_funds_rate =
pd.DataFrame(fred.get_series("FEDFUNDS",frequency='m'))
cpi = pd.DataFrame(fred.get_series('CPIAUCSL',frequency='m'))
unrate = pd.DataFrame(fred.get_series('UNRATE',frequency='q'))
```

```
dfs = [sp500, federal_funds_rate, cpi, unrate]
names = ['sp500','federal_funds_rate','cpi','unemployment']
dfs_names = list(zip(dfs, names))
print(dfs_names)
```

```
[(                0
2014-12-01      NaN
2015-01-01  2028.18
2015-02-01  2082.20
2015-03-01  2079.99
2015-04-01  2094.86
...             ...
2024-08-01  5478.21
2024-09-01  5621.26
2024-10-01  5792.32
2024-11-01  5929.92
2024-12-01      NaN

[121 rows x 1 columns], 'sp500'), (              0
1954-07-01 0.80
1954-08-01 1.22
1954-09-01 1.07
1954-10-01 0.85
1954-11-01 0.83
```

```
...           ...
2024-07-01 5.33
2024-08-01 5.33
2024-09-01 5.13
2024-10-01 4.83
2024-11-01 4.64

[845 rows x 1 columns], 'federal_funds_rate'), (                0
1947-01-01  21.48
1947-02-01  21.62
1947-03-01  22.00
1947-04-01  22.00
1947-05-01  21.95
...           ...
2024-07-01 313.53
2024-08-01 314.12
2024-09-01 314.69
2024-10-01 315.45
2024-11-01 316.44

[935 rows x 1 columns], 'cpi'), (              0
1948-01-01 3.70
1948-04-01 3.70
1948-07-01 3.80
1948-10-01 3.80
1949-01-01 4.70
...           ...
2023-10-01 3.70
2024-01-01 3.80
2024-04-01 4.00
2024-07-01 4.20
2024-10-01  NaN

[308 rows x 1 columns], 'unemployment')]
```

```
for i in dfs_names:
    i[0].sort_index(ascending=True, inplace=True)
    i[0].rename(columns={0:i[1]},inplace=True)
    i[0].index = pd.to_datetime(i[0].index)

sp500
```

```
             sp500
2014-12-01     NaN
2015-01-01 2028.18
2015-02-01 2082.20
2015-03-01 2079.99
2015-04-01 2094.86
...            ...
2024-08-01 5478.21
```

```
2024-09-01 5621.26
2024-10-01 5792.32
2024-11-01 5929.92
2024-12-01     NaN

[121 rows x 1 columns]

cpi

               cpi
1947-01-01  21.48
1947-02-01  21.62
1947-03-01  22.00
1947-04-01  22.00
1947-05-01  21.95
...           ...
2024-07-01 313.53
2024-08-01 314.12
2024-09-01 314.69
2024-10-01 315.45
2024-11-01 316.44

[935 rows x 1 columns]

unrate

            unemployment
1948-01-01          3.70
1948-04-01          3.70
1948-07-01          3.80
1948-10-01          3.80
1949-01-01          4.70
...                  ...
2023-10-01          3.70
2024-01-01          3.80
2024-04-01          4.00
2024-07-01          4.20
2024-10-01           NaN

[308 rows x 1 columns]
```

Step 2: Data Preperation

```
df = reduce(lambda left, right: pd.merge(left,right, left_index=True,
right_index=True,how='inner'), dfs)
# df = pd.concat(dfs, axis=1, join='inner')
df.sort_index(ascending=False)
```

	sp500	federal_funds_rate	cpi	unemployment
2024-10-01	5792.32	4.83	315.45	NaN
2024-07-01	5538.00	5.33	313.53	4.20
2024-04-01	5112.49	5.33	313.21	4.00
2024-01-01	4804.49	5.33	309.69	3.80
2023-10-01	4269.40	5.33	307.53	3.70
2023-07-01	4508.08	5.12	304.63	3.70
2023-04-01	4121.47	4.83	303.03	3.60
2023-01-01	3960.66	4.33	300.36	3.50
2022-10-01	3726.05	3.08	297.86	3.60
2022-07-01	3911.73	1.68	294.98	3.50
2022-04-01	4391.30	0.33	288.76	3.60
2022-01-01	4573.82	0.08	282.39	3.80
2021-10-01	4460.71	0.08	276.43	4.20
2021-07-01	4363.71	0.10	271.99	5.10
2021-04-01	4141.18	0.07	266.75	5.90
2021-01-01	3793.75	0.09	262.52	6.20
2020-10-01	3418.70	0.09	260.25	6.70
2020-07-01	3207.62	0.09	258.41	8.80
2020-04-01	2761.98	0.05	256.13	13.00
2020-01-01	3278.20	1.55	258.91	3.80
2019-10-01	2977.68	1.83	257.15	3.60
2019-07-01	2996.11	2.40	255.80	3.60
2019-04-01	2903.80	2.42	255.23	3.60
2019-01-01	2607.39	2.40	252.56	3.90
2018-10-01	2785.46	2.19	252.77	3.80
2018-07-01	2793.64	1.91	251.21	3.80
2018-04-01	2653.63	1.69	250.23	3.90
2018-01-01	2789.80	1.41	248.86	4.00
2017-10-01	2557.00	1.15	246.63	4.20
2017-07-01	2454.10	1.15	244.24	4.30
2017-04-01	2359.31	0.90	244.19	4.40
2017-01-01	2275.12	0.65	243.62	4.60
2016-10-01	2143.02	0.40	241.74	4.80
2016-07-01	2148.90	0.39	240.10	4.90
2016-04-01	2075.54	0.37	238.99	4.90
2016-01-01	1918.60	0.34	237.65	4.90
2015-10-01	2024.81	0.12	237.73	5.00
2015-07-01	2094.14	0.13	238.03	5.10
2015-04-01	2094.86	0.12	236.22	5.40
2015-01-01	2028.18	0.11	234.75	5.50

```
df.isna().sum()
```

```
sp500                 0
federal_funds_rate    0
cpi                   0
unemployment          1
dtype: int64
```

```
df.loc[df['unemployment'].isna()]
```

```
              sp500  federal_funds_rate    cpi  unemployment
2024-10-01 5792.32                4.83 315.45           NaN
```

```
df.tail(5)
```

```
              sp500  federal_funds_rate    cpi  unemployment
2023-10-01 4269.40                5.33 307.53          3.70
2024-01-01 4804.49                5.33 309.69          3.80
2024-04-01 5112.49                5.33 313.21          4.00
2024-07-01 5538.00                5.33 313.53          4.20
2024-10-01 5792.32                4.83 315.45           NaN
```

```
df.dropna(inplace=True)
```

```
df.tail(5)
```

```
              sp500  federal_funds_rate    cpi  unemployment
2023-07-01 4508.08                5.12 304.63          3.70
2023-10-01 4269.40                5.33 307.53          3.70
2024-01-01 4804.49                5.33 309.69          3.80
2024-04-01 5112.49                5.33 313.21          4.00
2024-07-01 5538.00                5.33 313.53          4.20
```

```
df[df.duplicated()].sum()
```

```
sp500                 0.00
federal_funds_rate    0.00
cpi                   0.00
unemployment          0.00
dtype: float64
```

```
df.index.unique()
```

```
DatetimeIndex(['2015-01-01', '2015-04-01', '2015-07-01', '2015-10-01',
               '2016-01-01', '2016-04-01', '2016-07-01', '2016-10-01',
               '2017-01-01', '2017-04-01', '2017-07-01', '2017-10-01',
               '2018-01-01', '2018-04-01', '2018-07-01', '2018-10-01',
               '2019-01-01', '2019-04-01', '2019-07-01', '2019-10-01',
               '2020-01-01', '2020-04-01', '2020-07-01', '2020-10-01',
               '2021-01-01', '2021-04-01', '2021-07-01', '2021-10-01',
               '2022-01-01', '2022-04-01', '2022-07-01', '2022-10-01',
               '2023-01-01', '2023-04-01', '2023-07-01', '2023-10-01',
               '2024-01-01', '2024-04-01', '2024-07-01'],
              dtype='datetime64[ns]', freq=None)
```

Step 3: Trends

```
df.info()
```

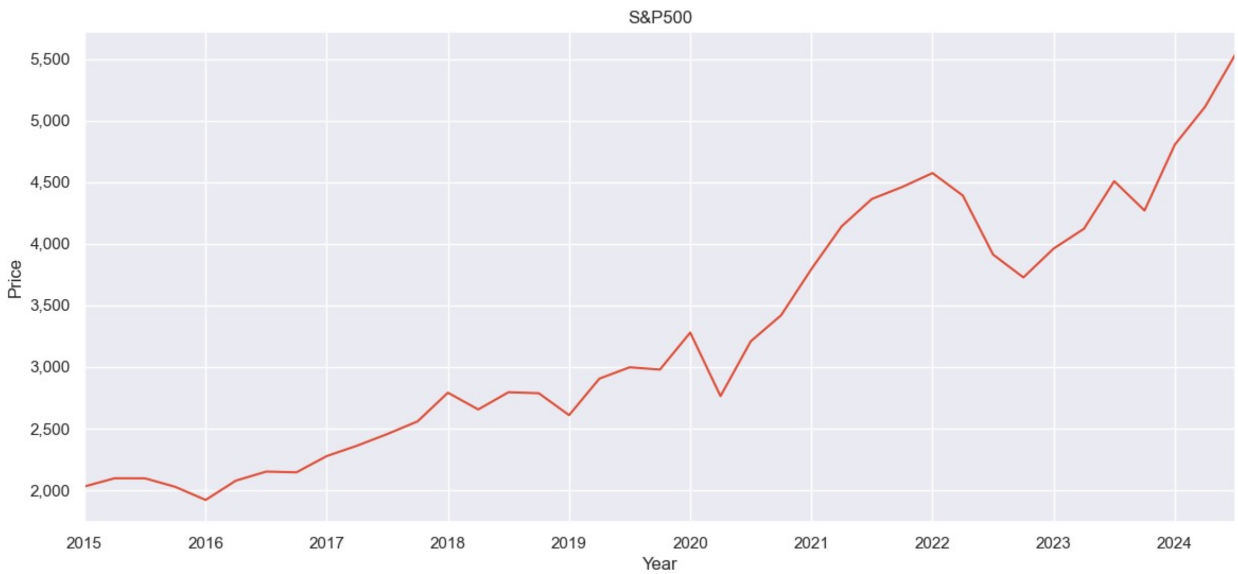
```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 39 entries, 2015-01-01 to 2024-07-01
Data columns (total 4 columns):
 #   Column              Non-Null Count  Dtype
---  ------              --------------  -----
 0   sp500               39 non-null     float64
 1   federal_funds_rate  39 non-null     float64
 2   cpi                 39 non-null     float64
 3   unemployment        39 non-null     float64
dtypes: float64(4)
memory usage: 2.6 KB
```

```
df.describe()
```

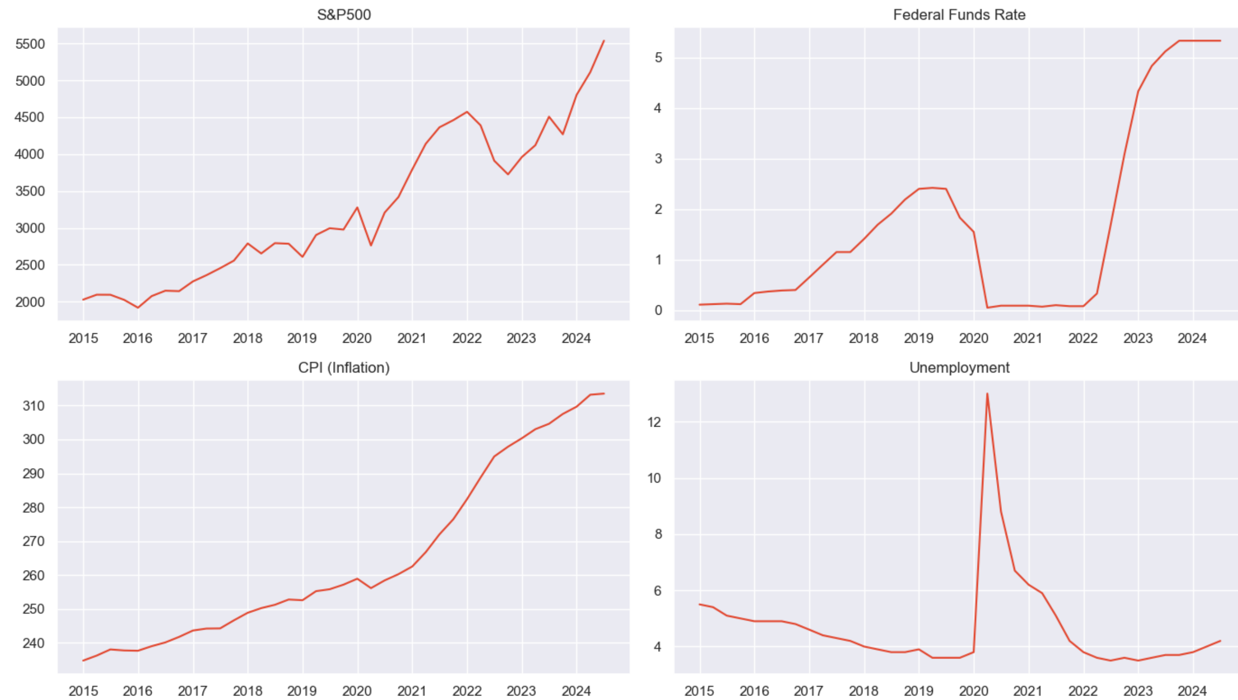
```
        sp500  federal_funds_rate    cpi  unemployment
count   39.00               39.00  39.00         39.00
mean  3257.04                1.67 265.00          4.69
std   1007.39                1.83  25.46          1.74
min   1918.60                0.05 234.75          3.50
25%   2406.70                0.12 244.22          3.75
50%   2977.68                1.15 256.13          4.20
75%   4131.33                2.40 285.58          4.95
max   5538.00                5.33 313.53         13.00
```

```
ax= df['sp500'].plot(kind='line',figsize=(14,6),title='S&P500')
ax.set_xlabel("Year")
ax.set_ylabel("Price")
ax.set_yticklabels(['{:,.0f}'.format(i) for i in plt.yticks()[0]])
plt.show()
```

```
C:\Users\abdal\AppData\Local\Temp\ipykernel_12040\2371682248.py:4:
UserWarning:

set_ticklabels() should only be used with a fixed number of ticks,
i.e. after set_ticks() or using a FixedLocator.
```



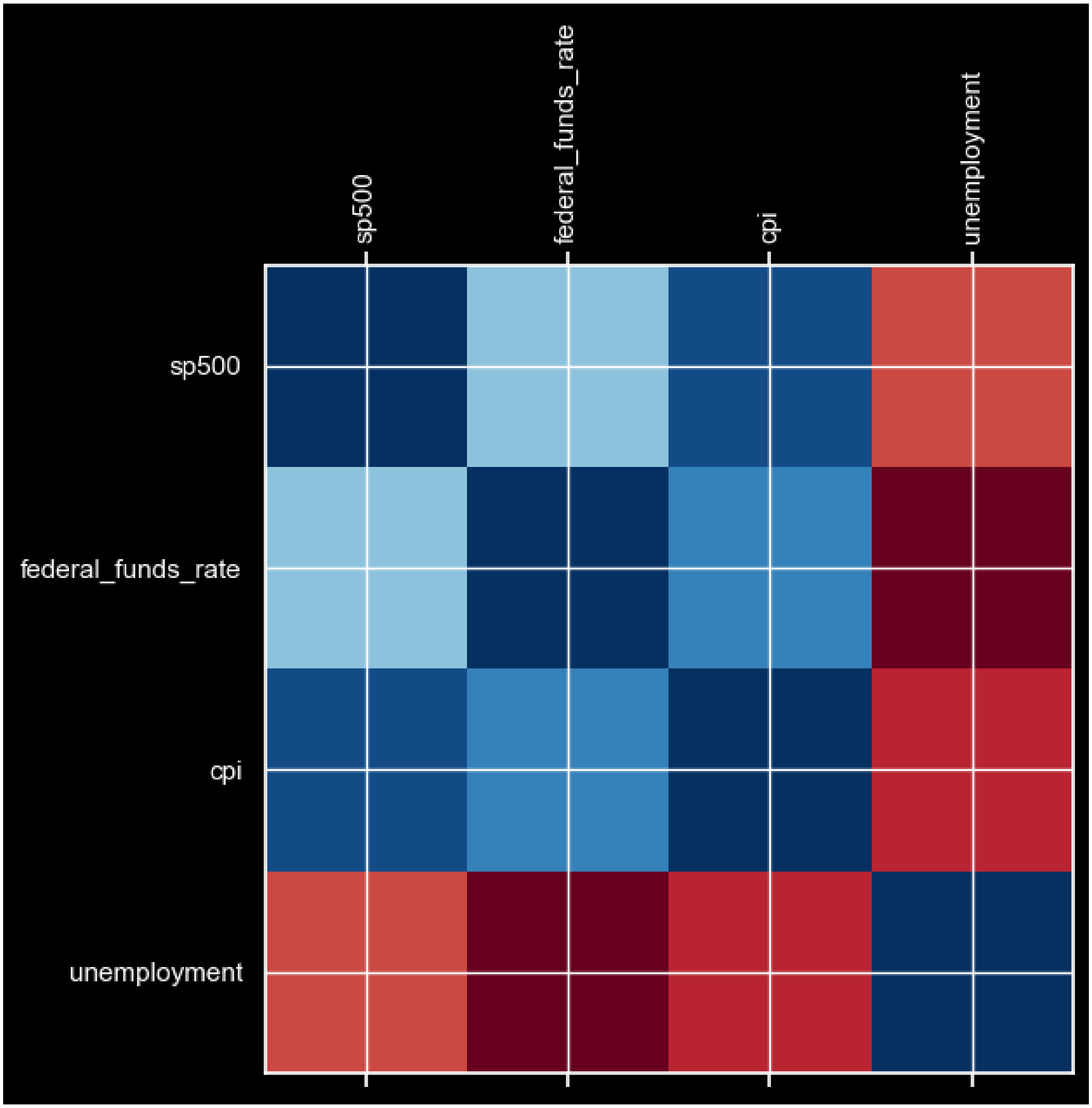

```
df.columns

Index(['sp500', 'federal_funds_rate', 'cpi', 'unemployment'],
dtype='object')

fig, ax = plt.subplots(2,2,figsize=(14,8))

ax[0,0].plot(df['sp500'])
ax[0,0].set_title('S&P500')
ax[0,1].plot(df['federal_funds_rate'])
ax[0,1].set_title('Federal Funds Rate')
ax[1,0].plot(df['cpi'])
ax[1,0].set_title('CPI (Inflation)')
ax[1,1].plot(df['unemployment'])
ax[1,1].set_title('Unemployment')

plt.tight_layout()
plt.show()
```




```
corr = df.corr()
corr
```

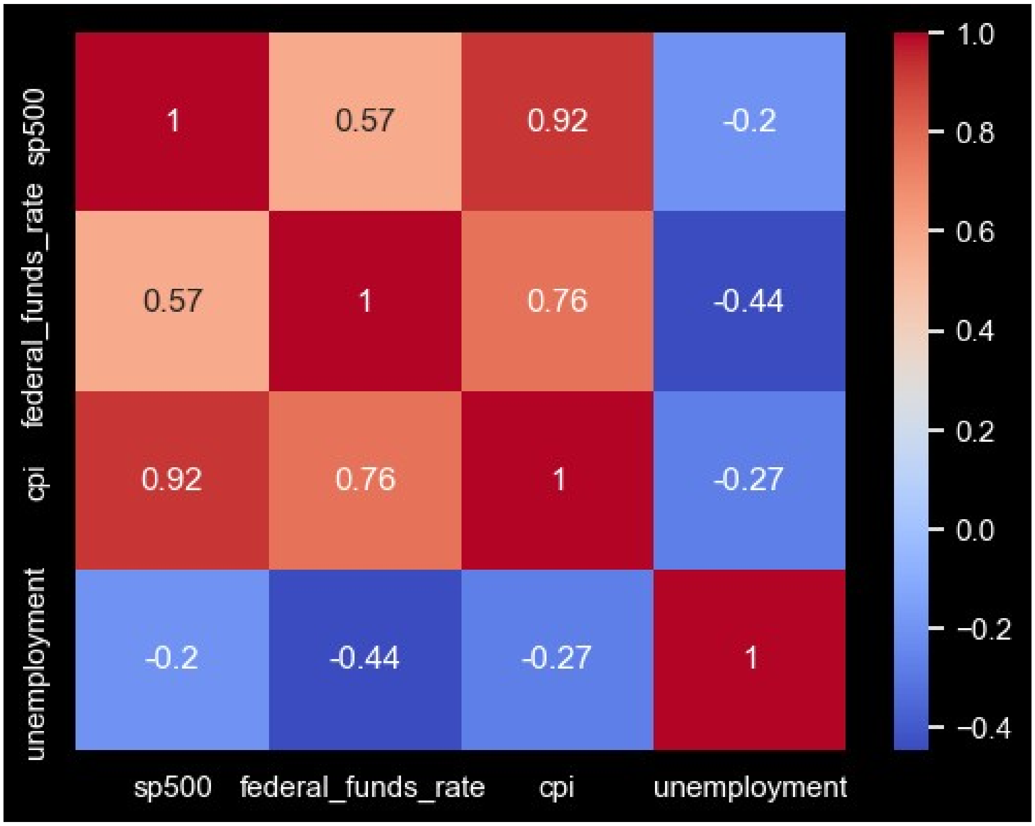
	sp500	federal_funds_rate	cpi	unemployment
sp500	1.00	0.57	0.92	-0.20
federal_funds_rate	0.57	1.00	0.76	-0.44
cpi	0.92	0.76	1.00	-0.27
unemployment	-0.20	-0.44	-0.27	1.00

```
plt.style.use('dark_background')
fig = plt.figure(figsize=(14,6))
plt.matshow(corr, cmap='RdBu',fignum=fig.number)
plt.xticks(range(len(corr.columns)),corr.columns,rotation='vertical')
plt.yticks(range(len(corr.columns)),corr.columns)
plt.show()
```

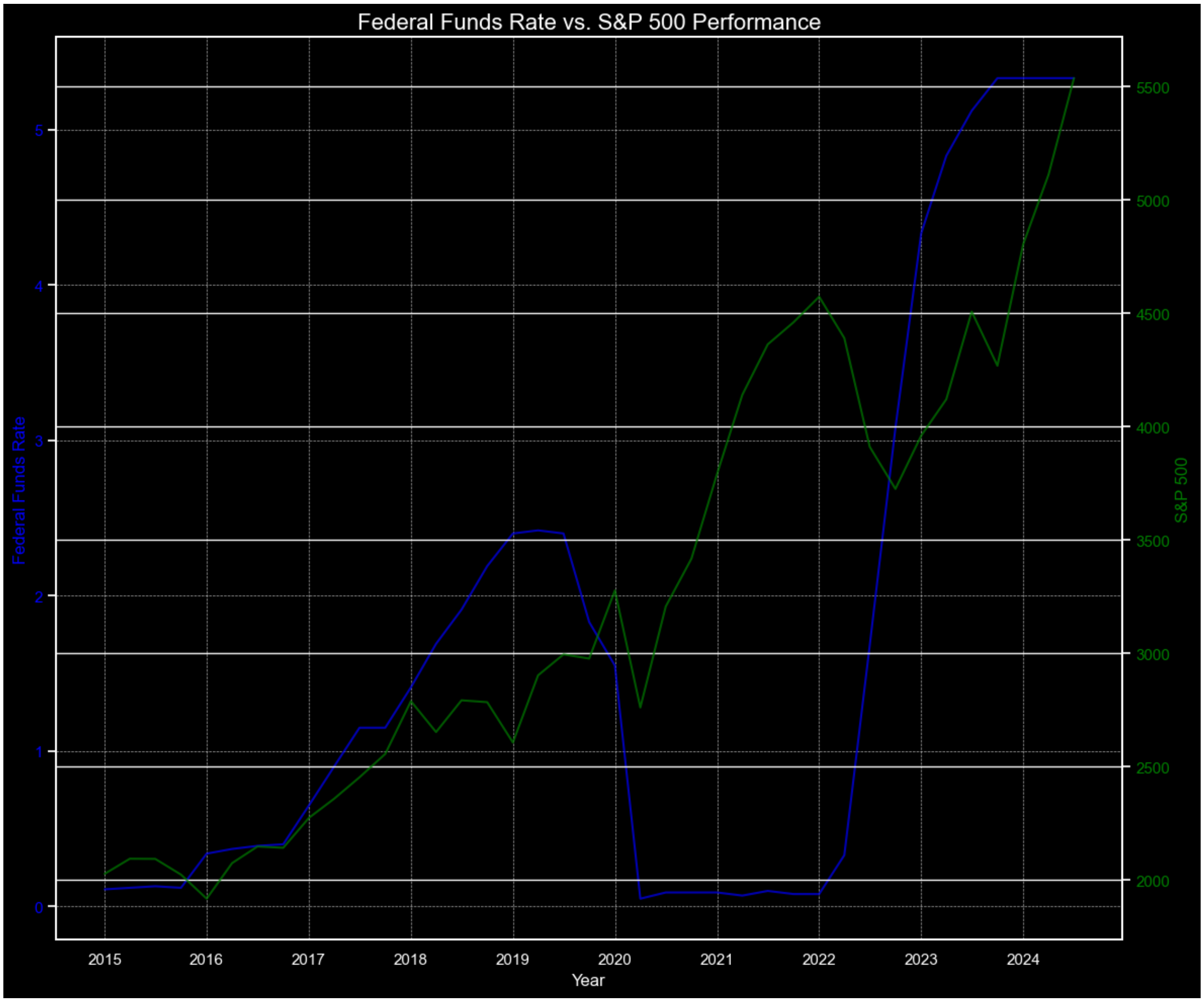



```
sns.heatmap(data=corr,annot=True,cmap='coolwarm')
```

```
<Axes: >
```




```
fig, ax1 = plt.subplots(figsize=(12, 10))

ax1.plot(df.index, df['federal_funds_rate'], label='Federal Funds
Rate', color='blue', alpha=0.7)
ax1.set_ylabel('Federal Funds Rate', fontsize=12, color='blue')
ax1.tick_params(axis='y', labelcolor='blue')
ax1.grid(True, which='both', linestyle='--', linewidth=0.5, alpha=0.7)

ax2 = ax1.twinx()
ax2.plot(df.index, df['sp500'], label='S&P 500', color='green',
alpha=0.7)
ax2.set_ylabel('S&P 500', fontsize=12, color='green')
ax2.tick_params(axis='y', labelcolor='green')

plt.title('Federal Funds Rate vs. S&P 500 Performance', fontsize=16)
ax1.set_xlabel('Year', fontsize=12)

fig.tight_layout()
plt.show()
```



Step 4: Statistics

To investigate whether inflation mediates the relationship between the federal funds rate (IV) and the S&P 500 (DV).

```
df
```

```
             sp500  federal_funds_rate     cpi  unemployment
2015-01-01 2028.18                0.11  234.75          5.50
2015-04-01 2094.86                0.12  236.22          5.40
2015-07-01 2094.14                0.13  238.03          5.10
2015-10-01 2024.81                0.12  237.73          5.00
2016-01-01 1918.60                0.34  237.65          4.90
2016-04-01 2075.54                0.37  238.99          4.90
2016-07-01 2148.90                0.39  240.10          4.90
2016-10-01 2143.02                0.40  241.74          4.80
2017-01-01 2275.12                0.65  243.62          4.60
```

```
2017-04-01 2359.31                0.90 244.19          4.40
2017-07-01 2454.10                1.15 244.24          4.30
2017-10-01 2557.00                1.15 246.63          4.20
2018-01-01 2789.80                1.41 248.86          4.00
2018-04-01 2653.63                1.69 250.23          3.90
2018-07-01 2793.64                1.91 251.21          3.80
2018-10-01 2785.46                2.19 252.77          3.80
2019-01-01 2607.39                2.40 252.56          3.90
2019-04-01 2903.80                2.42 255.23          3.60
2019-07-01 2996.11                2.40 255.80          3.60
2019-10-01 2977.68                1.83 257.15          3.60
2020-01-01 3278.20                1.55 258.91          3.80
2020-04-01 2761.98                0.05 256.13         13.00
2020-07-01 3207.62                0.09 258.41          8.80
2020-10-01 3418.70                0.09 260.25          6.70
2021-01-01 3793.75                0.09 262.52          6.20
2021-04-01 4141.18                0.07 266.75          5.90
2021-07-01 4363.71                0.10 271.99          5.10
2021-10-01 4460.71                0.08 276.43          4.20
2022-01-01 4573.82                0.08 282.39          3.80
2022-04-01 4391.30                0.33 288.76          3.60
2022-07-01 3911.73                1.68 294.98          3.50
2022-10-01 3726.05                3.08 297.86          3.60
2023-01-01 3960.66                4.33 300.36          3.50
2023-04-01 4121.47                4.83 303.03          3.60
2023-07-01 4508.08                5.12 304.63          3.70
2023-10-01 4269.40                5.33 307.53          3.70
2024-01-01 4804.49                5.33 309.69          3.80
2024-04-01 5112.49                5.33 313.21          4.00
2024-07-01 5538.00                5.33 313.53          4.20
```

Direct effect: How the independent variable (Federal Funds Rate) affects the dependent variable (S&P 500) directly.
Indirect effect: How the independent variable affects the dependent variable through the mediator (Inflation).

Independent Variable (IV): The variable I believe influences the outcome indirectly (e.g., Federal Funds Rate).
Mediator (M): The variable through which the IV is hypothesized to affect the outcome (e.g., Inflation).
Dependent Variable (DV): The outcome variable (e.g., S&P 500).

Step 1: Define Hypotheses

- H1: The Federal Funds Rate influences Inflation.
- H2: Inflation influences the S&P 500.
- H3: The Federal Funds Rate indirectly affects the S&P 500 through Inflation.

Step 2: Fit the Models

Path A (IV → Mediator): Regress the mediator on the independent variable to estimate how the IV influences the mediator.
$M = \beta 0 + \beta 1 \times IV + \epsilon$
Path B (Mediator → DV): Regress the dependent variable on both the mediator and the independent variable to estimate the mediator's effect on the DV, controlling for the IV.
$DV = \beta 0 + \beta 1 \times M + \beta 2 \times IV + \epsilon$
Total Effect (IV → DV): Regress the dependent variable on the independent variable to estimate the total effect.
$DV = \beta 0 + \beta 1 \times IV + \epsilon$

Step 3: Calculate the Effects

Direct Effect: The effect of the IV on the DV after accounting for the mediator (β2 in Path B).
Indirect Effect: The effect of the IV on the DV through the mediator. This is calculated as:
Indirect Effect = (β1 from Path A) × (β1 from Path B)
Total Effect: The sum of the direct and indirect effects.

Step 4: Test the Significance of the Indirect Effect

Use a statistical test to determine whether the indirect effect is significant:
Sobel Test: Tests the significance of the indirect effect using the standard errors of Path A and Path B.
Bootstrapping: Repeatedly samples the data to compute confidence intervals for the indirect effect (preferred for small sample sizes or non-normal data).

Step 5: Interpret Results

If the indirect effect is significant, the mediator explains part of the relationship between the IV and the DV.
Check the direct effect to determine whether the IV still has an influence on the DV after accounting for the mediator:
If the direct effect becomes non-significant, the mediation is full.
If the direct effect remains significant, the mediation is partial.

Step 6: Report Findings

Report the coefficients (β) for each path.
Discuss the significance of the direct, indirect, and total effects.
Include a mediation diagram to visualize the relationships.

IV: federal_funds_rate
Mediator (M): cpi (proxy for inflation)
DV: sp500

```
import statsmodels.api as sm
from statsmodels.formula.api import ols
from statsmodels.stats.mediation import Mediation
```

Path A: IV → Mediator

This estimates how federal_funds_rate (IV) influences cpi (Mediator):

```
model_a_fitted = ols("cpi ~ federal_funds_rate", data=df).fit()
print("Path A: IV → Mediator")
print(model_a_fitted.summary())

Path A: IV → Mediator
                            OLS Regression Results                            

======================================================================
========
Dep. Variable:                    cpi   R-squared:                       
0.583
Model:                            OLS   Adj. R-squared:                  
0.572
Method:                 Least Squares   F-statistic:                     
51.81
Date:                Mon, 16 Dec 2024   Prob (F-statistic):           
1.55e-08
Time:                        00:44:10   Log-Likelihood:                
-164.01
No. Observations:                  39   AIC:                             
332.0
Df Residuals:                      37   BIC:                             
335.4
Df Model:                           1                                         

Covariance Type:            nonrobust                                         

======================================================================
================
                         coef    std err          t      P>|t|      
[0.025      0.975]
----------------------------------------------------------------------
----------------
Intercept            247.2845      3.630     68.130      0.000     
239.930     254.639
federal_funds_rate    10.6354      1.478      7.198      0.000       
7.641      13.629
======================================================================
========
Omnibus:                        4.377   Durbin-Watson:                   
0.076
Prob(Omnibus):                  0.112   Jarque-Bera (JB):                
4.010
Skew:                           0.721   Prob(JB):                        
0.135
Kurtosis:                       2.377   Cond. No.                        
```

```
3.62
==============================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

Path B: Mediator + IV → DV

This estimates how cpi (Mediator) and federal_funds_rate (IV) jointly influence sp500 (DV):

```
model_b_fitted = ols("sp500 ~ cpi + federal_funds_rate",
data=df).fit()
print("\nPath B: Mediator + IV → DV")
print(model_b_fitted.summary())
```

```
Path B: Mediator + IV → DV
                            OLS Regression Results

==============================================================================
========
Dep. Variable:                  sp500   R-squared:
0.896
Model:                            OLS   Adj. R-squared:
0.891
Method:                 Least Squares   F-statistic:
155.7
Date:                Mon, 16 Dec 2024   Prob (F-statistic):
1.90e-18
Time:                        00:44:14   Log-Likelihood:
-280.31
No. Observations:                  39   AIC:
566.6
Df Residuals:                      36   BIC:
571.6
Df Model:                           2

Covariance Type:            nonrobust

==============================================================================
================
                         coef    std err          t      P>|t|
[0.025      0.975]
------------------------------------------------------------------------------
----------------
Intercept          -8707.0086    816.347    -10.666      0.000   -
1.04e+04   -7051.380
```

```
cpi                         46.2607      3.288     14.069      0.000
39.592      52.929
federal_funds_rate  -177.1566     45.787     -3.869      0.000    -
270.016     -84.297
======================================================================
========
Omnibus:                        0.980   Durbin-Watson:
0.545
Prob(Omnibus):                  0.613   Jarque-Bera (JB):
0.578
Skew:                          -0.298   Prob(JB):
0.749
Kurtosis:                       3.024   Cond. No.
4.08e+03
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 4.08e+03. This might indicate that
there are
strong multicollinearity or other numerical problems.
```

Total Effect: IV → DV

This estimates the total effect of federal_funds_rate (IV) on sp500 (DV) without accounting for the mediator:

```
model_total = ols("sp500 ~ federal_funds_rate", data=df).fit()
print("\nTotal Effect: IV → DV")
print(model_total.summary())
```

```
Total Effect: IV → DV
                            OLS Regression Results

======================================================================
========
Dep. Variable:                  sp500   R-squared:
0.327
Model:                            OLS   Adj. R-squared:
0.308
Method:                 Least Squares   F-statistic:
17.95
Date:                Mon, 16 Dec 2024   Prob (F-statistic):
0.000144
Time:                        00:44:17   Log-Likelihood:
-316.81
```

```
No. Observations:                      39   AIC:
637.6
Df Residuals:                          37   BIC:
640.9
Df Model:                               1

Covariance Type:            nonrobust

======================================================================
================
                         coef    std err          t      P>|t|
[0.025      0.975]
----------------------------------------------------------------------
----------------
Intercept           2732.5363    182.540     14.970      0.000
2362.676    3102.397
federal_funds_rate   314.8456     74.313      4.237      0.000
164.274     465.418
======================================================================
========
Omnibus:                        5.722   Durbin-Watson:
0.120
Prob(Omnibus):                  0.057   Jarque-Bera (JB):
5.572
Skew:                           0.892   Prob(JB):
0.0617
Kurtosis:                       2.502   Cond. No.
3.62
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

Using the Mediation Package

The mediation package allows to calculate the indirect effect directly:

```
model_a = ols("cpi ~ federal_funds_rate", data=df)
model_b = ols("sp500 ~ cpi + federal_funds_rate", data=df)

mediation = Mediation(model_b, model_a, "federal_funds_rate", "cpi")
mediation_results = mediation.fit(n_rep=1000)  # Bootstrapping for
significance testing

print(mediation_results.summary())
```

```
                          Estimate  Lower CI bound  Upper CI bound  P-
value
```

```
ACME (control)                  480.25          115.57          855.24
0.01
ACME (treated)                  480.25          115.57          855.24
0.01
ADE (control)                  -175.89         -262.71          -89.39
0.00
ADE (treated)                  -175.89         -262.71          -89.39
0.00
Total effect                    304.36          -83.60          687.71
0.11
Prop. mediated (control)          1.53           -2.34            5.20
0.10
Prop. mediated (treated)          1.53           -2.34            5.20
0.10
ACME (average)                  480.25          115.57          855.24
0.01
ADE (average)                  -175.89         -262.71          -89.39
0.00
Prop. mediated (average)          1.53           -2.34            5.20
0.10
```

Indirect Effect: 480.25 (ACME)
Direct Effect: -175.89 (ADE)
Total Effect: 304.36 (Total effect)
Proportion Mediated: 1.53 (Prop. mediated)

Manual Calculation of Effects

Path A Coefficient ($\beta IV \to M$):

```
beta_a = model_a_fitted.params["federal_funds_rate"]
```

Path B Coefficient ($\beta M \to DV$):

```
beta_b = model_b_fitted.params["cpi"]
```

Direct Effect ($\beta IV \to DV | M$):

```
direct_effect = model_b_fitted.params["federal_funds_rate"]
```

Indirect Effect ($\beta a \times \beta b$):

```
indirect_effect = beta_a * beta_b
```

Total Effect

```
total_effect = model_total.params["federal_funds_rate"]
```

```python
print(f"\nDirect Effect: {direct_effect:.4f}")
print(f"Indirect Effect: {indirect_effect:.4f}")
print(f"Total Effect: {total_effect:.4f}")
```

```
Direct Effect: -177.1566
Indirect Effect: 492.0021
Total Effect: 314.8456
```

```python
proportion_mediated = indirect_effect / total_effect
print(f"Proportion Mediated: {proportion_mediated:.4f}")
```

```
Proportion Mediated: 1.5627
```

Interpretation

Indirect Effect (ACME): 492.0021

The average indirect effect is 492.0021, which means the federal funds rate affects the S&P 500 through its impact on CPI. The confidence interval for ACME suggests that this effect is statistically significant, as it does not include 0.

The indirect effect measures the part of the relationship that is mediated by CPI. A positive indirect effect (492.0021) suggests that:

- An increase in the federal funds rate leads to changes in CPI, which in turn leads to an increase in the S&P 500.
- This could indicate that changes in inflation (CPI) caused by interest rates have a positive influence on stock market performance, possibly through mechanisms like nominal growth or price adjustments.

Direct Effect (ADE): -177.1566

The average direct effect is -177.1566, meaning that, holding CPI constant, the federal funds rate has a negative effect on the S&P 500. The confidence interval for ADE also suggests this effect is statistically significant.

The direct effect is the part of the relationship between the federal funds rate and the S&P 500 that is not mediated by CPI. A negative direct effect (-177.1566) suggests that:

- When CPI is held constant, an increase in the federal funds rate leads to a decrease in the S&P 500.
- This aligns with economic intuition, as higher interest rates typically discourage investment and reduce stock market valuations.

Total Effect: 314.8456

The total effect is 314.8456, which combines both the direct and indirect effects. The total effect is the sum of the direct and indirect effects:
Total Effect=Direct Effect+Indirect Effect

In this case: $314.8456 = -177.1566 + 492.0021$

The positive total effect suggests that:

- Overall, the federal funds rate has a net positive impact on the S&P 500 when both direct and mediated pathways are considered.

Proportion Mediated:

The proportion mediated (1.53) indicates that the mediation through CPI explains more than 100% of the total effect. This might seem counterintuitive, but it can happen when the indirect effect is larger than the direct effect, and suggests that the indirect pathway plays a dominant role in explaining the relationship between the federal funds rate and the S&P 500.

Key Takeaways

Competing Pathways:

The direct effect suggests that higher federal funds rates have a negative impact on the S&P 500, consistent with standard economic theory. The indirect effect through CPI offsets this, leading to a net positive impact. This could reflect unique economic dynamics or interactions in dataset.

Dominance of the Indirect Path:

The indirect effect (492.0021) is larger in magnitude than the direct effect (-177.1566), indicating that the mediation through CPI is a significant driver of the relationship.