MATH-F-427 students

Coxeter groups

Course notes

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ULB

Contents

Part I

Coxeter groups

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Introduction

This chapter is based on the first chapter of [MKS04]. This chapter will be an introduction of what groups are and how they are generated.

We recall in group theory that a group $(G, \cdot)$ is a non-empty set G of elements with a binary operation $\cdot$ for which the next axioms are satisfied:

- **Closure:** For all $a, b \in G$, c such that $a \cdot b = c$ implies that $c \in G$.
- **Associativity:** The operation $\cdot$ is associative, which means that for any elements $a, b, c \in G$:
$$(ab)c = a(bc)$$
- **Identity element:** There exists an element of G noted 1 for which:
$$a \cdot 1 = 1 \cdot a = a$$
- **Inverse element:** For any $a \in G$ there exists an *element* a^{-1} for which:
$$a \cdot a^{-1} = a^{-1} \cdot a = 1$$

We know two ways of defining a group; defining a *symmetry* of a set and if it is presented by generators and relators.

1.1 Symmetric groups

Definition 1.1. *The symmetric group on the set G is the group whose elements are permutations of the elements of G and its operation is the permutation composition. If $G = \{1, \ldots, n\}$ we call it S_n.*

1.2 Presentation of groups

A group

References

[MKS04] *Wilhelm Magnus, Abraham Karrass, and Donald Solitar.* Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations*. Dover Publications, Mineola, NY, unabridged and unaltered republication of the 2. rev. dover ed edition, 2004. OCLC: 254445530.*