ULB | Université Libre de Bruxelles

Thin strip graphs

Characterization and complexity

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Interval graphs

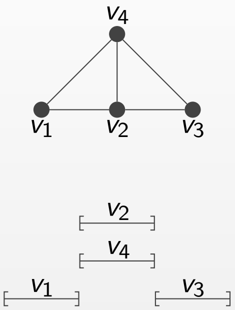


Figure: A unit interval graph with a realization.

Interval graphs

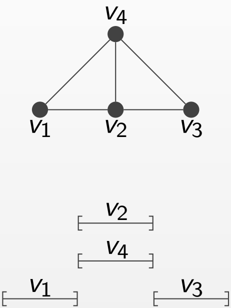


Figure: A unit interval graph with a realization.

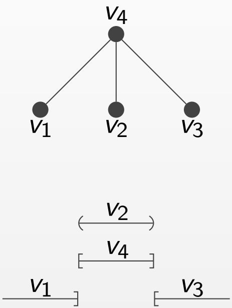


Figure: Representation of $K_{1,3}$ as a MUIG.

Unit disk graphs

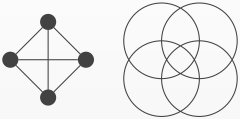


Figure: Realization of a UDG.

Unit disk graphs

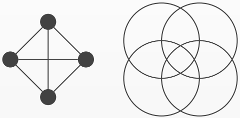
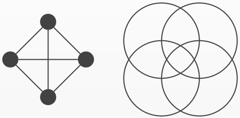


Figure: Realization of a UDG.

Theorem

CLIQUE problem is $\mathcal{NP}$-complete. Nevertheless, this problem is solved in polynomial time for unit disk graphs.

Unit disk graphs



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Theorem

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Theorem

Unit disk graph recognition is $\exists\mathbb{R}$.

c-strip graphs

Definition (c-strip graph)

A c-strip graph (SG(c)) is a unit disk graph such that the centers of the disks belong to $\{(x, y) : -\infty < x < \infty, 0 \leq y \leq c\}$.

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Remark

SG(0) = unit interval graph
SG(∞) = unit disk graph

Thin strip graphs

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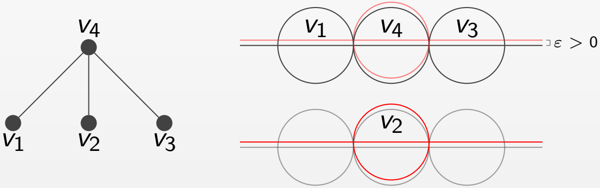

Figure: Proof that $\mathrm{TSG} \neq \mathrm{UIG}$.

Properties of thin strip graphs

Theorem

MUIG $\subsetneq$ *TSG* $\subsetneq$ *UUIG.*

Theorem

There is no constant t *such that* $SG(t) =$ *TSG.*

Properties of thin strip graphs

Theorem

$MUIG \subsetneq TSG \subsetneq UUIG$.

Theorem

There is no constant t such that $SG(t) = TSG$.

Remark

To prove these theorems, some forbidden induced subgraphs have been found.

Open questions

Forbidden induced subgraphs of TSGs

In order to study this graph, a characterization in terms of forbidden induced subgraphs has to be given. An exhaustive family of forbidden subgraphs could be researched.

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Complexity of TSG recognition

Is the recognition of thin strip graphs $\mathcal{NP}$?

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Complexity of TSG recognition

Is the recognition of thin strip graphs $\mathcal{NP}$?

Complexity of other graph-theoretic problems

What can we say about the complexity of other graph-theoretic problems applied to thin strip graphs?

Thanks for listening

Slides and resources can be found here:

`https://github.com/Abde5/memo201718`