

Characterization and complexity of Thin Strip Graphs

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ABSTRACT

Abstract

1 Graphs and disks

1.1 Graphs

A graph G is defined as $G = (V, E)$, where V is the set of vertices and E the set of edges. A vertex $v \in V$ is the fundamental unit of a graph. An edge $e \in E$ is a structures that links two vertices. The vertices $vw \in V$ that $e \in E$ links are called the *endpoints*.

Definition 1 *An embedding of a graph G is a representation of this graph on the plane.*

Definition 2 *A graph G is planar iff there is an embedding of this graph that doesn't have any crossing between the edges.*

1.2 Intersection graphs

Given a geometric construction with multiple objects, an intersection graph is a graph that maps the objects into vertices and every intersection between objects is an edge between the corresponding vertices.

Definition 3 *A graph G is a comparability graph if there is a partial order... (check c-strip article, good definition).*

1.2.1 Interval graphs

Definition of interval Graphs

Properties

Definition of MIXED interval graphs

1.2.2 Unit disk graphs

Definition of UID.

Definition of a realization.

2 Complexity

Problem solving is based on the complexity of a problem and not only a particular algorithm that solves it [6].

Definition 4 Let Σ be a finite alphabet, Σ^* every word derived from Σ , $L \subseteq \Sigma^*$ is a decision problem.

Definition 5 The algorithm A decides problem $L \subseteq \Sigma^*$ if for all word $w \in \Sigma^*$:

- A finishes and returns *TRUE* if $w \in L$.
- A finishes and returns *FALSE* if $w \notin L$.

Definition 6 A problem is decidable if there's an algorithm that decides it.

Definition 7 A problem is decidable if there's an algorithm that decides it.

2.1 P vs NP

Definition 8 A problem $L \in \mathcal{P}$ if L can be decided in polynomial time $\mathcal{O}(n^k)$.

Definition 9 A problem $L \in \mathcal{NP}$ if L can be verified in polynomial time $\mathcal{O}(n^k)$. Thus, $\mathcal{P} \subseteq \mathcal{NP}$.

2.2 $\exists\mathbb{R}$ complexity class

$\exists\mathbb{R}$ is the class that describes the problems such that they can be reduced to *the existential theory of the reals*[1]. The existential theory of the reals

2.2.1 Problems in $\exists\mathbb{R}$

The art gallery problem is $\exists\mathbb{R}$ -complete.[2]

Recognition of Unit Disk Graphs is $\exists\mathbb{R}$ -complete. (corollary of graph realizability problem)[4]

Stretchability is $\exists\mathbb{R}$ -complete.

3 Geometry

4 Disk Graph studies

4.1 Stabbing disks

Definition of stabbing.

Stabbing geometric structures.[5]

4.2 Thin Strip Graphs

c -strip graphs are unit disk graphs such that the centers of the disks are delimited on the area $\{(x, y) : -\infty < x < \infty, 0 < y \leq c\}$ and its class noted $\text{SG}(c)$. We can say that $\text{SG}(0) = \text{UIG}$ and $\text{SG}(\infty) = \text{UDG}$. [3]

Thin strip graphs are defined as $\bigcap_{c>0} \text{SG}(c)$.

$K_{1,3}$ in TSG, which is not possible for UIG.

$\text{MUIG} \subsetneq \text{TSG}$.

Denote that there's not constant t such that $\text{SG}(t) = \text{TSG}$.

Unfettered unit interval graphs = UUIF

Definition 10 *Find forbidden induced graphs for TSG.*

References

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- [5] L.M. Schlipf. *Stabbing and Covering Geometric Objects in the Plane*.
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