Université Libre de Bruxelles

Thin strip graphs

Characterization and complexity

Jean Cardinal

Abdeselam El-Haman Abdeselam

Context

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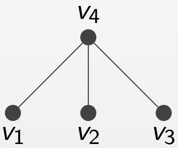

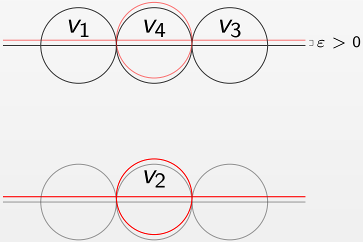

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- H. Breu. Algorithmic aspects of constrained unit disk graphs. Thesis, 1996

Mixed unit interval graphs

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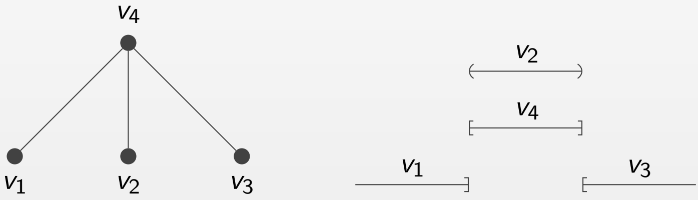

Mixed unit interval graphs

Theorem (Joos, 2014)

G is a MUIG if and only if it is a $\{F\} \cup \mathcal{R} \cup \mathcal{S} \cup \mathcal{S}'' \cup \mathcal{T}$-free interval graph.

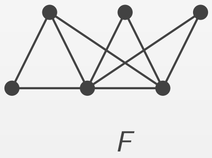

Figure: The graph F.

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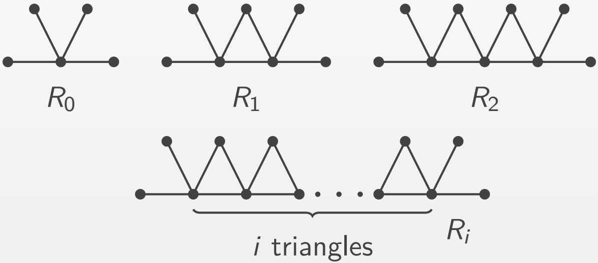

Figure: The family $\mathcal{R}$.

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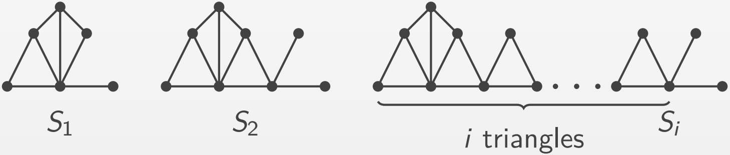

Figure: The family $\mathcal{S}$.

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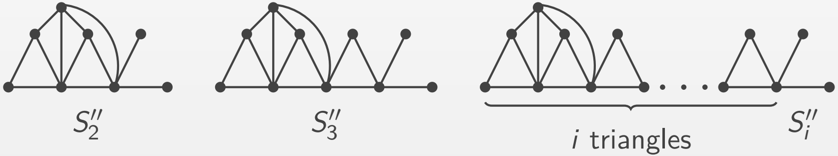

Figure: The family $\mathcal{S}''$.

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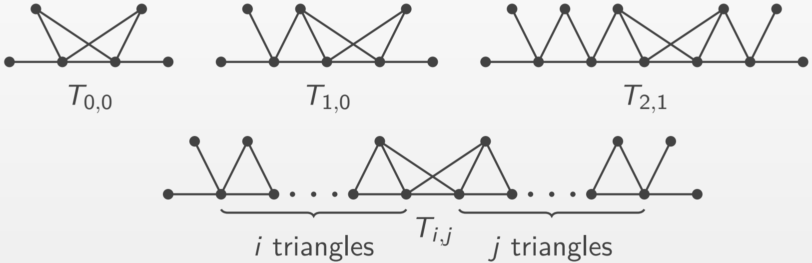

Figure: The family $\mathcal{T}$.

Unfettered unit interval graphs

An *unfettered unit interval graph* is an intersection graph of unit intervals in the real line. However, if two intervals *kiss*, we can choose whether they are adjacent or not.

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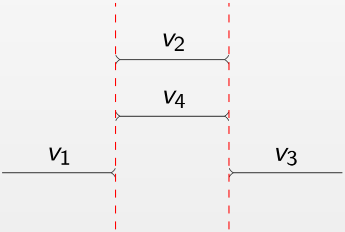

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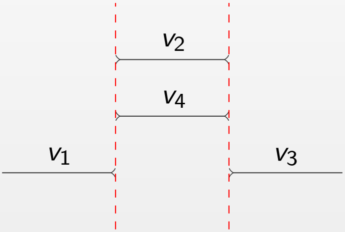

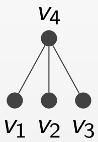

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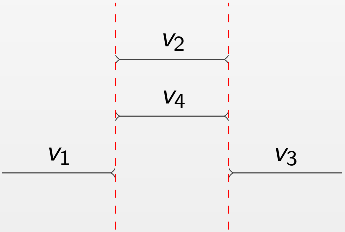

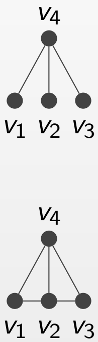

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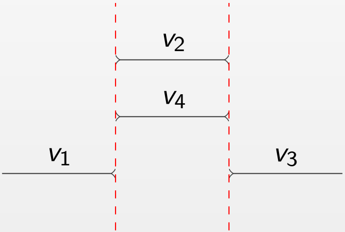

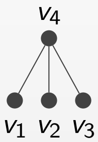

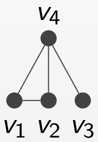

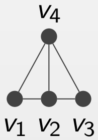

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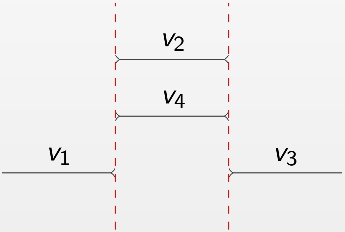

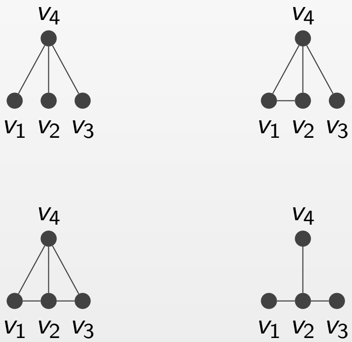

Unfettered unit interval graphs

This class has been completely characterized by its structure.

Theorem (Hayashi *et al.*, 2017)

A graph G is a unfettered unit interval graph if and only if it has a level structure where every level is a clique.

Definition

A *level structure* of a graph $G = (V, E)$ is a partition $L = \{L_i : i \in \{1, \ldots, t\}\}$ of V such that

$$v \in L_k \Rightarrow N(v) \subseteq L_{k-1} \cup L_k \cup L_{k+1}$$

where $L_0 = L_{t+1} = \varnothing$.

c-strip graphs

A *c-strip graph* - or SG(c) is a unit disk graph such that the centers of each disk belong to $\{(x, y) : -\infty < x < \infty, 0 \leq y \leq c\}$

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Remark

$SG(0) = UIG$.

Remark

$SG(\infty) = UDG$.

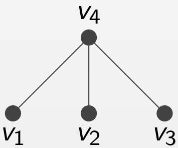
Remark

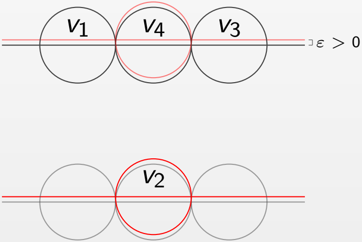
$SG(k) \subseteq SG(l)$ with $k < l$.

Thin strip graphs

Definition

The class of *thin strip graphs* is defined as $\mathrm{TSG} = \cap_{c>0} \mathrm{SG}(c)$.





Thin strip graphs

Hayashi *et al.* introduced the class of *thin strip graphs*. They also found these important results about c-strip graphs and thin strip graphs.

Theorem (Hayashi *et al.*, 2017)

There is no constant t such that $SG(t) = TSG$.

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In order to prove these theorems, they proved that a forbidden subgraph of MUIG is also forbidden in TSG.

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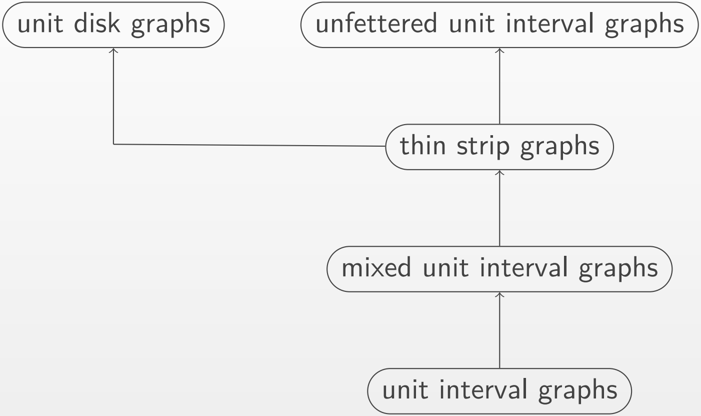
Theorem (Hayashi *et al.*, 2017)

Mixed unit interval graphs is a subclass of thin strip graphs.

Theorem (Hayashi *et al.*, 2017)

Thin strip graphs is a subclass of unfettered unit interval graphs.

Thin strip graphs



Results

The main result of this thesis is the representation of the forbidden subgraphs of MUIG as TSG. However, it has been proven that $\mathcal{R}$ is a forbidden subgraph family of TSG. However, we know that TSG $\subseteq$ UUIG.

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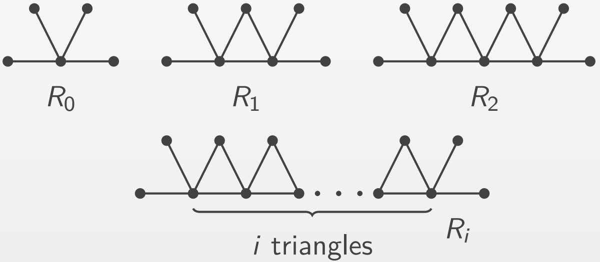

Results

Theorem

$\mathcal{R}$ is a family of forbidden induced subgraphs of UUIG.

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Proof.

By induction on i. $\square$

Figure: The graph R_0.

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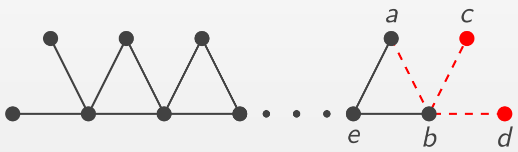
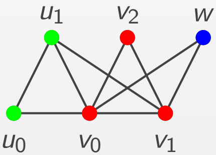
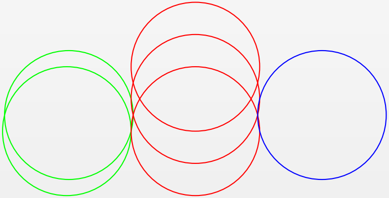


Figure: The graph R_{i+1}. You can see that the red edges and vertices are what differ from R_i.

Results

On the other hand, every other minimal forbidden subgraph of MUIG is a TSG.





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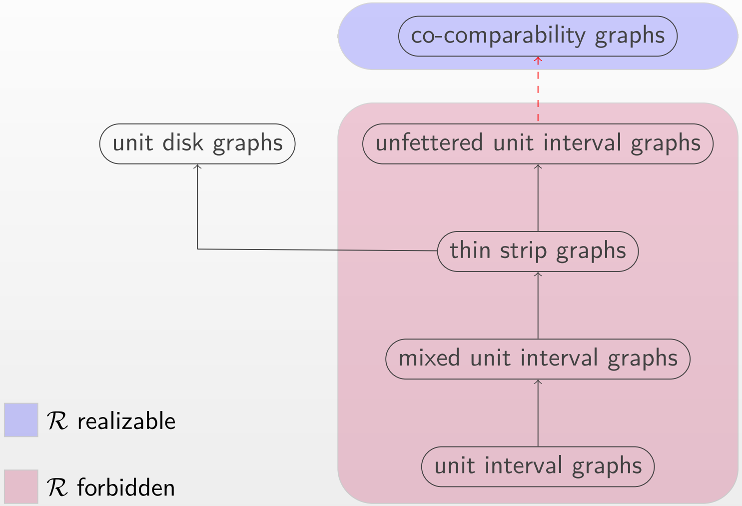
co-comparability graphs
unit disk graphs
unfettered unit interval graphs
thin strip graphs
mixed unit interval graphs
unit interval graphs
$\mathcal{R}$ realizable
$\mathcal{R}$ forbidden

Future work

Some questions are still open in this subject.

- Complete characterization of TSG.

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- Complete characterization of TSG.
- Recognition of TSG and UUIG.
- Two-level graph recognition and characterization.

Thanks for listening.