Université Libre de Bruxelles

# Thin strip graphs

## Characterization and complexity

*Jean Cardinal*

*Abdeselam El-Haman Abdeselam*

# Context

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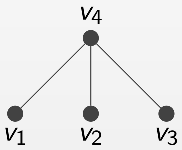


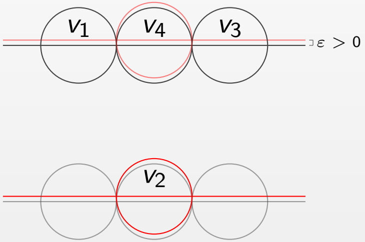

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The research in this paper is based on three papers.

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- H. Breu. Algorithmic aspects of constrained unit disk graphs. Thesis, 1996

# Mixed unit interval graphs

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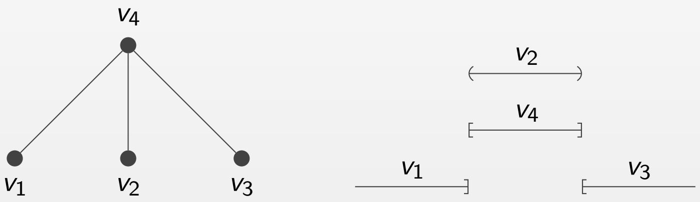

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Joos characterizes mixed unit interval graphs with an exhaustive list of families of minimal forbidden subgraphs.

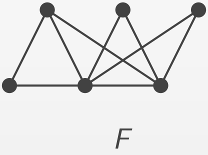


Figure: The graph $F$.

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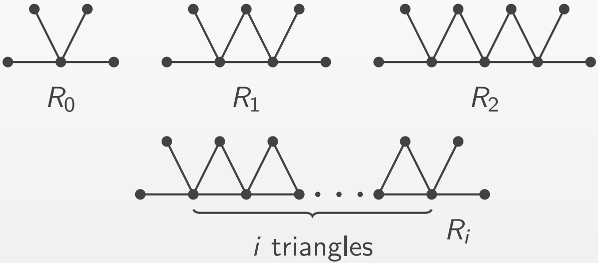


Figure: The family $\mathcal{R}$.

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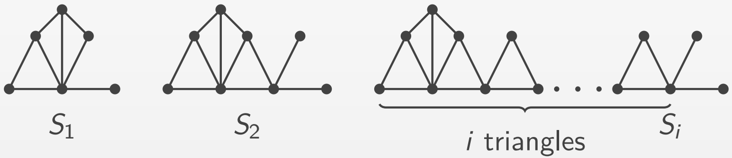


Figure: The family $\mathcal{S}$.

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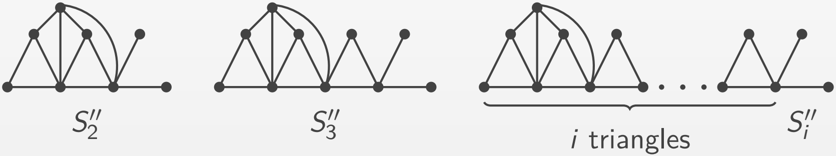


Figure: The family $\mathcal{S}''$.

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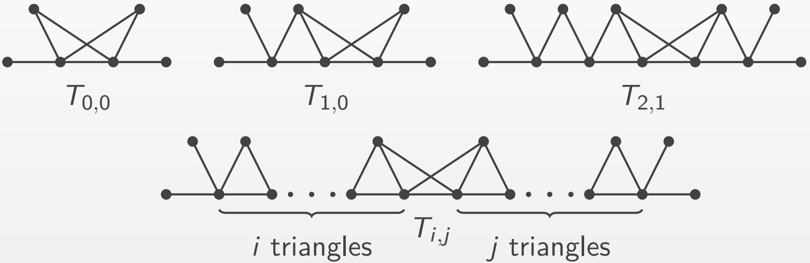


Figure: The family $\mathcal{T}$.

# Unfettered unit interval graphs

An *unfettered unit interval graph* is an intersection graph of unit intervals in the real line. However, if two intervals *kiss*, we can choose whether they are adjacent or not.

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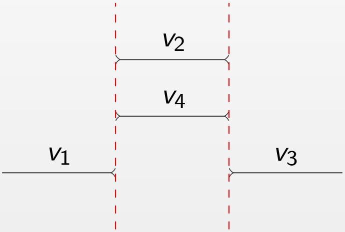

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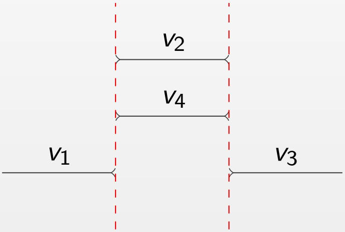


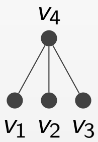

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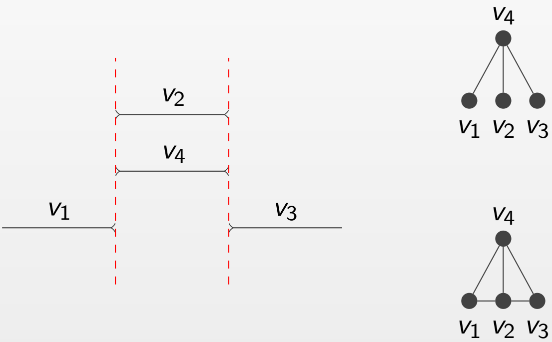

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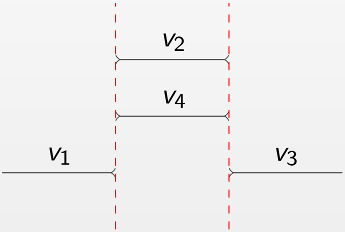


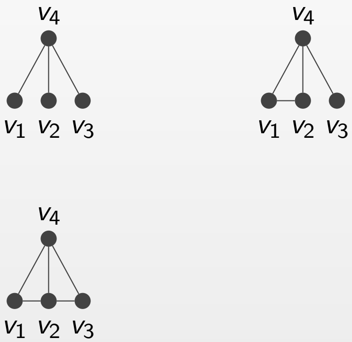

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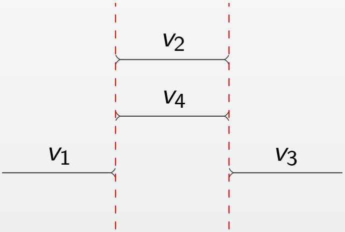


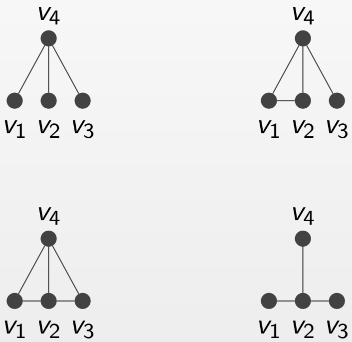

# Unfettered unit interval graphs

This class has been completely characterized by its structure.

**Theorem (Hayashi)**

*A graph $G$ is a unfettered unit interval graph if and only if it has a level structure where every level is a clique.*

**Definition**

A *level structure* of a graph $G = (V, E)$ is a partition $L = \{L_i : i \in \{1, \ldots, t\}\}$ of $V$ such that

$$v \in L_k \Rightarrow N(v) \subseteq L_{k-1} \cup L_k \cup L_{k+1}$$

where $L_0 = L_{t+1} = \varnothing$.

# $c$-strip graphs

A *c-strip graph* - or SG($c$) is a unit disk graph such that the centers of each disk belong to $\{(x, y) : -\infty < x < \infty, 0 \leq y \leq c\}$

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**Remark**

$SG(0) = UIG$.

**Remark**

$SG(\infty) = UDG$.

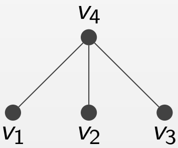
**Remark**

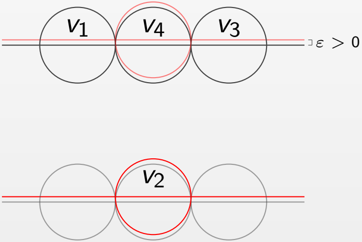
$SG(k) \subseteq SG(l)$ *with* $k < l$.

# Thin strip graphs

## Definition

The class of *thin strip graphs* is defined as $\mathrm{TSG} = \cap_{c>0} \mathrm{SG}(c)$.



# Thin strip graphs

Hayashi *et al.* introduced the class of *thin strip graphs*. They also found these important results about $c$-strip graphs and thin strip graphs.

**Theorem**

*There is no constant* $t$ *such that* $SG(t) = TSG$.

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In order to prove these theorems, they proved that a forbidden subgraph of MUIG is also forbidden in TSG.

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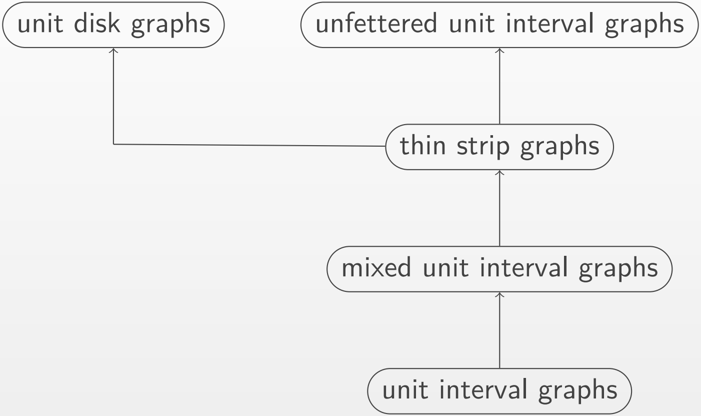
**Theorem**

*Mixed unit interval graphs is a subclass of thin strip graphs.*

**Theorem**

*Thin strip graphs is a subclass of unfettered unit interval graphs.*

# Thin strip graphs



# Results

The main result of this thesis is the representation of the forbidden subgraphs of MUIG as TSG. However, it has been proven that $\mathcal{R}$ is a forbidden subgraph family of TSG. However, we know that TSG $\subseteq$ UUIG.

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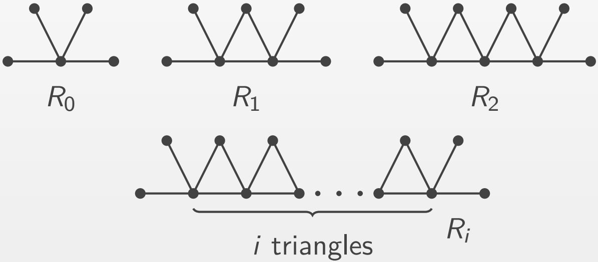

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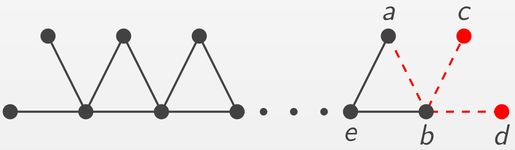
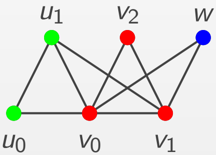
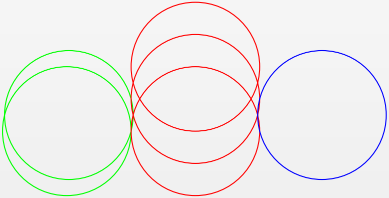



Figure: The graph $R_{i+1}$. You can see that the red edges and vertices are what differ from $R_i$.

# Results

On the other hand, every other minimal forbidden subgraph of MUIG is a TSG.



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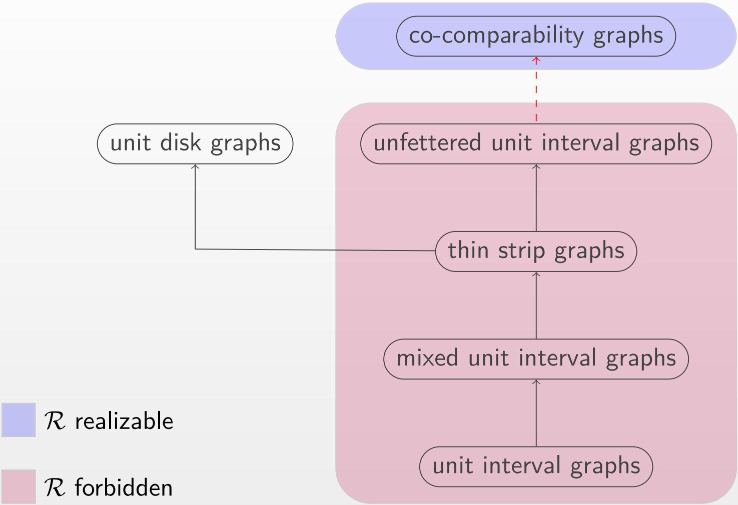
co-comparability graphs
unit disk graphs
unfettered unit interval graphs
thin strip graphs
mixed unit interval graphs
unit interval graphs
$\mathcal{R}$ realizable
$\mathcal{R}$ forbidden

# Future work

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- Two-level graph recognition and characterization.

Thanks for listening.