Characterization and complexity of Thin Strip Graphs

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May 7, 2014

ABSTRACT

Abstract

1 Graphs and disks

1.1 Graphs

A graph G is defined as $G = (V, E)$, where V is the set of vertices and E the set of edges. A vertex $v \in V$ is the fundamental unit of a graph. An edge $e \in E$ is a structures that links two vertices. The vertices $vw \in V$ that $e \in E$ links are called the *endpoints*.

Definition 1 *An embedding of a graph G is a representation of this graph on the plane.*

Definition 2 *A graph G is planar iff there is an embedding of this graph that doesn't have any crossing between the edges.*

1.2 Intersection graphs

Given a geometric construction with multiple objects, an intersection graph is a graph that maps the objects into vertices and every intersection between objects is an edge between the corresponding vertices.

Definition 3 *A graph G is a comparibility graph if there is a partial order... (check c-strip article, good definition).*

1.2.1 Interval graphs

Definition of interval Graphs
Properties
Definition of MIXED interval graphs

1.2.2 Unit disk graphs

Definition of UID.
Definition of a realization.

2 Complexity

Problem solving is based on the complexity of a problem and not only a particular algorithm that solves it [6].

Definition 4 *Let Σ be a finite alphabet, Σ^* every word derived from Σ, $L \subseteq \Sigma^*$ is a decision problem.*

Definition 5 *The algorithm A decides problem $L \subseteq \Sigma^*$ if for all word $w \in \Sigma^*$:*

- *A finishes and returns TRUE if $w \in L$.*
- *A finishes and returns FALSE if $w \notin L$.*

Definition 6 *A problem is decidable if there's an algorithm that decides it.*

Definition 7 *A problem is decidable if there's an algorithm that decides it.*

2.1 P vs NP

Definition 8 *A problem $L \in \mathcal{P}$ if L can be decided in polynomial time $\mathcal{O}(n^k)$.*

Definition 9 *A problem $L \in \mathcal{NP}$ if L can be verified in polynomial time $\mathcal{O}(n^k)$. Thus, $\mathcal{P} \subseteq \mathcal{NP}$.*

2.2 $\exists\mathbb{R}$ complexity class

$\exists\mathbb{R}$ is the class that describes the problems such that they can be reduced to *the existential theory of the reals*[1]. The existential theory of the reals

2.2.1 Problems in $\exists\mathbb{R}$

The art gallery problem is $\exists\mathbb{R}$-complete.[2]

Recognition of Unit Disk Graphs is $\exists\mathbb{R}$-complete. (corollary of graph realizability problem)[4]

Stretchability is $\exists\mathbb{R}$-complete.

3 Geometry

4 Disk Graph studies

4.1 Stabbing disks

Definition of stabbing.

Stabbing geometric structures.[5]

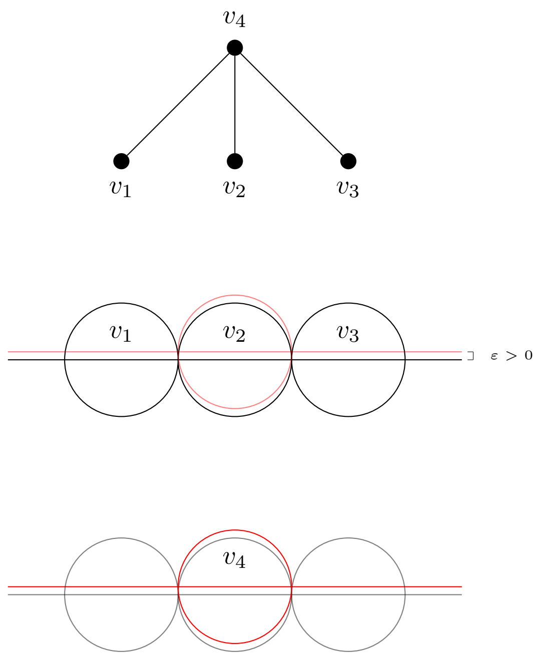

Figure 1: A construction of $K_{1,3}$ with a disk realization, being this graph a TSG.

4.2 Thin Strip Graphs

c-strip graphs are unit disk graphs such that the centers of the disks are delimited on the area $\{(x, y) : -\infty < x < \infty, 0 < y \leq c\}$ and its class noted SG(c). We can say that SG(0) = UIG and SG(∞) = UDG. [3]

Definition 10 *Thin strip graphs are defined as TSG = $\bigcap_{c>0}$ SG(c).*

Remark 11 *SG(0) $\neq$ TSG. This can be seen*

$K_{1,3}$ in TSG, which is not possible for UIG.
MUIG $\subsetneq$ TSG.

Denote that there's not constant t such that SG(t) = TSG.

Unfettered unit interval graphs = UUIG

MUIG $\subsetneq$ TSG $\subsetneq$ UUIG

Definition 12 *Find forbidden induced graphs for TSG.*

References

[1] Existential Theory of the Reals. In *Algorithms in Real Algebraic Geometry*, volume 10, pages 505–532. Springer Berlin Heidelberg.

[2] Mikkel Abrahamsen, Anna Adamaszek, and Tillmann Miltzow. The Art Gallery Problem is $\exists \mathbb{R}$-complete.

[3] Takashi Hayashi, Akitoshi Kawamura, Yota Otachi, Hidehiro Shinohara, and Koichi Yamazaki. Thin Strip Graphs. 216:203–210.

[4] Marcus Schaefer. Realizability of Graphs and Linkages. In Jnos Pach, editor, *Thirty Essays on Geometric Graph Theory*, pages 461–482. Springer New York.

[5] L.M. Schlipf. *Stabbing and Covering Geometric Objects in the Plane.*

[6] Michael Sipser. *Introduction to the Theory of Computation.* Course Technology, second edition.