# Characterization and complexity of Thin Strip Graphs

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*ABSTRACT*

Abstract

## 1 Graphs and disks

### 1.1 Graphs

A graph $G$ is defined as $G = (V, E)$, where $V$ is the set of vertices and $E$ the set of edges. A vertex $v \in V$ is the fundamental unit of a graph. An edge $e \in E$ is a structures that links two vertices. The vertices $vw \in V$ that $e \in E$ links are called the *endpoints*.

**Definition 1** *An embedding of a graph $G$ is a representation of this graph on the plane.*

**Definition 2** *A graph $G$ is planar iff there is an embedding of this graph that doesn't have any crossing between the edges.*

### 1.2 Intersection graphs

Given a geometric construction with multiple objects, an intersection graph is a graph that maps the objects into vertices and every intersection between objects is an edge between the corresponding vertices.

**Definition 3** *A graph $G$ is a comparibility graph if there is a partial order... (check c-strip article, good definition).*

#### 1.2.1 Interval graphs

Definition of interval Graphs
Properties
Definition of MIXED interval graphs

#### 1.2.2 Unit disk graphs

Definition of UID.
Definition of a realization.

# 2 Complexity

Problem solving is based on the complexity of a problem and not only a particular algorithm that solves it [6].

**Definition 4** *Let $\Sigma$ be a finite alphabet, $\Sigma^*$ every word derived from $\Sigma$, $L \subseteq \Sigma^*$ is a decision problem.*

**Definition 5** *The algorithm $A$ decides problem $L \subseteq \Sigma^*$ if for all word $w \in \Sigma^*$:*

- *$A$ finishes and returns TRUE if $w \in L$.*
- *$A$ finishes and returns FALSE if $w \notin L$.*

**Definition 6** *A problem is decidable if there's an algorithm that decides it.*

**Definition 7** *A problem is decidable if there's an algorithm that decides it.*

## 2.1 P vs NP

**Definition 8** *A problem $L \in \mathcal{P}$ if $L$ can be decided in polynomial time $\mathcal{O}(n^k)$.*

**Definition 9** *A problem $L \in \mathcal{NP}$ if $L$ can be verified in polynomial time $\mathcal{O}(n^k)$. Thus, $\mathcal{P} \subseteq \mathcal{NP}$.*

## 2.2 $\exists\mathbb{R}$ complexity class

$\exists\mathbb{R}$ is the class that describes the problems such that they can be reduced to *the existential theory of the reals*[1]. The existential theory of the reals

### 2.2.1 Problems in $\exists\mathbb{R}$

The art gallery problem is $\exists\mathbb{R}$-complete.[2]

Recognition of Unit Disk Graphs is $\exists\mathbb{R}$-complete. (corollary of graph realizability problem)[4]

Stretchability is $\exists\mathbb{R}$-complete.

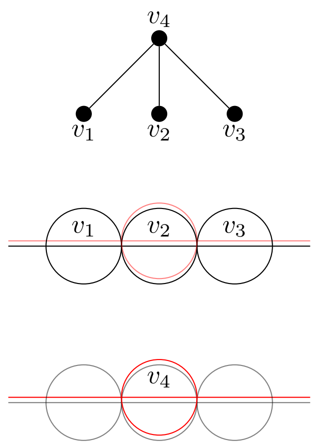
# 3 Geometry

# 4 Disk Graph studies

## 4.1 Stabbing disks

Definition of stabbing.

Stabbing geometric structures.[5]




### 4.2 Thin Strip Graphs

$c$-strip graphs are unit disk graphs such that the centers of the disks are delimited on the area $\{(x, y) : -\infty < x < \infty, 0 < y \leq c\}$ and its class noted SG($c$). We can say that SG(0) = UIG and SG($\infty$) = UDG. [3]

**Definition 10** *Thin strip graphs are defined as TSG* $= \bigcap_{c>0}$ *SG(c).*

**Remark 11** *SG(0)* $\neq$ *TSG. This can be seen*

$K_{1,3}$ in TSG, which is not possible for UIG.
MUIG $\subsetneq$ TSG.

Denote that there's not constant $t$ such that SG($t$) = TSG.

Unfettered unit interval graphs = UUIG

MUIG $\subsetneq$ TSG $\subsetneq$ UUIG

**Definition 12** *Find forbidden induced graphs for TSG.*

## References

[1] Existential Theory of the Reals. In *Algorithms in Real Algebraic Geometry*, volume 10, pages 505–532. Springer Berlin Heidelberg.

[2] Mikkel Abrahamsen, Anna Adamaszek, and Tillmann Miltzow. The Art Gallery Problem is $\exists \mathbb{R}$-complete.

[3] Takashi Hayashi, Akitoshi Kawamura, Yota Otachi, Hidehiro Shinohara, and Koichi Yamazaki. Thin Strip Graphs. 216:203–210.

[4] Marcus Schaefer. Realizability of Graphs and Linkages. In Jnos Pach, editor, *Thirty Essays on Geometric Graph Theory*, pages 461–482. Springer New York.

[5] L.M. Schlipf. *Stabbing and Covering Geometric Objects in the Plane.*

[6] Michael Sipser. *Introduction to the Theory of Computation.* Course Technology, second edition.