Disk graphs (provisional)

by

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Chapter 1

Graphs and disks

1.1 Graphs

A graph G is defined as $G(V, E)$, where V is the set of vertices and E the set of edges. A vertex $v \in V$ is the fundamental unit of a graph. An edge $e \in E$ is a structures that links two vertices. The vertices $vw \in V$ that $e \in E$ links are called the *endpoints*.

Definition 1.1 *An embedding of a graph G is a representation of this graph on the plane.*

Definition 1.2 *A graph G is planar iff there is an embedding of this graph that doesn't have any crossing between the edges.*

1.2 Intersection graphs

Given a geometric construction with multiple objects, an intersection graph is a graph that maps the objects into vertices and every intersection between objects is an edge between the corresponding vertices.

Definition 1.3 *A graph G is a comparibility graph if there is a partial order... (check c-strip article, good definition).*

1.2.1 Interval graphs

1.2.2 Unit disk graphs

Chapter 2

Complexity

Problem solving is based on the complexity of a problem and not only a particular algorithm that solves it [?].

Definition 2.1 *Let Σ be a finite alphabet, Σ^* every word derived from Σ, $L \subseteq \Sigma^*$ is a decision problem.*

Definition 2.2 *The algorithm A decides problem $L \subseteq \Sigma^*$ if for all word $w \in \Sigma^*$:*

- *A finishes and returns TRUE if $w \in L$.*
- *A finishes and returns FALSE if $w \notin L$.*

Definition 2.3 *A problem is decidable if there's an algorithm that decides it.*

Definition 2.4 *A problem is decidable if there's an algorithm that decides it.*

2.1 P vs NP

Definition 2.5 *A problem $L \in \mathcal{P}$ if L can be decided in polynomial time $\mathcal{O}(n^k)$.*

Definition 2.6 *A problem $L \in \mathcal{NP}$ if L can be verified in polynomial time $\mathcal{O}(n^k)$. Thus, $\mathcal{P} \subseteq \mathcal{NP}$.*

2.2 $\exists\mathbb{R}$ complexity class

$\exists\mathbb{R}$ is the class that describes the problems such that they can be reduced to *the existential theory of the reals*[?]. The existential theory of the reals

2.2.1 Problems in $\exists\mathbb{R}$

The art gallery problem is $\exists\mathbb{R}$-complete.[?]

Recognition of Unit Disk Graphs is $\exists\mathbb{R}$-complete. (corollary of graph realizability problem)[?]

Stretchability is $\exists\mathbb{R}$-complete.

Chapter 3

Geometry

Chapter 4

Disk Graph studies

4.1 Stabbing disks

Definition of stabbing.

Stabbing geometric structures.[**?**]

4.2 Thin Strip Graphs

c-strip graphs are unit disk graphs such that the centers of the disks are delimited on the area $\{(x, y) : -\infty < x < \infty, 0 < y \leq c\}$ and its class noted SG(c). We can say that SG(0) = UIG and SG(∞) = UDG. [**?**]

Thin strip graphs are defined as $\bigcap_{c>0} \mathrm{SG}(c)$