Disk graphs (provisional)

by

Abdeselam El-Haman Abdeselam

Submitted to the Department of Computer Science
in partial fulfillment of the requirements for the degree of

Master of Science in Computer Science

at the

UNIVERSITE LIBRE DE BRUXELLES

May 2018

Supervised by . . . .
Prof. Jean Cardinal
Thesis Supervisor

Contents

Chapter 1

Graphs and disks

1.1 Graphs

A graph G is defined as $G = (V, E)$, where V is the set of vertices and E the set of edges. A vertex $v \in V$ is the fundamental unit of a graph. An edge $e \in E$ is a structures that links two vertices. The vertices $vw \in V$ that $e \in E$ links are called the *endpoints*.

Definition 1.1 *An embedding of a graph G is a representation of this graph on the plane.*

Definition 1.2 *A graph G is planar iff there is an embedding of this graph that doesn't have any crossing between the edges.*

1.2 Intersection graphs

Given a geometric construction with multiple objects, an intersection graph is a graph that maps the objects into vertices and every intersection between objects is an edge between the corresponding vertices.

Definition 1.3 *A graph G is a comparibility graph if there is a partial order... (check c-strip article, good definition).*

1.2.1 Interval graphs

Definition of interval Graphs

Properties

Definition of MIXED interval graphs

1.2.2 Unit disk graphs

Definition of UID.

Definition of a realization.

Chapter 2

Complexity

Problem solving is based on the complexity of a problem and not only a particular algorithm that solves it [6].

Definition 2.1 *Let Σ be a finite alphabet, Σ^* every word derived from Σ, $L \subseteq \Sigma^*$ is a decision problem.*

Definition 2.2 *The algorithm A decides problem $L \subseteq \Sigma^*$ if for all word $w \in \Sigma^*$:*

- *A finishes and returns TRUE if $w \in L$.*
- *A finishes and returns FALSE if $w \notin L$.*

Definition 2.3 *A problem is decidable if there's an algorithm that decides it.*

Definition 2.4 *A problem is decidable if there's an algorithm that decides it.*

2.1 P vs NP

Definition 2.5 *A problem $L \in \mathcal{P}$ if L can be decided in polynomial time $\mathcal{O}(n^k)$.*

Definition 2.6 *A problem $L \in \mathcal{NP}$ if L can be verified in polynomial time $\mathcal{O}(n^k)$. Thus, $\mathcal{P} \subseteq \mathcal{NP}$.*

2.2 $\exists\mathbb{R}$ complexity class

$\exists\mathbb{R}$ is the class that describes the problems such that they can be reduced to *the existential theory of the reals*[1]. The existential theory of the reals

2.2.1 Problems in $\exists\mathbb{R}$

The art gallery problem is $\exists\mathbb{R}$-complete.[2]

Recognition of Unit Disk Graphs is $\exists\mathbb{R}$-complete. (corollary of graph realizability problem)[4]

Stretchability is $\exists\mathbb{R}$-complete.

Chapter 3

Geometry

Chapter 4

Disk Graph studies

4.1 Stabbing disks

Definition of stabbing.

Stabbing geometric structures.[5]

4.2 Thin Strip Graphs

c-strip graphs are unit disk graphs such that the centers of the disks are delimited on the area $\{(x, y) : -\infty < x < \infty, 0 < y \leq c\}$ and its class noted SG(c). We can say that SG(0) = UIG and SG(∞) = UDG. [3]

Thin strip graphs are defined as $\bigcap_{c>0}$ SG(c).

$K_{1,3}$ in TSG, which is not possible for UIG.

MUIG $\subsetneq$ TSG.

Denote that there's not constant t such that SG(t) = TSG.

Unfettered unit interval graphs = UUIF

Definition 4.1 *Find forbidden induced graphs for TSG.*

Bibliography

[1] Existential Theory of the Reals. In *Algorithms in Real Algebraic Geometry*, volume 10, pages 505–532. Springer Berlin Heidelberg.

[2] Mikkel Abrahamsen, Anna Adamaszek, and Tillmann Miltzow. The Art Gallery Problem is $\exists \mathbb{R}$-complete.

[3] Takashi Hayashi, Akitoshi Kawamura, Yota Otachi, Hidehiro Shinohara, and Koichi Yamazaki. Thin Strip Graphs. 216:203–210.

[4] Marcus Schaefer. Realizability of Graphs and Linkages. In JÃąnos Pach, editor, *Thirty Essays on Geometric Graph Theory*, pages 461–482. Springer New York.

[5] L.M. Schlipf. *Stabbing and Covering Geometric Objects in the Plane.*

[6] Michael Sipser. *Introduction to the Theory of Computation.* Course Technology, second edition.