Université Libre de Bruxelles

Thin strip graphs

Characterization and complexity

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Introduction

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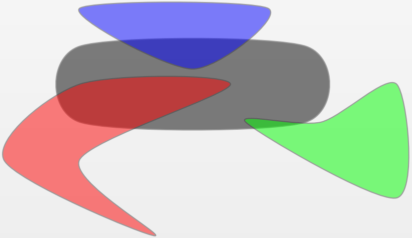

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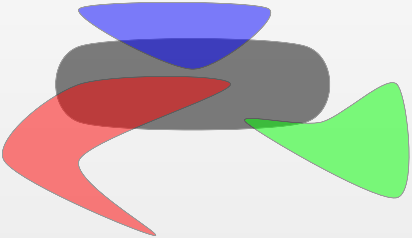

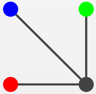

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Example (Kuratowski)

A graph G is planar if it does not contain $K_{3,3}$ or K_5 as a minor.

State of the art: interval graphs

An *interval graph* is an intersection graph of closed intervals in the real line. If the length of the intervals are the same, then it is an *unit interval graph*.

There exists a characterization of unit interval graphs for interval graphs.

Theorem (Roberts)

An interval graph is an unit interval graph if and only if it has no induced subgraph $K_{1,3}$.

State of the art: mixed unit interval graphs

A *mixed unit interval graph* is an intersection graph of unitary intervals in the real line. The interval of mixed unit interval graphs can be open, closed, open-closed or closed-open.

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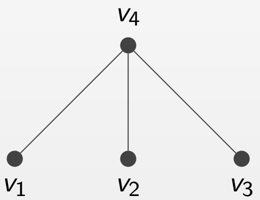

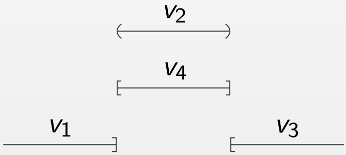

State of the art: mixed unit interval graphs

Joos characterizes mixed unit interval graphs with an exhaustive list of families of forbidden subgraphs.

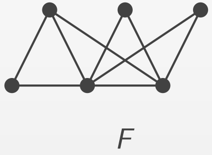

Figure: The graph F.

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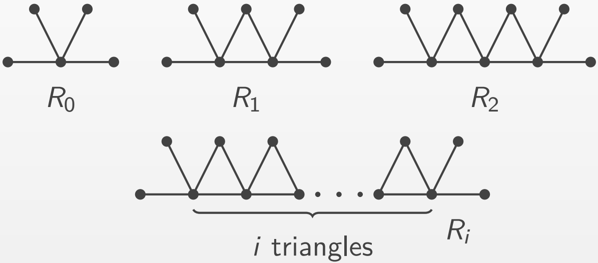

Figure: The family $\mathcal{R}$.

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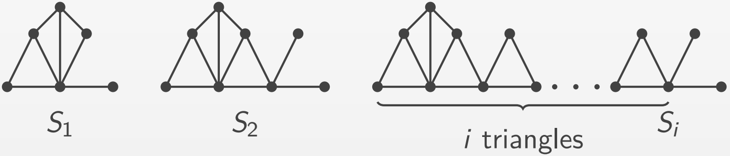

Figure: The family $\mathcal{S}$.

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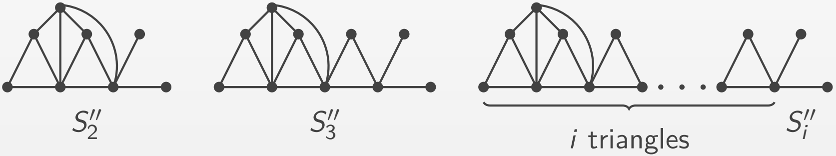

Figure: The family $\mathcal{S}''$.

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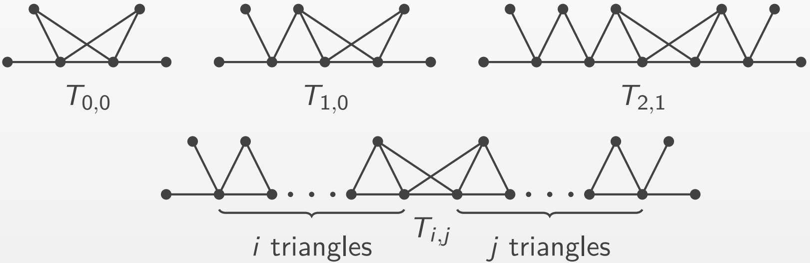

Figure: The family $\mathcal{T}$.

State of the art: unfettered unit interval graphs

An *unfettered unit interval graph* is an intersection graph of unitary intervals in the real line. However, if two intervals *kiss*, we can choose whether they are adjacent or not.

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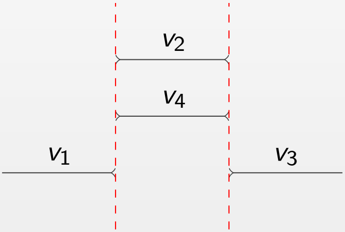

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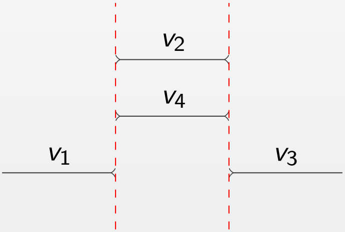

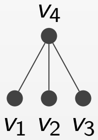

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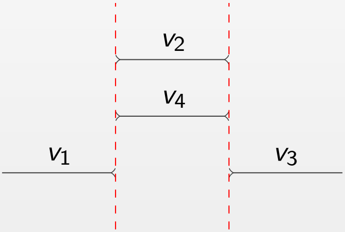

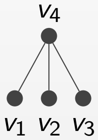

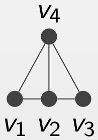

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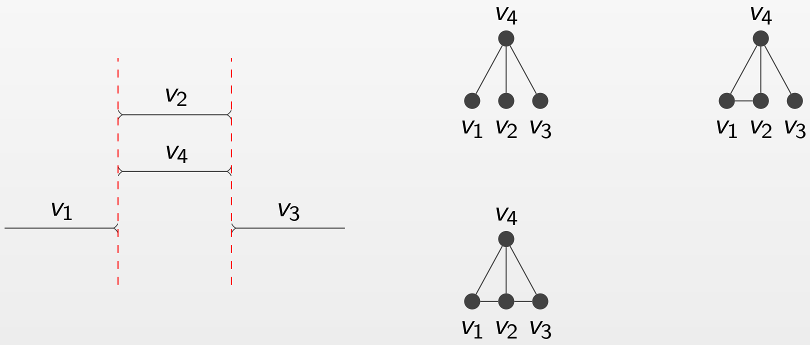

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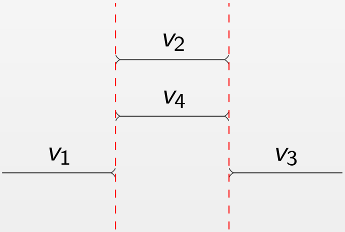
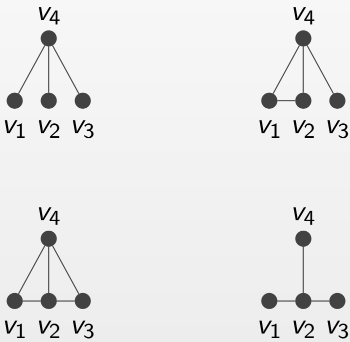




State of the art: unfettered unit interval graphs

This class has been completely characterized by its structure.

Theorem (Hayashi)

A graph G is a unfettered unit interval graph if and only if it has a level structure where every level is a clique.

Definition

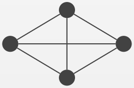
A *level structure* of a graph $G = (V, E)$ is a partition $L = \{L_i : i \in \{1, \dots, t\}\}$ of V such that

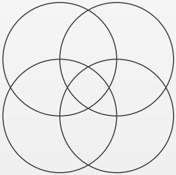
$$v \in L_k \Rightarrow N(v) \subseteq L_{k-1} \cup L_k \cup L_{k+1}$$

where $L_0 = L_{t+1} = \varnothing$.

State of the art: unit disk graphs

A *disk graph* is an intersection graph of closed intervals in the real line. If the length of the intervals are the same, then it is an *unit disk graph*.





State of the art: c-strip graphs

A *c-strip graph* - or SG(c) is a unit disk graph such that the centers of each disk belong to $\{(x, y) : -\infty < x < \infty, 0 \leq y \leq c\}$

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A *c-strip graph* - or SG(c) is a unit disk graph such that the centers of each disk belong to $\{(x, y) : -\infty < x < \infty, 0 \leq y \leq c\}$

Remark

$SG(0) = UIG$.

Remark

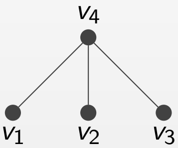
$SG(\infty) = UDG$.

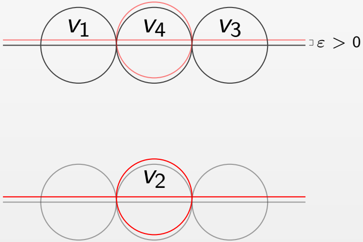
Remark

$SG(k) \subseteq SG(l)$ with $k < l$.

State of the art: thin strip graphs

The class of *thin strip graphs* is the intersection of every c-strip graph with $c > 0$. Thus, a ε-strip graph with ε arbitrarily small.





State of the art: thin strip graphs

Hayashi *et al.* introduced the class of *thin strip graphs*. They also found these important results about c-strip graphs and thin strip graphs.

Theorem

There is no constant t such that $SG(t) = TSG$.

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Theorem

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In order to prove these theorems, they proved that a forbidden subgraph of MUIG is also forbidden in TSG.

State of the art: thin strip graphs

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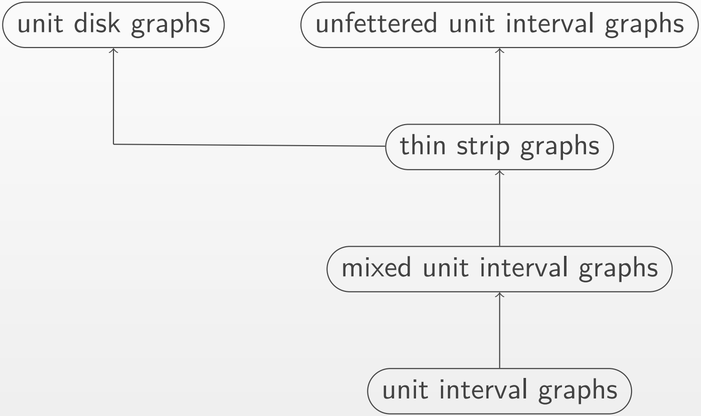
Theorem

Mixed unit interval graphs is a subclass of thin strip graphs.

Theorem

Thin strip graphs is a subclass of unfettered unit interval graphs.

State of the art: thin strip graphs



Results

The main result of this thesis is the representation of the forbidden subgraphs of MUIG as TSG. However, it has been proven that $\mathcal{R}$ is a forbidden subgraph family of TSG. However, we know that TSG $\subseteq$ UUIG.

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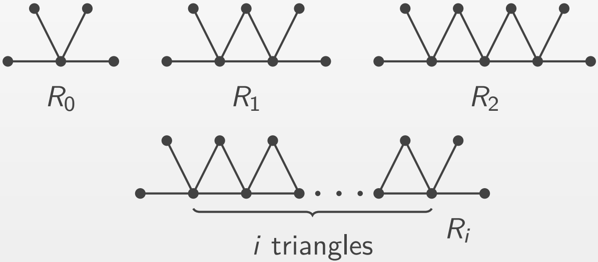

Results

Theorem

$\mathcal{R}$ is a family of forbidden subgraphs of UUIG.

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Proof.

By induction on i. □

Figure: The graph R_0.

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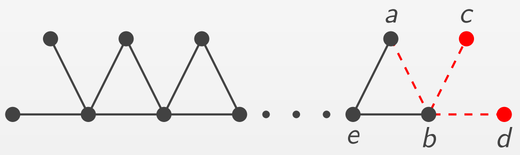

Figure: The graph R_{i+1}. You can see that the red edges and vertices are what differ from R_i.

Thanks for listening.