Thin strip graphs

Characterization and complexity

Jean Cardinal

Abdeselam El-Haman Abdeselam

Table of contents

Interval graphs

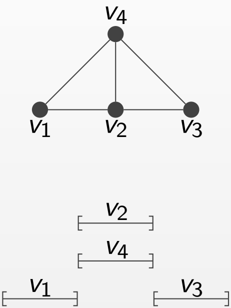

Figure: A unit interval graph with a realization.

Interval graphs

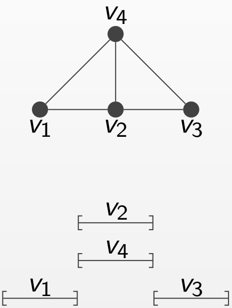

Figure: A unit interval graph with a realization.

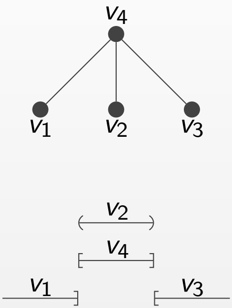

Figure: Representation of $K_{1,3}$ as a MUIG.

Unit disk graphs

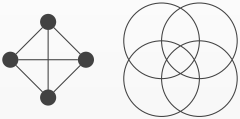

Figure: Realization of a UDG.

Unit disk graphs

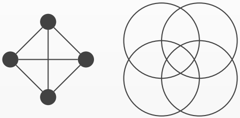

Figure: Realization of a UDG.

Theorem

CLIQUE problem is $\mathcal{NP}$-complete. Nevertheless, this problem is solved in polynomial time for unit disk graphs.

Unit disk graphs

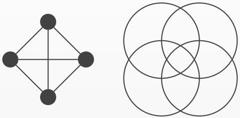

Figure: Realization of a UDG.

Theorem

CLIQUE problem is $\mathcal{NP}$-complete. Nevertheless, this problem is solved in polynomial time for unit disk graphs.

Theorem

Unit disk graph recognition is $\exists\mathbb{R}$.

c-strip graphs

Definition (c-strip graph)

A c-strip graph ($\mathrm{SG}(c)$) is a unit disk graph such that the centers of the disks belong to $\{(x, y) : -\infty < x < \infty, 0 \leq y \leq c\}$.

c-strip graphs

Definition (c-strip graph)

A c-strip graph (SG(c)) is a unit disk graph such that the centers of the disks belong to $\{(x, y) : -\infty < x < \infty, 0 \leq y \leq c\}$.

Remark

SG(0) = unit interval graph
SG(∞) = unit disk graph

Thin strip graphs

Definition (Thin strip graph)

Thin strip graphs are defined as $\mathrm{TSG} = \bigcap_{c>0} \mathrm{SG}(c)$.

Thin strip graphs

Definition (Thin strip graph)

Thin strip graphs are defined as $\mathrm{TSG} = \bigcap_{c>0} \mathrm{SG}(c)$.

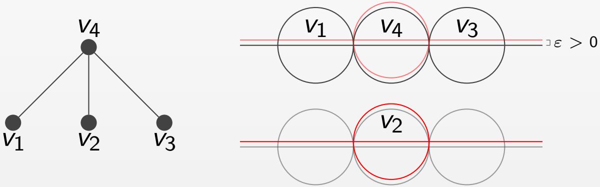

Figure: Proof that $\mathrm{TSG} \neq \mathrm{UIG}$.

Properties of thin strip graphs

Theorem

$MUIG \subsetneq TSG \subsetneq UUIG$.

Theorem

There is no constant t such that $SG(t) = TSG$.

Properties of thin strip graphs

Theorem

$MUIG \subsetneq TSG \subsetneq UUIG$.

Theorem

There is no constant t such that $SG(t) = TSG$.

Remark

To prove these theorems, some forbidden induced subgraphs have been found.

Open questions

Forbidden induced subgraphs of TSGs

In order to study this graph, a characterization in terms of forbidden induced subgraphs has to be given. An exhaustive family of forbidden subgraphs could be researched.

Open questions

Forbidden induced subgraphs of TSGs

In order to study this graph, a characterization in terms of forbidden induced subgraphs has to be given. An exhaustive family of forbidden subgraphs could be researched.

Complexity of TSG recognition

Is the recognition of thin strip graphs $\mathcal{NP}$?

Open questions

Forbidden induced subgraphs of TSGs

In order to study this graph, a characterization in terms of forbidden induced subgraphs has to be given. An exhaustive family of forbidden subgraphs could be researched.

Complexity of TSG recognition

Is the recognition of thin strip graphs $\mathcal{NP}$?

Complexity of other graph-theoretic problems

What can we say about the complexity of other graph-theoretic problems applied to thin strip graphs?

Thanks for listening

Slides and resources can be found here:

`https://github.com/Abde5/memo201718`