

CS 61B: Lecture 24
Wednesday, March 20, 2013

Today's reading: Goodrich & Tamassia, Chapter 7.

ROOTED TREES =====

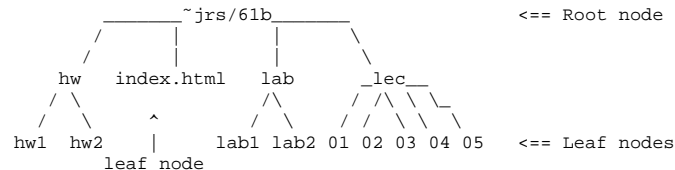
A `_tree_` consists of a set of nodes and a set of edges that connect pairs of nodes. A tree has the property that there is exactly one path (no more, no less) between any two nodes of the tree. A `_path_` is a sequence of one or more nodes, each consecutive pair being connected by an edge.

In a `_rooted_` tree, one distinguished node is called the `_root_`. Every node `c`, except the root, has exactly one `_parent_` node `p`, which is the first node traversed on the path from `c` to the root. `c` is `p`'s `_child_`. The root has no parent. A node can have any number of children.

Some other definitions:

- A `_leaf_` is a node with no children.
- `_Siblings_` are nodes with the same parent.
- The `_ancestors_` of a node `d` are the nodes on the path from `d` to the root. These include `d`'s parent, `d`'s parent's parent, `d`'s parent's parent's parent, and so forth up to the root. Technically, the ancestors of `d` also include `d` itself, which makes you wonder about `d`'s sex life. The root is an ancestor of every node in the tree.
- If `a` is an ancestor of `d`, then `d` is a `_descendant_` of `a`.
- The `_length_` of a path is the number of edges in the path.
- The `_depth_` of a node `n` is the length of the path from `n` to the root. (The depth of the root is zero.)
- The `_height_` of a node `n` is the length of the path from `n` to its deepest descendant. (The height of a leaf node is zero.)
- The height of a tree is the depth of its deepest node = height of the root.
- The `_subtree_` rooted at node `n` is the tree formed by `n` and its descendants.
- A `_binary_tree_` is a tree in which no node has more than two children, and every child is either a `_left_child_` or a `_right_child_`, even if it's the only child its parent has.

A commonly encountered application of trees is the directory structure of a file system.



Representing Rooted Trees

Goodrich and Tamassia present a data structure in which each node has three references: one reference to an item, one reference to the node's parent, and one reference to the node's children, which can be stored in any reasonable data structure like a linked list. Directories are typically stored this way, but the lists they use are represented very differently than our list ADTs.

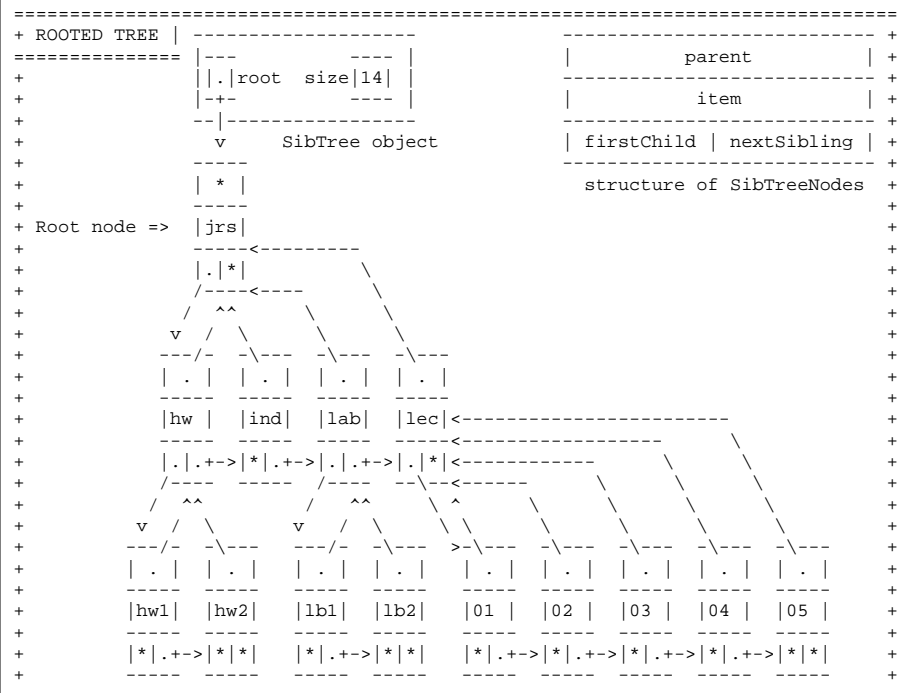
Another popular tree representation spurns separately encapsulated linked lists so that siblings are directly linked. It retains the "item" and "parent" references, but instead of referencing a list of children, each node references just its leftmost child. Each node also references its next sibling to the right. The "nextSibling" references are used to join the children of a node in a singly-linked list, whose head is the node's "firstChild".

I'll call this tree a "SibTree", since siblings are central to the representation. The nodes I call "SibTreeNode".

```

class SibTreeNode {
    Object item;
    SibTreeNode parent;
    SibTreeNode firstChild;
    SibTreeNode nextSibling;
}

class SibTree {
    SibTreeNode root;
    int size;
}
  
```



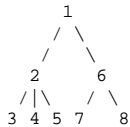
Tree Traversals

A `_traversal_` is a manner of `_visiting_` each node in a tree once. What you do when visiting any particular node depends on the application; for instance, you might print a node's value, or perform some calculation upon it. There are several different traversals, each of which orders the nodes differently.

Many traversals can be defined recursively. In a `_preorder_` traversal, you visit each node before recursively visiting its children, which are visited from left to right. The root is visited first.

```
class SibTreeNode {
    public void preorder() {
        this.visit();
        if (firstChild != null) {
            firstChild.preorder();
        }
        if (nextSibling != null) {
            nextSibling.preorder();
        }
    }
}
```

Suppose your `visit()` method numbers the nodes in the order they're visited. A preorder traversal visits the nodes in this order.



Each node is visited only once, so a preorder traversal takes $O(n)$ time, where n is the number of nodes in the tree. All the traversals we will consider take $O(n)$ time.

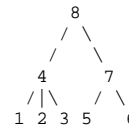
A preorder traversal is a natural way to print a directory's structure:

```
~jrs/61b
hw
  hw1
  hw2
index.html
lab
  lab1
  lab2
lec
  01
  02
  03
  04
  05
```

In a `_postorder_` traversal, you visit each node's children (in left-to-right order) before the node itself.

```
public void postorder() {
    if (firstChild != null) {
        firstChild.postorder();
    }
    this.visit();
    if (nextSibling != null) {
        nextSibling.postorder();
    }
}
```

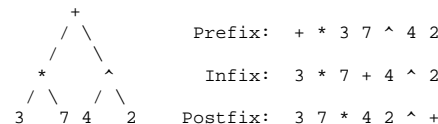
A postorder traversal visits the nodes in this order.



The `postorder()` code is trickier than it looks. The best way to understand it is to draw a depth-two tree on paper, then pretend you're the computer and execute the algorithm carefully. Trust me on this. It's worth your time.

A postorder traversal is the natural way to sum the total disk space used in the root directory and its descendants. In the example above, a postorder traversal would begin by summing the sizes of the files in `hw1/` and `hw2/`; then it would visit `hw/` and sum its two children. The last thing it would do is determine the total disk space at the root `~jrs/61b/`, which sums all the files in the tree.

Binary trees allow for an `_inorder_` traversal: recursively traverse the root's left subtree (rooted at the left child), then the root itself, then the root's right subtree. The preorder, inorder, and postorder traversals of an expression tree will print a `_prefix_`, `_infix_`, or `_postfix_` expression, respectively.



In a `_level_order_` traversal, you visit the root, then all the depth-1 nodes (from left to right), then all the depth-2 nodes, et cetera. The level order traversal of our expression tree is `" + * ^ 3 7 4 2 "` (which doesn't mean much).

Unlike the three previous traversals, a level order traversal is not straightforward to define recursively. However, a level order traversal can be done in $O(n)$ time. Use a queue, which initially contains only the root. Then repeat the following steps:

- Dequeue a node.
- Visit it.
- Enqueue its children (in order from left to right).

Continue until the queue is empty.

A final thought: if you use a stack instead of a queue, and push each node's children in reverse order--from right to left (so they pop off the stack in order from left to right)--you perform a preorder traversal. Think about why.