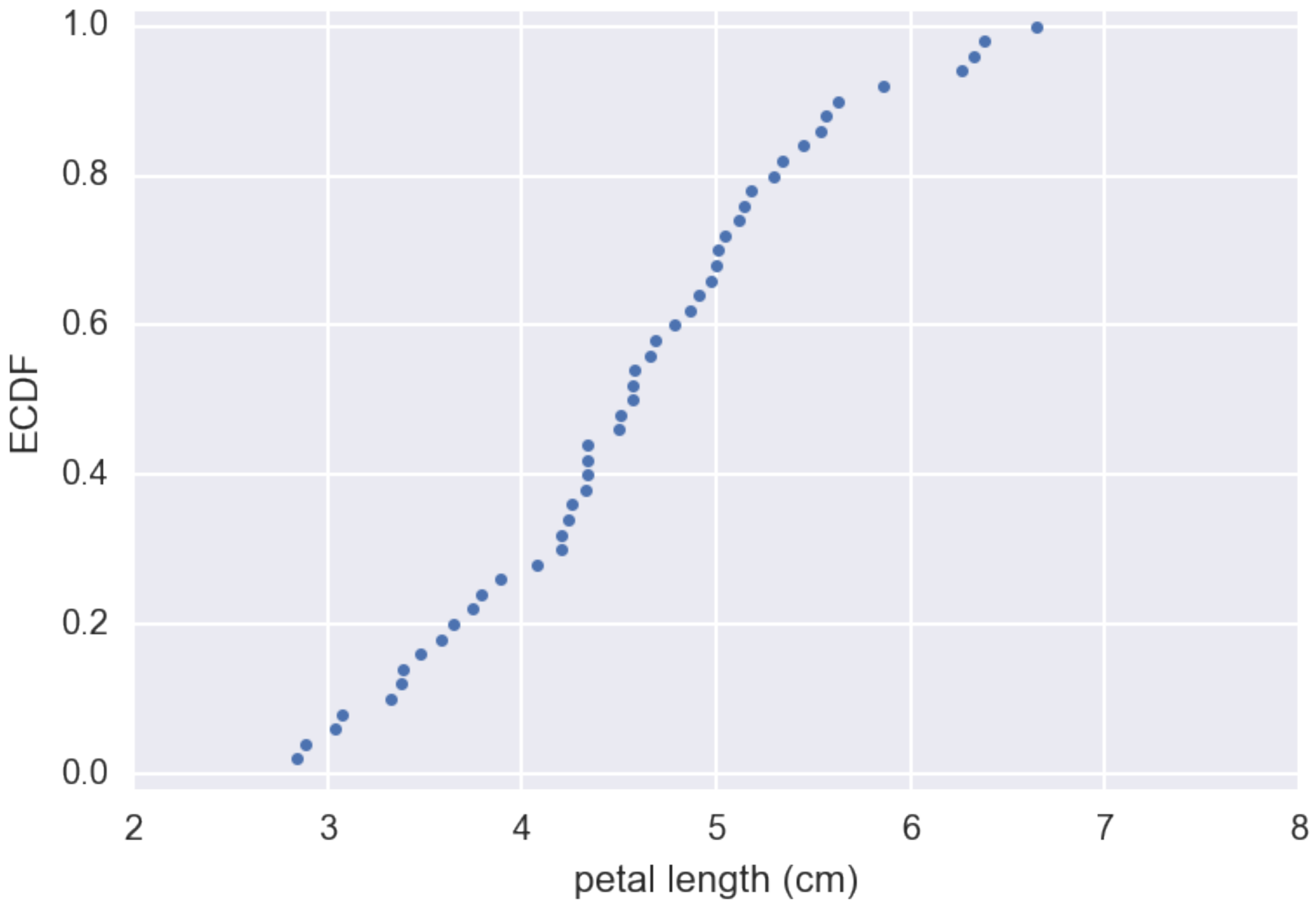
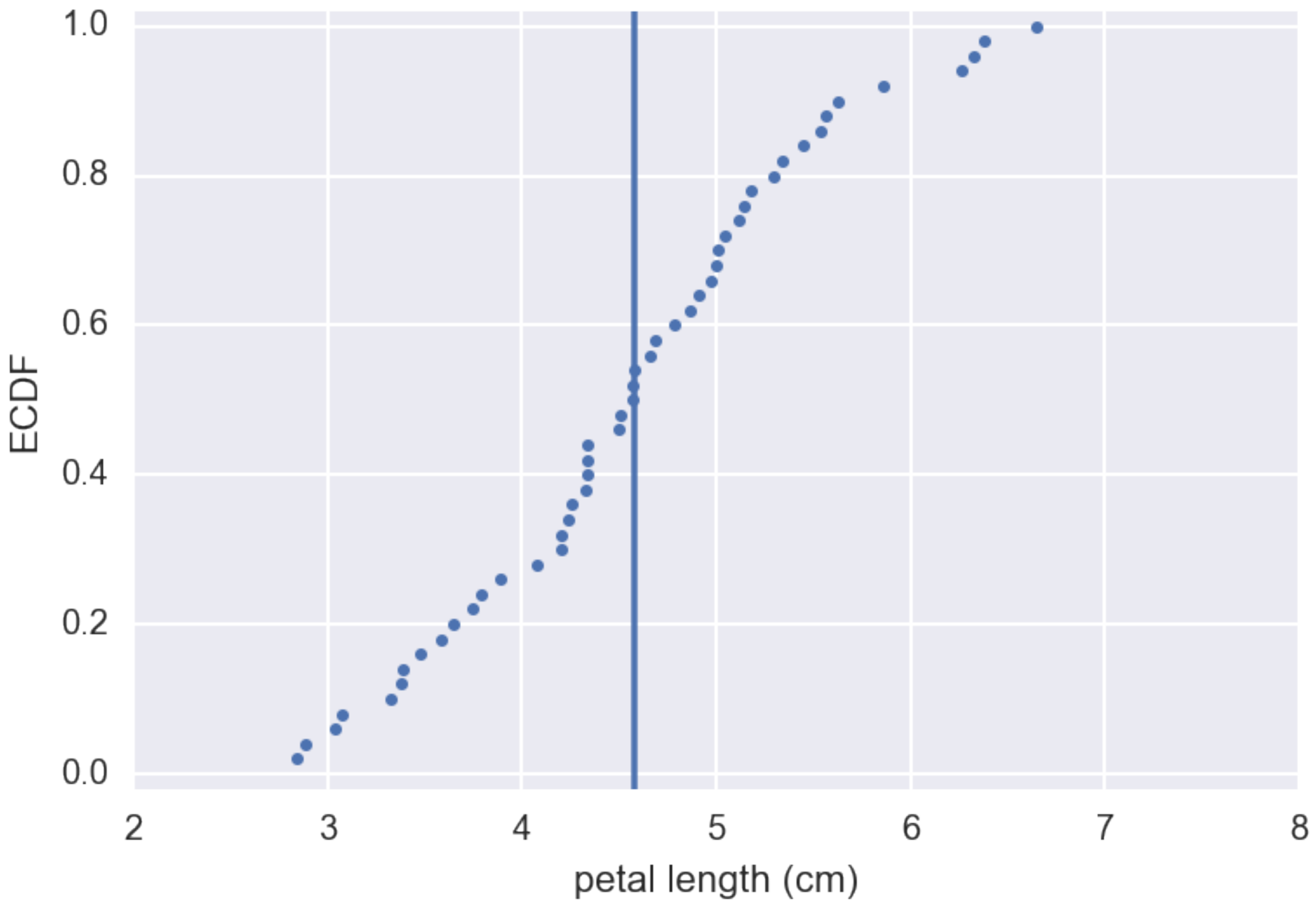
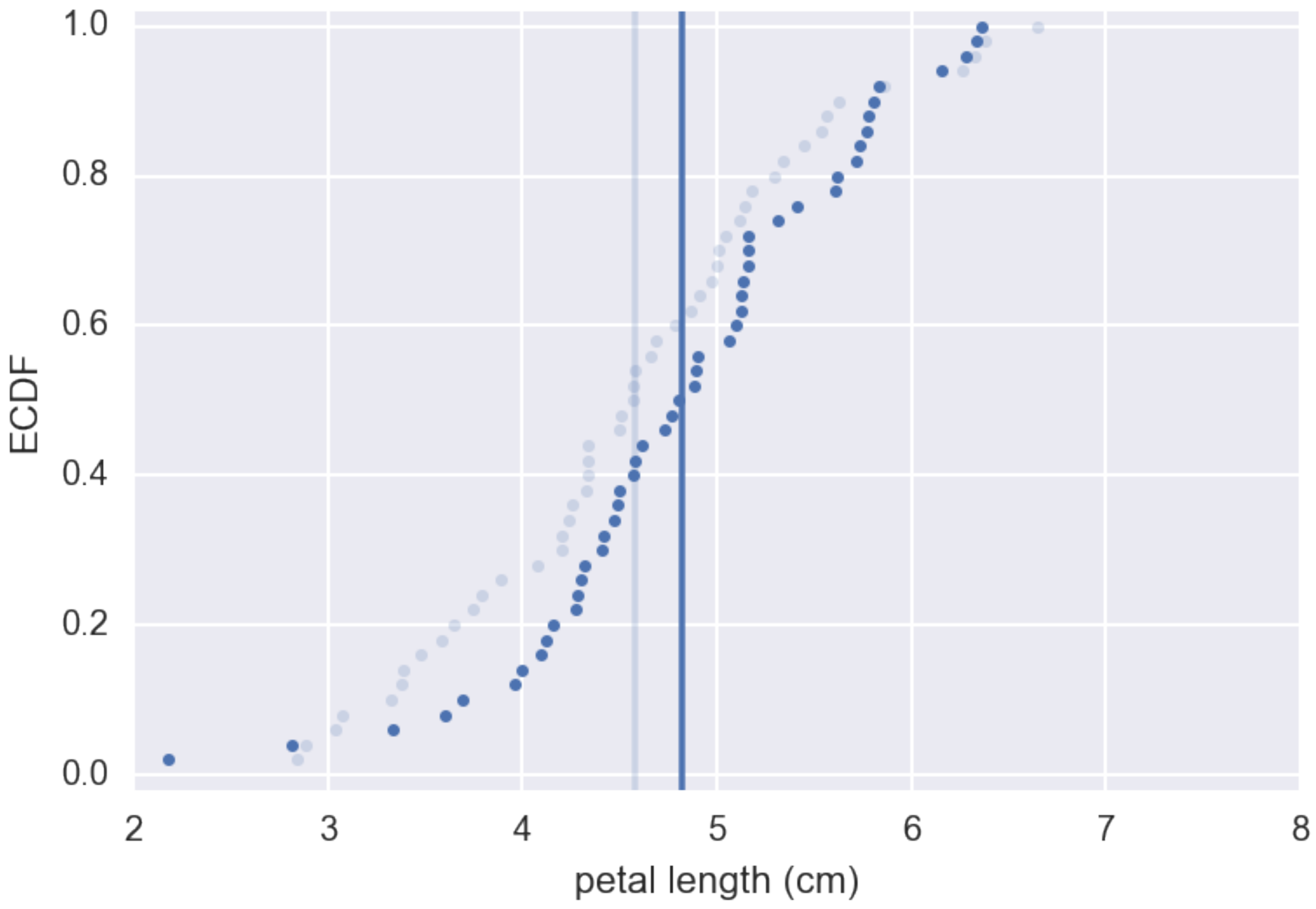
50 measurements of petal length



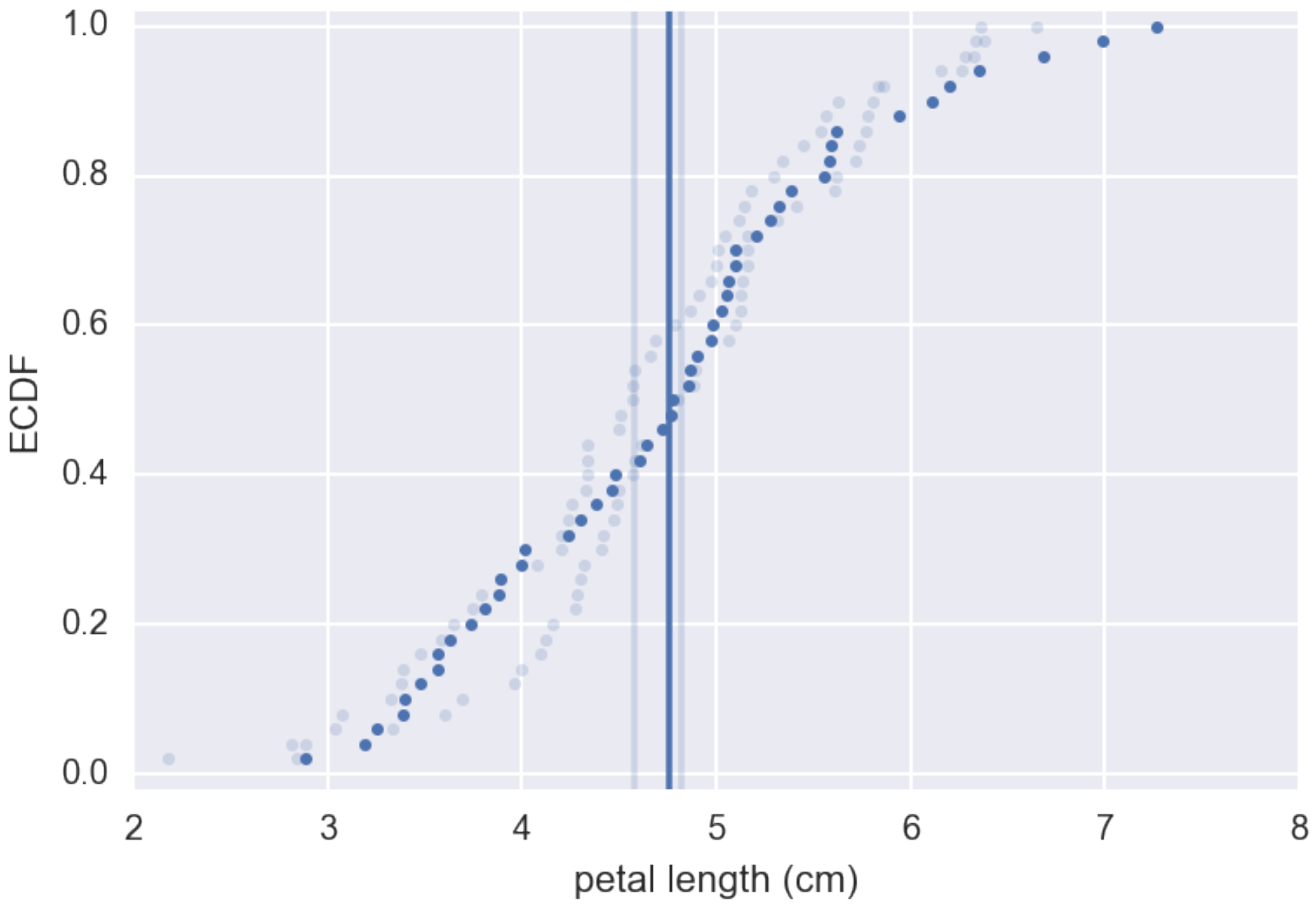
50 measurements of petal length



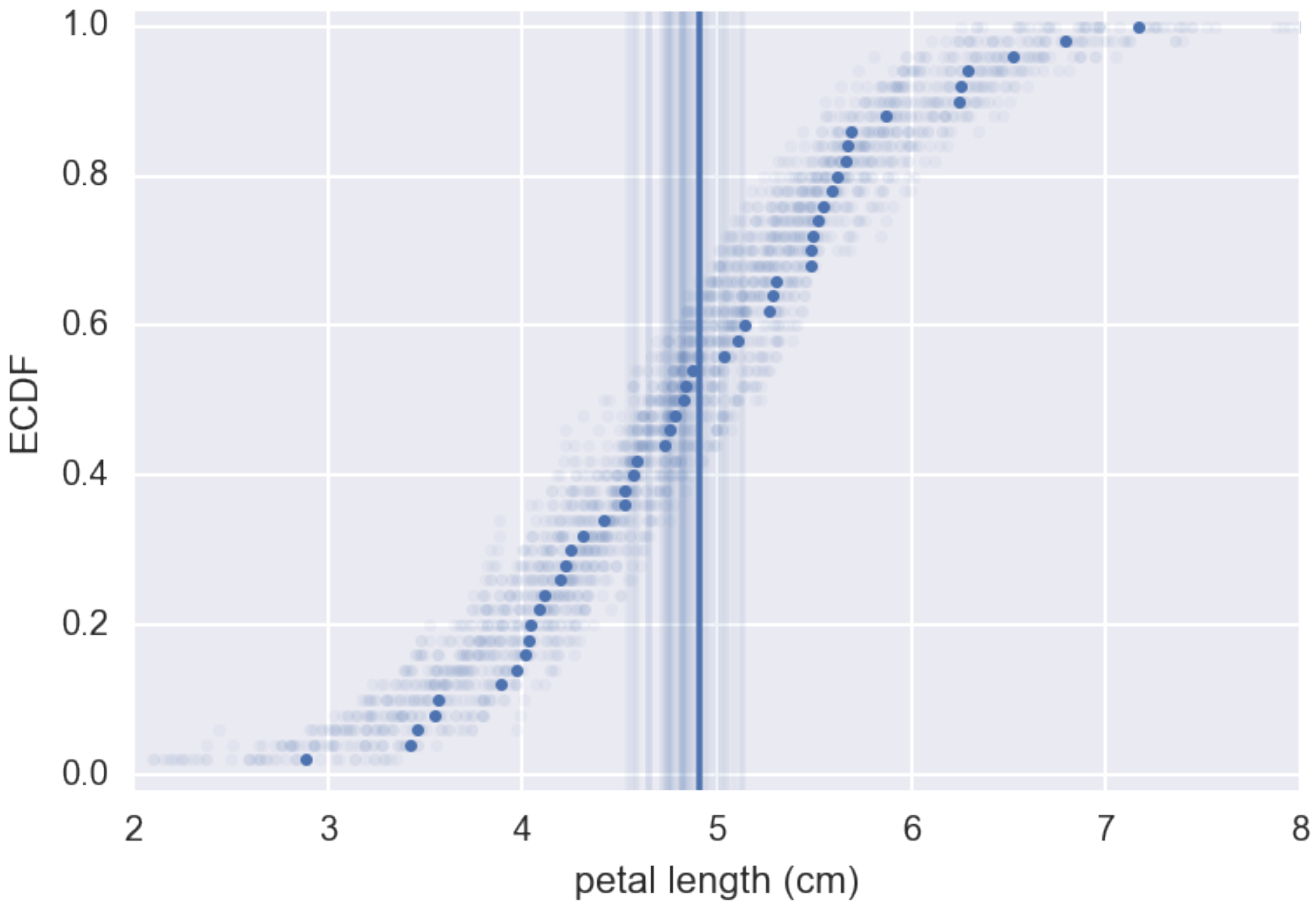
50 measurements of petal length



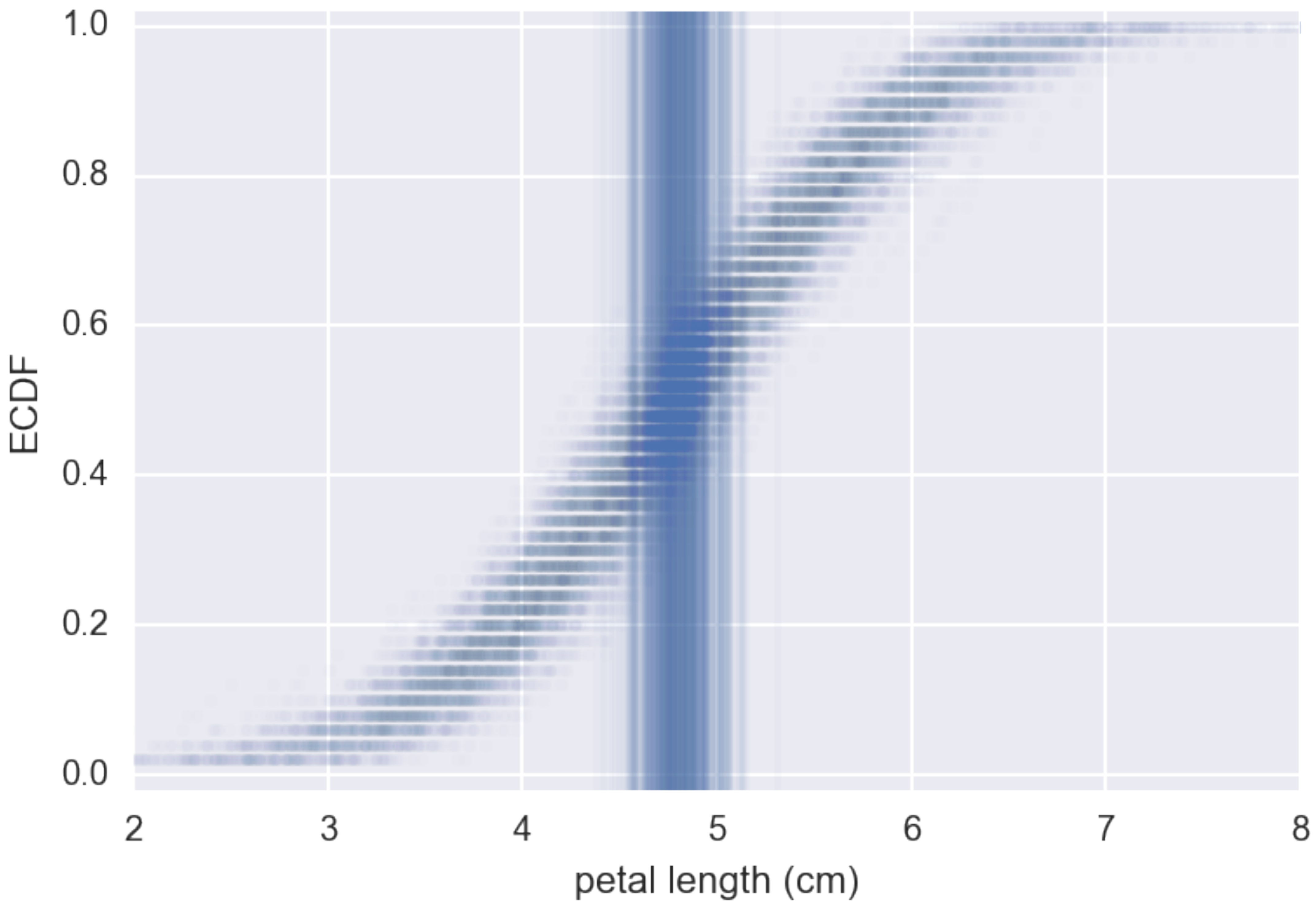
50 measurements of petal length



50 measurements of petal length



Repeats of 50 measurements of petal length



Hacker statistics

- Uses simulated repeated measurements to compute probabilities.

Image: artist unknown

Image: Heritage Auction

The `np.random` module

- Suite of functions based on *random number generation*
- `np.random.random()`:

    draw a number between 0 and 1

The `np.random` module

- Suite of functions based on *random number generation*
- `np.random.random()`:

    draw a number between 0 and 1

< 0.5 ⟶

≥ 0.5 ⟶

Images: Heritage Auction

Bernoulli trial

- An experiment that has two options, "success" (True) and "failure" (False).

Random number seed

- Integer fed into random number generating algorithm
- Manually seed random number generator if you need reproducibility
- Specified using `np.random.seed()`

Simulating 4 coin flips

```
In [1]: import numpy as np

In [2]: np.random.seed(42)

In [3]: random_numbers = np.random.random(size=4)

In [4]: random_numbers
Out[4]: array([ 0.37454012,  0.95071431,  0.73199394,
0.59865848])

In [5]: heads = random_numbers < 0.5

In [6]: heads
Out[6]: array([ True, False, False, False], dtype=bool)

In [7]: np.sum(heads)
Out[7]: 1
```

Simulating 4 coin flips

```
In [1]: n_all_heads = 0  # Initialize number of 4-heads trials

In [2]: for _ in range(10000):
   ...:     heads = np.random.random(size=4) < 0.5
   ...:     n_heads = np.sum(heads)
   ...:     if n_heads == 4:
   ...:         n_all_heads += 1
   ...:
   ...:

In [3]: n_all_heads / 10000
Out[3]: 0.0621
```

Hacker stats probabilities

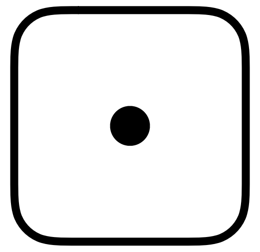
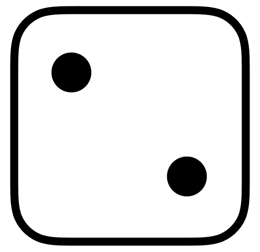
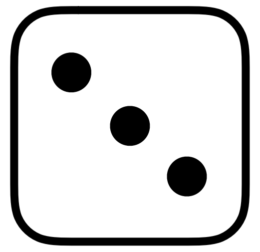
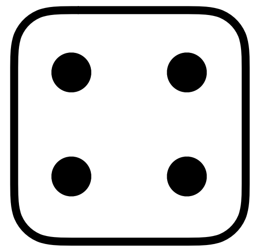
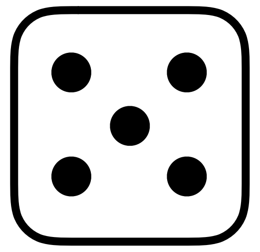
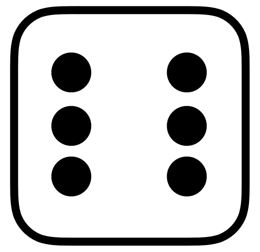
- Determine how to simulate data
- Simulate many many times
- Probability is approximately fraction of trials with the outcome of interest

Probability mass function (PMF)

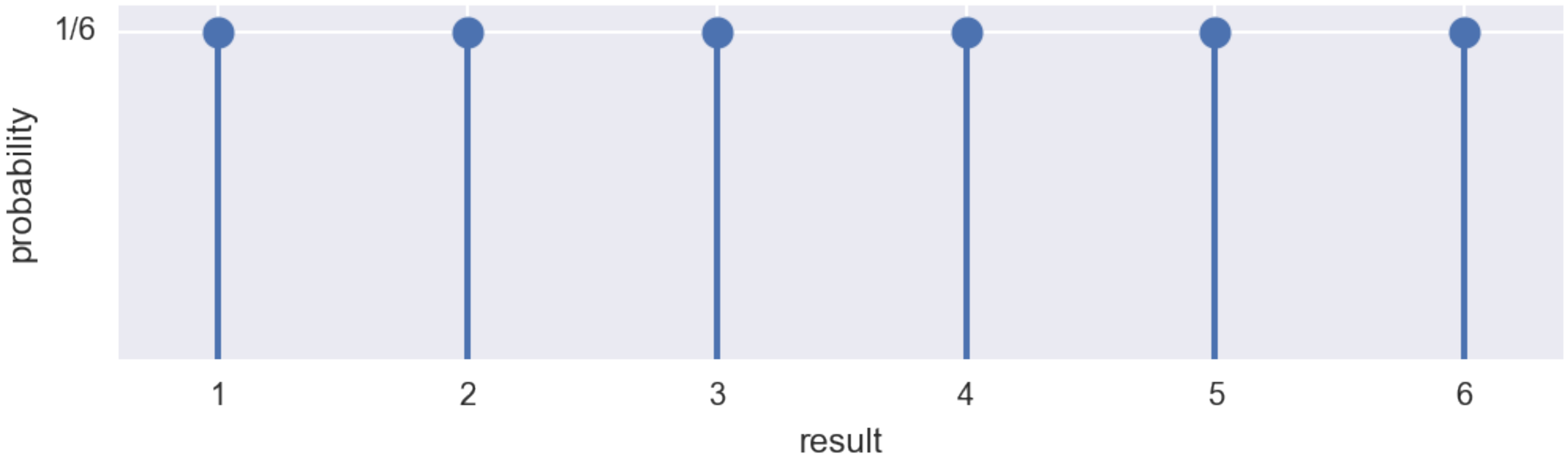
- The set of probabilities of discrete outcomes

Discrete Uniform PMF

Tabular

					
1/6	1/6	1/6	1/6	1/6	1/6

Graphical



Probability distribution

- A mathematical description of outcomes

Discrete Uniform distribution: the story

- The outcome of rolling a single fair die is Discrete Uniformly distributed.

Binomial distribution: the story

- The number r of successes in n Bernoulli trials with probability p of success, is Binomially distributed

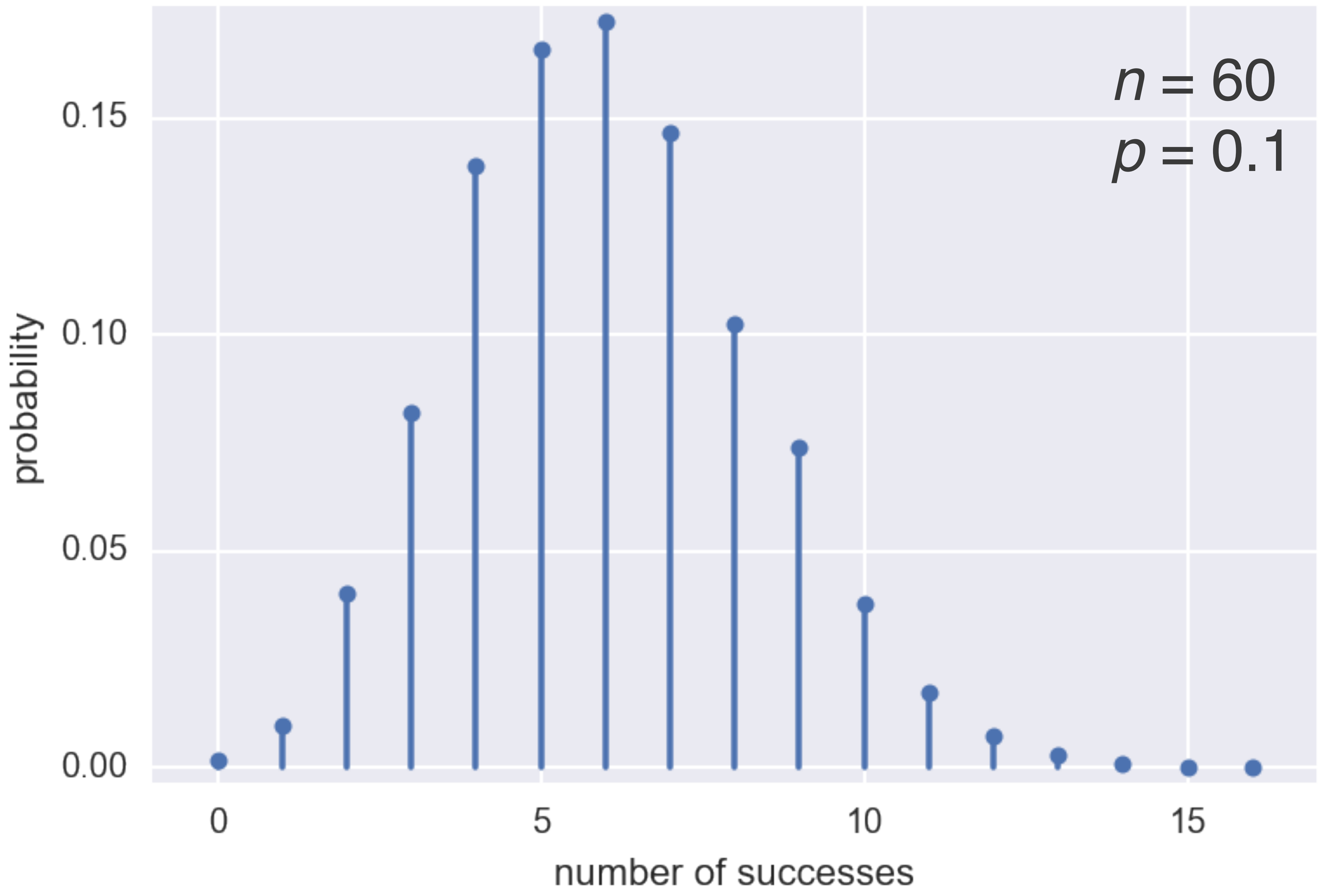
- The number r of heads in 4 coin flips with probability 0.5 of heads, is Binomially distributed

Sampling from the Binomial distribution

```
In [1]: np.random.binomial(4, 0.5)
Out[1]: 2

In [2]: np.random.binomial(4, 0.5, size=10)
Out[2]: array([4, 3, 2, 1, 1, 0, 3, 2, 3, 0])
```

The Binomial PMF

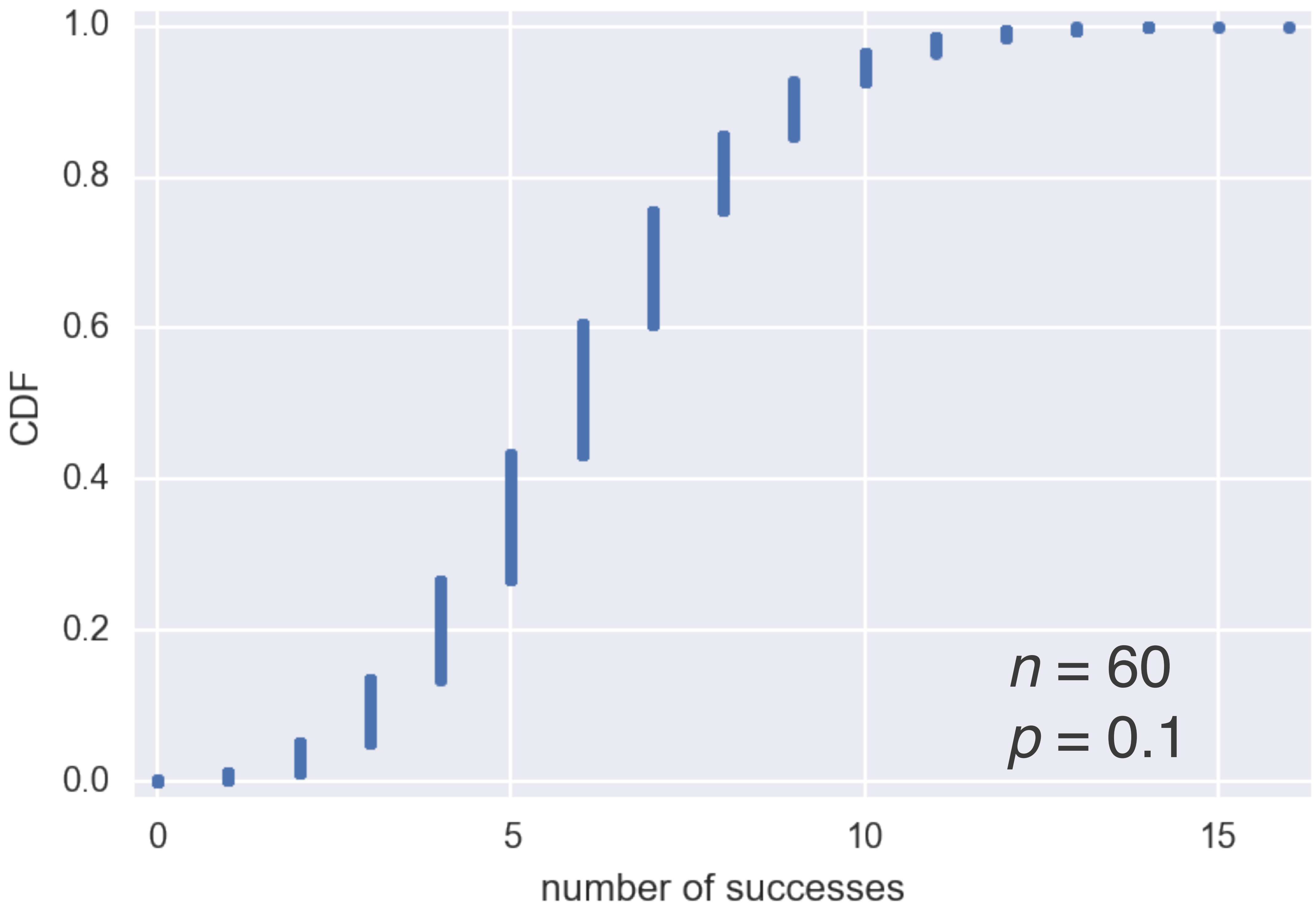
```
In [1]: samples = np.random.binomial(60, 0.1, size=10000)
```



The Binomial CDF

```
In [1]: import matplotlib.pyplot as plt

In [2]: import seaborn as sns

In [3]: sns.set()

In [4]: x, y = ecdf(samples)

In [5]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [6]: plt.margins(0.02)

In [7]: _ = plt.xlabel('number of successes')

In [8]: _ = plt.ylabel('CDF')

In [9]: plt.show()
```

The Binomial CDF



Poisson process

- The timing of the next event is completely independent of when the previous event happened

Examples of Poisson processes

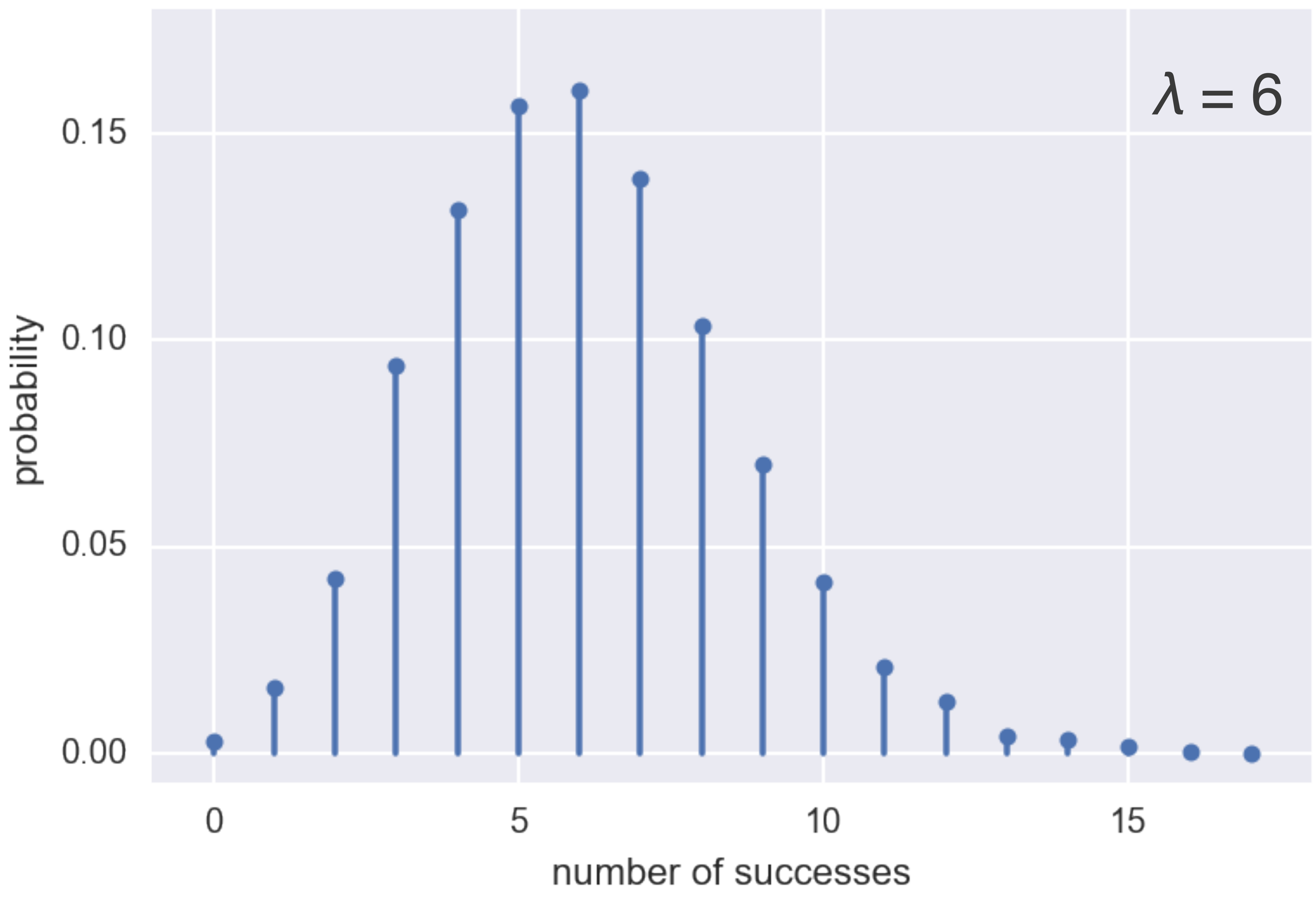
- Natural births in a given hospital
- Hit on a website during a given hour
- Meteor strikes
- Molecular collisions in a gas
- Aviation incidents
- Buses in Poissonville

Poisson distribution

- The number *r* of arrivals of a Poisson process in a given time interval with average rate of λ arrivals per interval is Poisson distributed.

- The number *r* of hits on a website in one hour with an average hit rate of 6 hits per hour is Poisson distributed.

Poisson PMF



$\lambda = 6$
0.15
0.10
0.05
0.00
probability
0
5
10
15
number of successes

Poisson Distribution

- Limit of the Binomial distribution for low probability of success and large number of trials.

- That is, for rare events.

The Poisson CDF

```
In [1]: samples = np.random.poisson(6, size=10000)

In [2]: x, y = ecdf(samples)

In [3]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [4]: plt.margins(0.02)

In [5]: _ = plt.xlabel('number of successes')

In [6]: _ = plt.ylabel('CDF')

In [7]: plt.show()
```

The Poisson CDF