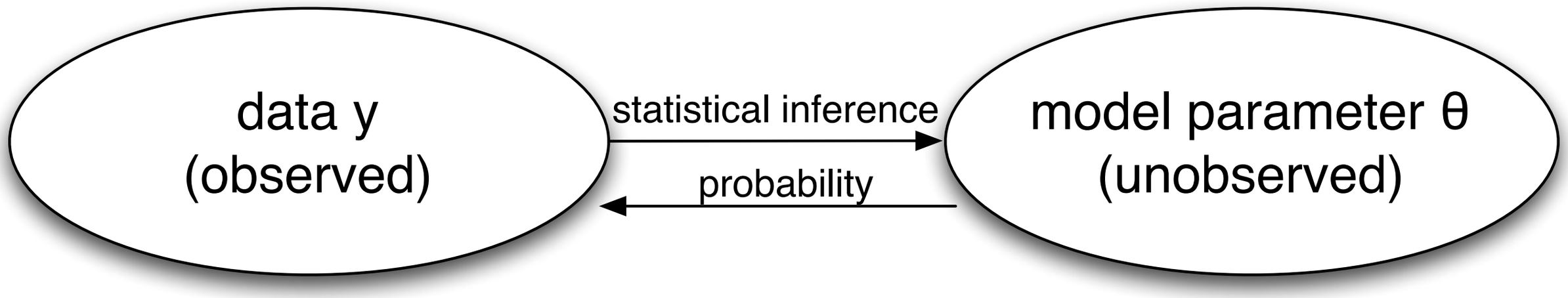
# CS109/Stat121/AC209/E-109
# Data Science
## Statistical Models

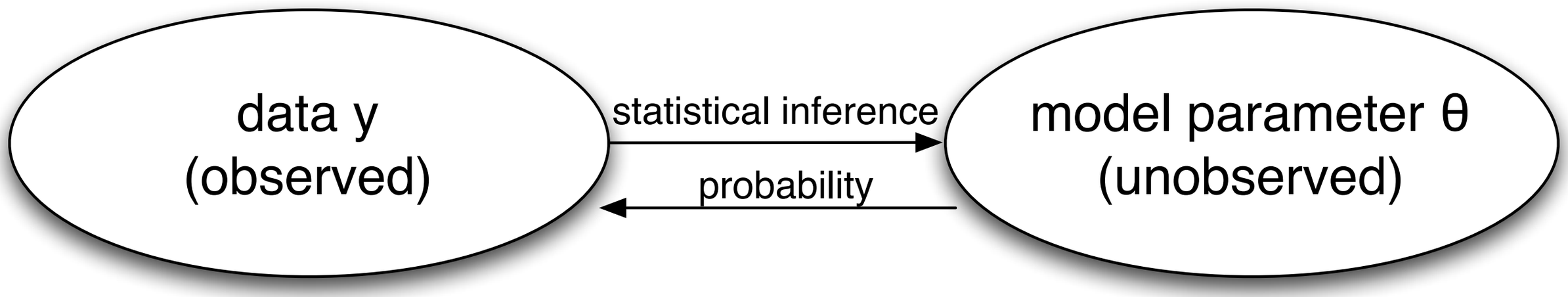


# Drowning in data, but starved for information

source: http://extensionengine.com/drowning-in-data-the-biggest-hurdle-for-mooc-proliferation/

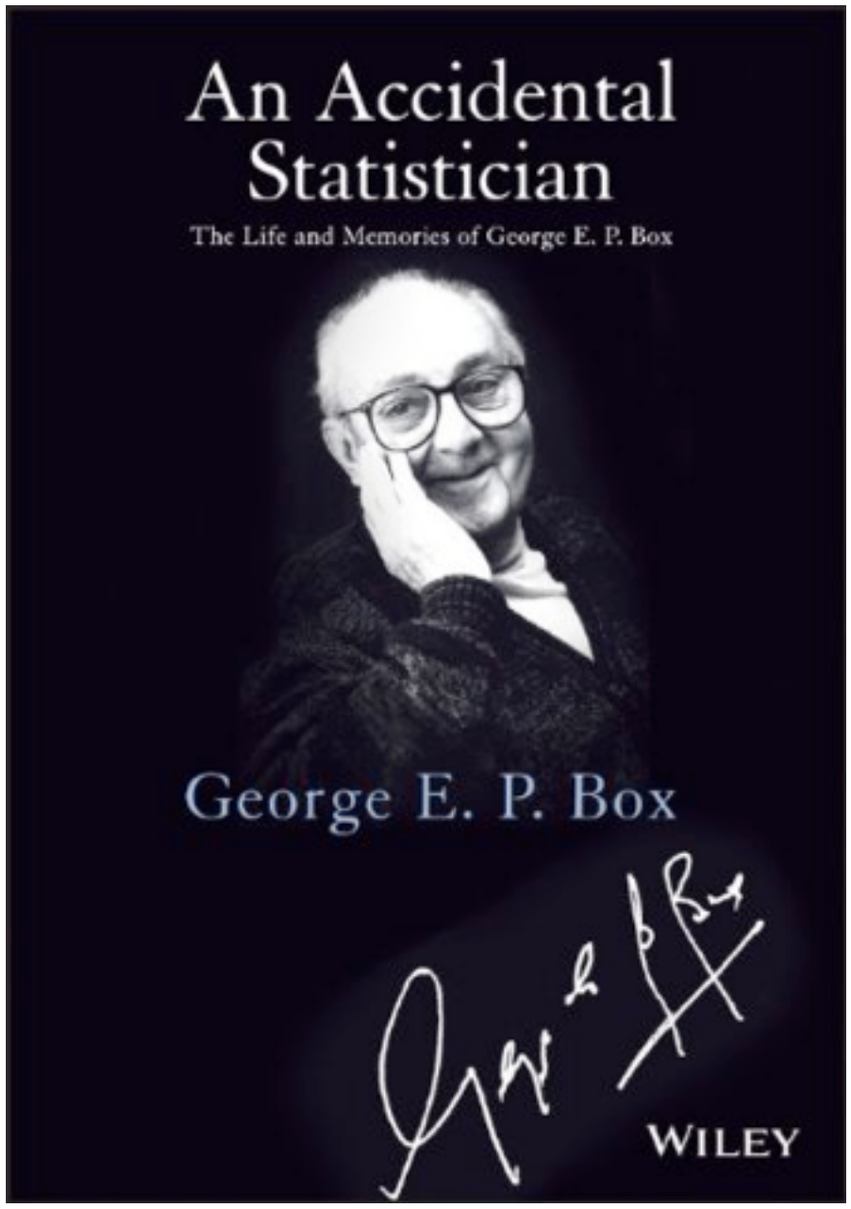
# What is a statistical model?

- a *family* of distributions, indexed by *parameters*
- sharpens distinction between *data* and *parameters*, and between *estimators* and *estimands*
- parametric (e.g., based on Normal, Binomial) vs. nonparametric (e.g., methods like bootstrap, KDE)



# What *good* is a statistical model?

"All models are wrong, but some models are useful."
– George Box (1919-2013)

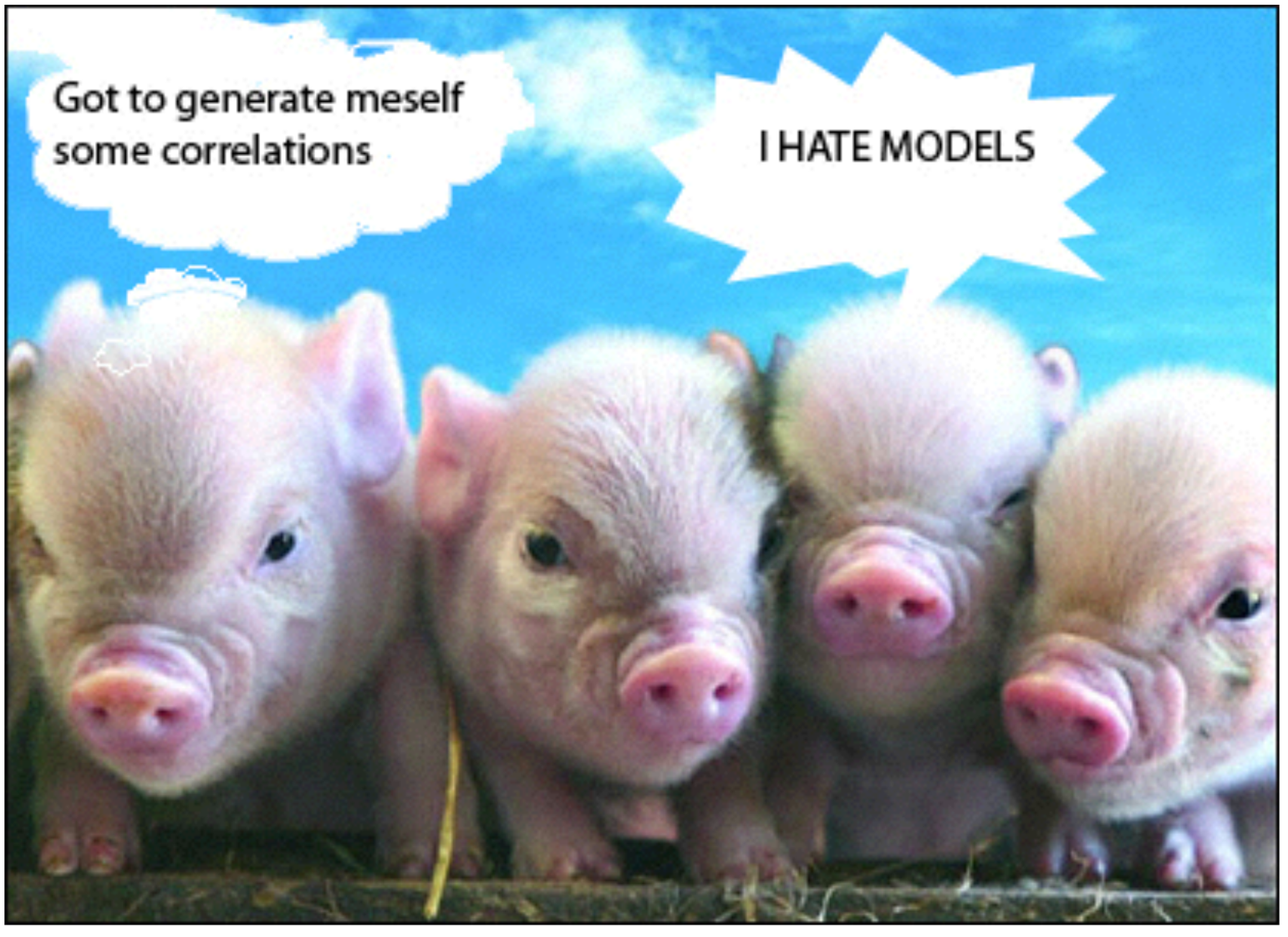


# Jorge Luis Borges,
# “On Exactitude in Science”

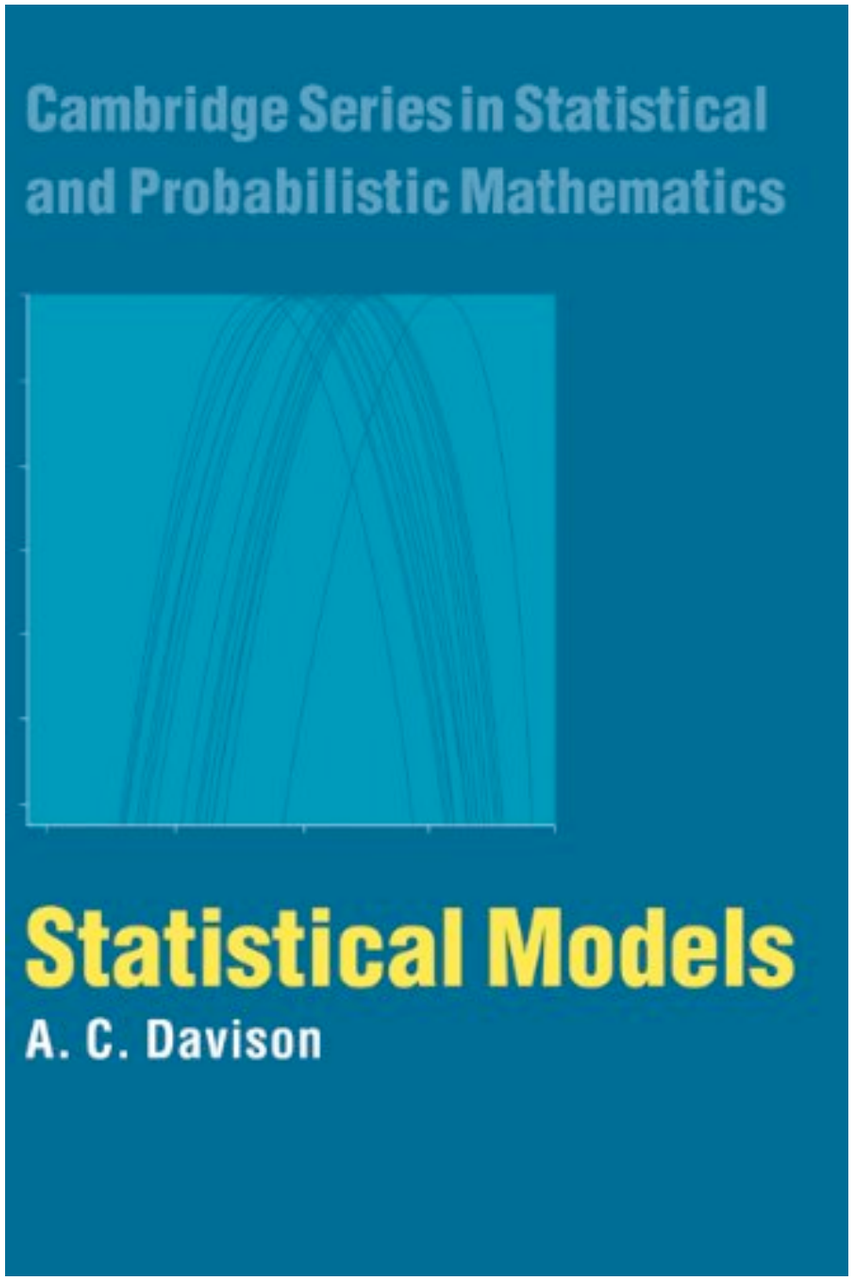
In that Empire, the Art of Cartography attained such Perfection that the map of a single Province occupied the entirety of a City, and the map of the Empire, the entirety of a Province. In time, those Unconscionable Maps no longer satisfied, and the Cartographers Guild struck a Map of the Empire whose size was that of the Empire, and which coincided point for point with it.

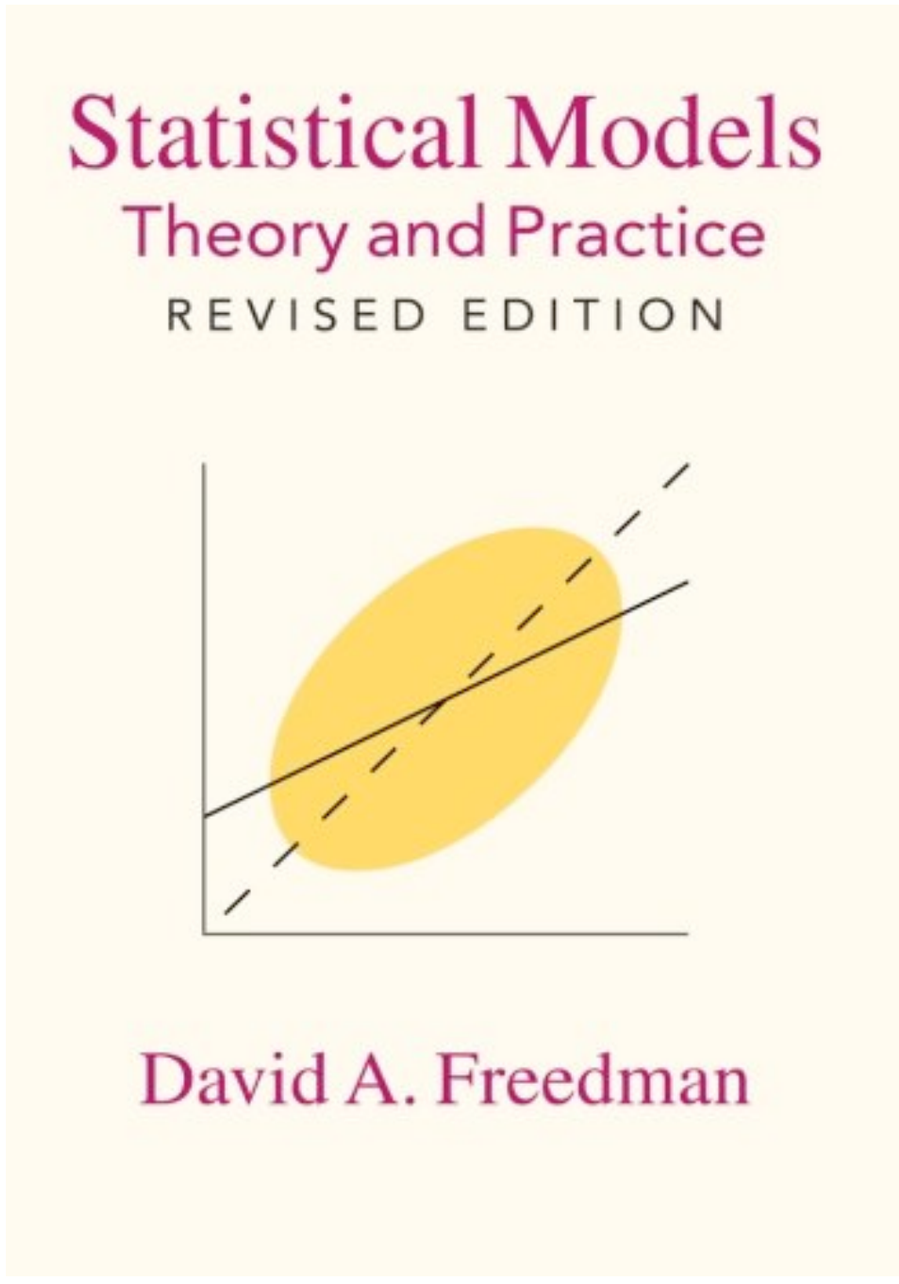
Borges Google Doodle

# “Big Data vs. Pig Data”: https://scensci.wordpress.com/2012/12/14/big-data-or-pig-data/



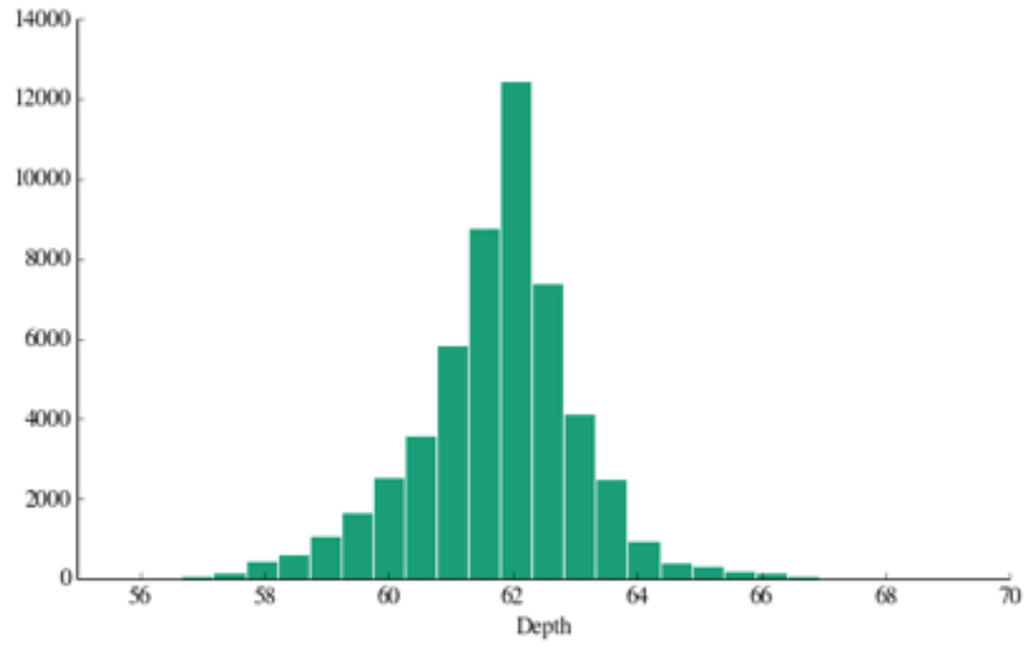
# Statistical Models: Two Books



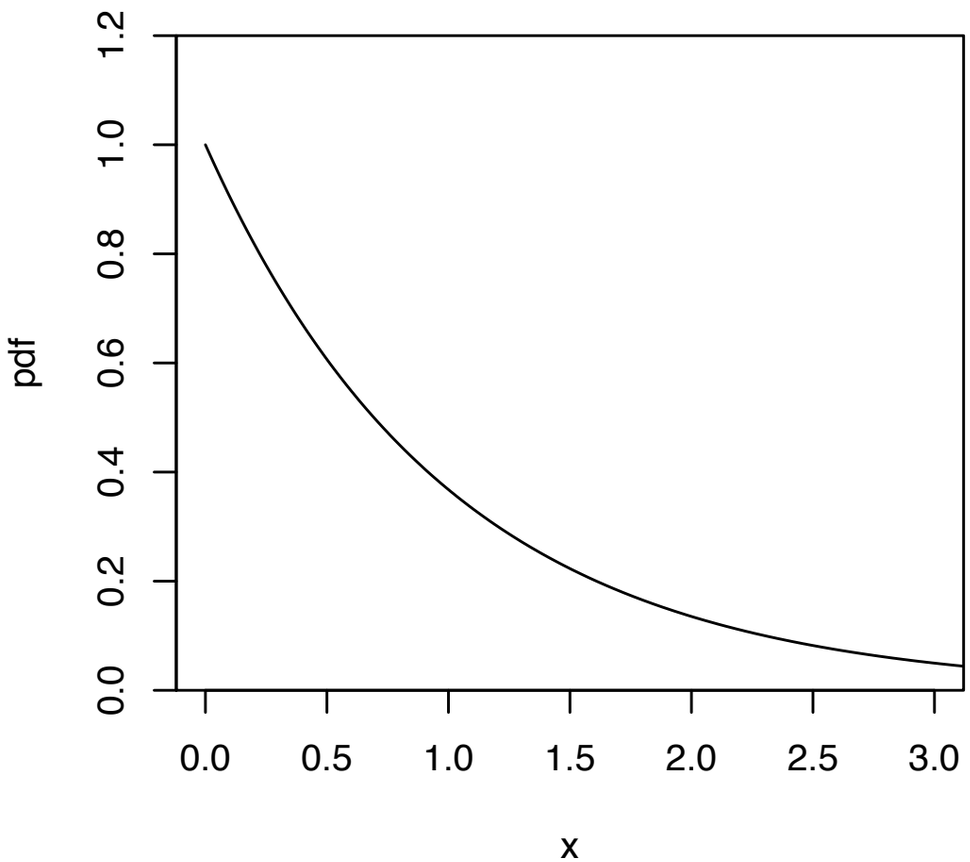
# Parametric vs. Nonparametric

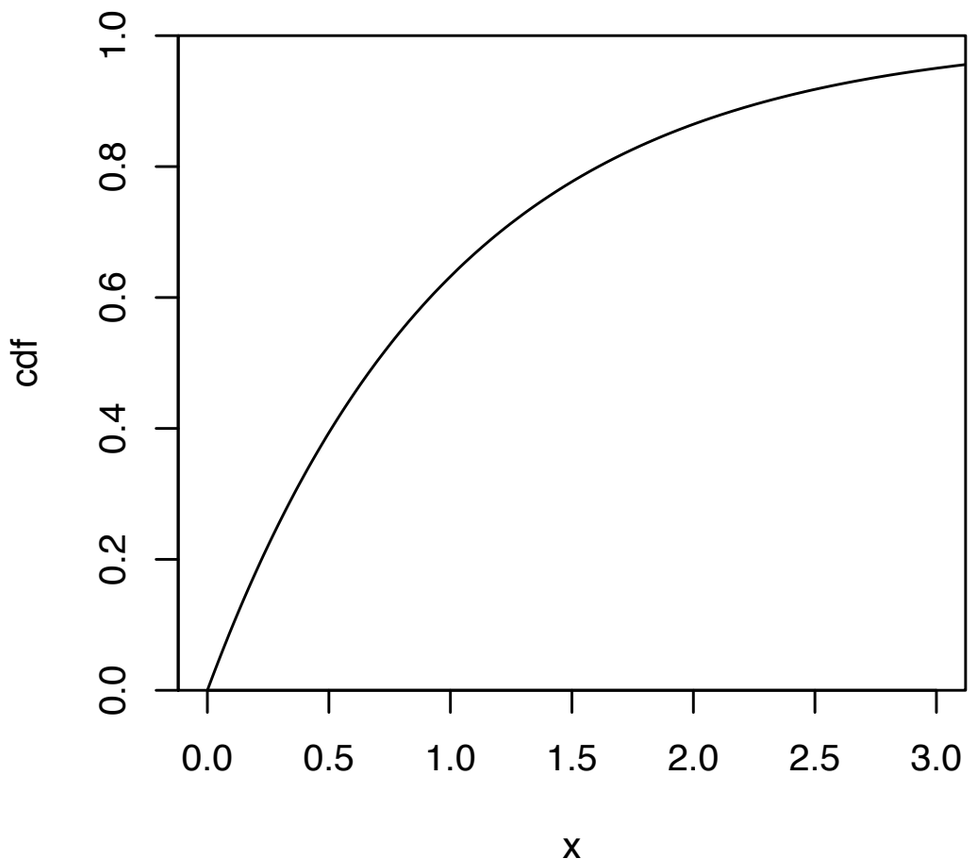
- parametric: finite-dimensional parameter space (e.g., mean and variance for a Normal)
- nonparametric: infinite-dimensional parameter space
- is there anything in between?
- nonparametric is very general, but no free lunch!
- remember to plot and explore the data!



# Parametric Model Example: Exponential Distribution

$$f(x) = \lambda e^{-\lambda x}, x > 0$$




Remember the memoryless property!

# Exponential Distribution

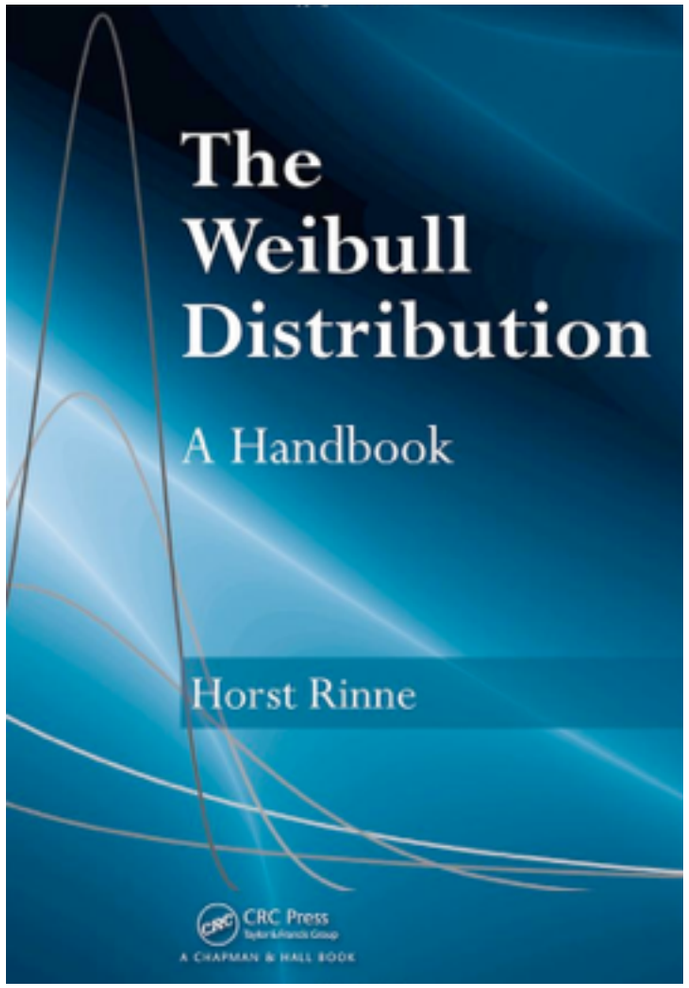
$$f(x) = \lambda e^{-\lambda x}, x > 0$$

- Exponential is *characterized* by memoryless property
- all models are wrong, but some are useful...
- iterate between exploring, the data model-building, model-fitting, and model-checking
- key building block for more realistic models

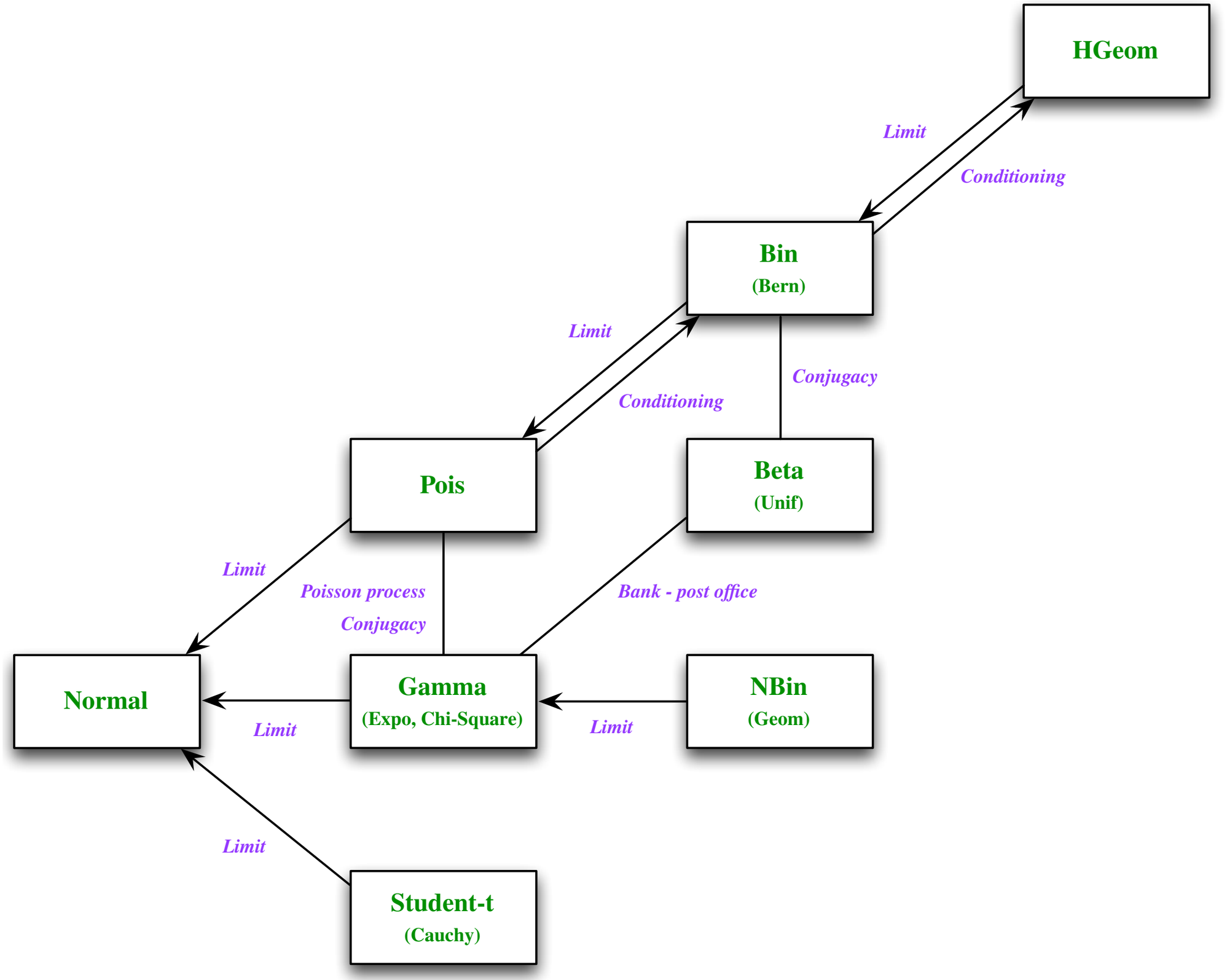
Remember the memoryless property!

# The Weibull Distribution

- Exponential has constant hazard function
- Weibull generalizes this to a hazard that is t to a power
- much more flexible and realistic than Exponential
- *representation*: a Weibull is an Expo to a power



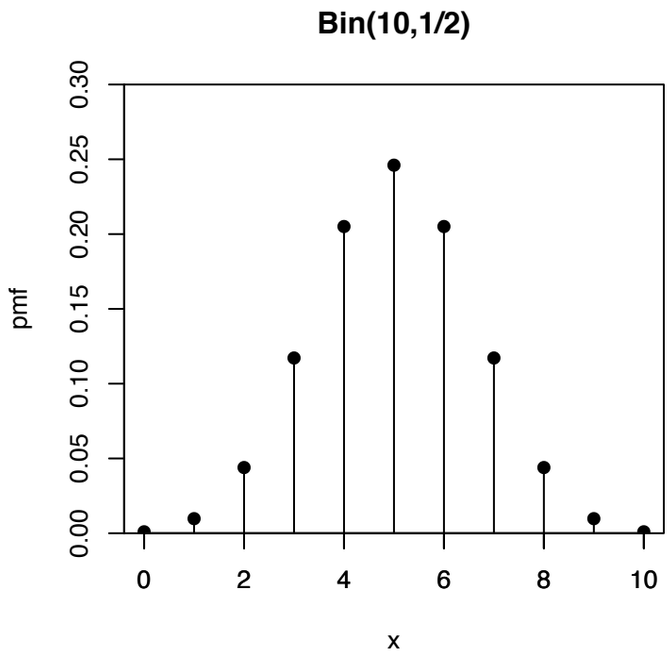
# Family Tree of Parametric Distributions

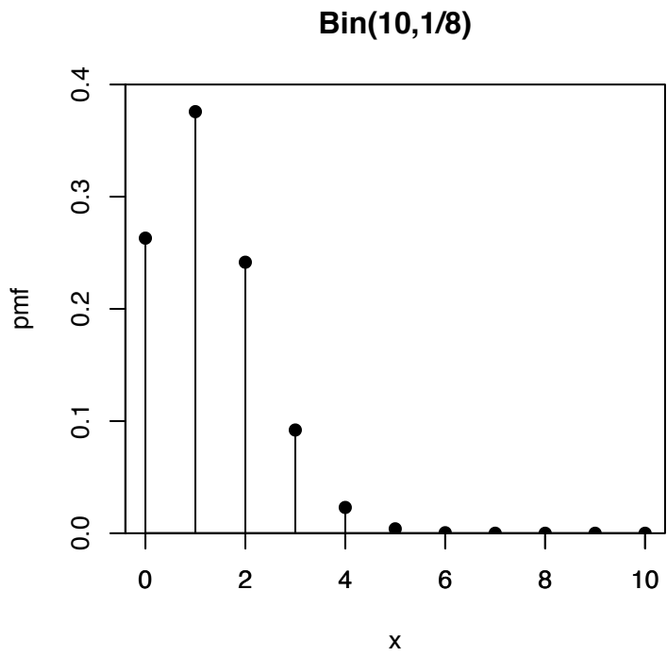


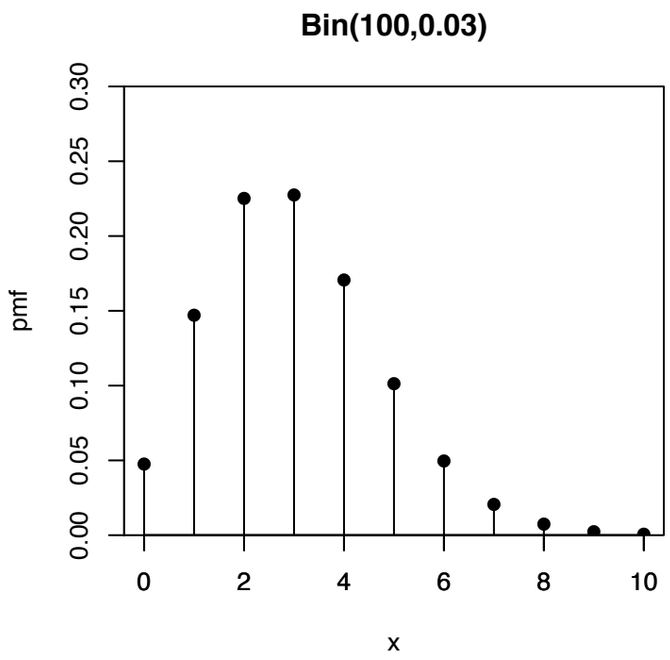

Blitzstein-Hwang, Introduction to Probability

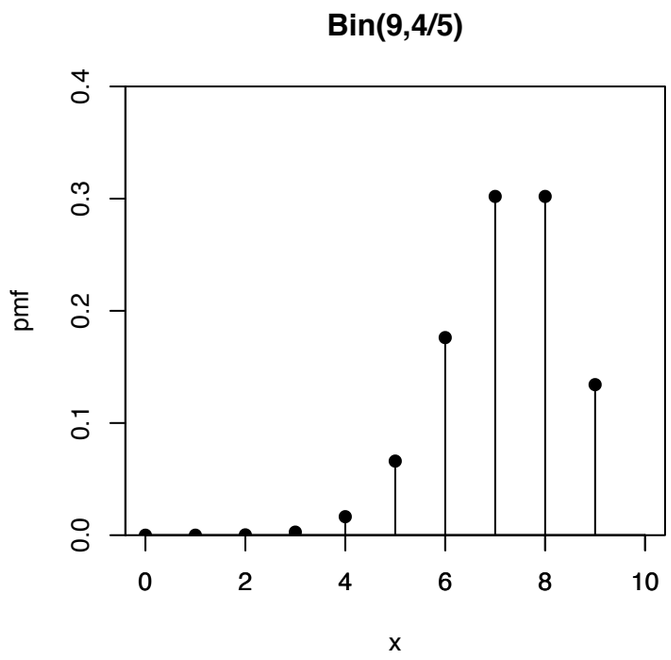
# Binomial Distribution

story: X~Bin(n,p) is the number of successes in n independent Bernoulli(p) trials.



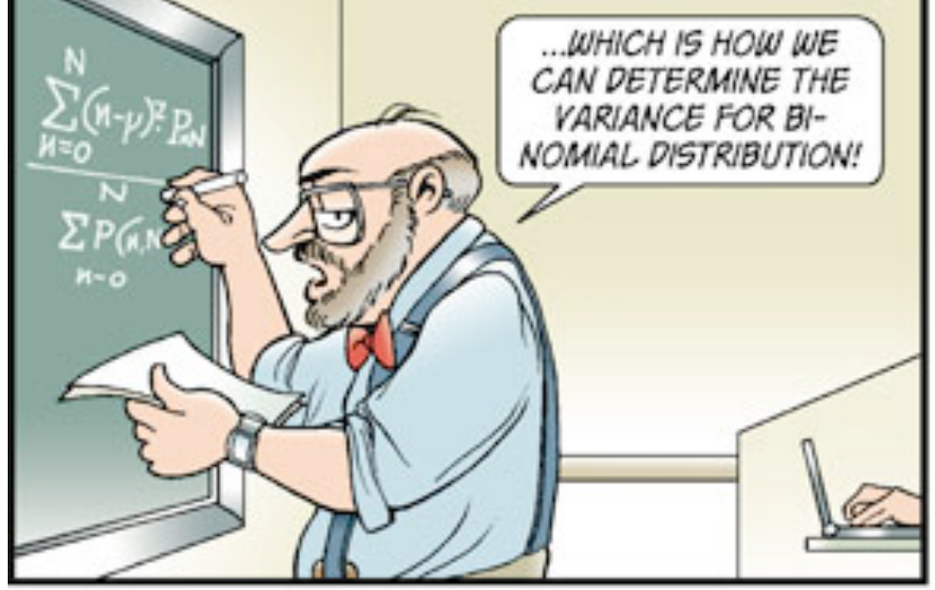
# Binomial Distribution

story: X~Bin(n,p) is the number of successes in n independent Bernoulli(p) trials.

Example: # votes for candidate A in election with n voters, where each independently votes for A with probability p

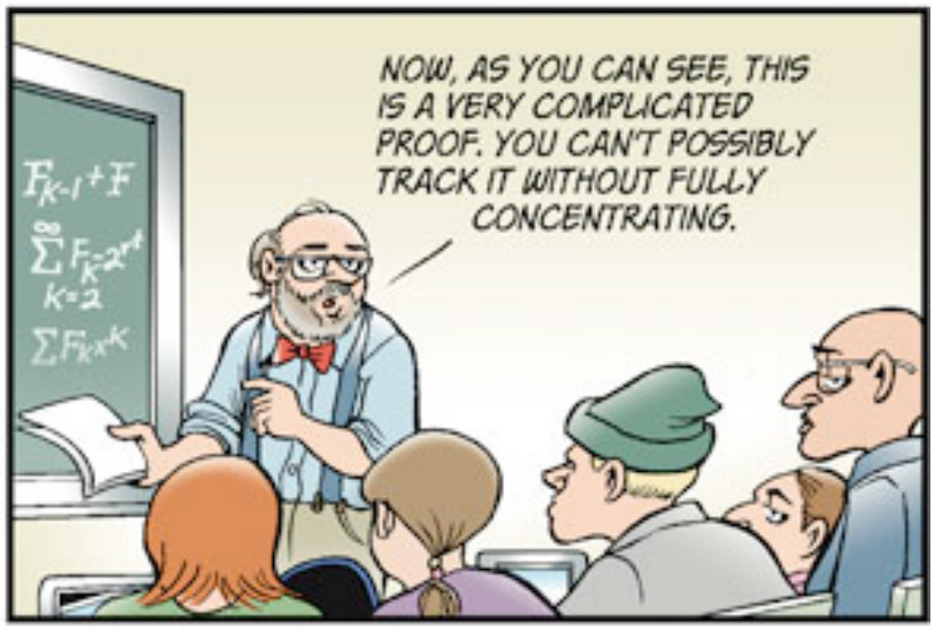
mean is np (by story and linearity of expectation: E(X+Y)=E(X)+E(Y))

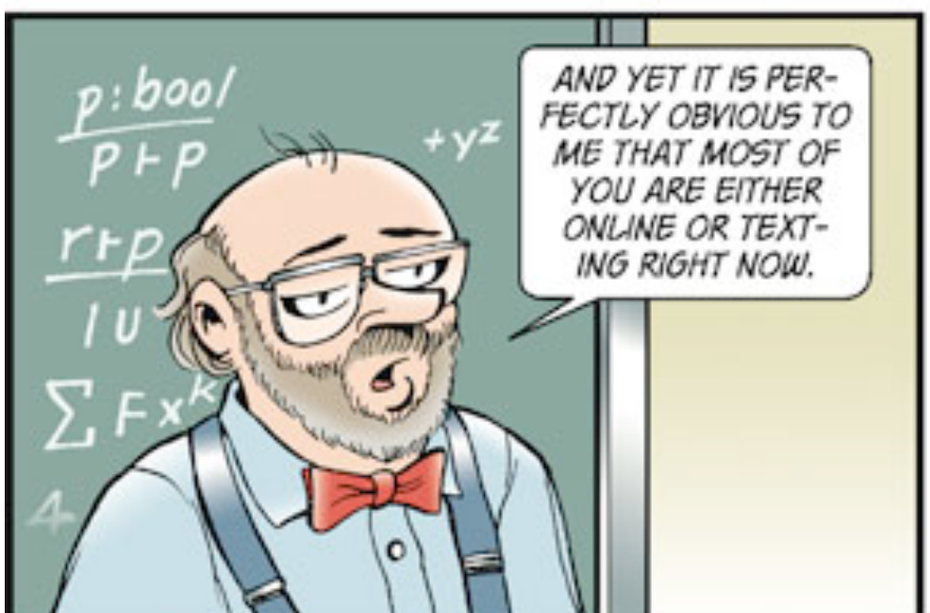
variance is np(1-p) (by story and the fact that Var(X+Y)=Var(X)+Var(Y) if X,Y are uncorrelated)

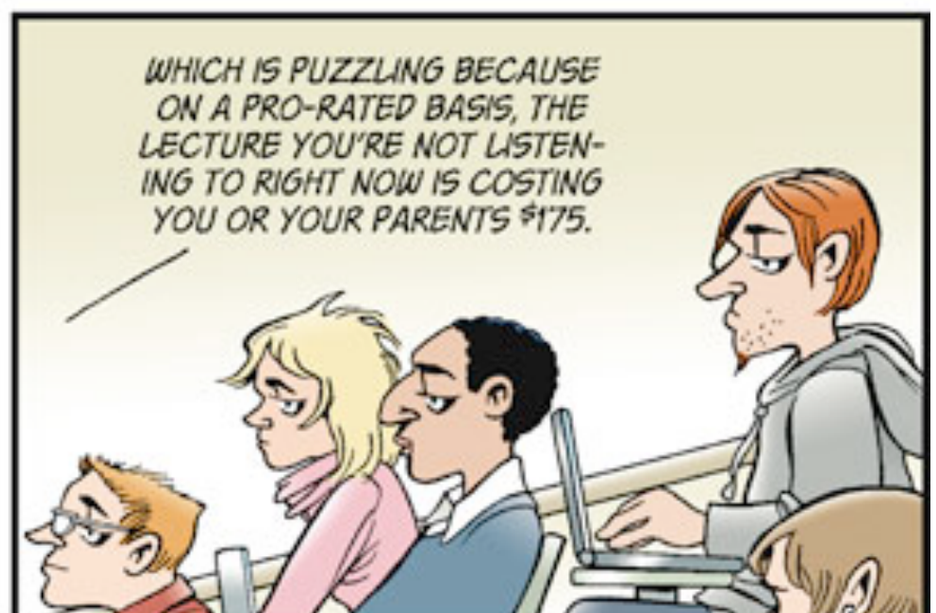

...WHICH IS HOW WE CAN DETERMINE THE VARIANCE FOR BI-NOMIAL DISTRIBUTION!


PRETTY GREAT, RIGHT? ANY-ONE SEE **THAT** COMING?


THE VALIDITY OF PROPOSITION 3 FOR L≥4 IS THUS CONFIRMED IN A MOST ELEGANT FASHION!


NOW, AS YOU CAN SEE, THIS IS A VERY COMPLICATED PROOF. YOU CAN'T POSSIBLY TRACK IT WITHOUT FULLY CONCENTRATING.


AND YET IT IS PER-FECTLY OBVIOUS TO ME THAT MOST OF YOU ARE EITHER ONLINE OR TEXT-ING RIGHT NOW.


WHICH IS PUZZLING BECAUSE ON A PRO-RATED BASIS, THE LECTURE YOU'RE NOT LISTEN-ING TO RIGHT NOW IS COSTING YOU OR YOUR PARENTS $175.

(Doonesbury)

# Poisson Distribution

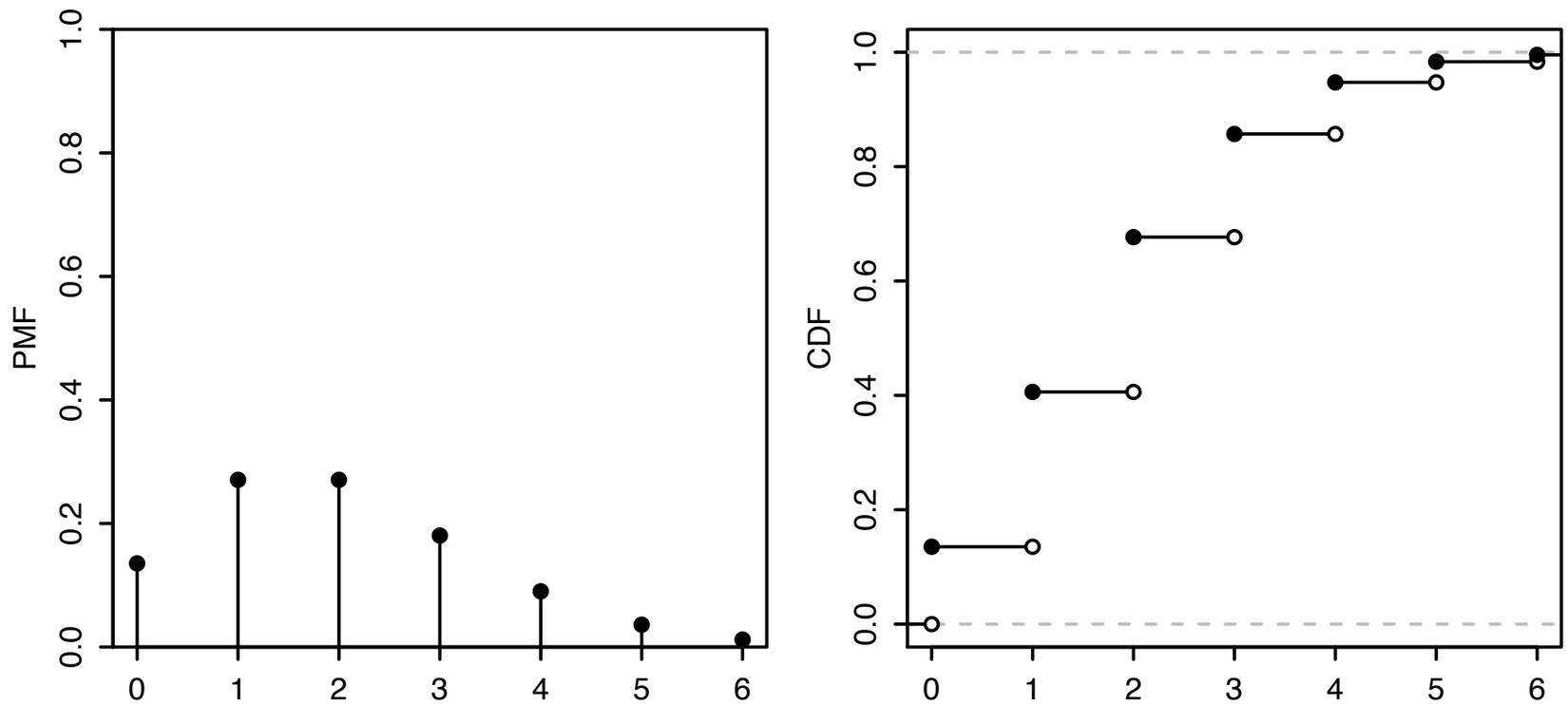
story: *count* number of events that occur, when there are a large number of independent *rare events*.

Examples: # mutations in genetics, # of traffic accidents at a certain intersection, # of emails in an inbox
mean = variance for the Poisson

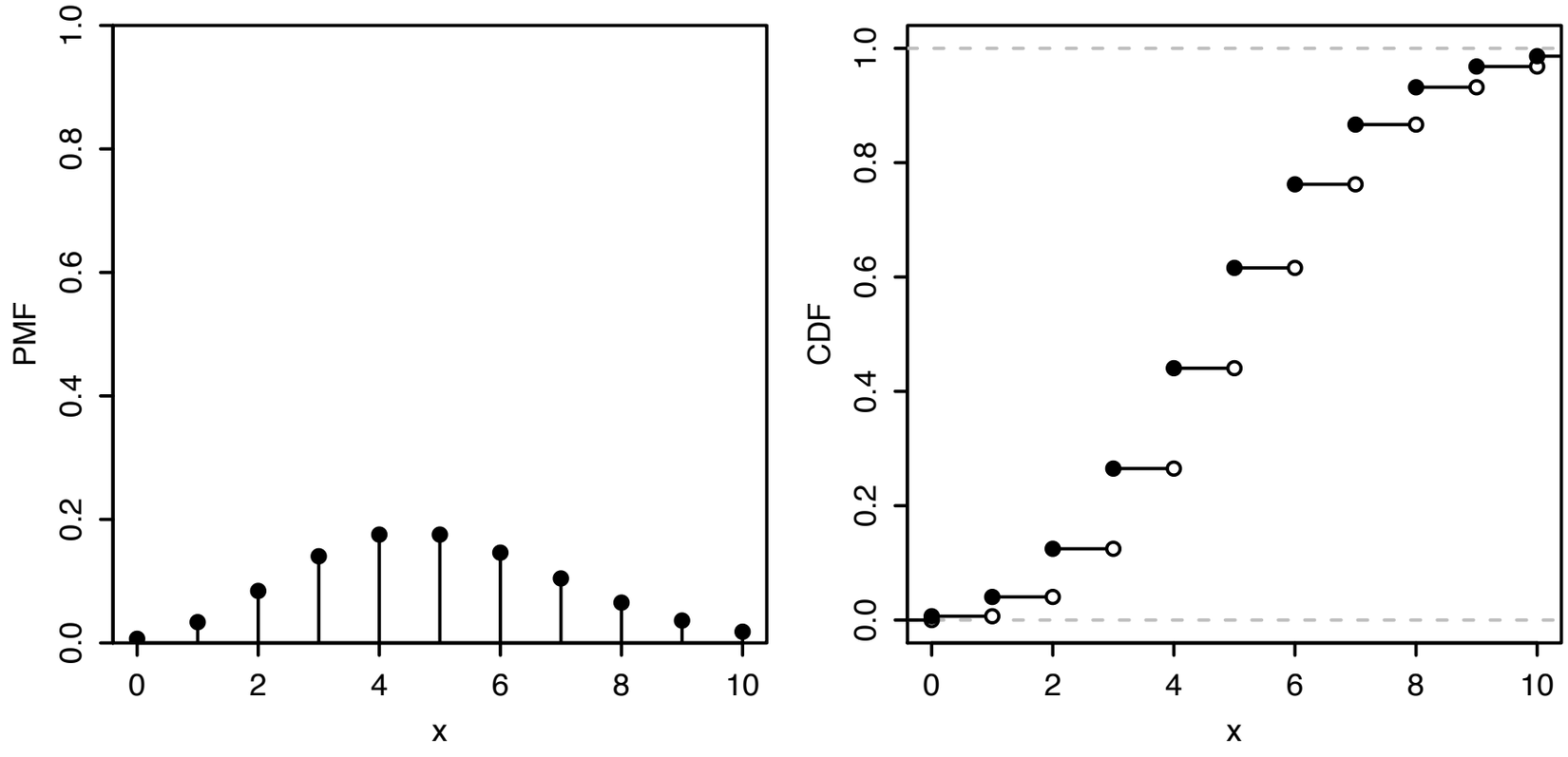
$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots.$$

# Poisson PMF, CDF

Pois(2)




Pois(5)



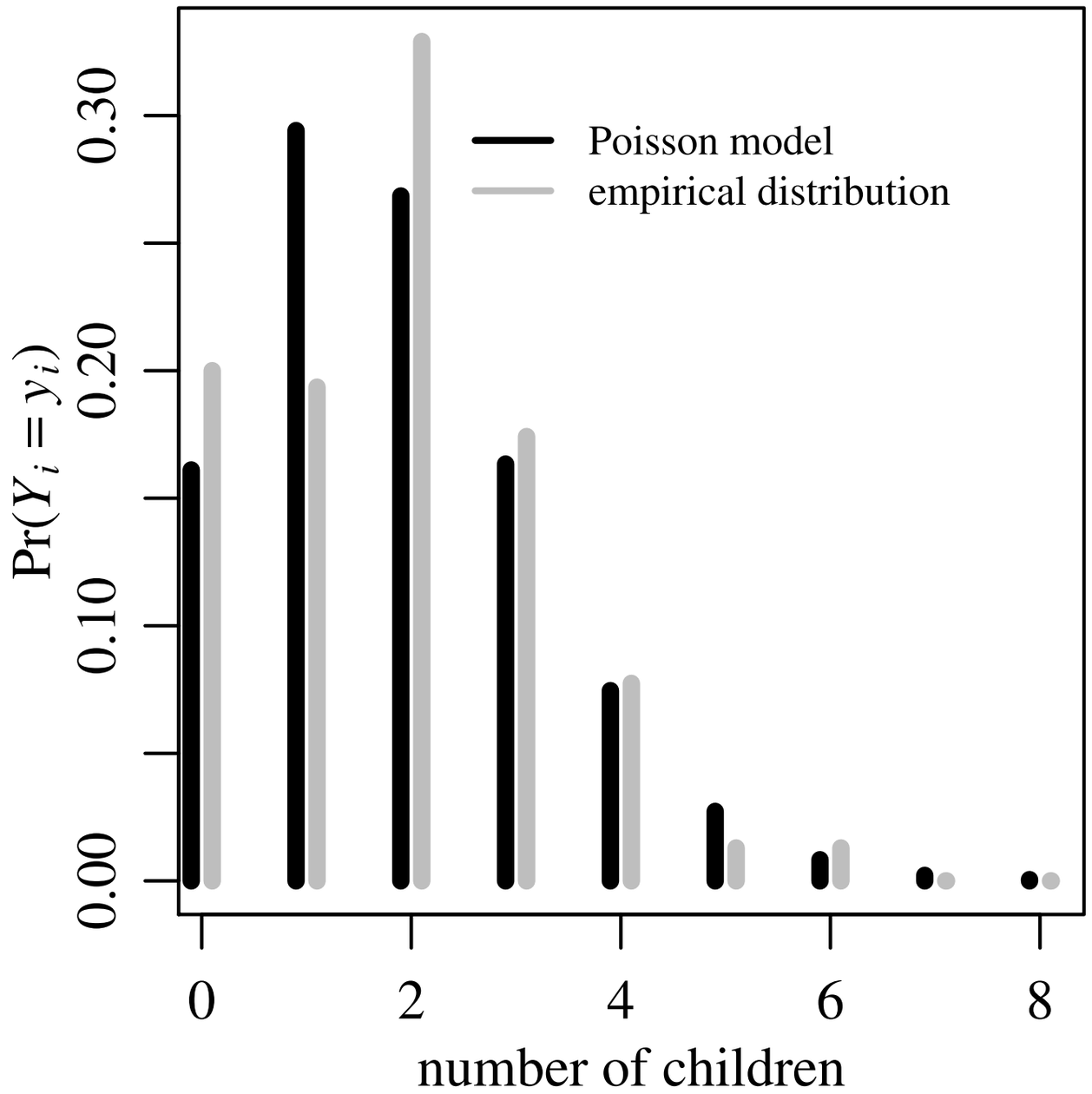
# Poisson Approximation

$$|P(X \in B) - P(N \in B)| \leq \min\left(1, \frac{1}{\lambda}\right) \sum_{j=1}^{n} p_j^2.$$

if $X$ is the number of events that occur, where the $i$th event has probability $p_i$, and $N \sim \text{Pois}(\lambda)$, with $\lambda$ the average number of events that occur.

Example: in matching problem, the probability of no match is approximately 1/e.

# Poisson Model Example

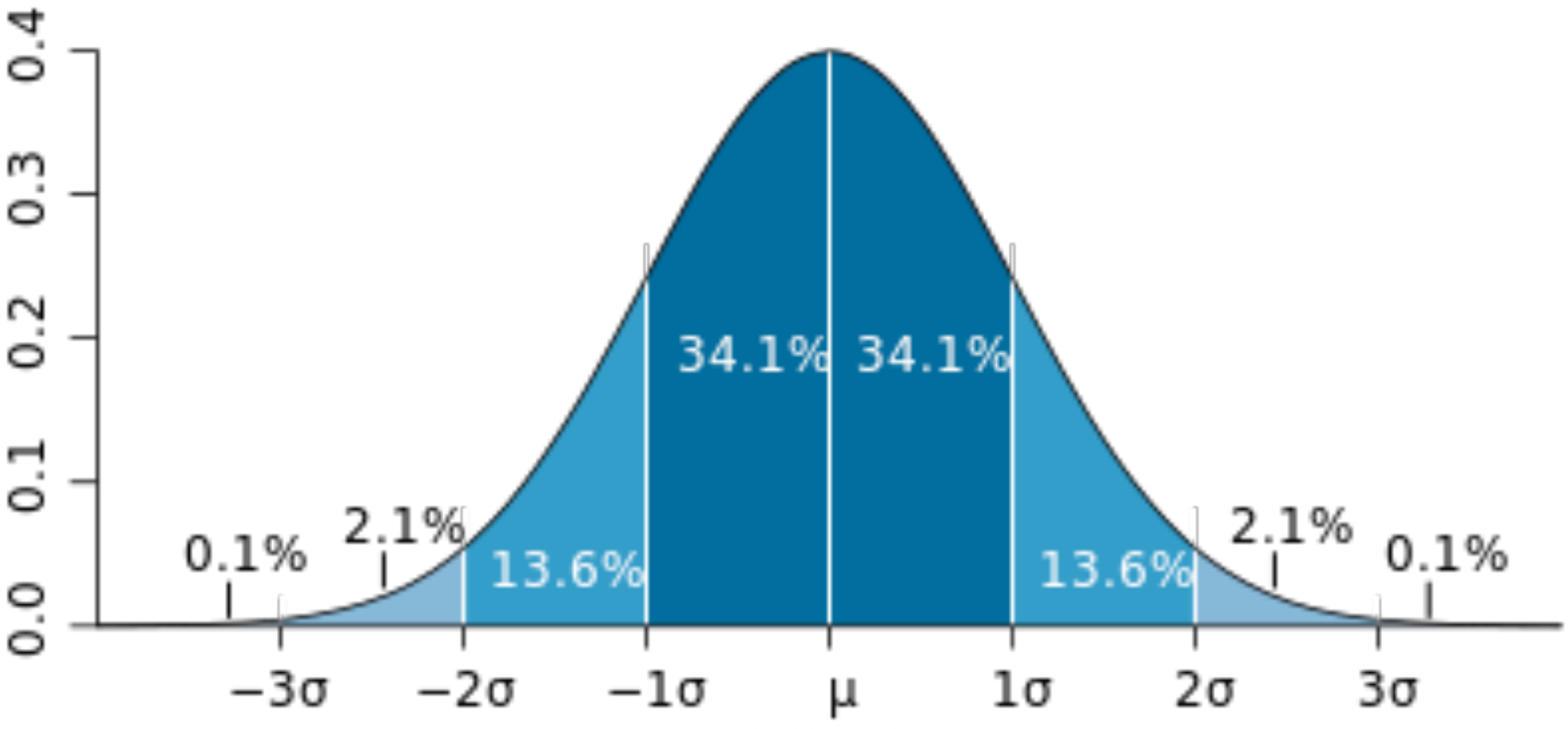



source: Hoff, A First Course in Bayesian Statistical Methods

Extensions: Zero-Inflated Poisson, Negative Binomial, …

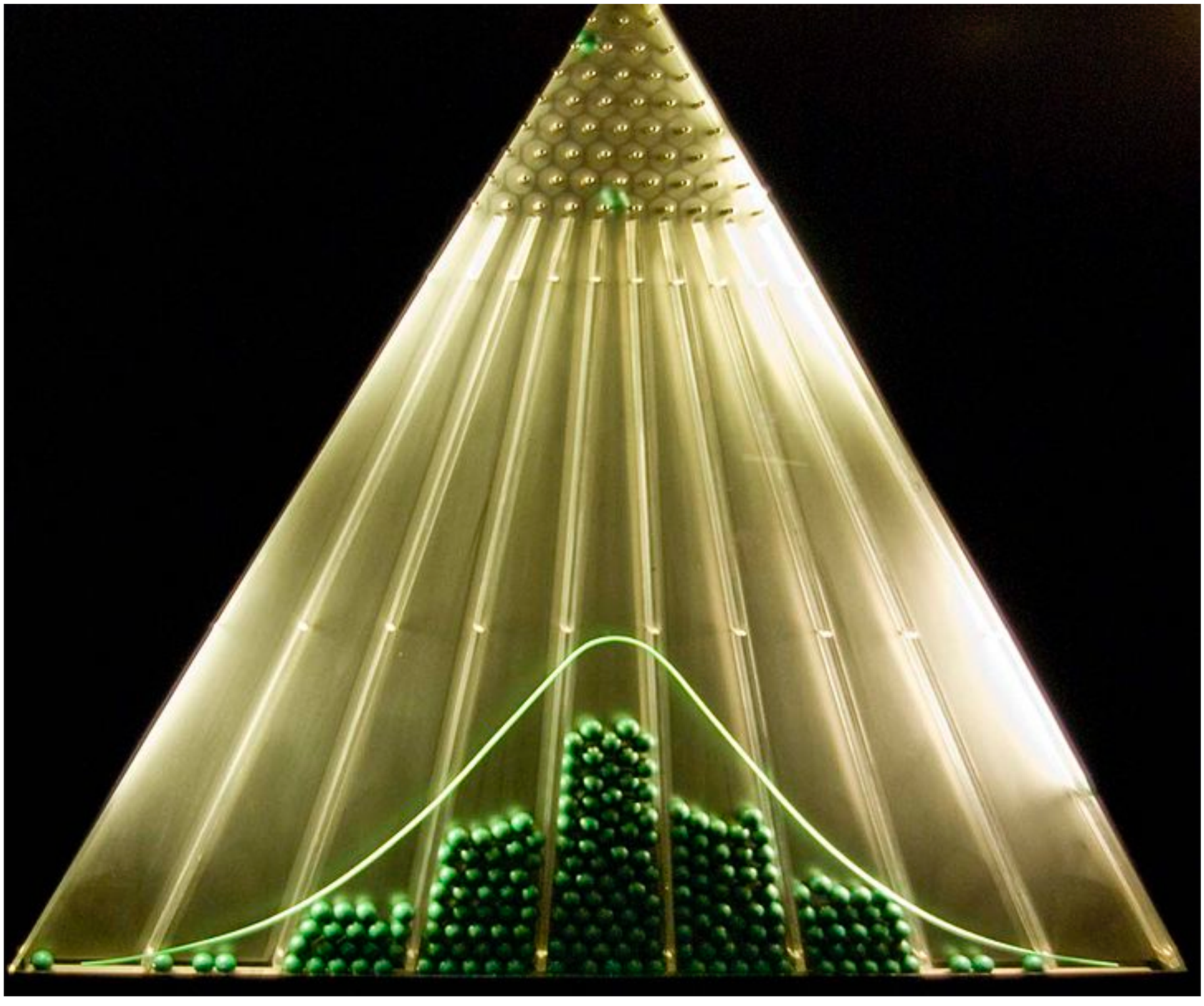
# Normal (Gaussian) Distribution

- symmetry
- central limit theorem
- characterizations (e.g., via entropy)
- 68-95-99.7% rule




Wikipedia

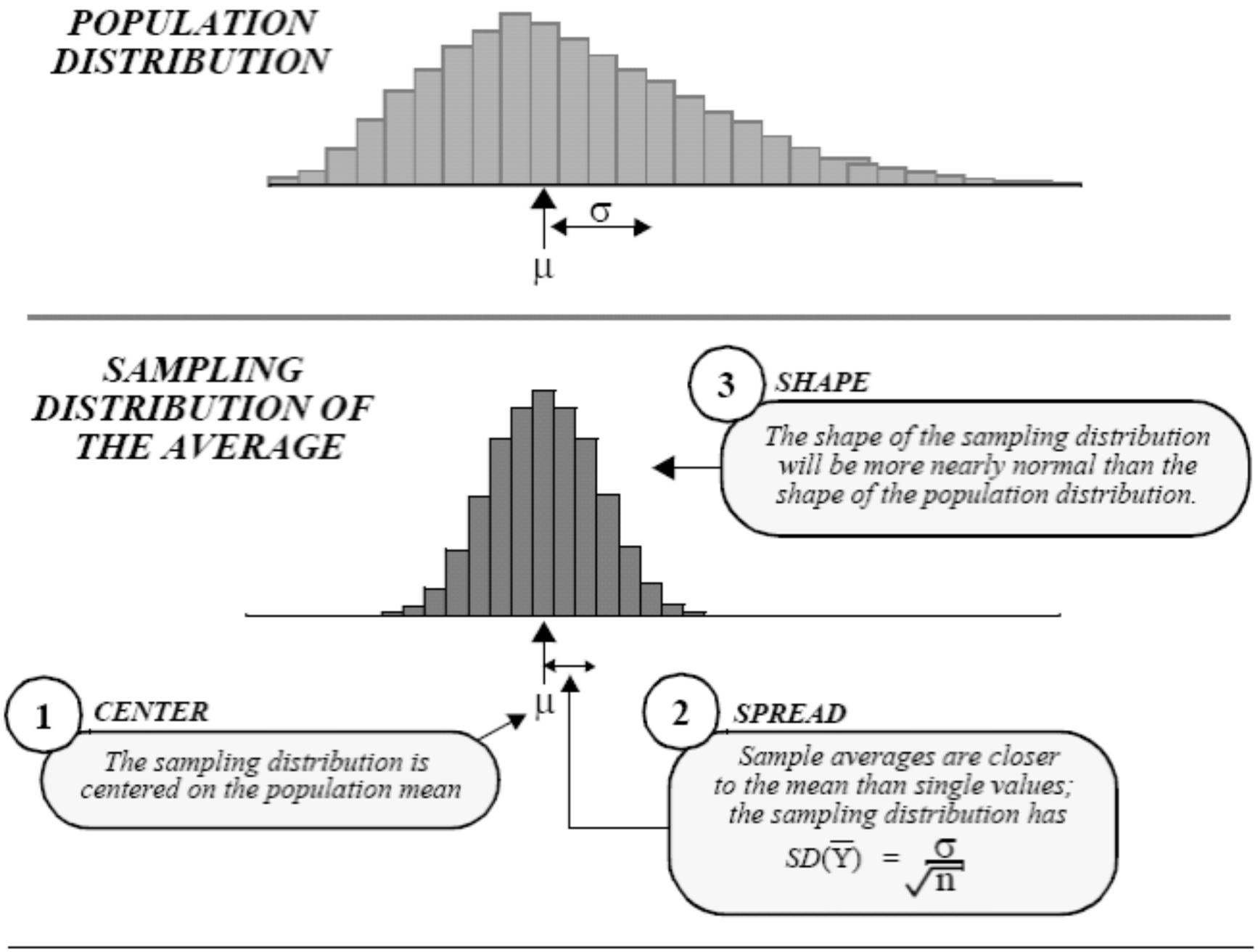
# Normal Approximation to Binomial



Wikipedia

# The Magic of Statistics

**The relationship between the population distribution and the sampling distribution of the average in random sampling**




source: Ramsey/Schafer, The Statistical Sleuth

# The Evil Cauchy Distribution

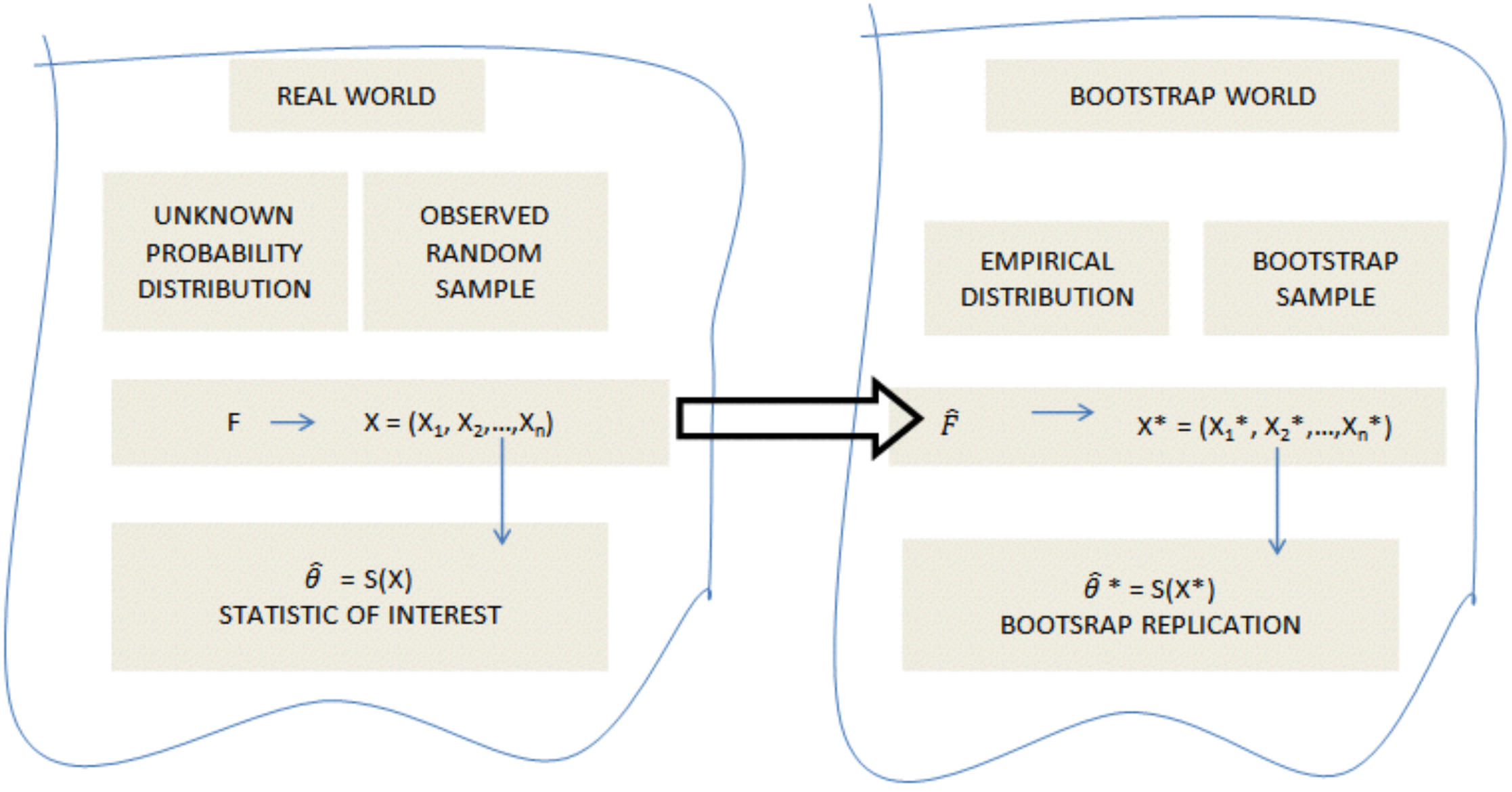
http://www.etsy.com/shop/NausicaaDistribution

# Bootstrap (Efron, 1979)

| | | | | | |
|---|---|---|---|---|---|
| data: | 3.142 | 2.718 | 1.414 | 0.693 | 1.618 |
| | 1.414 | 2.718 | 0.693 | 0.693 | 2.718 |
| | 1.618 | 3.142 | 1.618 | 1.414 | 3.142 |
| reps | 1.618 | 0.693 | 2.718 | 2.718 | 1.414 |
| | 0.693 | 1.414 | 3.142 | 1.618 | 3.142 |
| | 2.718 | 1.618 | 3.142 | 2.718 | 0.693 |
| | 1.414 | 0.693 | 1.618 | 3.142 | 3.142 |

resample *with replacement*, use empirical distribution to approximate true distribution

# Bootstrap World




source: http://pubs.sciepub.com/ijefm/3/3/2/
which is based on diagram in Efron-Tibshirani book