Master REOC: 2023 - 2024

Low Power Networks for the Internet of Things

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GitHub Repository : `https://github.com/Abdel211/LORA_REOC.git`

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1 Introduction

In this Exercice, we are presented with a geometric problem involving a triangle with sides of lengths a, b, and c. The context involves the placement of LoRaWAN gateways at each vertex of the triangle, with a fixed transmission range of 1 for each gateway. The primary objective is to establish the relationship between the side lengths of the triangle and the transmission range to achieve a compact coverage of the plane by the three gateways. The analysis aims to determine the throughput, which represents the rate of successful transmissions to at least one gateway across the entire covered area.

2 Geometric Conditions for Optimal LoRaWAN Gateway Placement

At the outset of our investigation, our curiosity was piqued by the possibility of an intersection among the three circles. This prompted us to draw lines passing through the midpoints of each side of the triangle. To our intrigue, we made the remarkable discovery that these three lines intersect at a specific point. This point, known as the center of the circumcircle of the triangle (the circle passing through all triangle vertices), was identified as the radius of this circumcircle.

We created a visualization using GeoGebra to validate our assertions. The GeoGebra file is available on the Git repository, and the corresponding figure is presented below:

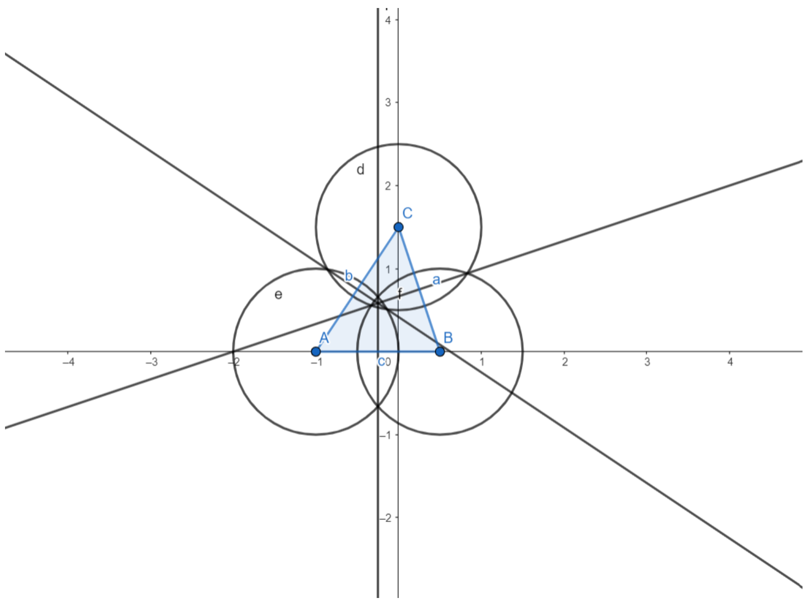

Figure 1: Mediatrix Construction

Considering this configuration, we inferred that the circumcircle radius (R) must be greater than or equal to the minimum of the radii of the three circles generated by the triangle's vertices. This condition can be expressed mathematically as $R \geq \min(R_a, R_b, R_c)$ [1], signifying that the circumcircle must have a radius larger than or equal to the smallest radius among the three circles generated by the triangle's vertices. This requirement ensures that the circumcircle is sufficiently large to encompass the circles drawn around each vertex of the triangle.

[1] We denote R_a as the radius of the circle centered at point A, R_b as the radius of the circle centered at point B, R_c as the radius of the circle centered at point C , and R as the radius of the circumcircle centered at the intersection point

We had previously studied a formula of Heron during our second year at INSA, which allows us to relate the radius of the circumcircle to the sides of the triangle. The formula for the radius is as follows:

$$r = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

where s is the semi-perimeter. For a triangle with sides of length a, b, and c, the semi-perimeter s is calculated as:

$$s = \frac{a+b+c}{2}$$

The formula can also be found in the link provided in the annex section.

We will simplify the formula so that we can find expression with only a,b,c.

$$\begin{aligned}
r^2 &= \frac{a^2b^2c^2}{16s(s-a)(s-b)(s-c)} \\
16s(s-a)(s-b)(s-c) &= \frac{a^2b^2c^2}{r^2} \\
s(s-a)(s-b)(s-c) &= \frac{a^2b^2c^2}{16r^2} \\
r &= \frac{abc}{4\sqrt{\frac{a^2b^2c^2}{16r^2}}} \\
r &= \frac{abc}{4\sqrt{a^2b^2c^2/r^2}} \\
r &= \frac{abc}{4\sqrt{a(a-b)(a-c)(b-c)}}
\end{aligned}$$

Thus, this formula $r = \frac{abc}{4\sqrt{a(a-b)(a-c)(b-c)}}$ provides an alternative means to establish the relationship between the radius of the circumcircle and the sides of the triangle, complementing our previous exploration.

We previously determined that R should be greater than $\min(R_a, R_b, R_c)$, where $R_a = R_b = R_c = 1$. Therefore, $r > 1$, and we deduce that $\frac{abc}{4\sqrt{a(a-b)(a-c)(b-c)}} > 1$. To ensure that the three gateways cover a compact set in the plane, it is necessary that $\boxed{abc > 4\sqrt{a(a-b)(a-c)(b-c)}}$

3 Throughput Analysis Across the Entire Coverage Area of Three LoRaWAN Gateways

We commence our exploration by employing the formula $S_L(\Omega)$ as presented in slide 15/86 of the course material:

$$S_L(\Omega) = p\mu \sum_{(\Delta\Gamma\cap\Omega\neq\emptyset)} A(\Delta_\Gamma \cap \Omega) \sum_{l=L}^{|\Gamma|} (-1)^{l-L} \binom{l-1}{L-1} \sum_{\substack{\Theta\subseteq\Gamma \\ |\Theta|=l}} Q(\mathcal{U}_\Theta)$$

We have the formula $S_1(\Omega)$ for $L = 1$ that we want to expand. The basic formula is as follows:

$$S_1(\Omega) = p\mu \sum_{(\Delta\Gamma\cap\Omega\neq\emptyset)} A(\Delta_\Gamma \cap \Omega) \sum_{l=1}^{|\Gamma|} (-1)^{l-1} \binom{l-1}{0} \sum_{\substack{\Theta\subseteq\Gamma \\ |\Theta|=l}} Q(\mathcal{U}_\Theta)$$

The condition $\Delta\Gamma \cap \Omega \neq \emptyset$ means that we only consider sets Γ such that $\Delta_\Gamma \cap \Omega \neq \emptyset$, which can be expressed as $|\Gamma| > 1$ and $\Delta_\Gamma \cap P \neq \emptyset$.

Therefore, we modified the sets Γ included in G to reflect this condition. The possible sets are $\{G1\}, \{G2\}, \{G3\}, \{G1, G2\}, \{G1, G3\}, \{G3, G2\}, G$.

Expanding the sum using these sets, we get the modified expression for $S_1(\Omega)$:

$$\begin{aligned} S_1(\Omega) =& p\mu\Big[A(\Delta_{G1} \cap \Omega)Q(\mathcal{U}_{G1}) - A(\Delta_{G2} \cap \Omega)Q(\mathcal{U}_{G2}) \\ &+ A(\Delta_{G3} \cap \Omega)Q(\mathcal{U}_{G3}) - A(\Delta_{G1\cap G2} \cap \Omega)Q(\mathcal{U}_{G1,G2}) \\ &+ A(\Delta_{G1\cap G3} \cap \Omega)Q(\mathcal{U}_{G1,G3}) - A(\Delta_{G3\cap G2} \cap \Omega)Q(\mathcal{U}_{G3,G2}) \\ &+ A(\Delta_G \cap \Omega)Q(\mathcal{U}_G)\Big] \end{aligned}$$

This reflects the expansion of $S_1(\Omega)$ taking into account the specific conditions on the sets Γ.