Machine Learning

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   - Linear Regression, Logistic Regression, Support Vector Machines, Trees, Random Forests, Boosting, Artificial Neural Networks
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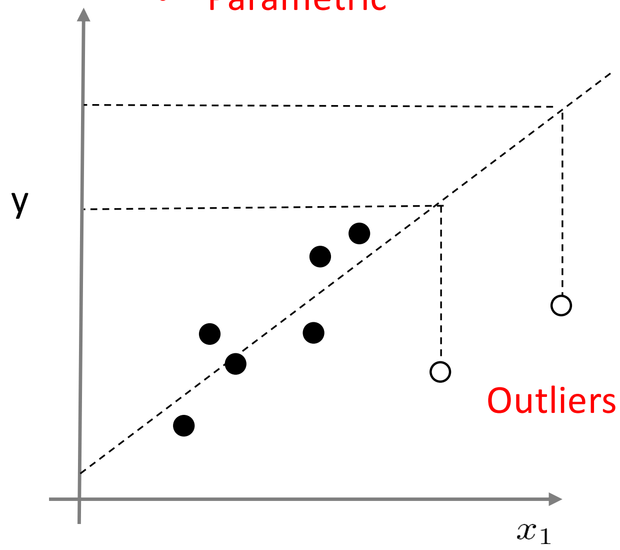
Supervised Learning

- Linear Regression
- Logistic Regression
- Support Vector Machines
- **Trees**
- Random Forests
- Boosting
- Artificial Neural Networks

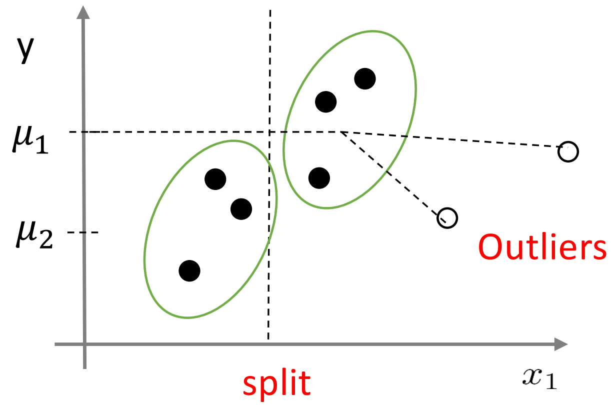
Classification and Regression Trees (CART)

Regression

Linear regression
- Linear models
- Parametric



Regression trees
- Non Linear model
- Non-Parametric

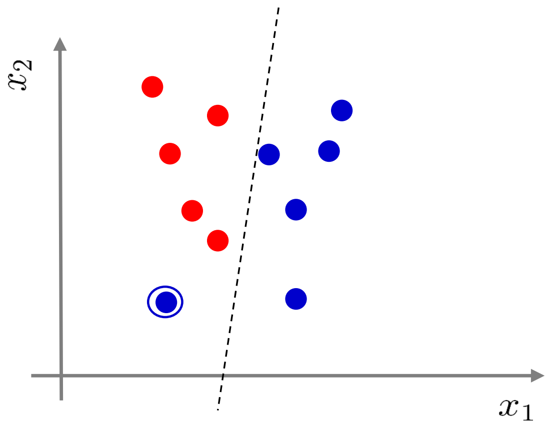


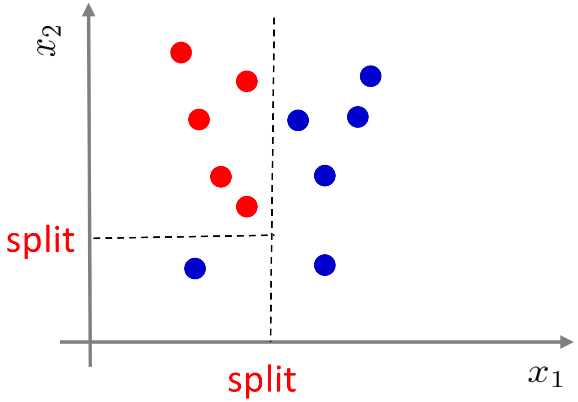
Classification and Regression Trees (CART)

Logistic regression
- Linear models
- Parametric

Classification trees
- Non Linear model
- Non-Parametric

Classification





Classification Trees (aka Decision Trees)

Example of Restaurant Data

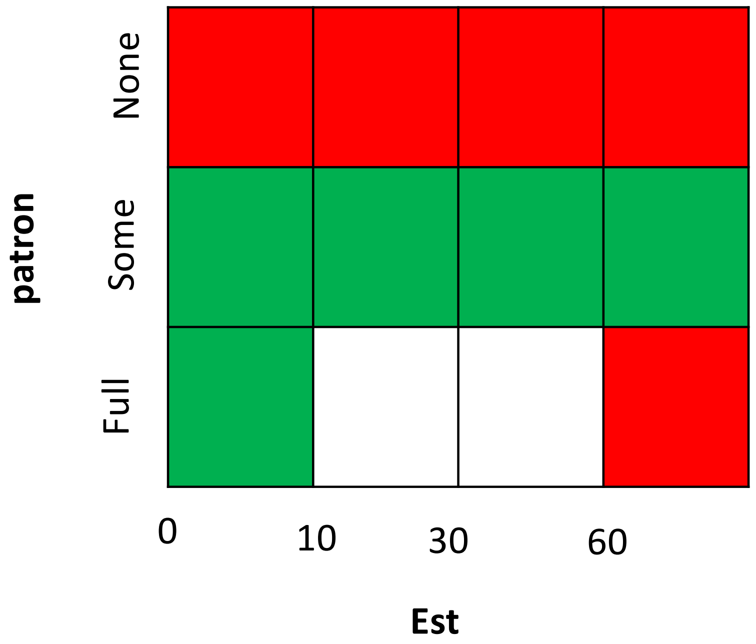
X	F										Y
Client	**Alt**	**Tea**	**Fri**	**Hun**	**Patron**	**Price**	**Rain**	**Res**	**Type**	**Est**	**Wait**
1	T	F	F	T	Some	$$$	F	T	Moroccan	0-10	T
2	T	F	F	T	Full	$	F	F	Chinese	30-60	F
3	F	T	F	F	Some	$	F	F	Burger	0-10	T
4	T	F	T	T	Full	$	F	F	Chinese	10-30	T
5	T	F	T	F	Full	$$$	F	T	Moroccan	>60	F
6	F	T	F	T	Some	$$	T	T	Italian	0-10	T
7	F	T	F	F	None	$	T	F	Burger	0-10	F
8	F	F	F	T	Some	$$	T	T	Chinese	0-10	T
9	F	T	T	F	Full	$	T	F	Burger	>60	F
10	T	T	T	T	Full	$$$	F	T	Italian	10-30	F
11	F	F	F	F	None	$	F	F	Chinese	0-10	F
12	T	T	T	T	Full	$	F	F	Burger	30-60	T

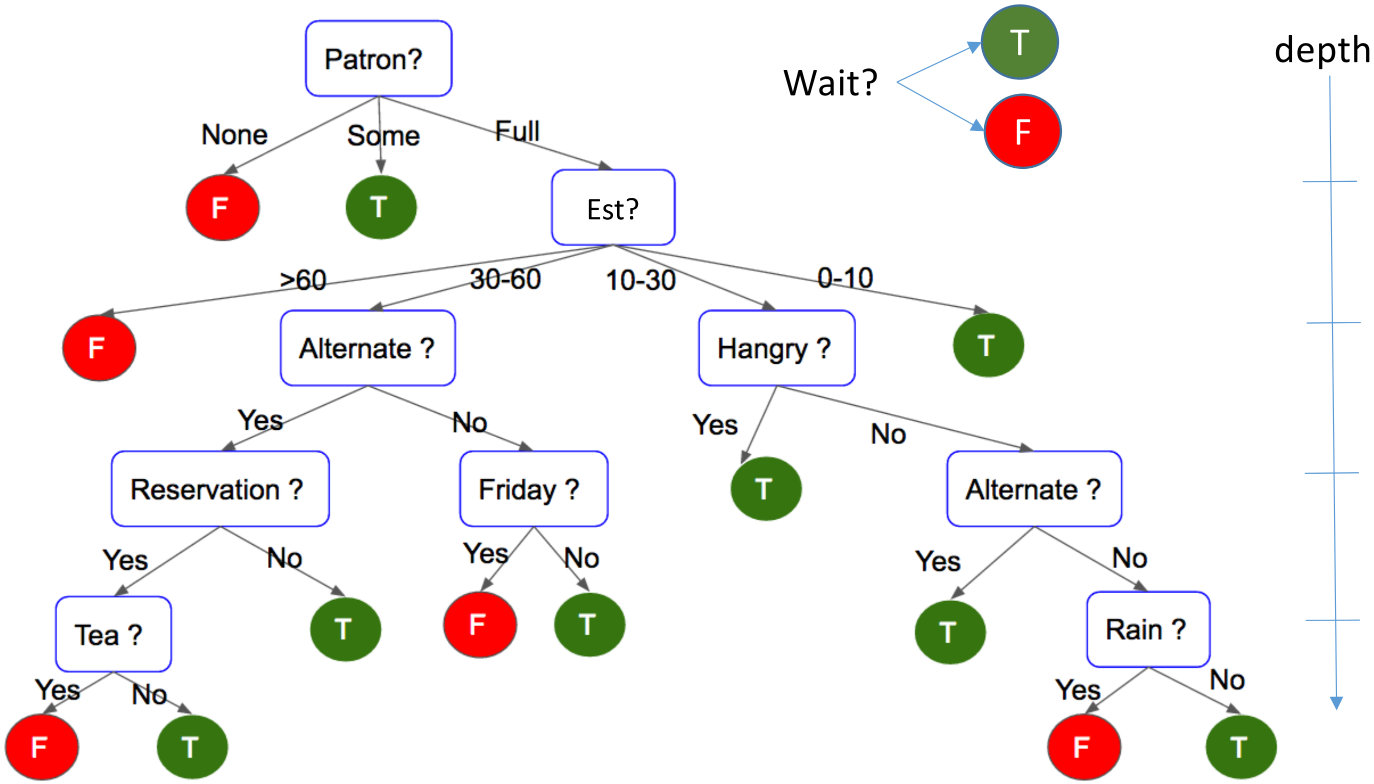
- Alt: is there any other alternative?
- Fri: is it Friday?
- Hun: is the client hungry?
- Patron: how many people are in the restaurant?
- Res: Restaurant
- Est: wait estimate

Most of the features are
Discrete (=categorical, =qualitative)

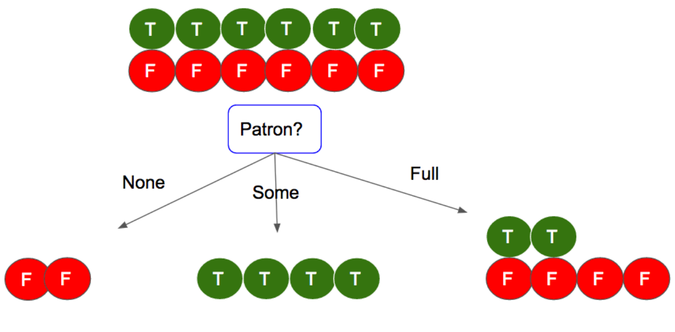
Classification Trees (aka Decision Trees)

Pick one feature at a time,
and make a binary decision

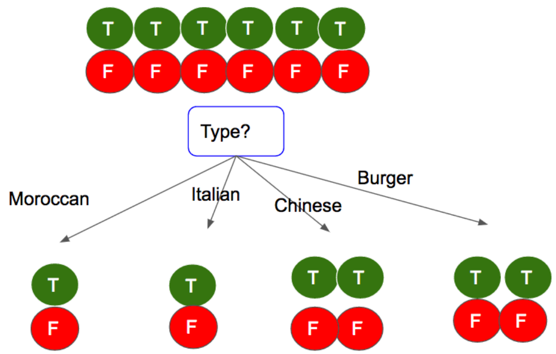




DT: Which feature to split with first?



More informative
Less impurity



Less informative
More impurity

Split with the feature F that maximizes the Information Gain

$$I(F) = H(S) - EH(F)$$

Information Gain = Parent entropy − Expected entropy

S: subset = {p positives and n negatives}

F: Feature

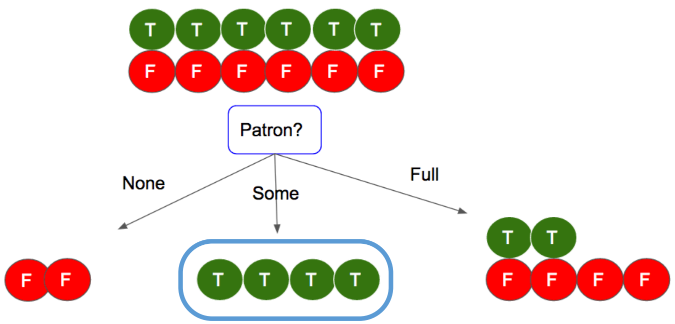
Entropy

$$H(S) = -\sum_{c} p_c \log_2(p_c)$$

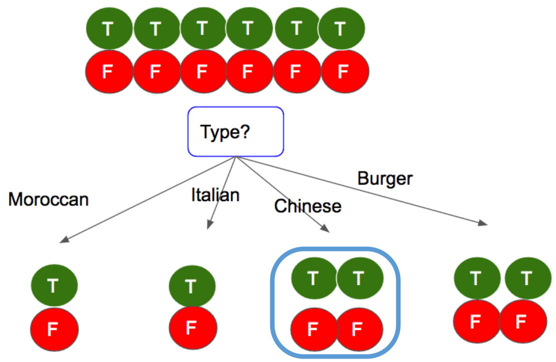
Python: np.log2()

p_c: probability of examples in class c
S : subset of data examples

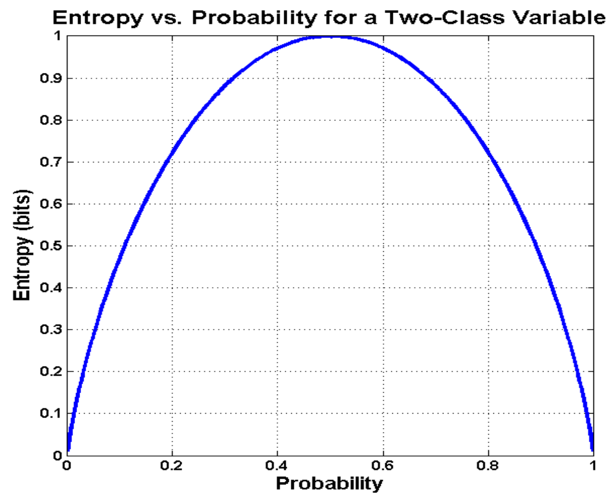
Interpretation: Measure of the impurity in a subset of examples



All examples in the same class
Entropy = 0



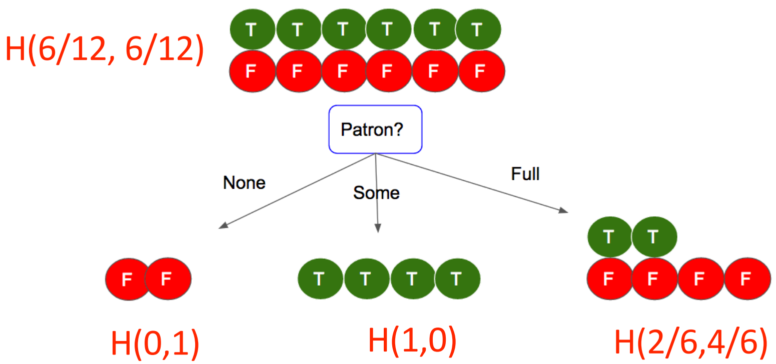
All examples evenly split between classes
Entropy = 1

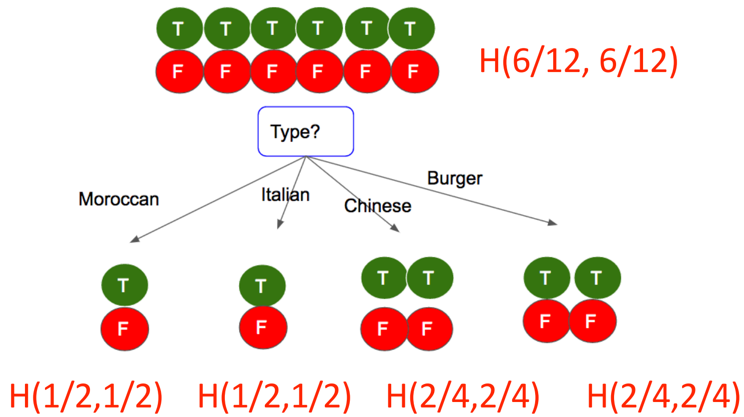


Entropy (binary classification)

$$H\left(\frac{p}{p+n},\frac{n}{p+n}\right) = -\frac{p}{p+n}\log(\frac{p}{p+n}) - \frac{n}{p+n}\log(\frac{n}{p+n})$$

p: positive
n: negative





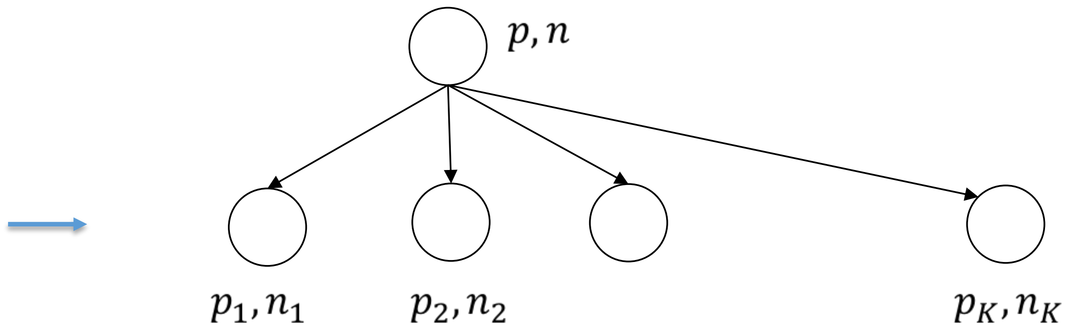
Expected Entropy

$$EH(F) = \sum_{i=1}^{K} \frac{p_i + n_i}{p + n} H\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

K = number of splits (regions) with Feature F
= number of children nodes

Expectation Entropy = weighted average of children entropy

children

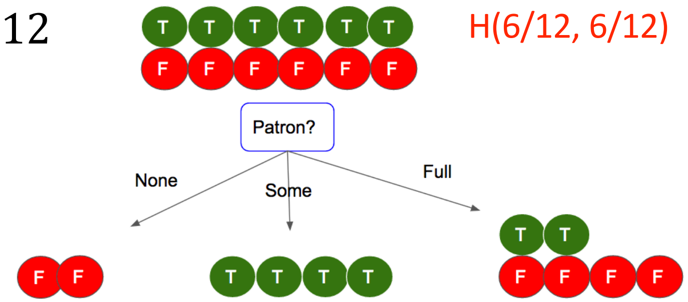


Expected Entropy (Example)

$$EH(F) = \sum_{i=1}^{K} \frac{p_i + n_i}{p + n} H\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

$p + n = 12$

H(6/12, 6/12)



$K = 3$

H(0,1) = 0

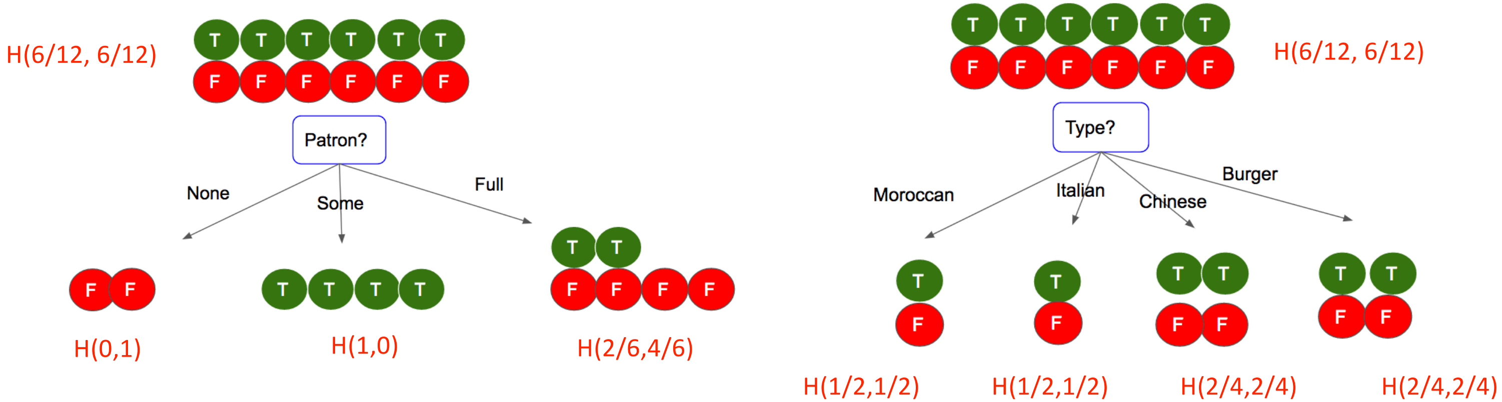
H(1,0) = 0

H(2/6,4/6) = .92

$p_i + n_i$	2	4	6

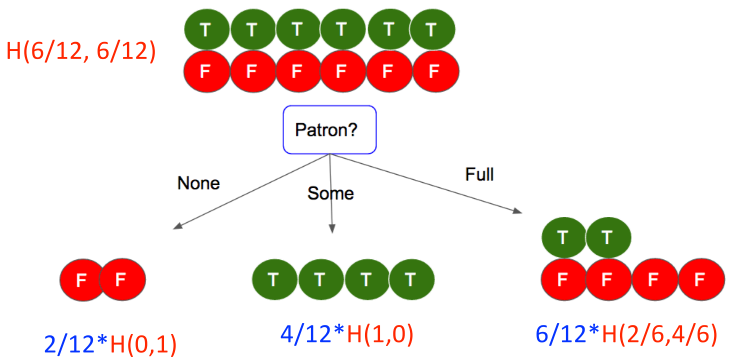
Information Gain

$$I(F) = H\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - EH(F)$$

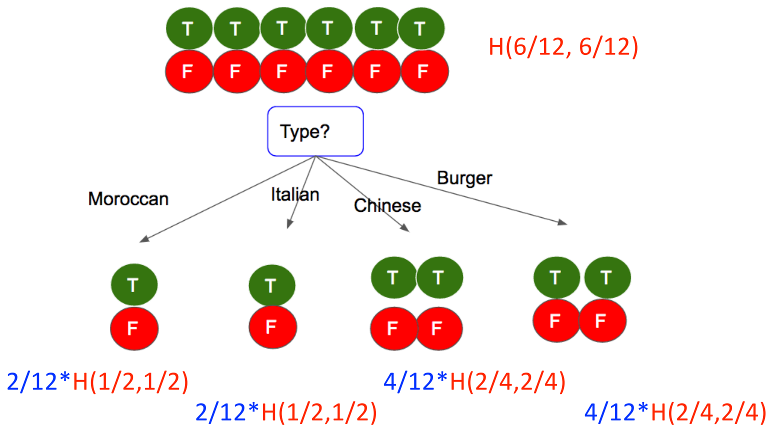


Information Gain

$$I(F) = H\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - EH(F)$$



I(Patron) = 0.54



I(Type) = 0.0

Other impurity measures

- p_c: probability of examples in class c
- S : subset of data examples

- CART algorithm uses the Entropy

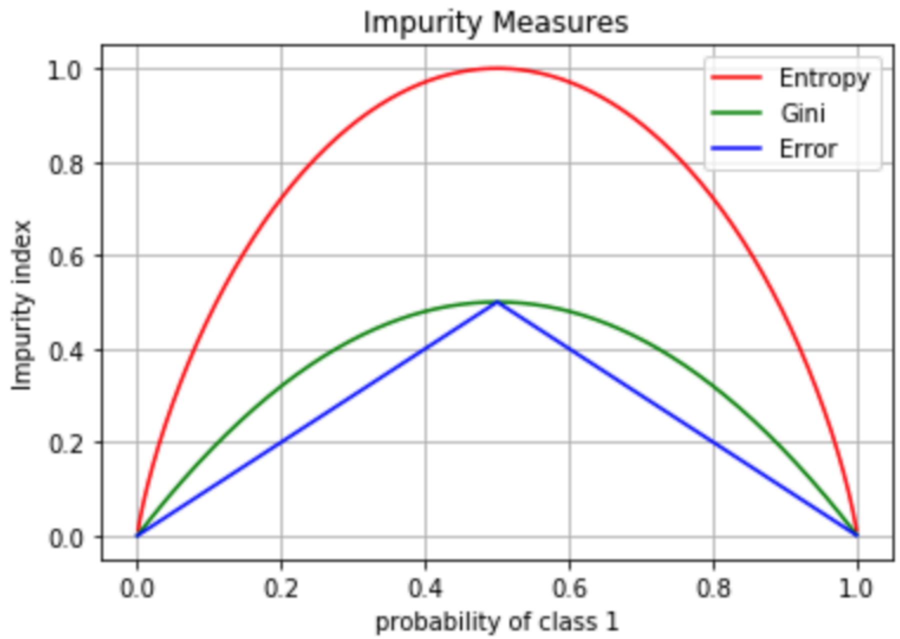
$$H(S) = -\sum_c p_c \log_2(p_c)$$

- Iterative Dichotomiser ID3 and C4.5 algorithms use Gini Index

$$G(S) = 1 - \sum_c p_c^2$$

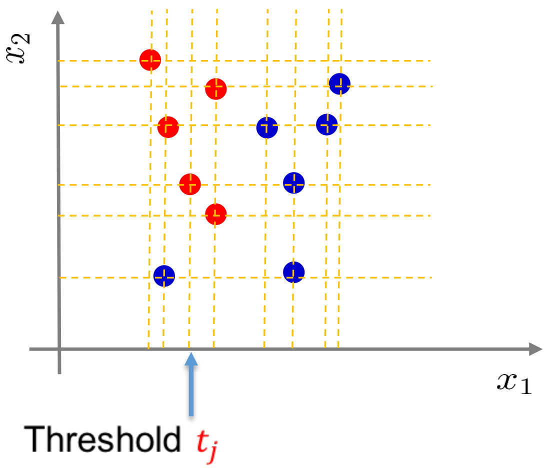
- One could also use the Class Error

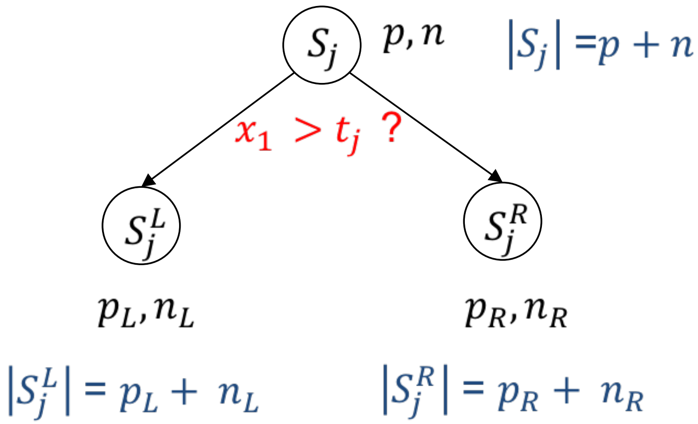
$$\mathrm{E}(S) = 1 - \max_c(p_c)$$



What if a feature is continuous (quantitative)?

$$I_j = H(S_j) - \sum_{i \epsilon \{L,R\}} \frac{|S_j^i|}{|S_j|} H(S_j^i)$$





Note: Doing so, trees are almost Binary! Even if a feature is categorical (qualitative)

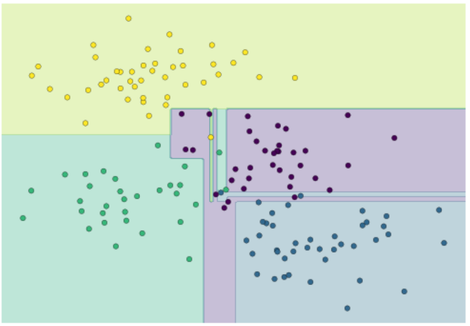
Decision Trees

- **Advantages**
  - Easy to interpret
  - Deals with non linearity
  - Handle qualitative features without the need to create fictive ones (one hot vector)
  - Provide most important features (in terms of information gain)

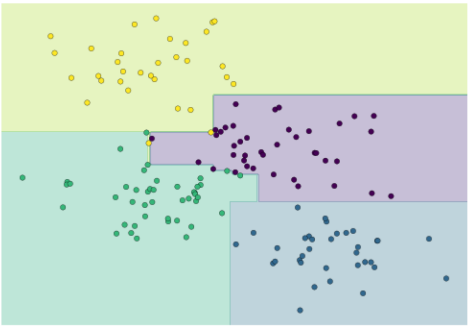
Decision Trees

- **Disadvantages**
  - Trees leads to overfitting (high variance): little change in little number of examples affect the whole tree.

DT on Data1



DT on Data2 = half of Data1

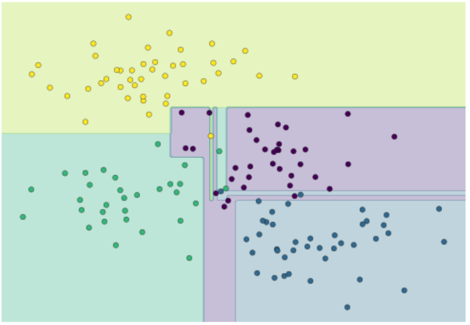


Decision Trees

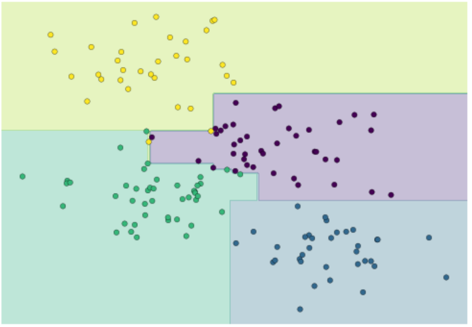
- **Disadvantages**
  - Trees leads to overfitting (high variance): little change in little number of examples affect the whole tree.

- **Solution: Ensemble Methods**
  - Bagging: Random Forest (Leo Breiman 2001) consists of combining multiple independent weak trees to reduce variance.
  - Boosting to reduce bias

DT on Data1



DT on Data2 = half of Data1

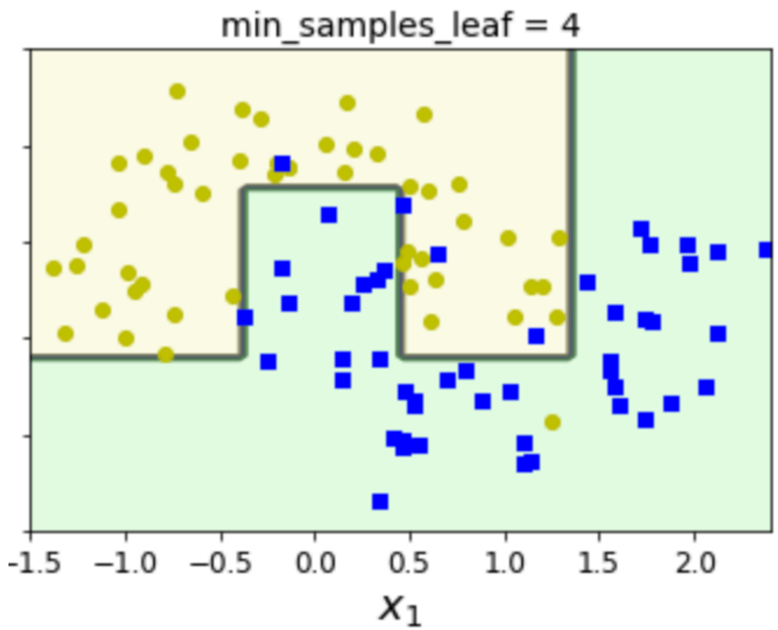


A tree alone will overfit.
However, it is clear that in some places, the two trees together produce consistent results

This idea comes from Bootstrapping (Brad Efron 1979):
Given a set of m independent observations $o_1.., o_m$, each with variance σ^2, the variance of the mean o of the observations is given by σ^2/m.

Classification And Regression Trees

Classification (Decision) Trees



Regression Trees