Math 404 Report3
Non Linear Optimization Algorithms

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Contents

1 1D Minimization Algorithms

1.1 Rosenbrock's Parabolic Problem

Rosenbrock's parabolic valley function starting from $\boldsymbol{X}_0 = (-1.2, 1.0)$ is given by:

$$f(x_1, x_2) = 100\left(x_2 - x_1^2\right)^2 + (1 - x_1)^2$$

The goal is to derive the one-dimensional function $f(\lambda)$ for the purpose of applying 1D minimization. To calculate S_0 for plotting, I compute the gradient of the function at the point $(0, 0)$. The obtained S_0 is then substituted into the expression $\lambda = x_0 + \lambda S_0$. Following this, I substitute λ back into f to express f solely as a function of λ. This process allows me to perform one-dimensional optimization on $f(\lambda)$.

This approach has been implemented using the provided MATLAB code for the Rosenbrock Function, and the function values are illustrated in Figure below.

```
f_Rose =

  function_handle with value:

    @(lamda)((lamda.*2.0+6.0./5.0).^2-1.0).^2.*1.0e+2+(lamda.*2.0+1.1e+1./5.0).^2
```

Figure 1: Matlab Rosenbrock Function

1.1.1 The Graphical Observations

The analysis of the below Graph indicates that the minimum of the function occurs within the lambda range of approximately -1.5 to 0.5. Additionally, the corresponding functional values lie in the range of 0 to 0.3×10^4.
This observation serves as a crucial test for the efficacy of my implemented 1D minimization algorithms. It provides a valuable benchmark, indicating their capability to successfully optimize the function and confirming their reliability in the optimization process.

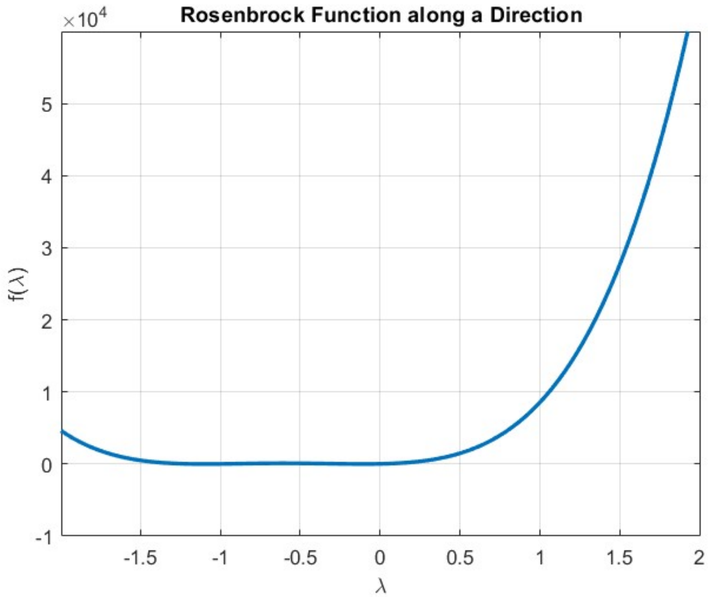

Figure 2: Rosenbrock Graph

1.1.2 The Results

```
-------------Applaying 1D minimization algorithms on Rosenbrock-----------
--------------------The Fibonacci Method------------------------
The final interval at which function is Min [0.000000 , 0.007692]
the Min value is 0.003846 and it functon value is 25.898029
--------------------The Golden Section Method--------------------
The final interval at which function is Min [0.000000 , 0.009015]
the Min value is 0.004507 and it functon value is 26.197921
--------------------The Quadratic interpolation Method--------------------
the min value is 0.006809 and it functon value is 27.260349
--------------------The Cubic interpolation Method--------------------
the min value is -0.102438 , it is functon value is 3.989984 and it is function dervative 0.235177
```

Figure 3: Rosenbrock Result

1.1.3 CPU time and Number of iteration Comparison

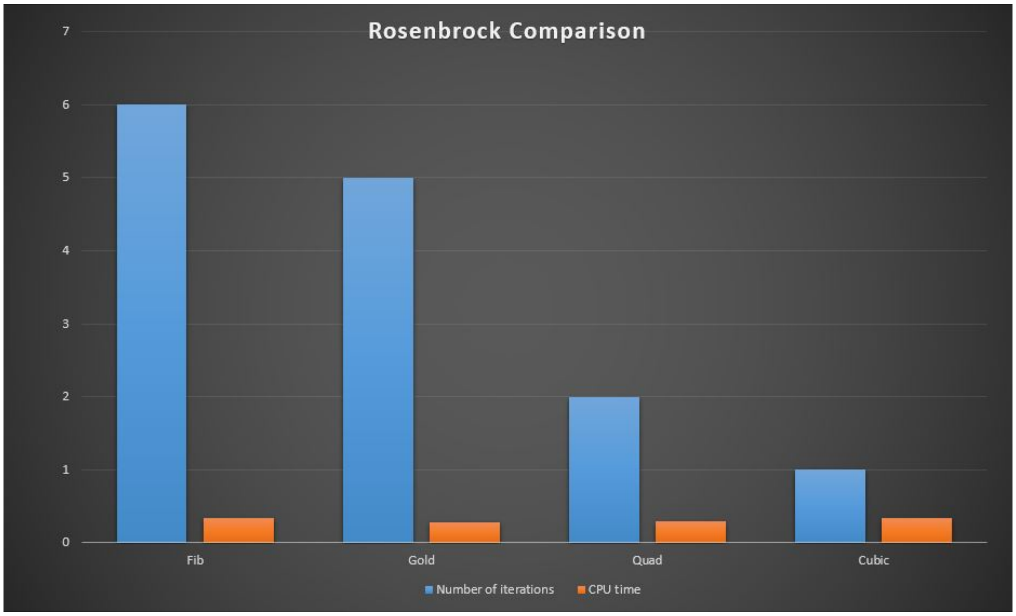

Figure 4: Rosenbrock Comparison

1.2 Powell's Quartic Problem

Powell's quartic function starting from $\boldsymbol{X}_0 = (3.0, -1.0, 0.0, 1.0)$

$$f(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$

I applied a similar strategy as in the Rosenbrock problem, adjusting the initial direction S_0 to $(1, 0, 0, 0)$.
This approach has been implemented using the provided MATLAB code for the Powell Function, and the function values are illustrated in Figure below.

```
f_powell =

  function_handle with value:

    @(lamda)(lamda.*2.0e+1-1.0).^4+(lamda.*4.0e+1-1.0).^2.*5.0+(lamda.*8.2e+1+2.0).^4.*1.0e+1+(lamda.*2.42e+2-7.0).^2
```

Figure 5: Matlab Powell Function

1.2.1 The Graphical Observations

Examining the graph reveals that the function attains its minimum within the lambda interval of approximately -0.5 to 0.5. Furthermore, the associated function values are confined to the range of 0 to 0.1×10^9.

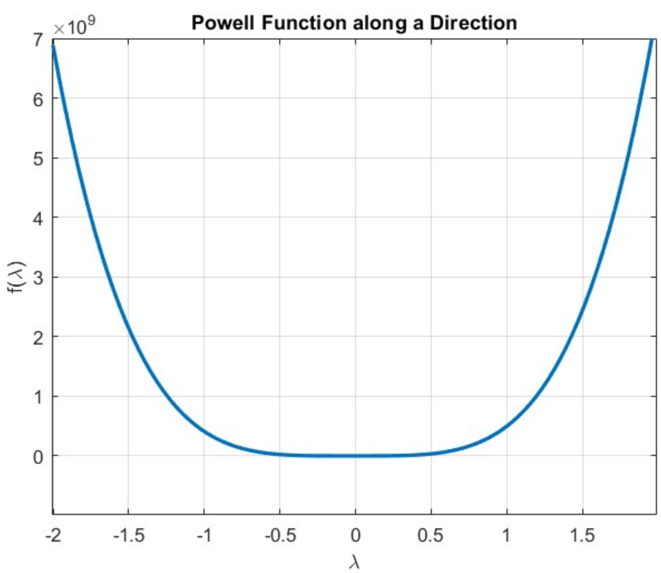

Figure 6: Powell Graph

1.2.2 The Results

```
-------------Applaying 1D minimization algorithms on Powell problem------------
--------------------The Fibonacci Method------------------------
The final interval at which function is Min [0.000000 , 0.007692]
the Min value is 0.003846 and it functon value is 328.545313
--------------------The Golden Section Method--------------------
The final interval at which function is Min [0.000000 , 0.009015]
the Min value is 0.004507 and it functon value is 354.244214
--------------------The Quadratic interpolation Method--------------------
the min value is -0.001045 and it functon value is 193.411612
--------------------The Cubic interpolation Method--------------------
the min value is -0.010106 , it is functon value is 119.990893 and it is function dervative -1.263027
```

Figure 7: Powell Result

1.2.3 CPU time and Number of iteration Comparison

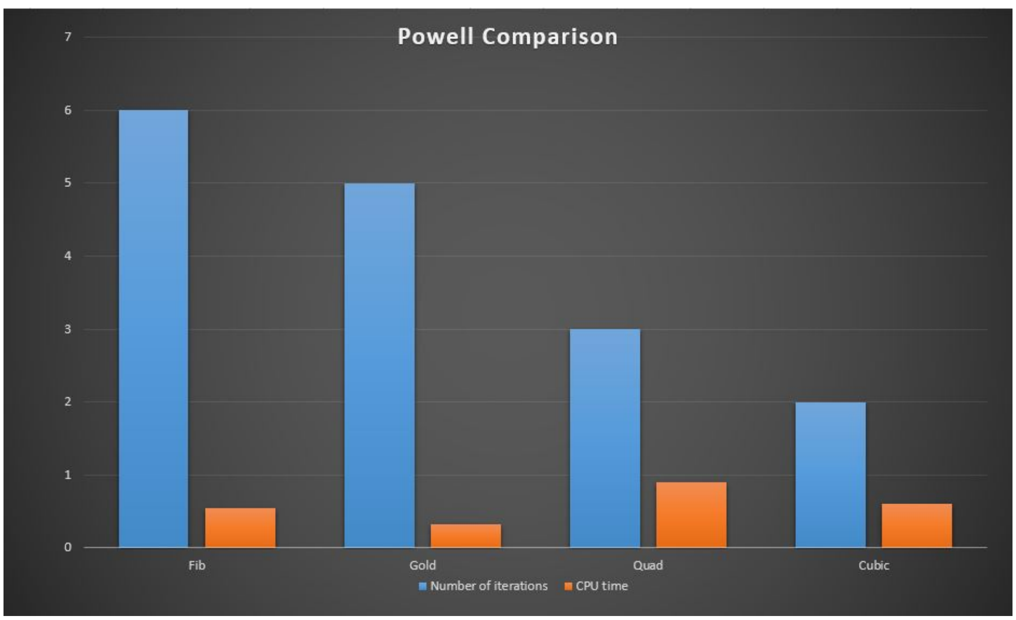

Figure 8: Powell Comparison

2 Fletcher-Reeves CG Method

I applied the algorithm provided in the lecture to solve both Rosenbrock's parabolic problem and Powell's quartic function problem. This algorithm is built into a MATLAB function named Fletcher-Reeves.

2.1 Results

```
-----------Applaying Fletcher-Reeves CG algorithms on Rosenbrock’s and Powell problem---------
---------------The Rosenbrock’s parabolic using Reeves CG Method----------------
The final value at which function is min [-1.026593,1.050180]
---------------The Powell’s quartic function using Reeves CG Method----------------
The final value at which function is min [1.909539,-0.075165,0.605828,1.524796]
```

Figure 9: Fletcher-Reeves CG Result

2.2 CPU time and Number of iteration Comparison

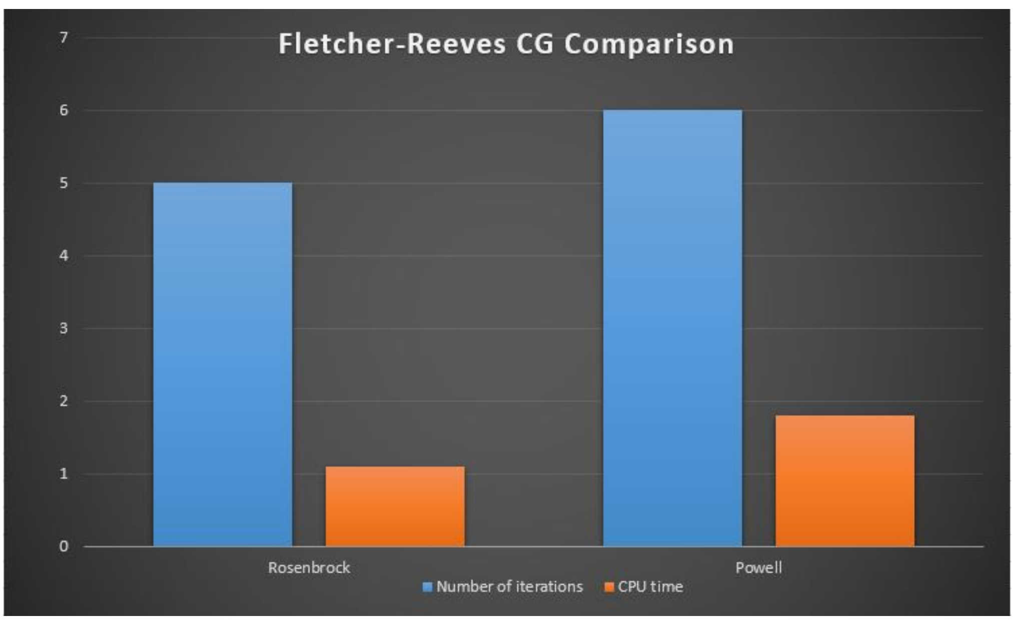

Figure 10: Fletcher-Reeves CG Comparison Comparison