A quick tour of Fourier Transform

with a short comparison with SVD

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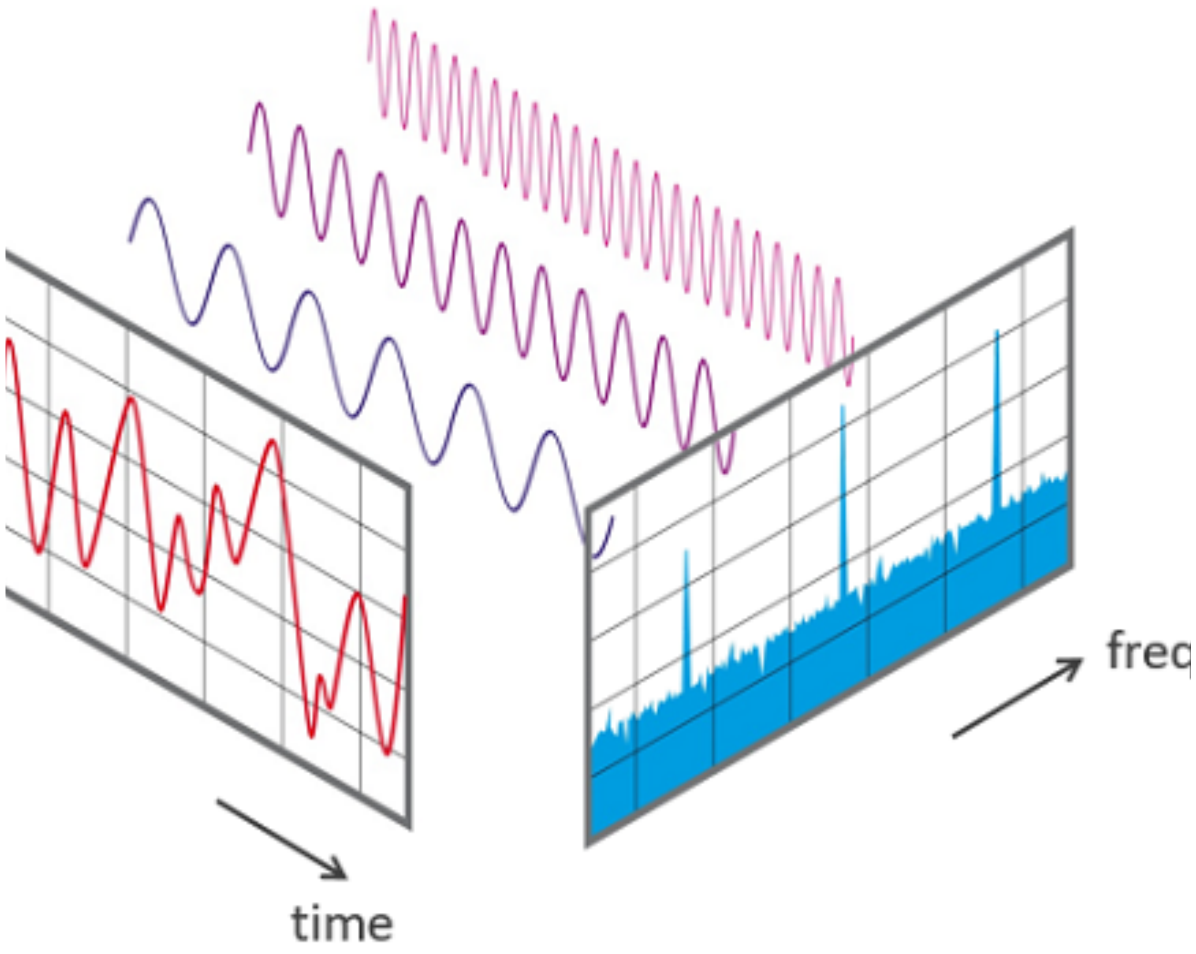
Fourier transform



- Formulated by Joseph Fourier to study the nature of heat transfer.
- Idea is to decompose any signals into basic sine waves
- Applied in ph

VIRTUALLY ANYTHING

Definitions

1D FOURIER TRANSFORM

$$\mathcal{F}: \mathbb{C} \to \mathbb{C}$$

$$\mathcal{F}(f)(\omega) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \omega \cdot t}\, dt$$

1D DISCRETE FT AND ITS INVERSE FT

$$\mathcal{F}: \mathbb{C}^N \to \mathbb{C}^N$$

$$\mathcal{F}(f)(k) = F(k) = \sum_{a=0}^{N-1} f(a) e^{-2\pi i \left(\frac{ka}{N}\right)}$$

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{-2\pi i \left(\frac{kx}{N}\right)}$$

Some properties

LINEARITY

- $\mathcal{F}(cf + dg) = c\mathcal{F}(f) + d\mathcal{F}(g), \quad \forall c, d \in \mathbb{C}$

SHIFTS

- $\mathcal{F}\left(f(a) \cdot e^{2\pi i\left(\frac{am}{N}\right)}\right)(k) = \mathcal{F}(f)(k - m) = F(k - m),$
- $\mathcal{F}(f(a - m))(k) = F(k) \cdot e^{-2\pi i\left(\frac{km}{N}\right)}$

CONVOLUTION

- $\mathcal{F}(f * K_N)(k) = F \cdot \mathcal{F}(K_N)(k), \quad \forall c, d \in \mathbb{C}$

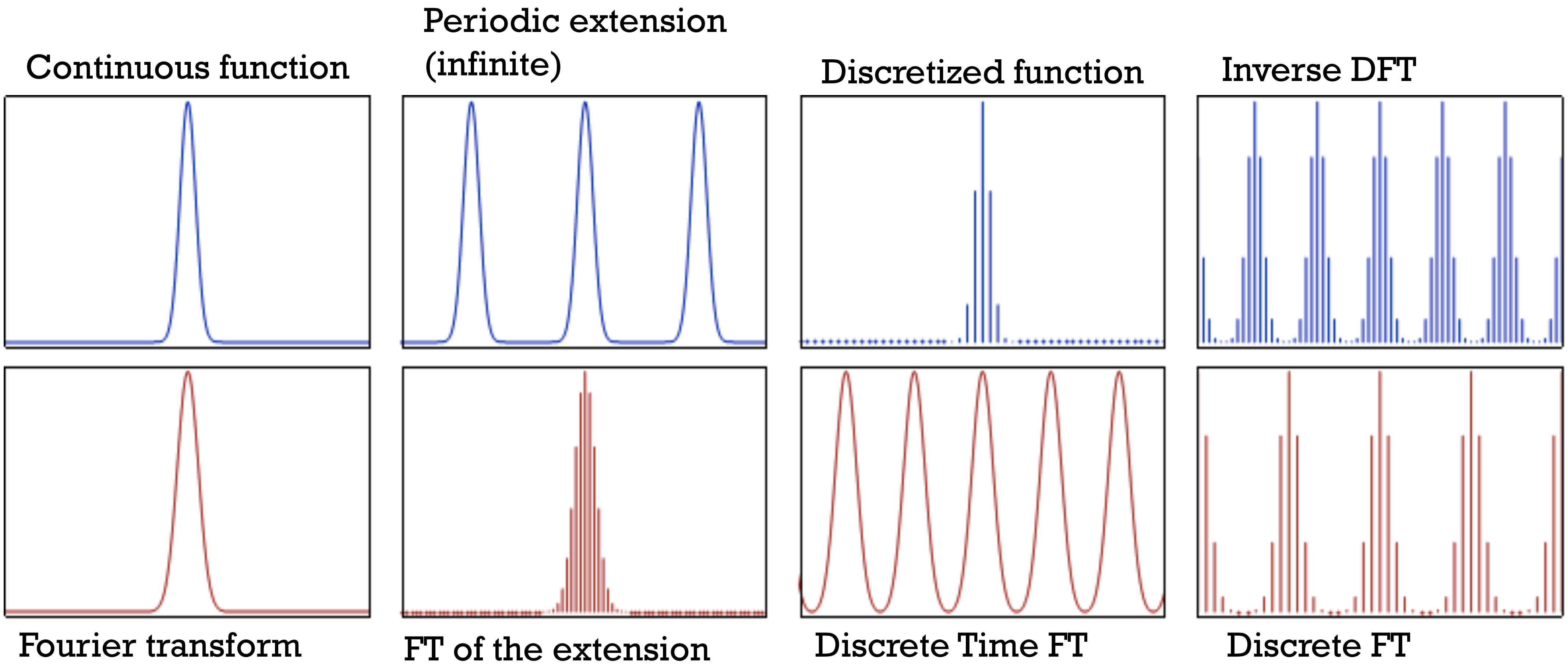
image

kernel

FT image

filter

Path to discrete setting



Fourier transfrom of images

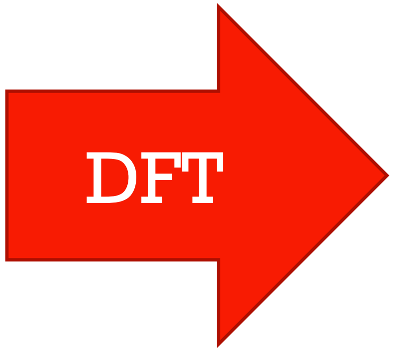
2D DISCRETE FT AND ITS INVERSE FT

$$F(k,l) = \sum_{a=0}^{N-1} \sum_{b=0}^{M-1} f(a,b) e^{-2\pi i\left(\frac{ka}{N}+\frac{lb}{M}\right)}$$

$$f(x,y) = \frac{1}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} F(k,l) e^{-2\pi i\left(\frac{kx}{N}+\frac{ly}{M}\right)}$$

Pixel-intensity space (spatial domain)

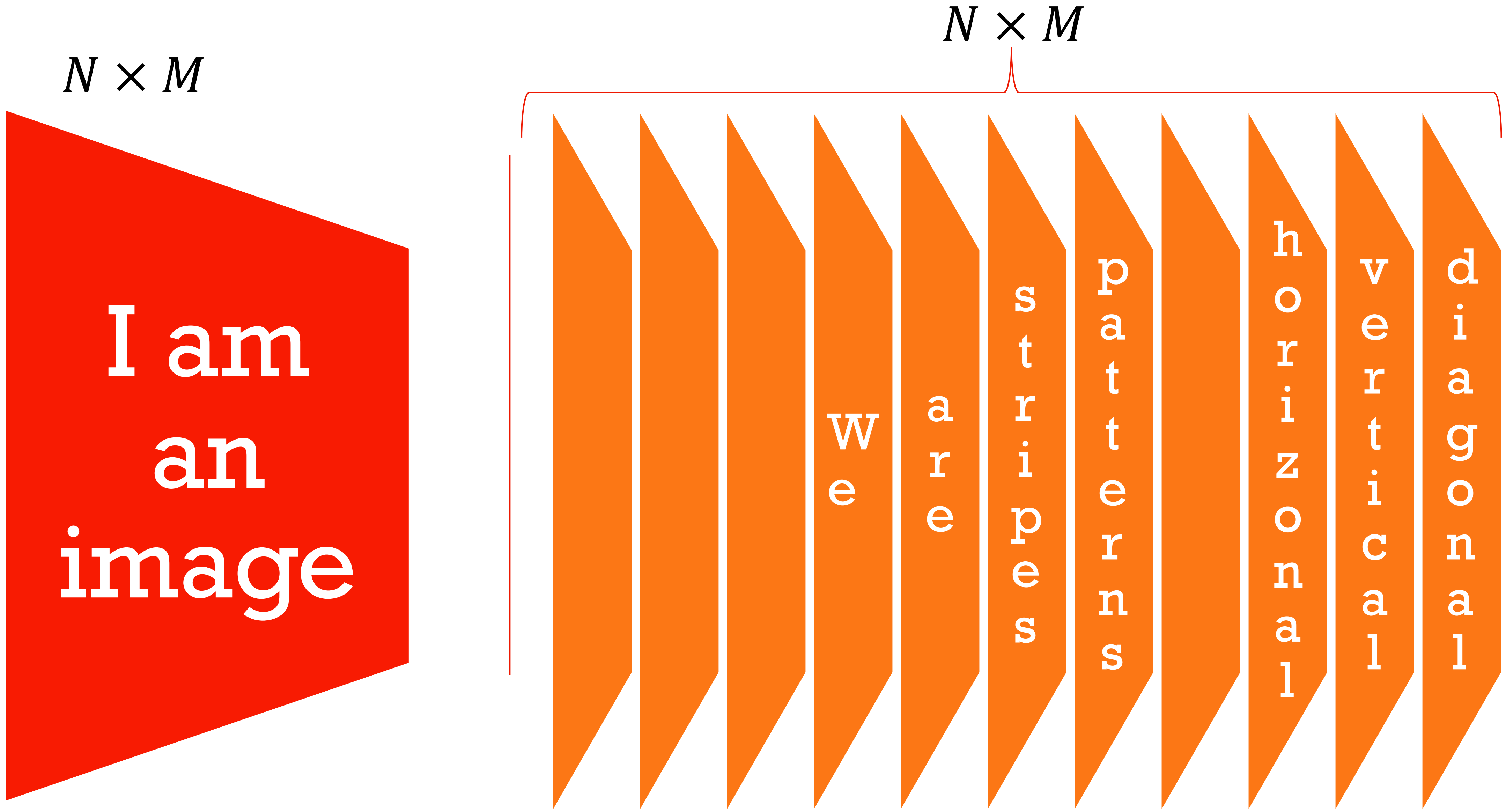
0	0	0	0	0	0
87	3	9	0	32	0
2	6	89	0	5	0
3	65	69	3	9	0
0	0	0	88	0	0
0	87	3	9	0	0



Frequency space (Fourier domain)

	0	1	2	3	4	5
0	1+0j	.5+.7j	2-6j	.2+3j		
1	-2+j					
2						
3						
4						
5						

* Fast Fourier Transform (FFT) is an efficient way to compute DFT.


$N \times M$
I am an image
$N \times M$
we
are
stripes
patterns
horizontal
vertical
diagonal

What is SVD?

- Singular Value Decomposition
- $A = U \Sigma V^T$
- SVD decomposes an $m \times n$ matrix A into products of orthogonal matrices U size $m \times m$, V size $n \times n$, and singular matrix Σ size $n \times m$.