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Regional Geoid Modelling

By classical classical remove-restore approach (Sans'o and Sideris 2013, ch. 7) for computing the geoid using classical gravity anomalies

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Abstract The geoid is an equipotential surface that coincides with the global mean sea level in the ocean and its extension through the continents. The geoid is used as a vertical datum for the measurement of precise surface elevations hence it is important to determine the geoid. This project focuses on geoid determination.There are several techniques for modelling of the geoid such as using ground gravity data,the geopotential models,the astro-geodetic components or a combination of them. A geoid model has been computed for USA(Nebraska) covering the region 40.0908N - 43.0189N and 95.8782W and 102.9637W.This was calculated by removing various topographical and low gegree global signals before restoration which is done after stokes' integration with the airborne gravity model of Nebraska,USA.The Gravity Data consists of 125431 Air-born gravity observations,281101 values of long-wavelength undulation (N_{GGM}) of the GGM model , 281101 values of long wavelength gravity anomaly($_{GGM}$).For geoid undulations spherical variant of the stokes kernel was used. Fast Fourier Transformation has been used in several places to increase the efficiency of computation.

Keywords Stokes kernel · Geoid Undulation · Fast Fourier Transformation

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1 Introduction

One of the major goal for physical geodesy is determination of geoid in cm accuracy.This may be easily demonstrated in lowland areas because of the dense gravity coverage but in hilly regions with insufficient gravity it is quite challenging.So,airborne gravity data and remove restore method id used to calculate this. The true shape of the earth is called the geoid.It is not a smooth surface and have varying density and different places.It is in not the same as its mathematically approximated shape that is the ellipsoid.The varying shape and the variation in density and terrain causes variation in density.This causes varied gravity can be used to estimate the bulge or contract at different places which then finally gives us the geoid.
There are also various anomalies that are present so various corrections such as free - air correction, removal of the global gravity field,correction to account for the gravitational attraction of the atmosphere,and gravimetric terrain reduction needs to be done.Later some additions such as the long wavelength gravity anomaly and the indirect effect needs to be done to get the final get the geoid.

2 Data-set

Multiple data-sets have been used majorly in the removal steps.The first data set comprises of the observed airborne gravity data of the (GRAV-D Block:CN03) Nebraska Block for the year 2014. This set comprises of 125431 $g_{observed}$ values between latitude 40.0908N and 43.0189N and longitudes 95.8782W and 102.9637W. For calculating rough estimates of orthometric height we get the data of long wave undulation(N_{GGM}) for coordinates enclosing Nebraska for WGS84 Reference System and GGM01C model(Link).It consists of 281101 data points.Similarly for the same GGM model we get the gravity anomaly data if is for coordinates enclosing Nebraska and it consists of 281101 data points.The Last set of data is used for the last step of Removal.A Digital Elevation Model (DEM) is used for coordinates enclosing Nebraska.This model is fulfill the height model requirements and evaluation of primary indirect effect.The data is given as a grid of heights of all the different points in the given coordinates the grid size that we get from the data is 6000*12000.
All of these of following data if required is converted into the desired resolution using:

DesiredData = griddata(GivenLat,GivenLon,GivenData,DesiredLat,DesiredLon);

The resolution we are working on is $\underline{0.01° \times 0.01°}$

3 Methodology

We constructed a regional geoid model with the resolution $0.01° \times 0.01°$ for the area 40.0908° N - 43.0189° N and 95.8782°W and 102.9637°W.

3.1 **Remove**

The free-air gravity anomalies(Δg_{FGA}) over the target area where derived from the data-sets described in the previous section using the following equation:

$$\Delta g_{FGA} = g_{observed} - \gamma + \delta g_{Freeair}. \tag{1}$$

Where $g_{Observed}$ is the observed gravity data,γ is the normal gravity on the ellipsoid,$\delta g_{FreeAir}$ is the free-air gravity correction.γ, $\Delta g_{Freeair}$can be computed as follows:

$$\gamma = \frac{a\gamma_a \cos^2(\phi) + b\gamma_b \sin^2(\phi)}{\sqrt{(a\cos^2(\phi) + b\sin^2(\phi))}} \tag{2}$$

Where a and b are the semi-major axis and semi minor axis respectively of the reference ellipsoid in meters,γ_a and γ_b are the normal l gravity values at the equator and at the poles, respectively, and ϕ is the geodetic/ellipsoidal latitude.

$$\delta g_{Freeair} = \frac{2\gamma_0}{a}(1 + f + m - 2f\sin^2(\phi))H_P - \frac{3\gamma_0}{a^2}H_p^2 \tag{3}$$

Where γ_0 is the normal gravity on the ellipsoid at a given latitude,a and b are the semi-major axis and semi minor axis respectively of the reference ellipsoid in meters.f is the flattening,ϕ is geodetic latitude,H is the orthometric height.

The effect of global global gravity field model(long wavelength gravity anomaly(g_{GGM})) which was downloaded as mentioned in the link above is removed from the free-air gravity anomaly which leaves us with the short and medium wavelength gravity anomaly.This step can be written by

$$\Delta f_{smw} = \Delta g_{Freeair} - \Delta g_{GGM} \tag{4}$$

Further we remove the gravity attraction due to the atmosphere which leaves us with the Δg_{smw}^{atm}.We can calculate the gravity anomaly after removal of gravitational attraction of the atmosphere by:

$$\Delta g_{smw}^{atm} = \Delta g_{smw} - \delta g_{atm}. \tag{5}$$

Here Δg_{smw} can be calculated by using equation (5) and δg_{atm} can be calculated by using the equation below:

$$\delta g_{atm} = 0.871 - 1.0298H + 5.3105\times10^{-9}H^2 - 2.1642\times10^{-13}H^3 + 9.5246\times10^{-18}H^4 - 2.2411\times10^{-22}H^5 \tag{6}$$

Where H is the rough estimate of orthometric height by adding long-wavelength undulation (N_{GGM}).
The Final removal step is terrain correction which gives us the Faye anomaly Δg_{Faye}.This is done to account for the real world topography surrounding a point.There are two methods to find the terrain correction one is through classical integration which is simple to interpret but might take several days of computational time while the other method make use of FFT (Fast Fourier Transform) which is faster.The difference between these techniques is negligible in flat regions and since Nebraska is fairly flat region it works well according to our requirements. The classical method integrates the gravitational effect produced by each prism constructed from the DEM.This method for any Point P can be mathematically written as:

$$\delta g_T(P) = \frac{G\rho}{2}\left[H_p^2 \iint\limits_{\Omega} \frac{1}{l^3}dx'dy' + \iint\limits_{\Omega} \frac{H_Q^2}{l^3}dx'dy' - 2H_p \iint\limits_{\Omega} \frac{H_Q}{l^3}dx'dy'\right] \tag{7}$$

Where $l = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P - z_Q)^2}$ is the distance between point of evaluation P and the data points surrounding it Q, and HP and HQ are the heights of the evaluation and data points respectively.H_P and H_Q can be found by adding N_GGM to the DEM that has been derived as described in the previous section. When the above convolution integral is written as Fourier transform we get,

$$\delta g_T(P) = \frac{G\rho}{2}\left[H_p^2\left(\mathcal{F}^{-1}(\mathcal{F}(1)\cdot\mathcal{F}(1/r_Q^3))\right) + \mathcal{F}^{-1}(\mathcal{F}(H_Q^2)\cdot\mathcal{F}(1/r_Q^3)) - 2H_p\left(\mathcal{F}^{-1}(\mathcal{F}(H_Q)\cdot\mathcal{F}(1/r_Q^3))\right)\right] \tag{8}$$

Where the F and $\mathcal{F}^{-1}$ is the e forward (analysis) and the inverse (synthesis) Fourier transforms.This terrain correction removed from Δg_{smw}^{atm} gives us g_{Faye}.

$$g_{Faye} = \Delta g_{smw}^{atm} - \delta g_T. \tag{9}$$

Finally the entire removal step where g_{Faye} is the final result in short can be written as:

$$g_{Faye} = (g_o bserved + \delta g_{FreeAir}) - \gamma - \Delta g_{GGM} - \delta g_{atm} - \delta g_T. \tag{10}$$

3.2 **Stokes' Integration**

We use the stokes integral for evaluating the disturbing potential which can further be used to find the geoid undulation using the buruns's formula as shown in equation[15].The Stokes's integral for evaluating the disturbing potential at a point on the geoid can be expressed as :

$$T_r = \frac{R}{4\pi}\iint_{\Omega} \Delta g_{faye} S(\psi) d\Omega. \tag{11}$$

Where R is the mean earth radius,Ω is the sphere of integration,ψ is the geocentric angle.S(ψ) is the stokes function.The Fourier transform of equation[11] can be expressed as :

$$T_r = \frac{R}{4\pi}\left[\mathcal{F}^{-}1\left(\mathcal{F}(\Delta g_{faye})\mathcal{F}(S(\psi)\right)\right]. \tag{12}$$

S(ψ) or the stokes' function may be written as:

$$S(\psi) = \frac{1}{\sin\left(\frac{\psi}{2}\right)} - 4 - 6\sin\left(\frac{\psi}{2}\right) + 10\sin^2\left(\frac{\psi}{2}\right) - \left[3 - 6\sin^2\left(\frac{\psi}{2}\right)\right] - ln\left[\sin\left(\frac{\psi}{2}\right) + \sin^2\left(\frac{\psi}{2}\right)\right] \tag{13}$$

Where,

$$\sin^2\left(\frac{\psi}{2}\right) = \sin^2\left(\frac{\phi_P - \phi}{2}\right) + \sin^2\left(\frac{\lambda_P - \lambda}{2}\right)\cos\phi_P\cos\phi \tag{14}$$

Here

3.3 **Restore**

The Disturbing potential calculated in the above section(see Sect. 3.2) can now be used to calculate the undulation using the Bruns's formula which is:

$$N_r = \frac{T_r}{\gamma}. \tag{15}$$

Where T_r is the disturbing potential and γ is the is the normal gravity on the ellipsoid.This $N_r$when added with the previously removed N_{GGM} this gives us the co-geoid which can be written as:

$$N_{cogeoid} = N_r + N_{GGM} \tag{16}$$

The Stokes' integration conbined with the Burns's formula did not provide us the geoid,but rather the cogeoid,because of the mathematical condensation of the topographic masses outside the geoid onto an infinite layer on the geoid. This discrepancy between the geoid and cogeoid is called the primary indirect effect (PIDE) and can be computed using DEM.So assumptions such as constant density of the earth along with planar assumptions have been made.This Indirect effect can be calculated by equation[17](Classical Integration) or equation[18](Fourier Tranform).

$$\delta N_{indirect}(P) = -\frac{G\rho}{\gamma}\left[\pi H_P^2 + \frac{R^2}{6}\iint_{\Omega}\frac{H_Q^3 - H_P^3}{l^3}d\Omega - \frac{3R^2}{40}\iint_{\Omega}\frac{H_Q^5 - H_P^5}{l^5}d\Omega\right] \tag{17}$$

Which can also be written as:

$$\begin{aligned}\delta N_{indirect}(P) = -\frac{G\rho}{\gamma}\Bigg[\pi H_P^2 + \frac{R^2}{6}\Bigg[\mathcal{F}^{-1}\left(\mathcal{F}(H_Q^3)\cdot\mathcal{F}(\frac{1}{r_Q^3})\right) - \mathcal{F}^{-1}\left(H_R^3\mathcal{F}(1)\cdot\mathcal{F}(\frac{1}{r_Q^3})\right)\Bigg] \\ - \frac{3R^2}{40}\left[\mathcal{F}^{-1}\left(\mathcal{F}(H_Q^5)\cdot\mathcal{F}(\frac{1}{r_Q^5})\right) - \mathcal{F}^{-1}\left(H_R^5\mathcal{F}(1)\cdot\mathcal{F}(\frac{1}{r_Q^5})\right)\right]\end{aligned} \tag{18}$$

On adding indirect effect with the $N_{cogeoid}$ we get N_{geoid}.The final equation to get N_{geoid} will be:

$$N_{geoid} = N_{cogeoid} + \delta N_{indirect}. \tag{19}$$

The entire restore step can be summed up as:

$$N_{geoid} = \frac{T_r}{\gamma} + N_{GGM} + \delta N_{indirect}. \tag{20}$$

4 Results and Discussions

As shown is section 3.3 The stokes scheme,finally gives us a model composed off three components(N_{GGM},N_r,$N_{indirect}$) as expressed in equation (19) summing these three gives us a readily available geoid model.The geoid model that we receive can be graphically represented as Fig4.

5 Conclusion

The Gravimetric geoid model has been improved significantly by incorporating the previous steps.The existence of aerial gravimatric data over a region can be used to improve improve the geoid computation.The increase in computational power and the increase in gravity data density will improve the quality of geoid models.Thus by incorporating airborne gravity data, introducing a laterally variable topographic density model, and refining our methodology we can significantly improve our model.

References

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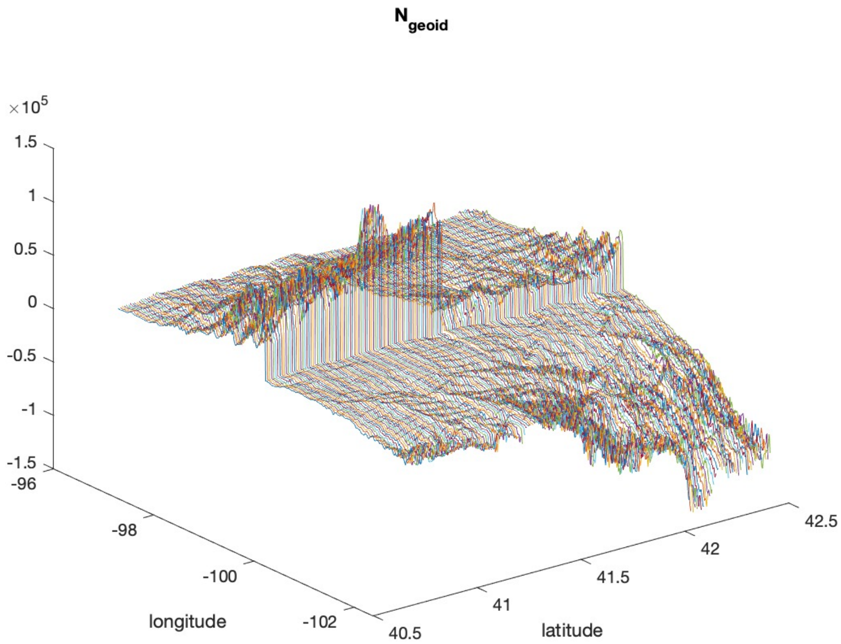
N geoid
×10^5
1.5
1
0.5
0
-0.5
-1
-1.5
-96
-98
-100
-102
longitude
40.5
41
41.5
42
42.5
latitude