Project work – Regional geoid modelling

CE678A Physical Geodesy | 2020-21 I

Instructor: Balaji Devaraju [dbalaji]

TA: Satyam Choubey [satyamkc]
Avadh B. Narayan [avadhbn]

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The geoid is an equipotential surface that coincides with the global mean sea level in the ocean and its extension through the continents. The geoid is used as a vertical datum for the measurement of precise surface elevations. This project focuses on geoid determination. The Airborne geoid mapping techniques is a very efficient technique to map geoid over large areas of the Earth. Therefore in this project students have to model regional geoid using airborne gravity data.

Based on the assumptions, there are several approaches to determine the geoid. Therefore, give reasons for choosing any approach and mention different assumptions.

1 Geoid modelling

There are many ways of modelling the geoid. Here, we will be following the classical remove-restore approach (Sansò and Sideris 2013, ch. 7) for computing the geoid using classical gravity anomalies.

1. Download observed airborne gravity data (g_{observe}) of any tile from `https://geodesy.noaa.gov/GRAV-D/data_products.shtml`

2. Calculate a rough estimate of orthometric height by adding long-wavelength undulation (N_{GGM}) of the GGM model to the Ellipsoidal height. Download long-wavelength undulation from one of the following links:
   `https://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/anomalies_dov.html`
   `http://icgem.gfz-potsdam.de/calcgrid`.

3. Calculate free air anomaly (medium wavelength).

$$\Delta g = (g_{\text{observed}} \;\text{[illegible]}\; \delta g_{\text{Free air}}) - \gamma$$

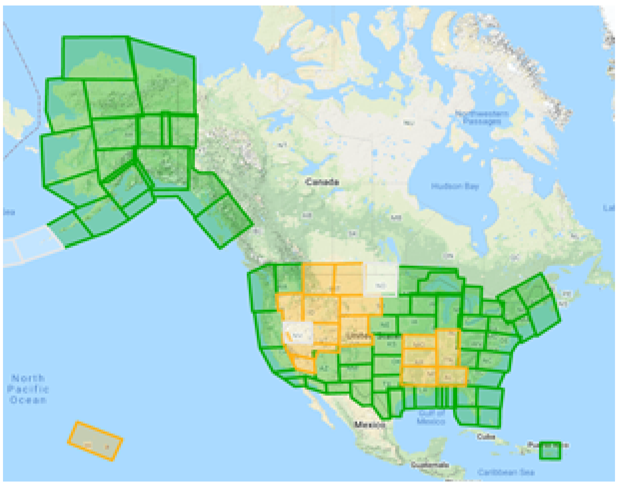

Figure 1: Tiles of acquisition of the airborne gravity data in the GRAV-D project. Source: NOAA

To calculate the normal gravity γ use the Somigliana-Pizzetti formula, which is given as

$$\gamma = \frac{a\,\gamma_a\,\cos^2\varphi\,+\,b\,\gamma_b\,\sin^2\varphi}{\sqrt{a\,\cos^2\varphi\,+\,b\,\sin^2\varphi}} \tag{1}$$

where a and b are the semi-major and semi-minor axes of the reference ellipsoid in metres, γ_a and γ_b are the normal gravity values at the equator and at the poles, respectively, and φ is the geodetic/ellipsoidal latitude.

4. From the free-air gravity anomaly, remove the effect of a global gravity field model, a long-wavelength gravity anomaly (Δg_{GGM}) .

$$\Delta g_{\mathrm{s\&mw}} = \Delta g - \Delta g_{\mathrm{GGM}}$$

The resultant reduced gravity anomaly $\Delta g_{\mathrm{s\&mw}}$ contains only the medium and the short wavelengths. In order to be consistent choose the same geopotential model that you chose for step 2. Download gravity anomaly of any GGM model from `http://icgem.gfz-potsdam.de/calcgrid`

5. A correction to account for the gravitational attraction of the attraction must be applied

$$\Delta g^{\mathrm{atm}}_{\mathrm{s\&mw}} = \Delta g_{\mathrm{s\&mw}} - \delta g_{\mathrm{atm}}$$

$$\begin{aligned}\delta g_{\mathrm{atm}} = 0.871 - 1.0298\times10^{-4}\,H\,+\,5.3105\times10^{-9}\,H^2\,-\,2.1642\times10^{-13}\,H^3\\ +\,9.5246\times10^{-18}\,H^4\,-\,2.2411\times10^{-22}\,H^5\end{aligned}$$

6. Convert the gravity anomaly $\Delta g^{\mathrm{atm}}_{\mathrm{s\&mw}}$ data points into a grid.

7. Apply gravimetric terrain reduction δg_{T} to compute the *Faye anomaly* Δg_{Faye}.

$$\Delta g_{\mathrm{Faye}} = \Delta g^{\mathrm{atm}}_{\mathrm{s\&mw}} - \delta g_{\mathrm{T}}\,,$$

where δg_{T} is the terrain correction. For a given point P the terrain correction at the point is given as

$$\delta g_{\mathrm{T}}(\mathrm{P}) = \frac{G\rho}{2}\left[H_{\mathrm{P}}^2\iint\limits_{\Omega}\frac{1}{l^3}\,\mathrm{d}x'\,\mathrm{d}y'\,+\,\iint\limits_{\Omega}\frac{H_{\mathrm{Q}}^2}{l^3}\,\mathrm{d}x'\,\mathrm{d}y'\,-\,2H\iint\limits_{\Omega}\frac{H_{\mathrm{Q}}}{l^3}\,\mathrm{d}x'\,\mathrm{d}y'\right]$$

where $l = \sqrt{(x_{\mathrm{P}}-x_{\mathrm{Q}})^2+(y_{\mathrm{P}}-y_{\mathrm{Q}})^2+(y_{\mathrm{P}}-y_{\mathrm{Q}})^2}$ is the distance between point of evaluation P and the data points Q, and H_{P} and H_{Q} are the heights of the evaluation and data points, respectively. The above integration is in essence a *convolution integral* (Forsberg 1985; McCubbine et al. 2017). It can be written as

$$\delta g_{\mathrm{T}}(\mathrm{P}) = \frac{G\rho}{2}\left[H_{\mathrm{P}}^2\left(1*\frac{1}{r_{\mathrm{Q}}^3}\right)+\left(H_{\mathrm{Q}}^2*\frac{1}{r_{\mathrm{Q}}^3}\right)-2H_{\mathrm{P}}\left(H_{\mathrm{Q}}*\frac{1}{r_{\mathrm{Q}}^3}\right)\right].$$

The convolution integral can be efficiently solved using FFT. Writing the above convolution integral as a Fourier transform

$$\begin{aligned}\delta g_{\mathrm{T}}(\mathrm{P}) = \frac{G\rho}{2}\Bigg[&H_{\mathrm{P}}^2\left(\mathcal{F}^{-1}\left(\mathcal{F}(1)\,\mathcal{F}\left(\frac{1}{r_{\mathrm{Q}}^3}\right)\right)\right)\\ &+\left(\mathcal{F}^{-1}\left(\mathcal{F}(H_{\mathrm{Q}}^2)\,\mathcal{F}\left(\frac{1}{r_{\mathrm{Q}}^3}\right)\right)\right)\\ &-2H_{\mathrm{P}}\left(\mathcal{F}^{-1}\left(\mathcal{F}(H_{\mathrm{Q}})\,\mathcal{F}\left(\frac{1}{r_{\mathrm{Q}}^3}\right)\right)\right)\Bigg].\end{aligned} \tag{2}$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the forward (analysis) and the inverse (synthesis) Fourier transforms. A Digital Elevation Model (DEM) is required for computing the height information. Download a DEM from `http://srtm.csi.cgiar.org/srtmdata/`

8. Calculate disturbing potential by using Stokes integral.

$$T_r = \frac{R}{4\pi} \iint\limits_{\Omega} \Delta g_{\text{Faye}}\, S(\psi)\, d\Omega\,,$$

where $S(\psi)$ is the Stokes kernel and the integration is over the entire sphere. For the regional case, the integral can be done on a planar cartesian grid.
Bonus: Instead of the standard Stokes kernel you can also apply any modified kernel.

9. By using Bruns's formula, calculate undulation.

$$N_r = \frac{T_r}{\gamma}$$

10. Restore the undulation (N_{GGM}) corresponding to the removed long-wavelength gravity anomaly of the GGM model (Δg_{GGM}) . You will get a co-geoid.

$$N_{\text{cogeoid}} = N_r + N_{\text{GGM}}$$

11. To get the geoid, add the indirect effect ($\delta N_{\text{indirect}}$) of the corresponding gravimetric terrain reduction (Δg_T) method.

$$N_{\text{geoid}} = N_{\text{cogeoid}} + \delta N_{\text{indirect}}\,,$$

where

$$\delta N_{\text{indirect}} = -\frac{G\rho}{\gamma}\left[\pi\, H_{\text{P}}^2 + \frac{R^2}{6}\iint\limits_{\Omega} \frac{H_{\text{Q}}^3 - H_{\text{P}}^3}{l^3}\text{d}\Omega - \frac{3R^2}{40}\iint\limits_{\Omega} \frac{H_{\text{Q}}^5 - H_{\text{P}}^5}{l^5}\text{d}\Omega\right]$$

For further details on the topic, please refer to (Sansò and Sideris 2013).

2 Technical report and software documentation

As per the protocol, you will need to submit a short report (maximum 5 pages including graphs and figures), software written as a toolbox and its documentation. The report must be structured as follows: abstract, introduction, mathemtical details, methods, results, discussion and conclusion. Use the manuscript submission format of International Association of Geodesy Symposia given at `https://www.springer.com/series/1345`. Your project will be evaluated for ease of use of the software, the clarity of your documentation and the scientific quality of your report. All results should be reproducible, and therefore, please submit your code that you used for creating your figures, tables and other results. Otherwise, the results will be considered plagiarised. Plagiarism of any form will be dealt with severely. Since it is a project work, you are also expected to do some literature review and study about the project topic at your end.

References

[1] René Forsberg. "Gravity field terrain effect computations by FFT". In: *Bulletin Géodésique* 59.4 (Dec. 1985), pp. 342–360. DOI: `10.1007/bf02521068`.

[2] J. C. McCubbine et al. “The New Zealand gravimetric quasigeoid model 2017 that incorporates nationwide airborne gravimetry”. In: *Journal of Geodesy* 92.8 (Dec. 2017), pp. 923–937. DOI: 10.1007/s00190-017-1103-1.

[3] Fernando Sansò and Michael G. Sideris, eds. *Geoid Determination – Theory and Methods*. Springer Berlin Heidelberg, 2013. DOI: 10.1007/978-3-540-74700-0.