

Assignment 2

DS

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Q1:

1- Decision Variables : x_{ij} $\begin{matrix} i, j \\ 1 \rightarrow 3 \quad 1 \rightarrow 4 \end{matrix}$
objective function

$$Z = 500x_{11} + 750x_{12} + 300x_{13} + 450x_{14} \\ + 650x_{21} + 800x_{22} + 400x_{23} + 600x_{24} \\ + 400x_{31} + 700x_{32} + 500x_{33} + 550x_{34}$$

Constraints

$$\left. \begin{aligned} \bullet x_{11} + x_{12} + x_{13} + x_{14} &= 12 \\ \bullet x_{21} + x_{22} + x_{23} + x_{24} &= 17 \\ \bullet x_{31} + x_{32} + x_{33} + x_{34} &= 11 \end{aligned} \right\} \text{Supply}$$

$$\left. \begin{aligned} \bullet x_{11} + x_{21} + x_{31} &= 10 \\ \bullet x_{12} + x_{22} + x_{32} &= 10 \\ \bullet x_{13} + x_{23} + x_{33} &= 10 \\ \bullet x_{14} + x_{24} + x_{34} &= 10 \end{aligned} \right\} \text{demand}$$

$$\bullet x_{ij} \geq 0$$

2-a) North west corner method

| From | 1 | 2 | 3 | 4 | Supp |
|--------|-----------|-----------|-----------|-----------|---------------|
| 1 | 500 10 | 750 2 | 300 | 450 | 12 20 |
| 2 | 650 | 800 8 | 900 9 | 600 | 17 90 |
| 3 | 400 | 200 | 500 1 | 550 10 | 11 100 |
| Demand | 100 | 1000 8 | 1000 1 | 1000 0 | total 4000 |

$$\begin{aligned} T.C. &= 10 \times 500 + 2 \times 750 + 8 \times 800 \\ &+ 9 \times 400 + 1 \times 500 + 10 \times 550 = 22500 \end{aligned}$$

b) Least cost method

| From | 1 | 2 | 3 | 4 | Supp |
|--------|--------|--------|--------|-------|----------|
| 1 | 500 | 750 | 10 300 | 2 450 | 12 250 |
| 2 | 650 | 10 800 | 400 | 7 600 | 17 200 |
| 3 | 10 400 | 900 | 500 | 1 550 | 11 150 |
| Demand | 10 | 10 | 10 | 8 | total 40 |

$$T.C = 10 \times 400 + 80 \times 800 + 10 \times 300 + 2 \times 450 + 7 \times 600 + 1 \times 550 = 20650$$

c) Vogel's APPROXIMATION method (VOM)

| From | 1 | 2 | 3 | 4 | Supp |
|--------|--------|-------|--------|--------|----------|
| 1 | 500 | 2 750 | 300 | 10 450 | 12 250 |
| 2 | 650 | 7 800 | 10 400 | 600 | 17 200 |
| 3 | 10 400 | 1 700 | 500 | 550 | 11 150 |
| Demand | 10 | 10 | 10 | 10 | Total 40 |

$$T.C = 10 \times 400 + 2 \times 750 + 7 \times 800 + 700 \times 1 + 10 \times 400 + 10 \times 450 = 20300$$

3- MOD I

$$\begin{matrix} -300 & 0 & -700 & -300 \\ U_1 & U_2 & U_3 & U_4 \end{matrix}$$

| From | 1 | 2 | 3 | 4 | Supp | |
|--------|-----|-----|-----|-----|-------|--------------|
| 1 | 500 | 200 | 300 | 450 | 12 | 750 U_1 |
| 2 | 650 | 800 | 400 | 600 | 17 | 800 U_2 |
| 3 | 400 | 700 | 500 | 550 | 11 | 200 U_3 |
| Demand | 10 | 10 | 10 | 10 | Total | 40 |

$$U_1 + V_4 = 10, U_2 + V_3 = 10, U_3 + V_2 = 10$$

$$U_1 + V_2 = 2, U_2 + V_2 = 7, U_3 + V_1 = 10$$

$$\text{let } V_2 = 0, U_1 = 2, U_2 = 7, U_3 = 10$$

$$U_1 + V_4 = 450, U_2 + V_3 = 400$$

$$U_1 + V_2 = 750, U_2 + V_2 = 800$$

$$U_3 + V_2 = 700, U_3 + V_1 = 400$$

$$\text{let } U_2 = 0 \therefore U_1 = 750$$

$$x_{11} = 500 - 750 + 300, U_2 = 800$$

$$x = 50, U_3 = 700$$

$$x_{21} = 650 + 300 - 800, V_1 = -300$$

$$x = 150$$

$$x_{13} = 300 + 300 - 750 = -150, V_3 = -400$$

$$x_{33} = 500 + 300 - 700 = 100, V_4 = -300$$

$$x$$

$$x_{24} = 600 + 300 - 800 = 100x$$

$$x_{34} = 550 + 300 - 700 = 150x$$

| | | القاربع: U_i | | | | الموضوع: | |
|--------|--|----------------|-------|-------|-------|----------|-------|
| | | V_1 | V_2 | V_3 | V_4 | | |
| From | | 1 | 2 | 3 | 4 | Supp | |
| 1 | | 500 ✓ | 750 | 300 | 450 | 12 | U_1 |
| | | | | 2 | 10 | | |
| 2 | | 650 | 800 ✓ | 400 | 600 | 17 | U_2 |
| | | | 9 | 8 | | | |
| 3 | | 400 | 800 ✓ | 500 | 550 | 11 | U_3 |
| | | 10 | 1 | | | | |
| Demand | | 10 | 10 | 10 | 10 | total | |
| | | | | | | 40 | |

$$U_1 + V_3 = 300, \quad U_2 + V_2 = 800$$

$$U_1 + V_4 = 450, \quad U_2 + V_3 = 400$$

$$U_3 + V_1 = 400, \quad U_3 + V_2 = 700$$

$$\text{let } U_1 = 0, \quad U_2 = 100$$

$$x_{11} = 500 - 400 = 100 \times$$

$$U_3 = 0$$

$$V_1 = 400$$

$$x_{21} = 650 - 400 - 100 = 150 \times$$

$$V_2 = 700$$

$$x_{12} = 750 - 700 = 50 \times$$

$$V_3 = 300$$

$$x_{33} = 500 - 300 - 200 = 0 \times$$

$$V_4 = 450$$

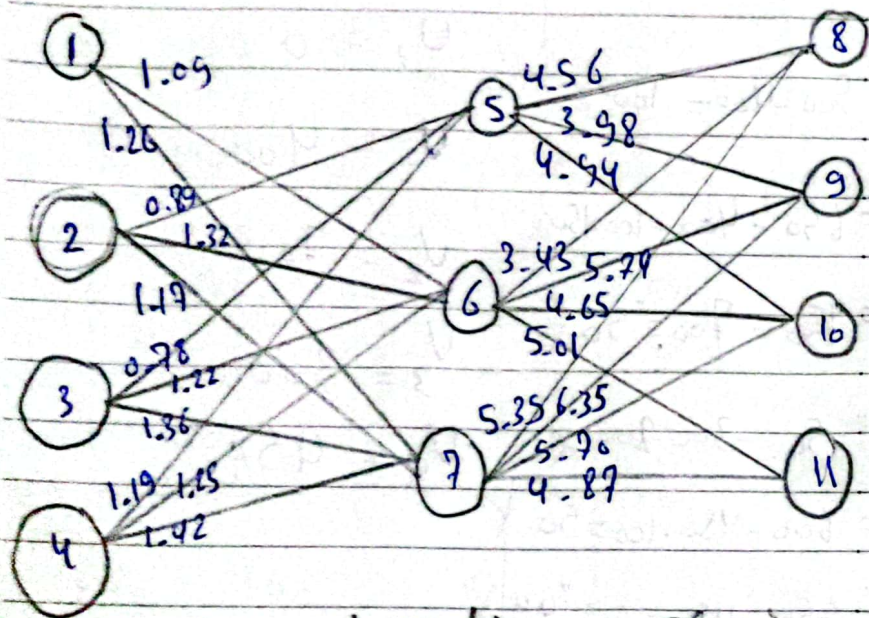
$$x_{24} = 600 - 450 - 100 = 50 \times$$

$$x_{34} = 550 - 450 - 0 = 100 \times$$

$$Z_{\min} = 2 \times 300 + 10 \times 450 + 9 \times 800 + 8 \times 400$$

$$+ 10 \times 400 + 1 \times 700 = 20200$$

Q2



Decision Variable

 x_{ij}

$$1 \rightarrow 5 \quad 5 \rightarrow 11$$

$$Z_{\min} = 1.09 x_{16} + 1.26 x_{17} + 0.89 x_{25} + 1.32 x_{26} + 1.17 x_{27} + 0.78 x_{35} + 1.22 x_{36} + 1.36 x_{37} + 1.19 x_{45} + 1.25 x_{46} + 1.42 x_{47}$$

$$\rightarrow$$

$$+ 4.56 x_{58} + 3.98 x_{59} + 4.94 x_{510} + 3.43 x_{68} + 5.74 x_{69} + 4.65 x_{610} + 5.01 x_{611} + 5.35 x_{78} + 6.35 x_{79} + 5.90 x_{710} + 4.87 x_{711}$$

Constraints:

- $x_{16} + x_{17} \leq 1600$
- $x_{26} + x_{27} + x_{28} \leq 1100$
- $x_{36} + x_{37} + x_{38} \leq 1400$
- $x_{45} + x_{46} + x_{47} \leq 1900$
- $x_{58} + x_{68} + x_{78} = 1200$

$$\bullet x_{59} + x_{69} + x_{79} = 900$$

$$\bullet x_{510} + x_{610} + x_{710} = 1100$$

$$\bullet x_{511} + x_{611} + x_{711} = 1500$$

$$\bullet x_{25} + x_{35} + x_{45} = x_{58} + x_{59} + x_{510}$$

$$\bullet x_{16} + x_{26} + x_{36} + x_{46} = x_{68} + x_{69} + x_{610} + x_{611}$$

$$\bullet x_{17} + x_{27} + x_{37} + x_{47} = x_{78} + x_{79} + x_{710} + x_{711}$$

$$\bullet x_{25} + x_{35} + x_{45} \leq 1800$$

$$\bullet x_{16} + x_{26} + x_{36} + x_{46} \leq 2200$$

$$\bullet x_{17} + x_{27} + x_{37} + x_{47} \leq 1600$$

$$\bullet x_{58} + x_{59} + x_{510} \leq 1800$$

$$\bullet x_{68} + x_{69} + x_{610} + x_{611} \leq 2200$$

$$\bullet x_{78} + x_{79} + x_{710} + x_{711} \leq 1600$$

$$\bullet x_{ij} \geq 0$$

Q3: Objective function

$$Z = 10x_{1A} + 12x_{1B} + 9x_{1C} + 11x_{1D}$$

$$5x_{2A} + 10x_{2B} + 7x_{2C} + 8x_{2D}$$

$$12x_{3A} + 14x_{3B} + 13x_{3C} + 11x_{3D}$$

$$8x_{4A} + 15x_{4B} + 11x_{4C} + 9x_{4D}$$

Constrains

$$x_{1A} + x_{1B} + x_{1C} + x_{1D} = 1$$

$$x_{2A} + x_{2B} + x_{2C} + x_{2D} = 1$$

$$x_{3A} + x_{3B} + x_{3C} + x_{3D} = 1$$

$$x_{4A} + x_{4B} + x_{4C} + x_{4D} = 1$$

$$x_{1A} + x_{2A} + x_{3A} + x_{4A} = 1$$

$$x_{1B} + x_{2B} + x_{3B} + x_{4B} = 1$$

$$x_{1C} + x_{2C} + x_{3C} + x_{4C} = 1$$

$$x_{1D} + x_{2D} + x_{3D} + x_{4D} = 1$$

$$x_{ij} = 1 \text{ or } x_{ij} = 0$$

| | | | | |
|---|--------------------|----|--------------------|---------------------|
| 1 | 10 | 12 | (9) _{min} | 11 |
| 2 | (5) _{min} | 10 | 7 | 8 |
| 3 | 12 | 14 | 13 | (11) _{min} |
| 4 | (8) _{min} | 15 | 11 | 9 |
| | A | B | C | D |

Row reduction \Rightarrow

| الموضوع: | الموضوع: | الموضوع: | الموضوع: |
|----------|----------|----------|----------|
| A | B | C | D |
| 1 | 1 | 3 | 2 |
| 2 | 0 | 5 | 3 |
| 3 | 1 | 3 | 6 |
| 4 | 7 | 3 | 1 |

Column reduction

| A | B | C | D |
|---|---|---|---|
| 1 | 0 | 0 | 2 |
| 2 | 0 | 2 | 3 |
| 3 | 0 | 2 | 0 |
| 4 | 4 | 3 | 1 |

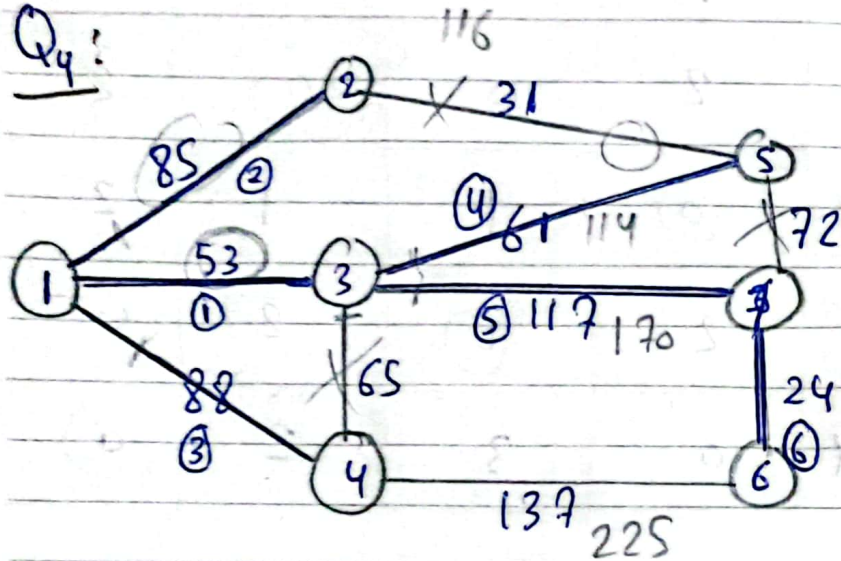
3 ≠ 4

| الموضوع: | الموضوع: | الموضوع: | الموضوع: |
|----------|----------|----------|----------|
| A | B | C | D |
| 1 | 2 | 0 | 2 |
| 2 | 0 | 1 | 2 |
| 3 | 2 | 0 | 0 |
| 4 | 0 | 3 | 0 |

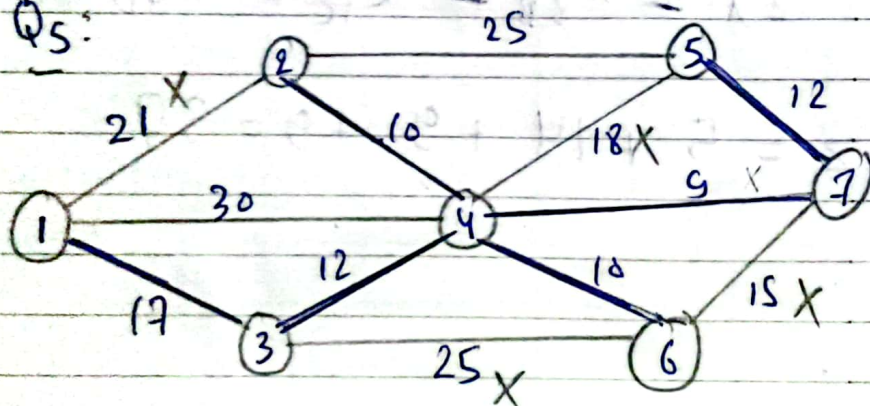
$$x_{2A} \neq x_{3B} \neq x_{1C} = x_{4D} = 1$$

$$Z = 5 + 14 + 9 + 9 = 37$$

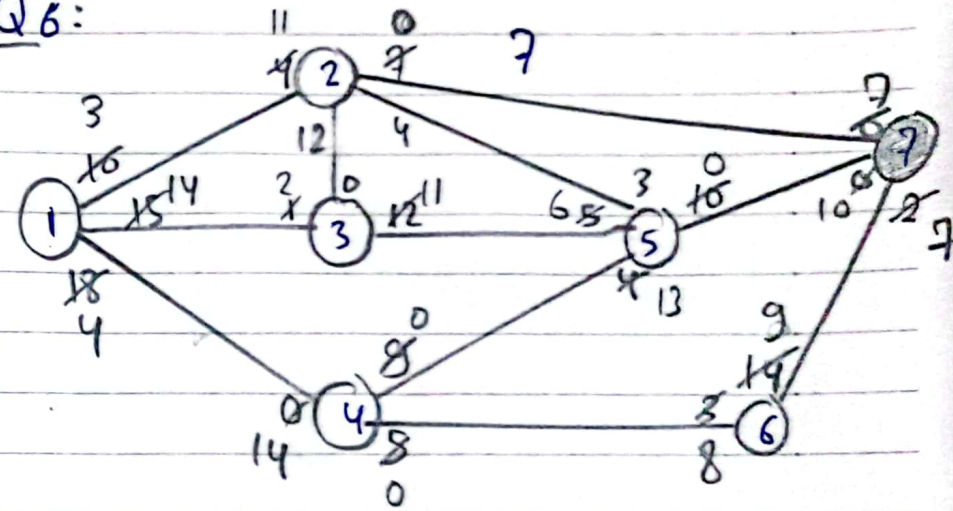
Q4:



Q5:



Q6:



$$1 - 2 - 7 \rightarrow 7$$

$$1 - 3 - 5 - 7 \rightarrow 1$$

$$1 - 4 - 5 - 7 \rightarrow 9$$

$$1 - 4 - 6 - 7 \rightarrow 5$$

$$7 + 1 + 9 + 5 = 22$$