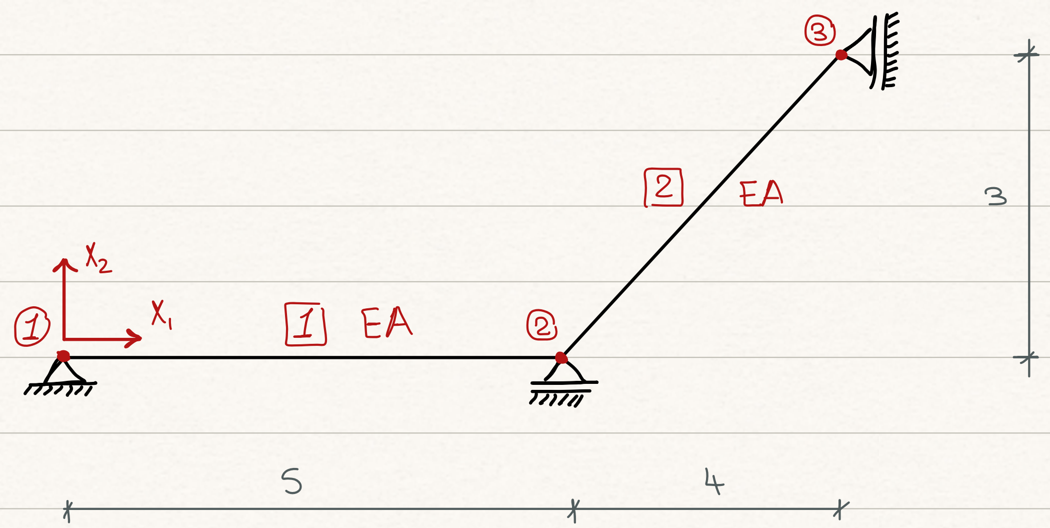
problem:

$EA = 250.$



Required:

1. Compute the internal force $\underline{f}^e_{int}$, $\underline{K}^e_T$, with the consideration of geometrical nonlinearity.

$$\underline{K}^e_T := \frac{\partial \underline{f}_{int}}{\partial \underline{u}_e}$$

Solution

Remember:

Shape functions. Normal force

$$\delta W^e_{int} = \int_{L^e} \delta u_{,X} \overbrace{N}^{:= N(\lambda)} \, \partial X = \delta \underline{u}^{e,T} \underbrace{\int_{L^e} \underline{N}^T_{,X} \cdot \underline{N} \, \partial X}_{\underline{f}^e_{int}}$$

$$\underline{K}^e_{int} = \frac{\partial \underline{f}_{int}}{\partial \underline{u}_e}$$

tangent matrix

$$N := N(\lambda) = \frac{EA}{\lambda} \underbrace{\frac{\lambda^2 - 1}{2}}$$

1D Green Lagrange strain measure (i.e. E_{11})

$$\lambda = \frac{\partial x}{\partial X} = \frac{\partial (X + u)}{\partial X} = 1 + \frac{\partial u}{\partial X} \Rightarrow \lambda = 1 + u_{,X}$$

$$E_{11} = \frac{\lambda^2 - 1}{2} = \frac{1}{2}\left[(1 + u_{,X})^2 - 1\right] = \frac{1}{2}\left[\cancel{1} + u^2_{,X} + 2u_{,X} - \cancel{1}\right] = \frac{1}{2} u^2_{,X} + u_{,X}$$

For $\underline{g}^e_{int}$:

$$\delta W^e_{int} = \int_{L^e} \delta u_{,X}\, EA\,\lambda \left(\frac{\lambda^2-1}{2}\right) dX$$

$u_{,X}$ is still continous

$$= \int_{L^e} \underline{N}_{,X}\, \delta \underline{u}_e \,(1+u_{,X})\, EA \left[\frac{1}{2} u^2_{,X} + u_{,X}\right] dX$$

$$= \delta \underline{u}^T_e \underbrace{\int_{L^e} \underline{N}^T_{,X} (1+\underline{N}_{,X}\, \underline{u}_e)\, EA \left[\frac{1}{2}(\underline{N}_{,X}\cdot \underline{u}_e)^2 + \underline{N}_{,X}\cdot \underline{u}_e\right] dX}_{\underline{g}^e_{int}}$$

Remember: LFEM

$$\underline{N}_{,X} = \frac{\partial \underline{N}}{\partial X} = \underbrace{\frac{\partial \underline{\xi}}{\partial X}}_{J^{-1}} \cdot \frac{\partial \underline{N}}{\partial \underline{\xi}} = \underline{J}^{-1} \cdot \underline{N}_{,\xi}$$

for 1D: $J^{-1} = \frac{2}{L}$ $\rightarrow$ $J = \frac{\text{Length in Physical}}{\text{Length in isoparametric}}$

Linear approximation: $\underline{N} = \left[\frac{1}{2}(1-\xi) \quad \frac{1}{2}(1+\xi)\right]$

$$\underline{N}_{,\xi} = \left[-\frac{1}{2} \quad \frac{1}{2}\right]$$

$$\therefore \underline{N}_{,X} = \underline{J}^{-1} \cdot \underline{N}_{,\xi} = \frac{2}{L}\left[\frac{-1}{2} \quad \frac{1}{2}\right] = \left[\frac{-1}{L} \quad \frac{1}{L}\right]$$

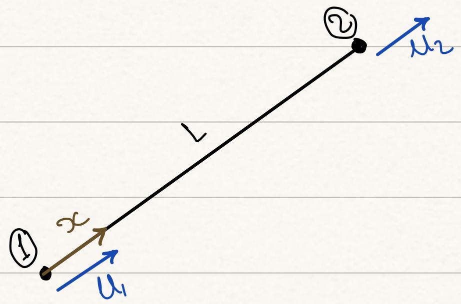
$$\underline{g}^e_{int} = \int_{-1}^{+1} \begin{bmatrix} -1/L^e \\ 1/L^e \end{bmatrix} \left\{1 + \left[\frac{-1}{L^e} \quad \frac{1}{L^e}\right] \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right\} EA \left\{\frac{1}{2}\left(\left[\frac{-1}{L^e} \quad \frac{1}{L^e}\right] \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right)^2 + \left[\frac{-1}{L^e} \quad \frac{1}{L^e}\right] \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right\} \frac{L}{2}\, d\xi$$

$$\left[\frac{-1}{L^e} \quad \frac{1}{L^e}\right] \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{-u_1}{L^e} + \frac{u_2}{L^e}$$

$$\frac{1}{2}\left(\begin{bmatrix} \frac{-1}{L^e} & \frac{1}{L^e} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right)^2 + \begin{bmatrix} \frac{-1}{L^e} & \frac{1}{L^e} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$= \frac{1}{2L^2}(u_2 - u_1)^2 + \frac{1}{L}(u_2 - u_1) = \frac{(u_2-u_1)^2 + 2L(u_2-u_1)}{2L^2} = \underbrace{\frac{L^2 \cancel{-2L\ell} + \ell^2 \cancel{+2L\ell} - 2L^2}{2L^2}}_{\frac{\ell^2 - L^2}{2L^2}}$$

Note:



$$\Delta u = u_2 - u_1 = \ell - L$$

$$= \begin{bmatrix} -1/L \\ 1/L \end{bmatrix} \frac{1}{L}(1 - u_1 + u_2)\, EA\, \frac{\ell^2 - L^2}{2L^2} \underbrace{\int_{-1}^{+1} \frac{1}{2}\, \partial \xi}_{1} = \underbrace{\begin{bmatrix} -1 \\ +1 \end{bmatrix} (1 - u_1 + u_2)\, EA\, (\ell^2 - L^2)}_{f^e_{int}} \rightarrow \#1$$

→ Q: is this enough? Local.

For K^e_{int}:

$$\underline{K}^e_{int} = \frac{\partial f^e_{int}}{\partial \underline{u}^e} = \frac{\partial}{\partial \underline{u}^e}\left(\int_{L^e} \underline{N}^T_{,X} \cdot N(\lambda)\, \partial X \right) = \int_{L^e} \underline{N}^T_{,X} \cdot \frac{\partial N}{\partial \lambda} \frac{\partial \lambda}{\partial u}\, \partial X$$

$$N = EA\lambda \underbrace{\left(\frac{\lambda^2 - 1}{2} \right)}_{E_{11} \rightarrow \text{1D Green lagrange strain.}}$$

$$= \int_{L^e} \underline{N}^T_{,X}\, EA \left[\frac{\lambda^2 - 1}{2} + \lambda^2 \right] \lambda_{,u}\, dX$$

$$= \underbrace{\int_{L^e} \underline{N}^T_{,X}\, EA \left(\frac{\lambda^2-1}{2}\right) \lambda_{,u}\, dX}_{\text{geometrical part (I)}} + \underbrace{\int_{L^e} \underline{N}^T_{,X}\, EA\, \lambda^2\, \lambda_{,u}\, dX}_{\text{material part (II)}}$$

$\underline{K}_{geo}$ $\qquad\qquad$ $\underline{K}_{mat}$

Q: Why K_{geo}, K_{mat}

For K_{geo}:

$$\underline{K}_{geo} = \int_{L^e} \underline{N}^T_{,X}\, EA\, \frac{\lambda^2-1}{2}\, \lambda_{,u}\, dX$$

Note:

$$\lambda = \frac{\partial x}{\partial X} = 1 + u_{,X} = 1 + \underline{N}_{,X} \cdot \underline{u}^e$$

$$\lambda_{,u} = \frac{\partial \lambda}{\partial \underline{u}^e} = \frac{\partial (1 + \underline{N}_{,X} \cdot \underline{u}^e)}{\partial \underline{u}^e} = 0 + \underline{N}_{,X} \cdot \underbrace{\frac{\partial \underline{u}^e}{\partial \underline{u}^e}}_{= \underline{\underline{I}}}$$

$$\underline{K}_{geo} = \int_{-1}^{+1} \begin{bmatrix} -1/L \\ 1/L \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} EA \left(\frac{l^2 - L^2}{2L^2}\right) \frac{L}{2}\, d\xi \quad \rightarrow dX = J\, d\xi$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \frac{EA}{L} \left(\frac{l^2 - L^2}{2L^2}\right)$$

non linear part.

For $\underline{K}_{mat}$:

$$\underline{K}_{mat} = \int_{L^e} \underline{N}_{,X}^T \, EA \, \underbrace{\lambda^2}_{(\underline{N}_{,X} \cdot \underline{u}^e)^2} \, \lambda_{,u} \, dX$$

$$= \int_{-1}^{+1} \begin{bmatrix} -1/L \\ 1/L \end{bmatrix} \cdot \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} EA \left[1 + \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \right]^2 \frac{L}{2} \, d\xi$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \frac{EA}{L} \underbrace{\left[1 + \frac{u_2 - u_1}{L} \right]^2}_{\text{nonlinear part}}$$