The system shown in Fig. 1 consists of two springs with stiffness k and undeformed length L, under the action of a vertical point force F. The springs are supported at their left and right end, respectively.

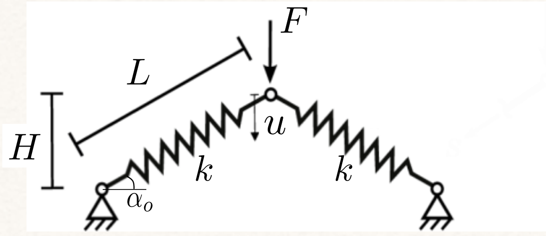


Figure 1: Two spring system(undeformed configuration).

The constitutive equation for the spring is assumed to be linear elastic as follows:

$$N = k\Delta l = EA\epsilon \tag{1}$$

where $k = \frac{EA}{L}$ is the stiffness of the spring, EA is constant, $\Delta l = l - L$ is the elongation of the spring, l is the deformed length, and $\epsilon = \frac{l-L}{L}$.

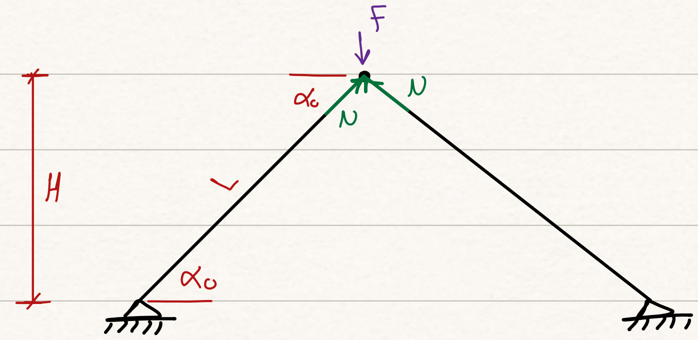
1. Write the equilibrium of the system and find an expression for the force F as a function of u assuming geometrical linear theory.

2. Write the equilibrium of the system and find an expression for the force F as a function of u assuming geometrical nonlinear theory.

3. Plot the load-displacement curves with the results obtained from (1) and (2) as F/k vs u. Assume the values of $L = 10$ and $H = 5$.

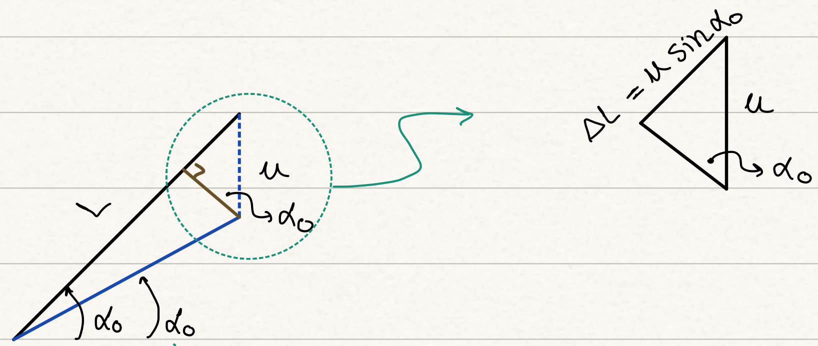
4. How does the system behaves under load control (increasing the load downward)?

Question 01:



Write Equilibrium in initial config:

$\Sigma F_y = 0 \Rightarrow F = 2N \sin\alpha_0$, $N = EA\,\varepsilon$, $\varepsilon = \dfrac{\ell - L}{L}$

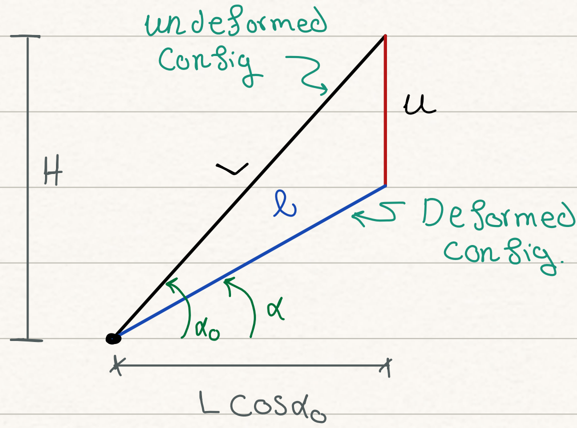


we are assuming Linear theory

$$\therefore\ N = EA\,\frac{\Delta L}{L} = \frac{EA}{L}\,u \sin\alpha_0$$

$$\boxed{\therefore\ F = 2\,\frac{EA}{L}\,u \sin^2\alpha_0} \longrightarrow \#$$

Question 02:



Write equilibrium equation in the current config:

$$F = 2N\sin\alpha = 2EA\varepsilon\sin\alpha$$

$$\varepsilon = \frac{l - L}{L} \quad , \quad \sin\alpha = \frac{H-u}{l}$$

$$l = \sqrt{(L\cos\alpha_0)^2 + (H-u)^2}$$

$$\therefore F = 2EA\,\frac{\sqrt{(L\cos\alpha_0)^2 + (H-u)^2} - L}{L} \cdot \frac{H-u}{\sqrt{(L\cos\alpha_0)^2 + (H-u)^2}}$$

$$\boxed{F = 2EA\left(\frac{H-u}{L}\right)\cdot\left[1 - \frac{L}{\sqrt{(L\cos\alpha_0)^2 + (H-u)^2}}\right]} \rightarrow \#$$

Question 03: