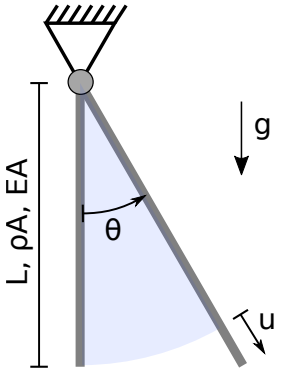
Homework 2: Generalized-Alpha method



$$\Delta t = 0.01\ s$$
$$L = 1\ m$$
$$\rho A = 1\ kg/m$$
$$EA = 500\ N$$
$$g = 9.8\ m/s^2$$

Equations of motion for an elastic pendulum with length L, mass density ρ, cross-sectional area A, elasticity modulus E and acceleration due to gravity g are given by:

$$\frac{\rho A L^3}{3}\ddot{\theta} + \frac{\rho A g L^2}{2} sin(\theta) = 0 \tag{1}$$

$$\frac{\rho A L}{3}\ddot{u} + \frac{EA}{L}u = 0 \tag{2}$$

Assuming small amplitude vibrations, perform the following tasks for the given initial conditions.

$$u_0 = -\frac{L}{5},\ \dot{u}_0 = 0,\ \theta_0 = 0,\ \dot{\theta}_0 = \sqrt{\frac{g}{6L}}$$

Tasks:

1. Write the semi-discrete equation of motion in the matrix form , assuming Rayleigh damping ($\mathbf{C} = \alpha_1\mathbf{M}+\alpha_2\mathbf{K}$).
2. Implement the generalized-alpha scheme in Matlab, where $\alpha_f, \alpha_m, \beta$ and γ are computed using ρ_∞.
3. Solve the system for 5 s and plot the dynamic response of the system for the following set of parameters:
   (a) $\alpha_1 = 1$, $\alpha_2 = 0$ and $\rho_\infty = 1.0$
   (b) $\alpha_1 = 0$, $\alpha_2 = 0$ and $\rho_\infty = 0.1$

   Explain briefly the differences and the sources of the differences obtained in the dynamic response for the two cases.
4. Using the error indicator ($\mathbf{e}$) by Zienkiewicz and Xie, extend your implementation to compute and store the error norm ($||\mathbf{e}||$), cumulative error norm, and the relative error (η) for each time step.
5. Extend your implementation of the generalized-alpha scheme to update the time step (Δt) at the end of each step depending on the relative error (η). Lower and upper bounds of the relative error are defined as $\nu_1\eta_e$ and $\nu_2\eta_e$, where $\nu_1 = 1.0, \nu_2 = 10.0$ and $\eta_e = 1.0 \times 10^{-3}$. The new time step has to be computed as $\Delta t_{new} = \Delta t_{old} \cdot \sqrt{\eta_e/\eta}$.
6. Solve the system for 5 s using the adaptive time stepping algorithm with $\alpha_1 = 1$, $\alpha_2 = 0$, and $\rho_\infty = 1.0$. Plot and compare the dynamic response as well as the evolution of the time step, relative error, and cumulative error norm with the one obtained in task-3a. For both cases, what is the cumulative error norm after 5 s and how many time steps did the algorithm?

Note: The solution has to be submitted in hardcopy by **Monday, 17th June,** in **IC-6/173**. Additionally, the Matlab files must be uploaded in moodle.