LECTURE 1_2
CHAPTER 9: RECURSION

PROBLEM SOLVING AND PROGRAM DESIGN IN C
7TH EDITION

FIGURE 9.1 SPLITTING A PROBLEM INTO SMALLER PROBLEMS

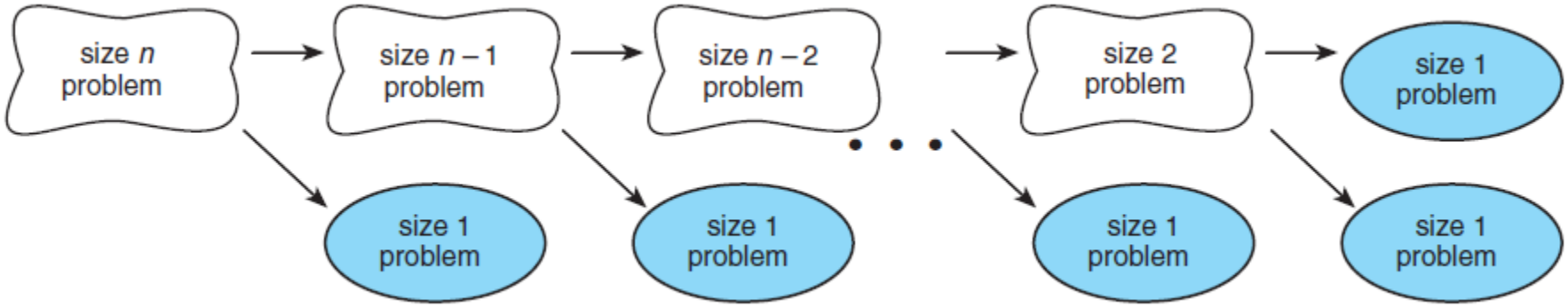


FIGURE 9.2 RECURSIVE FUNCTION MULTIPLY

```
1.  /*
2.   * Performs integer multiplication using + operator.
3.   * Pre:   m and n are defined and n > 0
4.   * Post:  returns m * n
5.   */
6.  int
7.  multiply(int m, int n)
8.  {
9.      int ans;
10.
11.     if (n == 1)
12.           ans = m;     /* simple case */
13.     else
14.           ans = m + multiply(m, n - 1); /* recursive step */
15.
16.     return (ans);
17. }
```

FIGURE 9.5 TRACE OF FUNCTION MULTIPLY

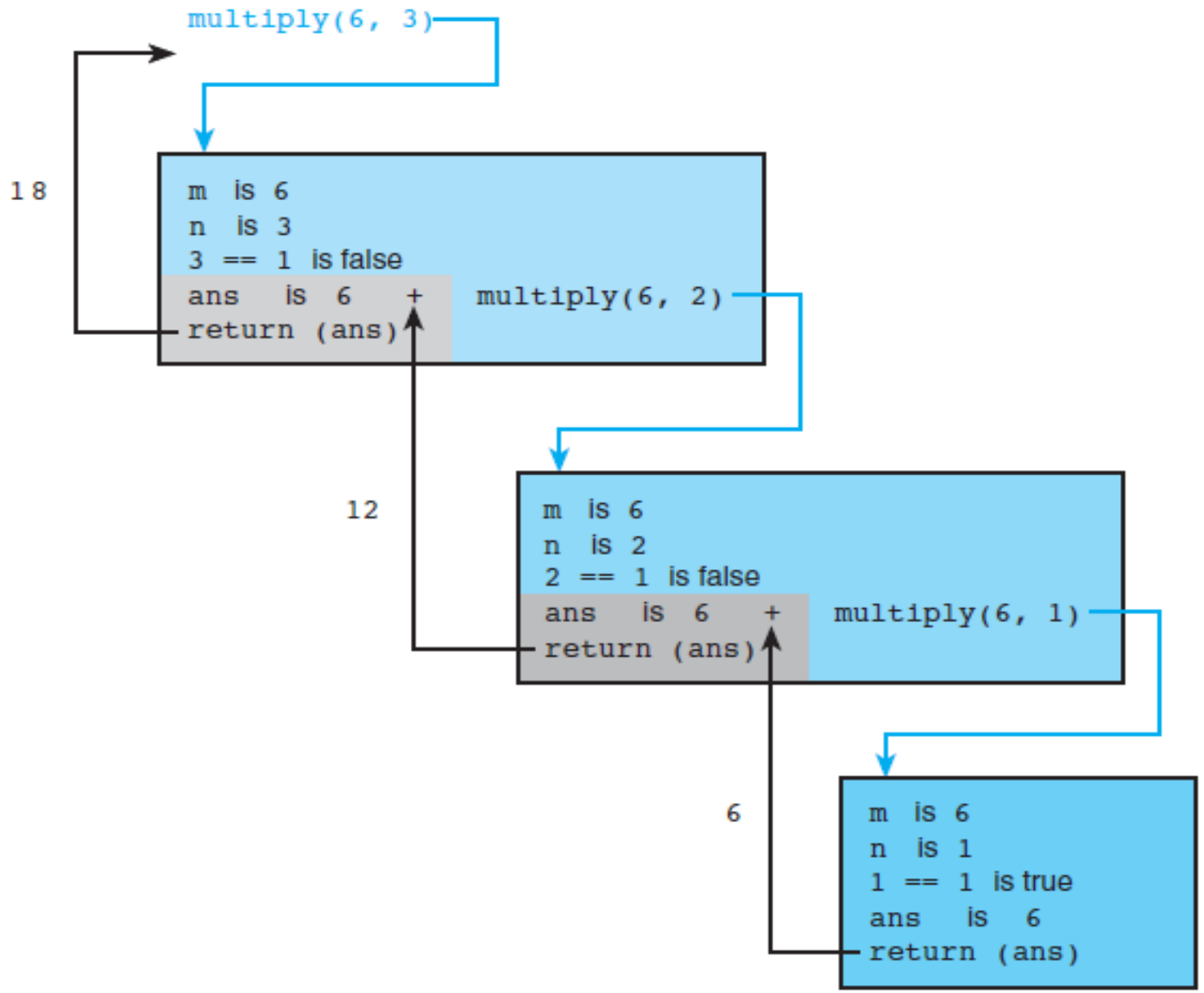


FIGURE 9.9 RECURSIVE FUNCTION MULTIPLY WITH PRINT STATEMENTS TO CREATE TRACE AND OUTPUT FROM MULTIPLY(8, 3)

```
1.  /*
2.   * *** Includes calls to printf to trace execution ***
3.   * Performs integer multiplication using + operator.
4.   * Pre: m and n are defined and n > 0
5.   * Post: returns m * n
6.   */
7.  int
8.  multiply(int m, int n)
9.  {
10.       int ans;
11.
12.   printf("Entering multiply with m = %d, n = %d\n", m, n);
13.
14.       if (n == 1)
15.             ans = m; /* simple case */
16.       else
17.             ans = m + multiply(m, n - 1); /* recursive step */
18.   printf("multiply(%d, %d) returning %d\n", m, n, ans);
19.
20.       return (ans);
21. }
22.
23. Entering multiply with m = 8, n = 3
24. Entering multiply with m = 8, n = 2
25. Entering multiply with m = 8, n = 1
26. multiply(8, 1) returning 8
27. multiply(8, 2) returning 16
28. multiply(8, 3) returning 24
```

FIGURE 9.10 RECURSIVE FACTORIAL FUNCTION

```
1.  /*
2.   * Compute n! using a recursive definition
3.   * Pre: n >= 0
4.   */
5.  int
6.  factorial(int n)
7.  {
8.        int ans;
9.
10.       if (n == 0)
11.             ans = 1;
12.       else
13.             ans = n * factorial(n - 1);
14.
15.       return (ans);
16. }
```

FIGURE 9.11 TRACE OF FACT = FACTORIAL(3);

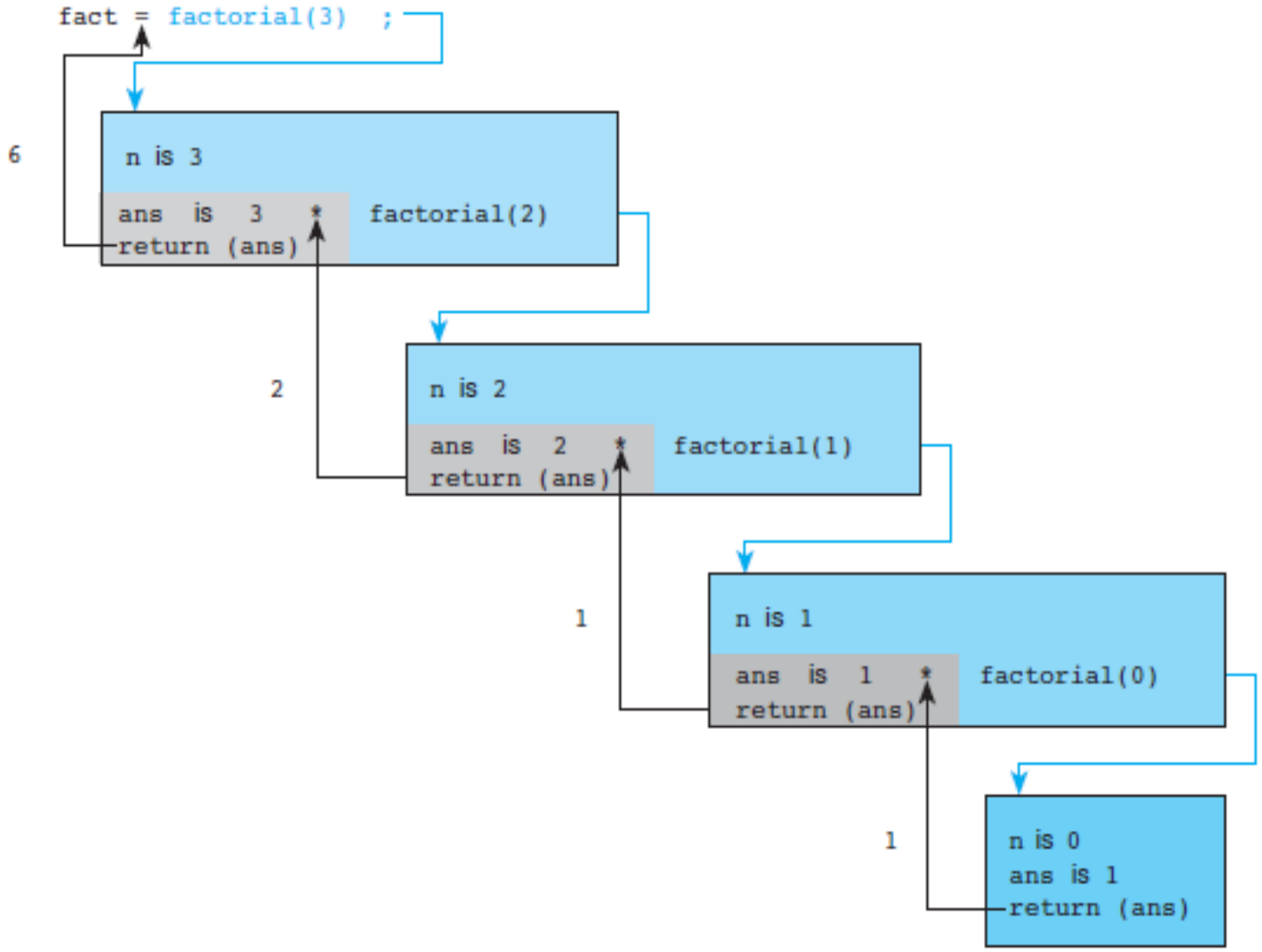


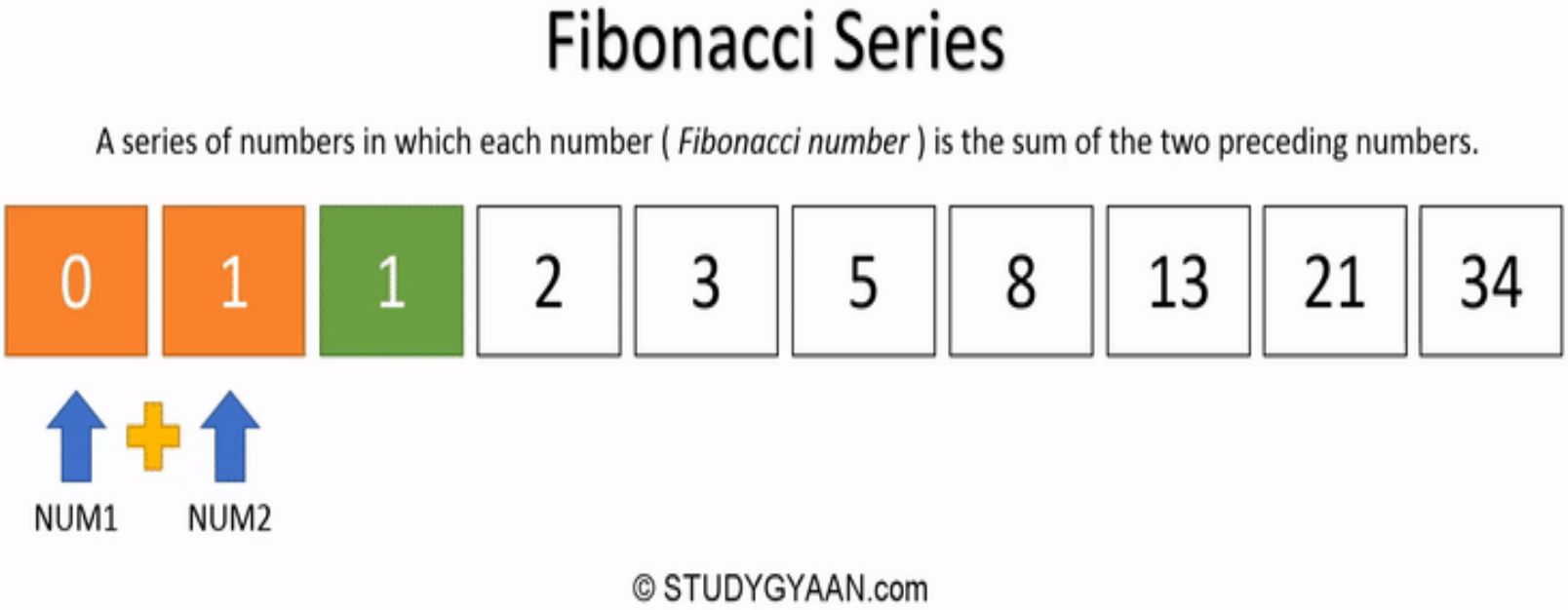
FIGURE 9.12 ITERATIVE FUNCTION FACTORIAL

```
1.  /*
2.   * Computes n!
3.   * Pre: n is greater than or equal to zero
4.   * /
5.  int
6.  factorial(int n)
7.  {
8.      int i,              /* local variables */
9.          product = 1;
10.
11.     /* Compute the product n x (n-1) x (n-2) x . . . x 2 x 1 */
12.     for (i = n; i > 1; --i) {
13.        product = product * i;
14.     }
15.
16.     /* Return function result */
17.     return (product);
18. }
```

FIGURE 9.13 RECURSIVE FUNCTION FIBONACCI

```
1.  /*
2.   * Computes the nth Fibonacci number
3.   * Pre: n > 0
4.   */
5.  int
6.  fibonacci(int n)
7.  {
8.        int ans;
9.
10.       if (n == 1 || n == 2)
11.             ans = 1;
12.       else
13.             ans = fibonacci(n - 2) + fibonacci(n - 1);
14.
15.       return (ans);
16. }
```

RECURSIVE FUNCTION FIBONACCI



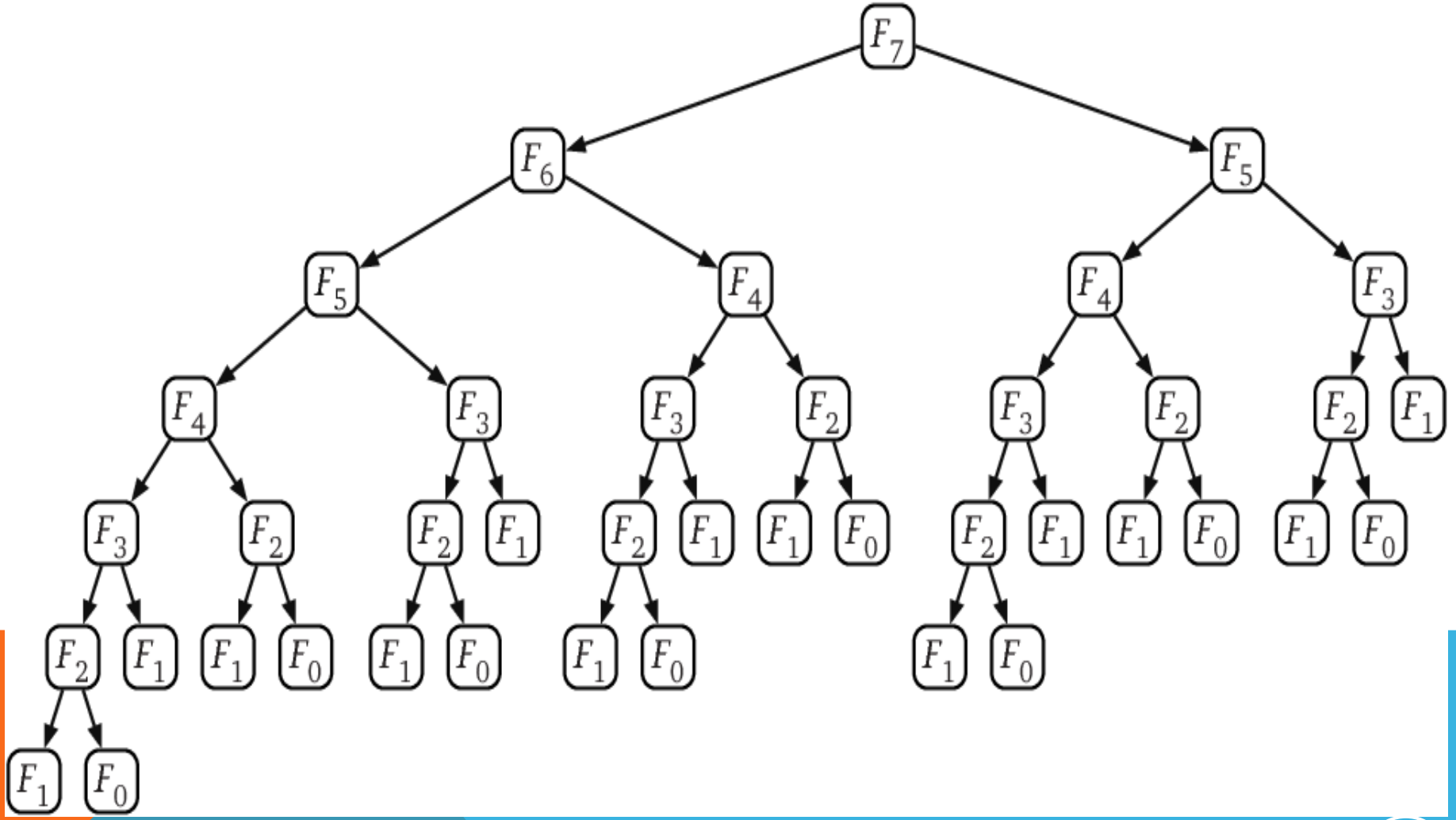
FIBONACCI FOR DYNAMIC PROGRAMMING

Running time?

```
FIBONACCI-DP(n)
1  fib[1] ← 1
2  fib[2] ← 1
3  for i ← 3 to n
4          fib[i] ← fib[i − 1] + fib[i − 2]
5  return fib[n]
```

$\Theta(n)$

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```


F_7
F_6 F_5
F_5 F_4 F_4 F_3
F_4 F_3 F_3 F_2 F_3 F_2 F_2 F_1
F_3 F_2 F_2 F_1 F_2 F_1 F_1 F_0 F_2 F_1 F_1 F_0 F_2 F_1 F_1 F_0
F_2 F_1 F_1 F_0 F_1 F_0 F_1 F_0 F_1 F_0 F_1 F_0 F_1 F_0
F_1 F_0