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Discrete Sheet 4

Q1)

1) $a_1 = 1/9$, $a_2 = 1/4$, $a_3 = 3/7$, $a_4 = 2/3$
2) $c_0 = 1$, $c_1 = -1/3$, $c_2 = 1/9$, $c_3 = -1/27$

Q2)

1) $A_i = i * (-1)^{i+1}$, $i >= 0$
2) $A_i = i / (i + 2)$, $i > 0$
3) $A_i = i / (i+1)^2$, $i > 0$

Q3)

1) $a_0 = 2$
2) $a_2 + a_4 + a_6 = -4$
3) $a_0 * a_1 * a_2 * a_3 * a_4 * a_5 * a_6 = 0$
4) $a_2 = -2$

Q4)

1) $\sum_{i=1}^{7} i^2 \cdot (-1)^{i+1}$
2) $\sum_{i=1}^{n} \frac{n-i+1}{i!}$

Q5)

a) **Base case**: (prove: 2! + 2 is divisible by 2)
   let n = 2
   2! + 2 = 4
   since 4 is a multiple of 2
   therefore the base case is proven

   Induction Hypothesis:
   assume that k! + 2 is divisible by 2 for k >= 2

Inductive Step: (prove: (k+1)! + 2 is divisible by 2)

from the hypothesis: (k! + 2 = 2m) where m is an arbitrary integer number

(k+1) * (k! + 2) = (k+1) * 2m

(k+1)(k!) + 2*(k+1) = 2mk + 2m

(k+1)! + 2k + 2 = 2mk + 2m

(k+1)! + 2 = 2mk * 2m – 2k

(k+1)! + 2 = 2(mk * m – k)

since L.H.S is a multiple of 2 then the inductive step is proven

b) n! = ($\prod_{i=1}^{n} i$) , n >= 2 and 1 < k <= n

from definition of factorial and k, k is always a factor of n!

therefore $\frac{(\prod_{i=1}^{n} i)}{k}$ is an integer

therefore n! + k = $k\left(\frac{n!}{T-1}+1\right)$ which is a multiple of k therefore it is divisible by k

Q6)

2 + 4 + 6 + …… + 2n = $2\sum_{i=1}^{n} i = 2\frac{(n)(n+1)}{2}$ = (n)(n+1) = n^2 + n

Q7)

Prove: $\sum_{i=1}^{n}(4i-3) = n(2n-1)$

Base Case:

n = 1

L.H.S: 4 * 1 – 3 = 1 , R.H.S: 1*(2*1 – 1) = 1

therefore base case holds

Induction Hypothesis:

assume: $\sum_{i=1}^{k}(4i-3) = k(2k-1)$

Inductive Step:

prove: $\sum_{i=1}^{k+1}(4i-3) = (k+1)(2(k+1)-1)$

L.H.S: $\sum_{i=1}^{k+1}(4i-3) = \left(\sum_{i=1}^{k}(4i-3)\right) + 4(k+1) - 3$

R.H.S:

$(k+1)(2(k+1)-1)$

$= 2(k+1)^2 – (k+1)$

$= 2(k^2 + 2k + 1) – (k+1)$

$= 2k^2 + 4k + 2 – k – 1$

$= (2k^2 – k) + 4k +1$

$= k(2k - 1) + 4k + 4 – 3$

$= k(2k – 1) + 4(k + 1) – 3$

from the inductive hypothesis L.H.S cancels R.H.S

therefore they are equal

Q8)

Prove: $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

Base Case:

let n = 0

L.H.S: $2^0 = 1$, R.H.S: $2^{0+1} – 1 = 1$

therefore base case holds

Induction Hypothesis:

assume: $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Inductive Step:

prove: $\sum_{i=0}^{k+1} 2^i = 2^{k+2} - 1$

L.H.S: $\sum_{i=0}^{k+1} 2^i = \sum_{i=0}^{k} 2^i + 2^{k+1}$

R.H.S: $2^{k+2} - 1$

$= 2(2^{k+1}) - 1$

$= 2^{k+1} + 2^{k+1} - 1$

$= (2^{k+1} - 1) + 2^{k+1}$

from the induction hypothesis, L.H.S = R.H.S

Q9)

1) **prove:** $\boldsymbol{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$

Base Case:

let n = 1

L.H.S: $1^2 = 1$, R.H.S: (1)(2)(3)/6 = 1

therefore base case holds

Induction Hypothesis:

assume: $\boldsymbol{\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}}$

Inductive Step:

prove: $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$

L.H.S:

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$$