Topic #3
The Transfer Function

Group 1
Abdelrahman Elsissy (03780605)
Richard Piekara (03689873)
Victor Cano (03781052)

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Abstract

This report covers the calculation with different approaches of the Transfer Function of a given circuit (RC first order High-Pass Filter). The first implemented approach was the theoretical one. Once the theoretical Transfer Function was obtained, the next step was the demonstration of it with several practical approaches. For the last part of the exercise a comparison between these practical methods was done. For this exercise, the calculations for the theoretical approach were done by Victor Cano. The building and simulation of the circuit was done by Richard Piekara and Abdelrahman Elsissy and the coding was developed as a teamwork.

Contents

1 Building the Filter

1.1 Assignment 1

The given circuit was a First Order Passive High-Pass Filter built with a resistor R of $2\,\mathrm{k\Omega}$ and a capacitor C of $100\,\mathrm{nF}$.

a) The theoretical calculation of the Transfer Function of this filter is shown in figure 1.

b) Using the following equation as a template of a Transfer Function H(s):

$$H(s) = \frac{K}{1 + \frac{s}{\omega_0}} \ . \tag{1}$$

Where K is the circuit's Gain and ω_0. Substituting this constants with the values of our circuit we get the following equivalences.

$$K = R * C \tag{2}$$

$$\omega_0 = \frac{1}{R * C} \tag{3}$$

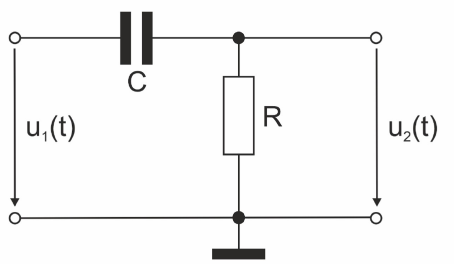
Which after substituting the values of R and C and transferring them to the time domain give the following results.

$$K = 1.25 \times 10^{-3} \tag{4}$$

$$\omega_0 = 795.77\,\mathrm{Hz} \tag{5}$$

c) Using MATLAB the Bode plot for this Filter was created (figure 2)

d) Figure 3 shows an schematic of the built filter. The Signal Generator represents the output of the computer going to the circuit and the Oscilloscopes represent the measurements of the signal, in green the input to the circuit (U_1) and in blue the output from the circuit (U_2).



$R = 2k\Omega \qquad C = 100nF$

Solving by Kirchhoff's law:

$$U_1(t) = V_C(t) + V_R(t) \qquad U_2(t) = V_R(t)$$

$$U_1(t) = \frac{1}{C}\int i(t)dt + Ri(t)$$

Applying Laplace transform:

$$U_1(s) = \frac{1}{sC} i(s) + Ri(s) \qquad U_2(s) = Ri(s) \quad => \quad i(s) = \frac{U_2(s)}{R}$$

$$U_1(s) = \frac{1}{sC}\left(\frac{U_2(s)}{R}\right) + R\left(\frac{U_2(s)}{R}\right)$$

$$U_1(s) = U_2(s)\left(1 + \frac{1}{RsC}\right)$$

Obtained Transfer Function:

$$\boldsymbol{H(s)} = \frac{\boldsymbol{U_2(s)}}{\boldsymbol{U_1(s)}} = \frac{\boldsymbol{RsC}}{\boldsymbol{1 + RsC}}$$

Figure 1: **Theoretical calculation of the Transfer Function of the circuit.**

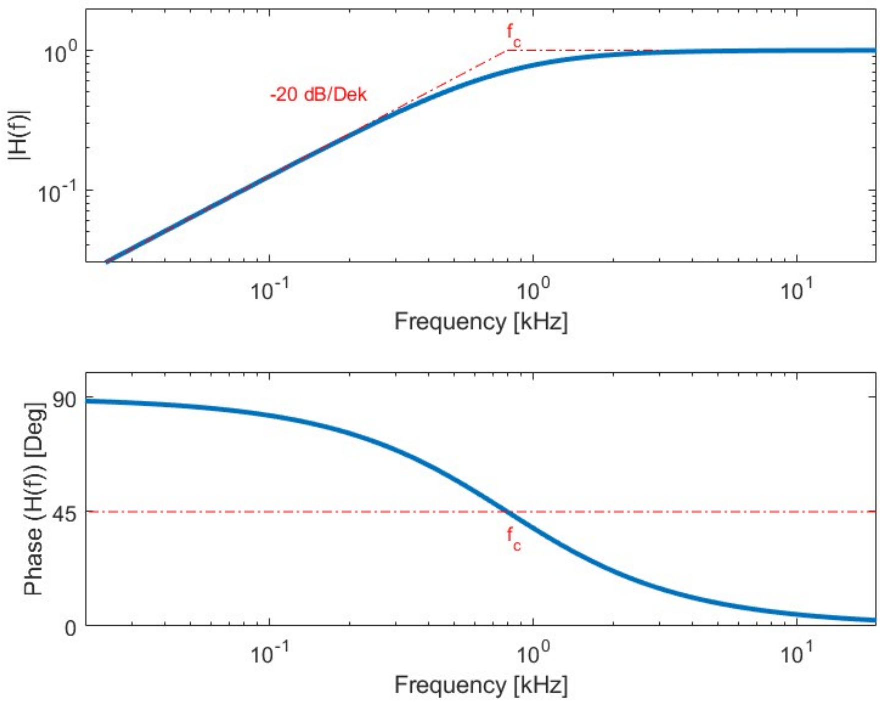


Figure 2: **Bode plot of Magnitude (above) and Phase (below) of the High-Pass Filter.**

2 How to retrieve the Transfer Function from the Impulse Response

As a first practical approach to calculate the Transfer Function, an Impulse Response (Delta Peak) was used.

2.1 Assignment 2

a) For this assignment an Impulse response of 4 s with its peak of $1 a.u.$ at 2 s was created. The input and output to the circuit are shown in figure 4.

b) After processing the signal, the Bode plot in figure 5 was calculated.

c) During the calculation of the Transfer Function by means of an Impulse response the team faced two major issues. The first one was the appearance of some noise at the output of the circuit, this because of a loose cable in the breadboard, which, before solving the wiring problem, was causing some distortions on the signal and the Transfer Function. The second one was the appearance of an Impulse Response with a negative amplitude. However, this effect was inevitable due to the fact that this Impulse comes from the hardware architecture and its physical limitations.

3 How to retrieve the Transfer Function from White Noise

For the second practical approach to calculate the Transfer Function, the chosen signal was a White Noise signal.

3.1 Assignment 3

a) The chosen section of the signal for this assignment was the central part of it. To get only this section, the start and the end of the signal, which was affected by the ramp filter, was removed.

b) The implemented signal for this assignment was a white noise signal. The input and output to the circuit are shown in figure 6.

c) The Bode obtained from the processing of the white noise signal is shown in figure 7

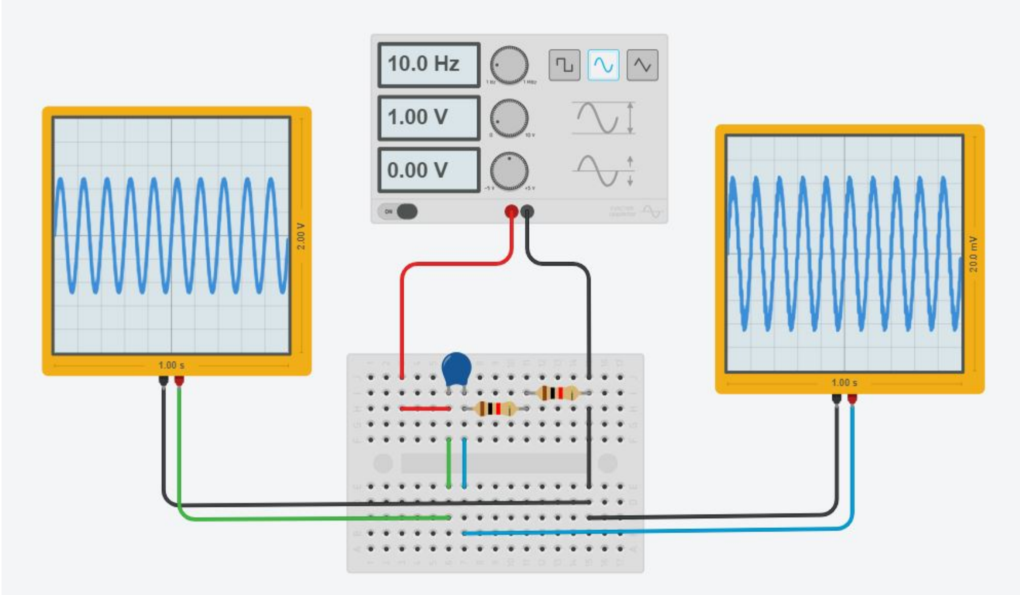


Figure 3: **Schematics of the circuit built in the Breadboard.**

4 How to retrieve the Transfer Function from a Linear Sweep

4.1 Assignment 4

In this assignment a Linear Sweep signal with a range from 100 Hz to 24 kHz was created.

a) The signal used to determine the Transfer Function before and after the filter are shown in figure 8.

b) The magnitude spectrum of both signals is shown in figure 9.

c) The Bode obtained from the processing of the linear sweep signal is shown in figure 10

5 How to retrieve the Transfer Function from a Log Sweep

5.1 Assignment 5

Similarly to the previous one, in this assignment a Log Sweep signal with a range from 100 Hz to 24 kHz was created.

a) The signal used to determine the Transfer Function before and after the filter are shown in figure 11.

b) The magnitude spectrum of both signals is shown in figure 12.

c) The Bode obtained from the processing of the linear sweep signal is shown in figure 13

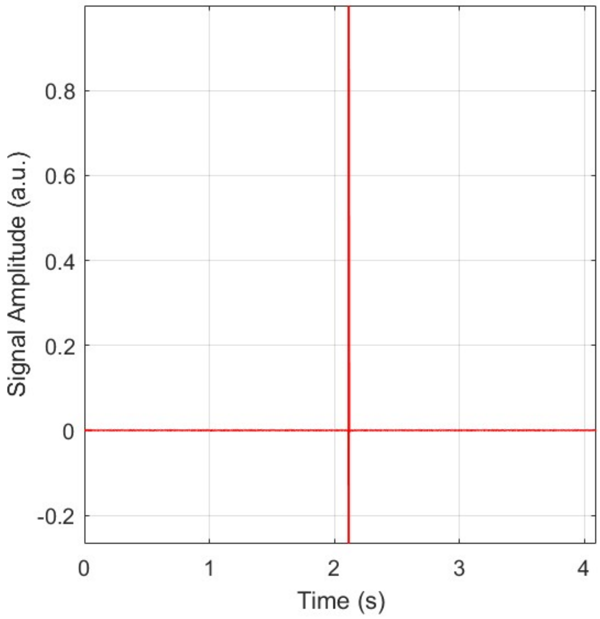
6 How to retrieve the Transfer Function from a Multisine

6.1 Assignment 6

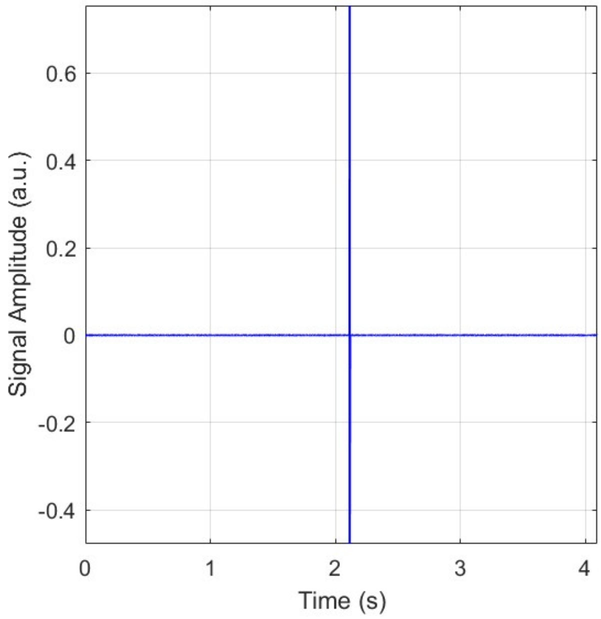
For this assignment a group of Cosine signals were generated. The frquencies used for were 10 Hz, 300 Hz, 100 Hz, 300 Hz, 1 kHz, 3 kHz, 10 kHz, 20 kHz.

a) and b) The signals used to determine the Transfer Functions of each one of them were split into two different figures for a better presentation of the signals. Figure 14 displays the inputs to the circuit, while figure 15 displays the outputs from the circuit. In both figures the signal is ordered such that the frequencies are increasing from top to bottom.

c) The Bode Plots obtained from each signal are displayed as follows: 10 Hz(figure 16), 30 Hz(figure 17), 100 Hz(figure 18), 300 Hz(figure 19), 1 kHz(figure 20), 3 kHz(figure 21), 10 kHz(figure 22),20 kHz(figure 23).



(a)



(b)

Figure 4: **Input and output of the circuit. (a)** Input to the circuit U_1. **(b)** Output from the circuit U_2.

6.2 Assignment 7

For this assignment, the MATLAB function provided by Professor Werner Hemmert for obtaining a Transfer Function from several sinusoidal signals was adapted to obtain the Bode Plot of our circuit.

b) The obtained Bode plot for our filter is displayed in figure 24.

7 Advantages and Disadvantages

7.1 Assignment 8

In Resume, obtaining the Transfer Function of a circuit is a task which can be achieved by many different methods, each one of them with their respective advantages and disadvantages. A summary of the techniques implemented during this exercise, as well as some Pros and Cons are listed below.

Theoretical Calculation
- Advantage: Gives accurate values of the Gain and Cut-off Frequency.
- Disadvantage: Mathematically complex to obtain (Lapace transform is required to simplify calculations).

TF from an Impuse Response
- Advantage: Simple signal do design.
- Disadvantage: If not well chosen can contain also many low frequency components. Prone to noise.

TF from White Noise
- Advantage: Good approach to the ideal Bode Plot with some additional components.
- Disadvantage: If the signal is not well conditioned (Windowing) the results may vary.

TF from a Linear Sweep
- Advantage: Good approach to the ideal Bode Plot without the components seen in the White Noise.
- Disadvantage: If the Amplitude of the signal is too high, the high frequency components are not well generated, which produces inconsistencies in the Transfer Function.

TF from a Log Sweep
- Advantage: Similarly as the Linear Sweep, good approach to the ideal Bode Plot without the components seen in the White Noise. The effect of the filter is drastically observed in both time and frequency domain.
- Disadvantage: As in the Linear Sweep, the Amplitude of the signal affects the generated signal resulting in an inaccurate Transfer Function.

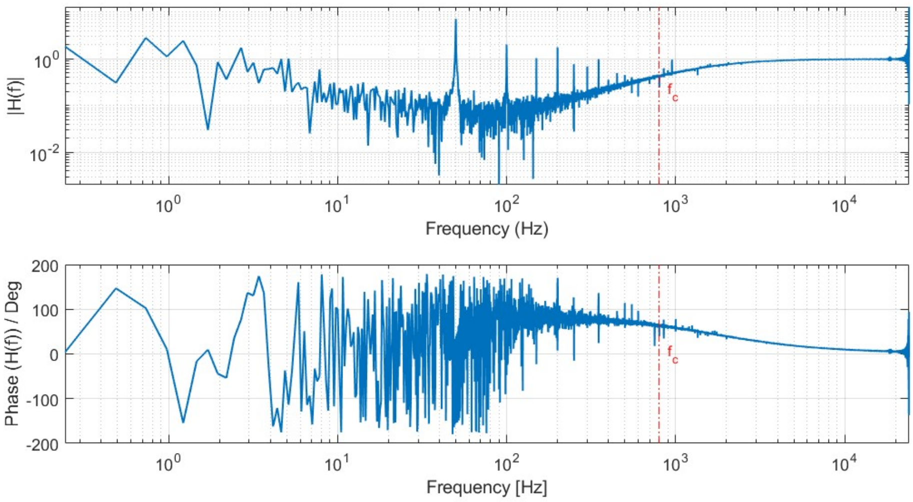


Figure 5: **Bode plot of the Transfer Function for an Impulse Response.**

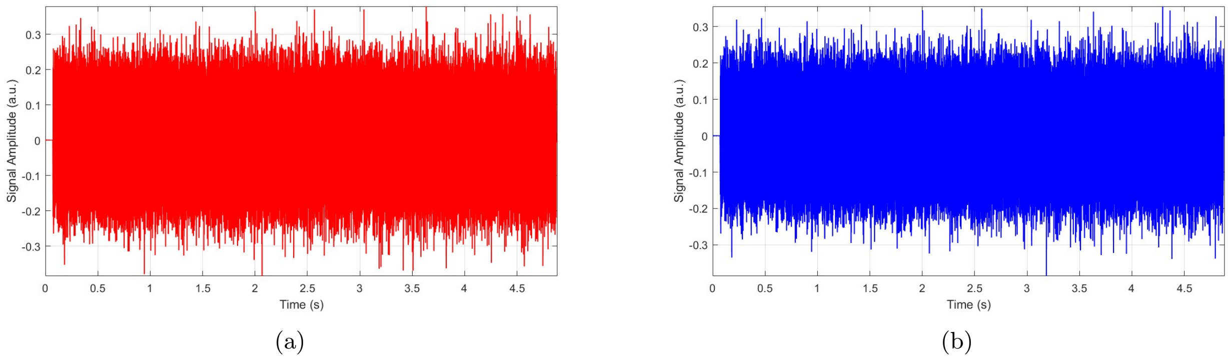


Figure 6: **Input and output of the circuit.** **(a)** Input to the circuit U_1. **(b)** Output from the circuit U_2.

TF from a Series of Cosine Signals
- Advantage: The effect of the filter is clearly visible in both Amplitude in time domain and in the Bode Plot.
- Disadvantage: The interpretation of a Bode plot for a single cosine signal is not enough to determine accurately the Transfer Function of the system.

TF from a Multisine
- Advantage: The calculation of the Transfer Function is very precise.
- Disadvantage: Computationally more complex than the other methods.

For all of the Methods implemented in this Exercise, the **cut-off frequency** was not the exact same as the theoretical one. Nevertheless, its value was very close to the one obtained with the mathematical approach.

Based on what we have learned so far, the reason for the deviation on the real cut-off frequency value from the calculated one could be produced by two factors. The first one, the intrinsic noise produced by the electronic components of the circuit and the whole processing hardware(UAI and PC). And second, because of manner in which MATLAB does the processing of the signals.

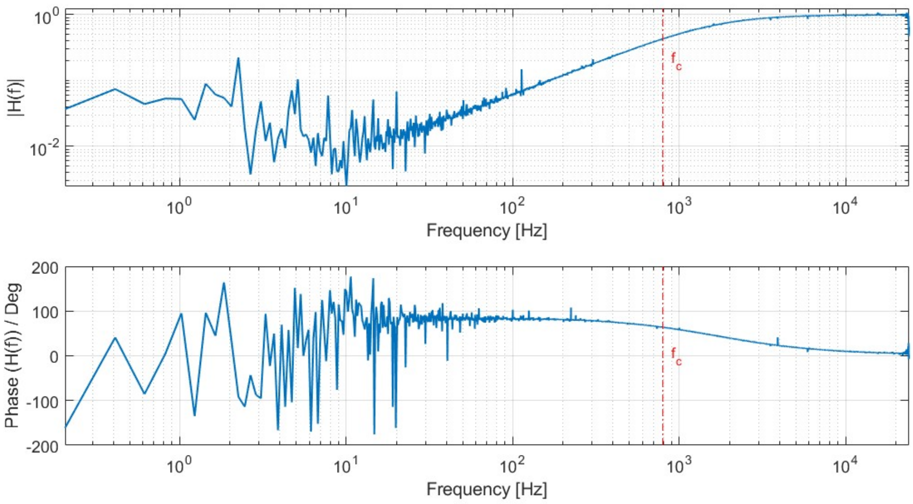
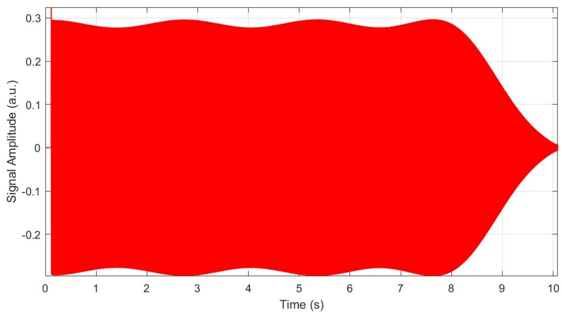
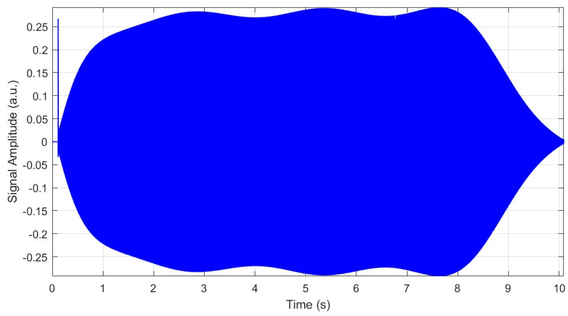


Figure 7: **Bode plot of the Transfer Function for a White Noise signal.**

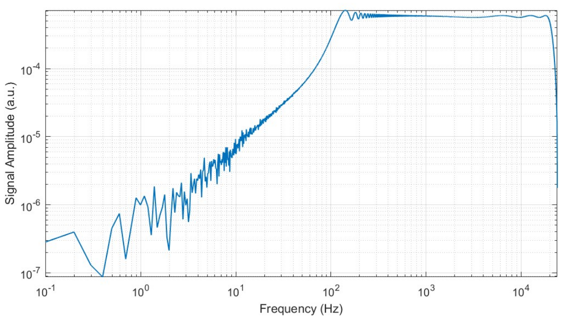


(a)

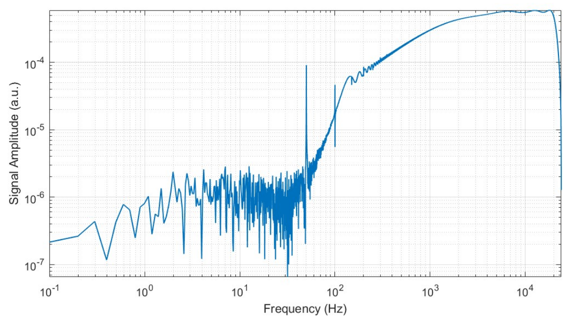


(b)

Figure 8: **Input and output of the circuit. (a)** Input to the circuit U_1. **(b)** Output from the circuit U_2.



(a)



(b)

Figure 9: **Magnitude spectrum of the input and output of the circuit. (a)** Magnitude spectrum for U_1. **(b)** Magnitude spectrum for U_2.

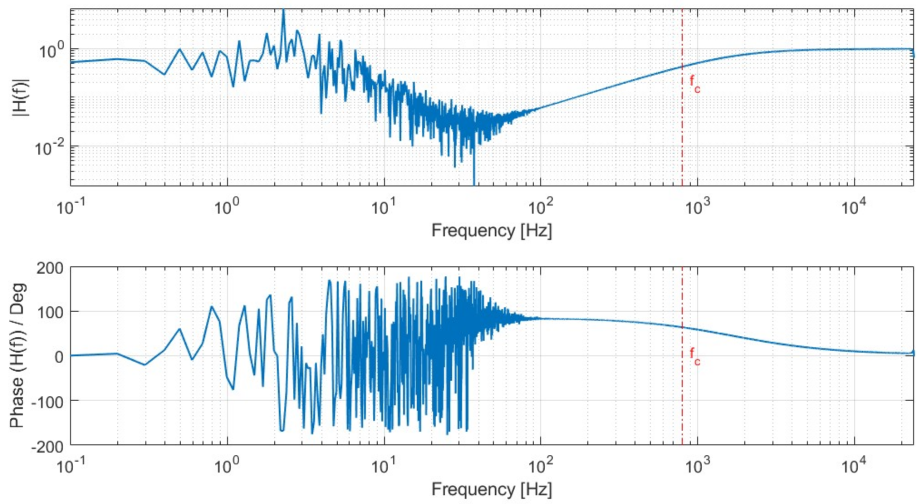
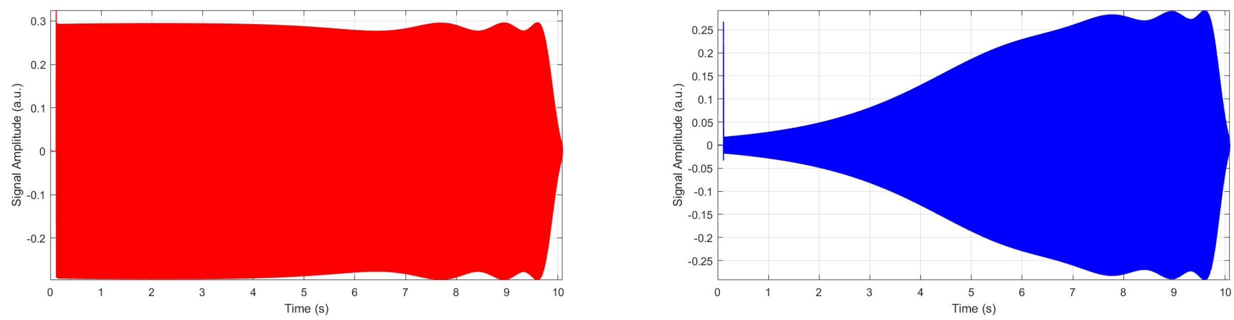
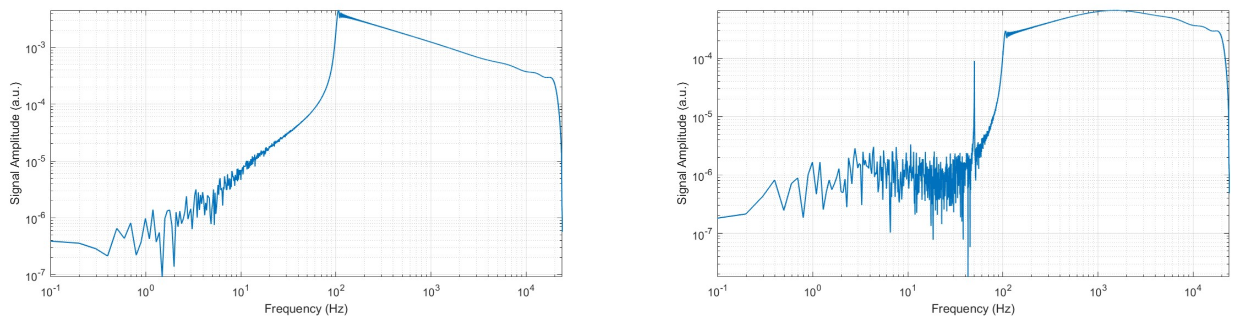


Figure 10: **Bode plot of the Transfer Function for a Linear Sweep signal.**



(a) (b)

Figure 11: **Input and output of the circuit. (a)** Input to the circuit U_1. **(b)** Output from the circuit U_2.



(a) (b)

Figure 12: **Magnitude spectrum of the input and output of the circuit. (a)** Magnitude spectrum for U_1. **(b)** Magnitude spectrum for U_2.

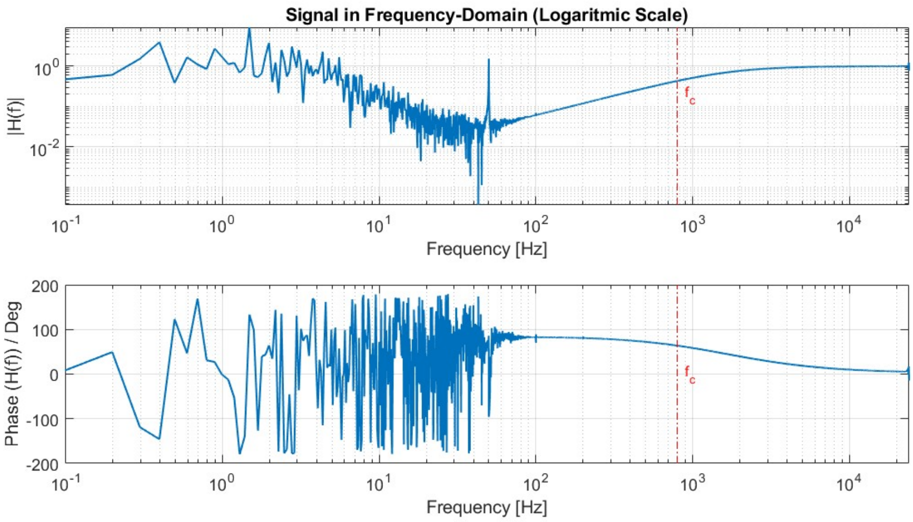


Figure 13: **Bode plot of the Transfer Function for a Log Sweep signal.**

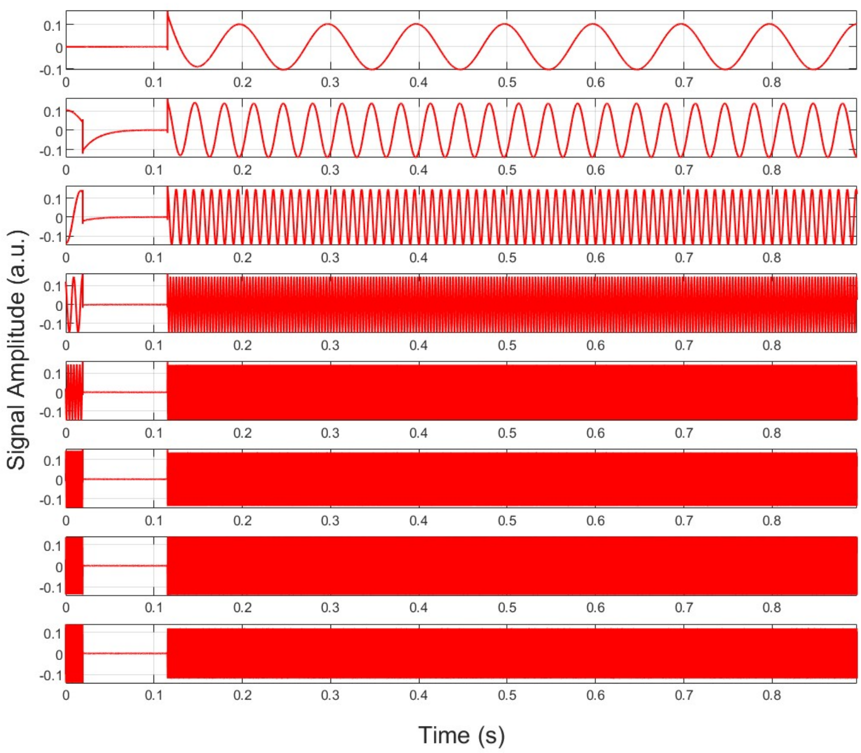


Figure 14: **Input signals in increasing frequency order (from top to bottom) (U_1).**

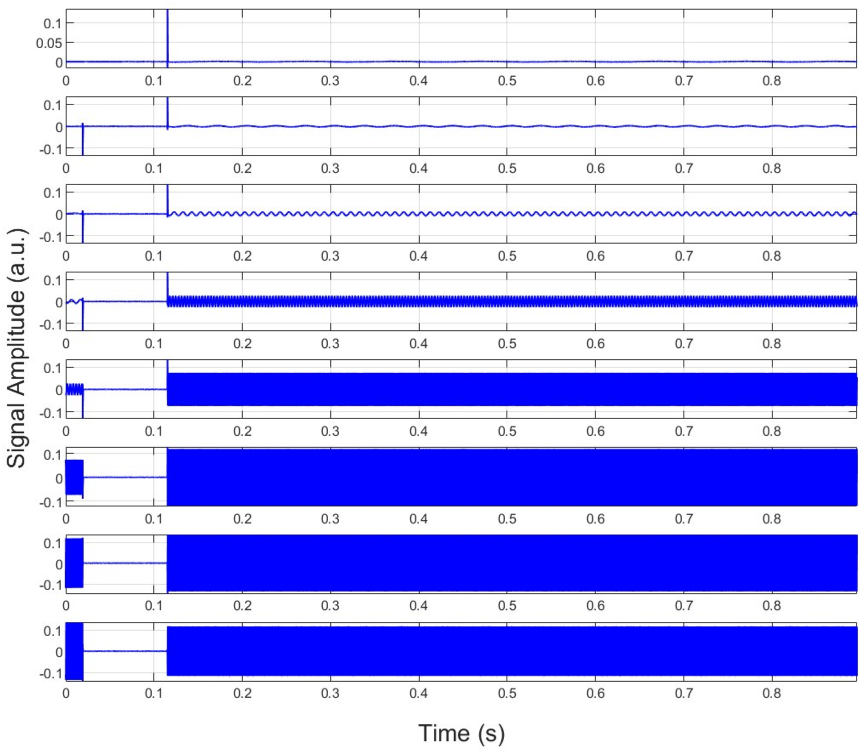


Figure 15: **Output signals in increasing frequency order (from top to bottom) (U_2).**

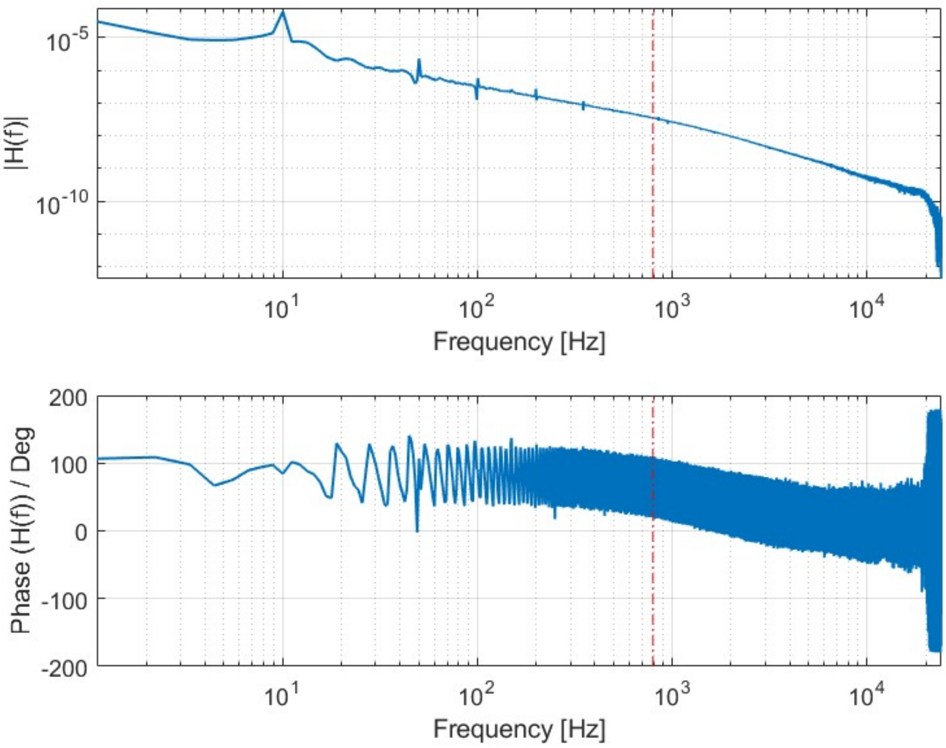


Figure 16: **Bode plot of the Transfer Function for a Cosine signal of 10 Hz.**

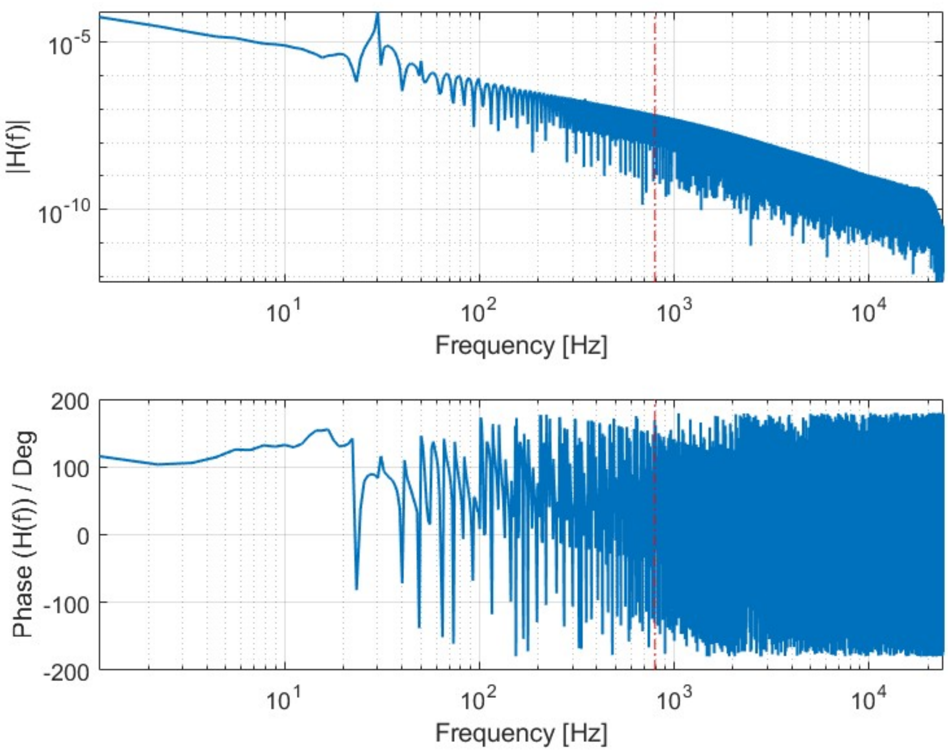


Figure 17: **Bode plot of the Transfer Function for a Cosine signal of** 30 Hz**.**

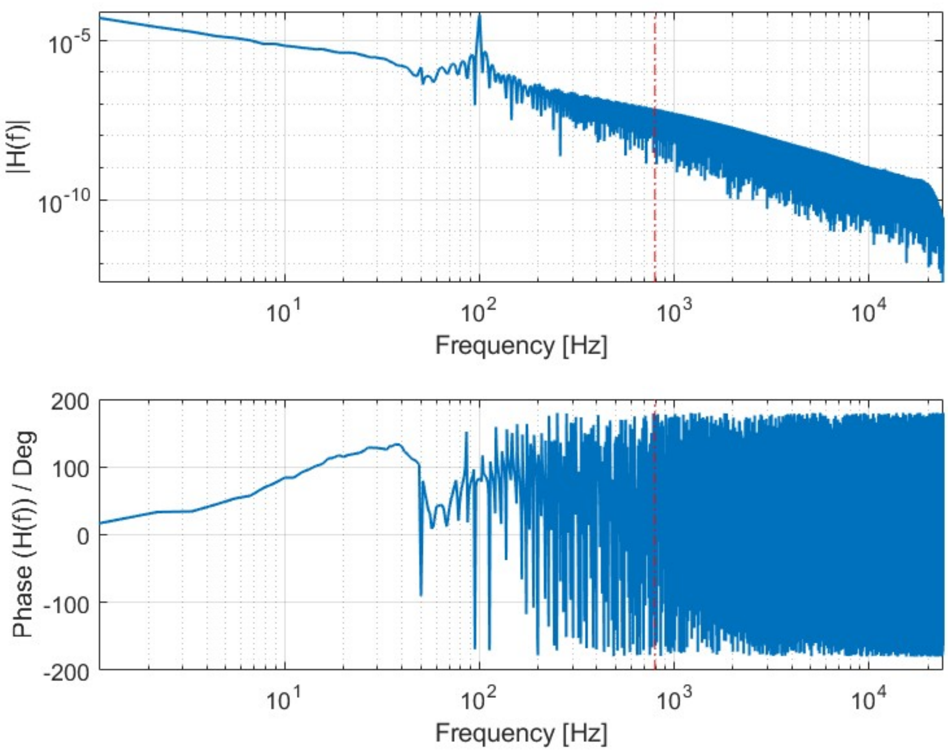


Figure 18: **Bode plot of the Transfer Function for a Cosine signal of** 100 Hz**.**

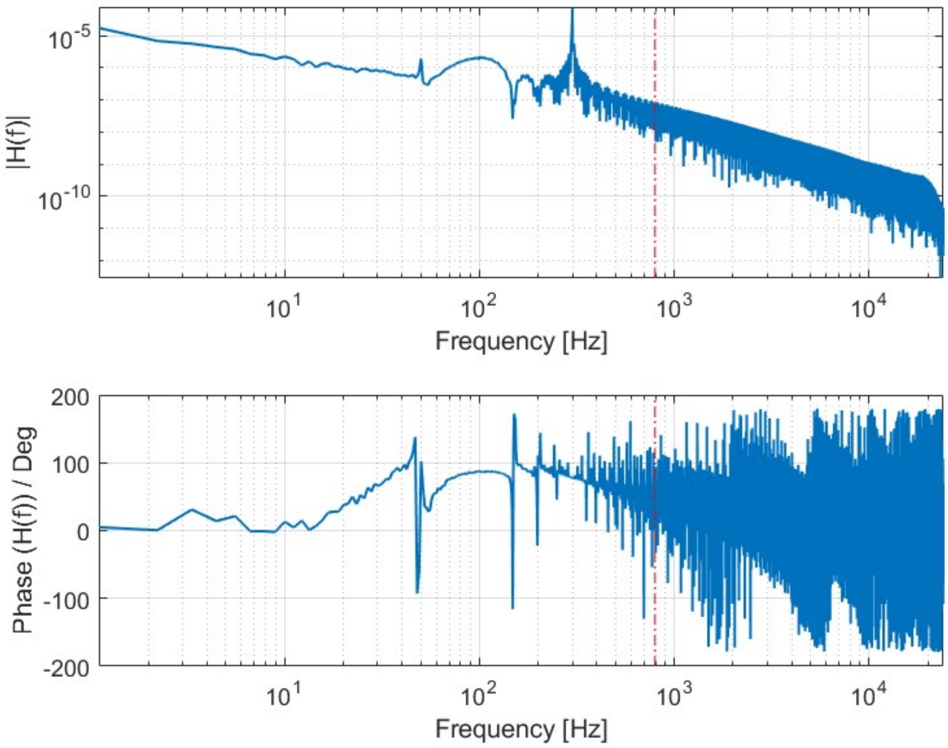


Figure 19: **Bode plot of the Transfer Function for a Cosine signal of** 300 Hz**.**

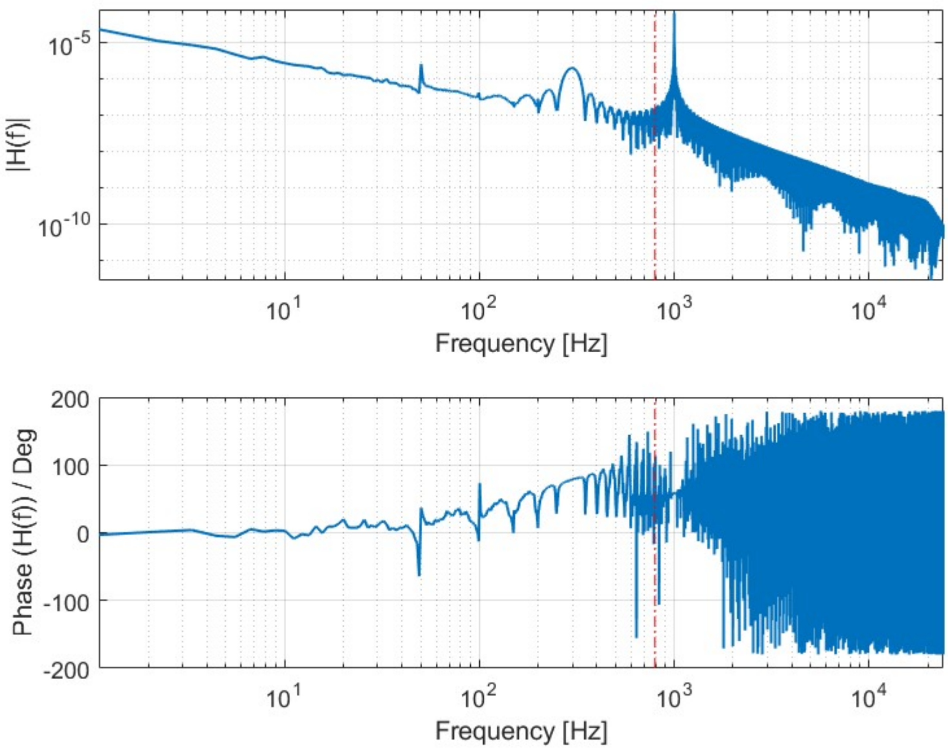


Figure 20: **Bode plot of the Transfer Function for a Cosine signal of** 1 kHz**.**

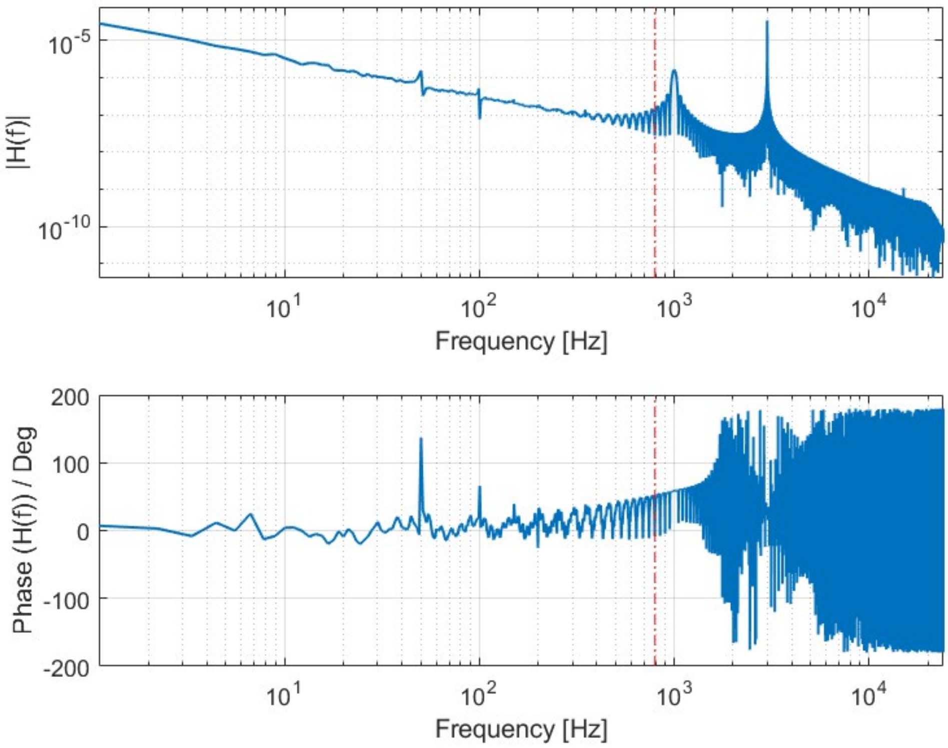


Figure 21: **Bode plot of the Transfer Function for a Cosine signal of** 3 kHz**.**

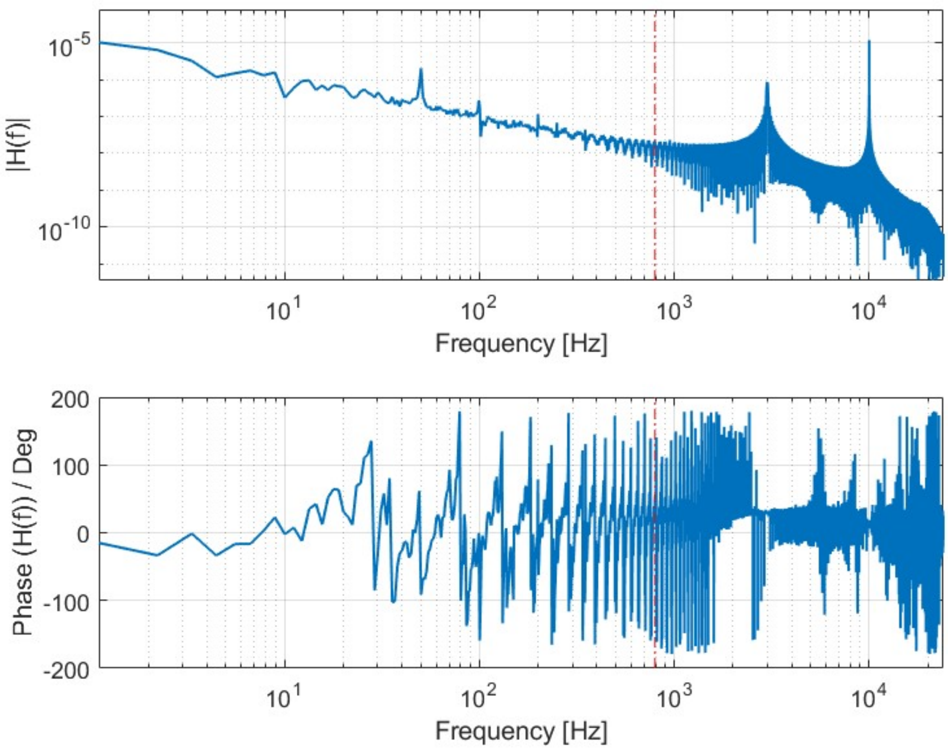


Figure 22: **Bode plot of the Transfer Function for a Cosine signal of** 10 kHz**.**

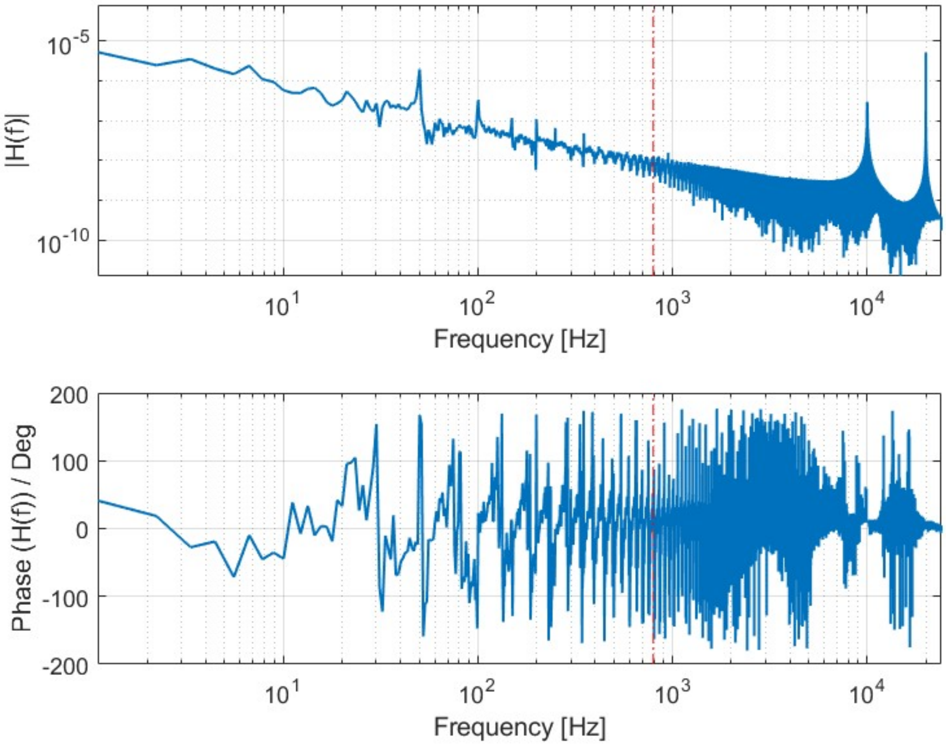


Figure 23: **Bode plot of the Transfer Function for a Cosine signal of** 20 kHz**.**

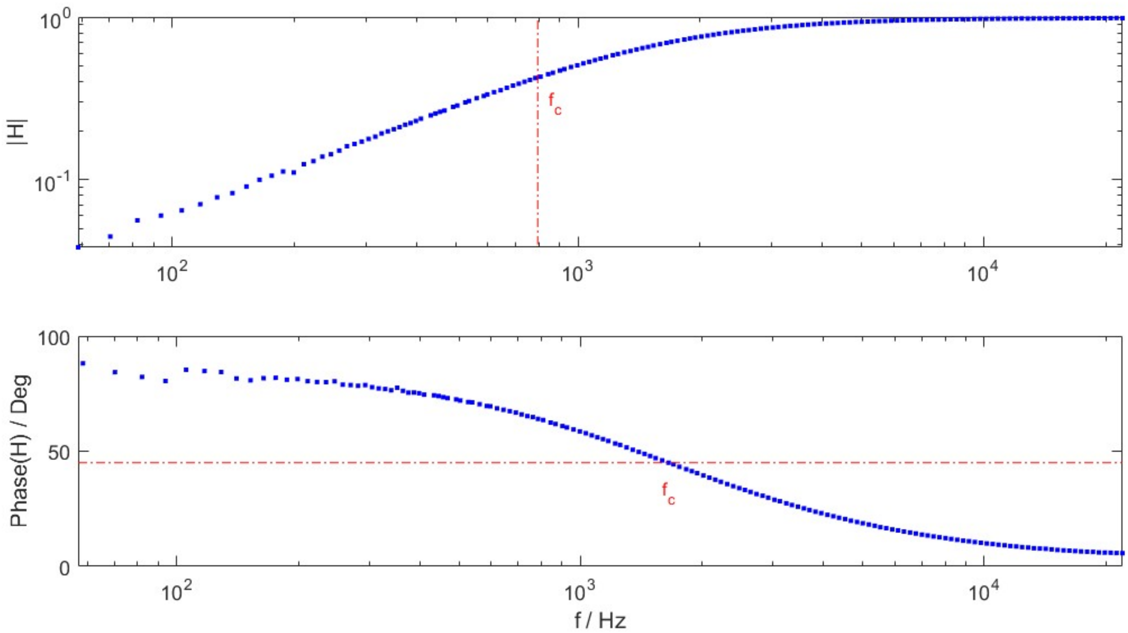


Figure 24: **Bode plot of the Transfer Function obtained from several sinusoidal signals.**