Topic #4

TF and Noise

Group 1

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March 12, 2024

Abstract

This report focuses on the effects of noise on the signal. Noise is the undesired or unpredictable signal in a measurement system. Noise can be present due to the effects of external or internal sources, such as the electrical components of a circuit. It is easier to eliminate the noise effects of external components. In this exercise, we will see the effect of a second-order low-pass filter on signals with noise components. We will also demonstrate the effects of averaging the signal on noise. As the signal duration doubles, the noise doubles as well, which means that the noise in the signal can be reduced by dividing the signal into several blocks and averaging the signal blocks.

Contents

1 Building the filter

1.1 Assignment 1

a) The theoretical calculation of the transfer function is shown in figure 1.

b) Using the following equation as a template of a Transfer Function H(s):

$$H(s) = \frac{K}{1 + \frac{s}{\omega_0}} \,. \tag{1}$$

Where K is the circuit's Gain and ω_0. Substituting this constant with the values of our circuit, we get the following equivalences.

$$K = R * C \tag{2}$$

$$\omega_0 = \frac{1}{R * C} \tag{3}$$

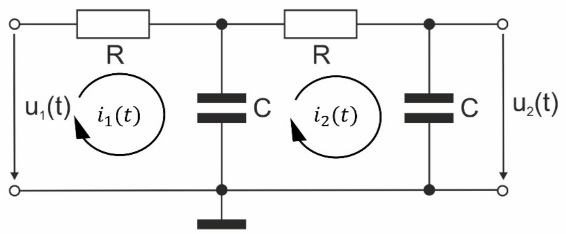
After substituting the values of R and C and transferring them to the time domain give the following results.

$$K = 1 \tag{4}$$

$$\omega_0 = 1591.54\,\text{Hz} \tag{5}$$

c) Using MATLAB, the Bode plot for this Filter was created (figure 2)

d) Figure 3 shows a schematic of the built filter. The Signal Generator represents the output of the computer going to the circuit and the Oscilloscopes represent the measurements of the signal, in green the input to the circuit (U_1) and in blue the output from the circuit (U_2).



$R = 1k\Omega$ $\qquad C = 100nF$

Solving by Kirchhoff's law:

For loop 1:

$U_1(t) = V_R(t) + V_C(t)$

$U_1(t) = Ri_1(t) + Z_C(i_1(t) - i_2(t))$

For loop 2:

$V_C(t) + V_R(t) + V_C(t) = 0 \qquad U_2(t)$

$Z_C i_2(t) + Ri_2(t) + Z_C(i_2(t) - i_1(t)) = 0$

Applying Laplace transform:

For loop 1:

$U_1(s) = i_1(s)\left[R + \frac{1}{sC}\right] - i_2(s)\frac{1}{sC}$ **(1)**

For loop 2:

$i_1(s) = i_2(s)[2 + RsC]$ **(2)**

$U_2(s) = i_2(s)\frac{1}{sC}$

Replacing (2) in (1):

$U_1(s) = i_2(s)[2 + RsC]\left[R + \frac{1}{sC}\right] - i_2(s)\frac{1}{sC}$

Obtained Transfer Function:

$$\boldsymbol{H(s) = \frac{U_2(s)}{U_1(s)} = \frac{1}{R^2 s^2 C^2 + 3RsC - 1}}$$

Figure 1: **Theoretical calculation of the Transfer Function of the circuit.**

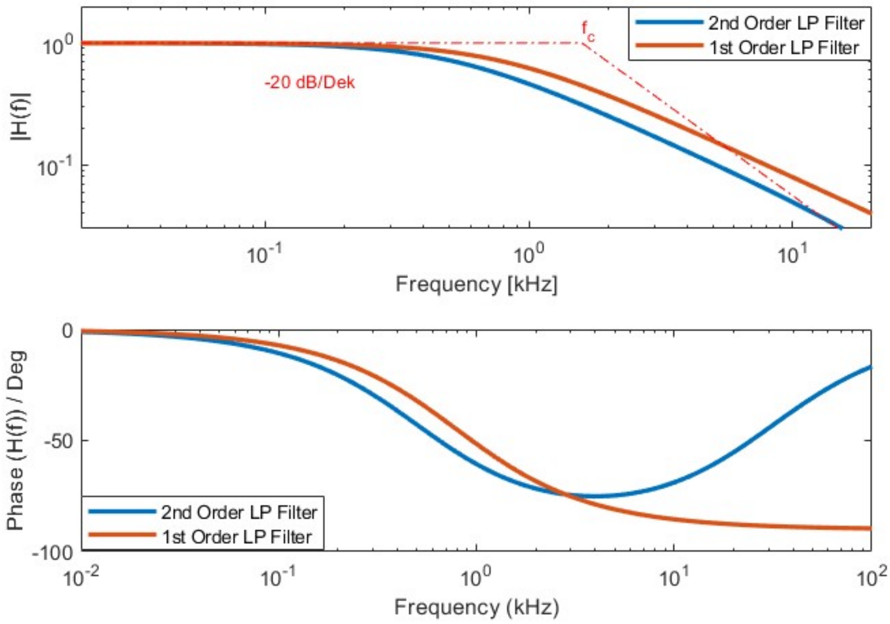


Figure 2: **Bode plot of Magnitude (above) and Phase (below) of the first and second-order Low-Pass Filter.**

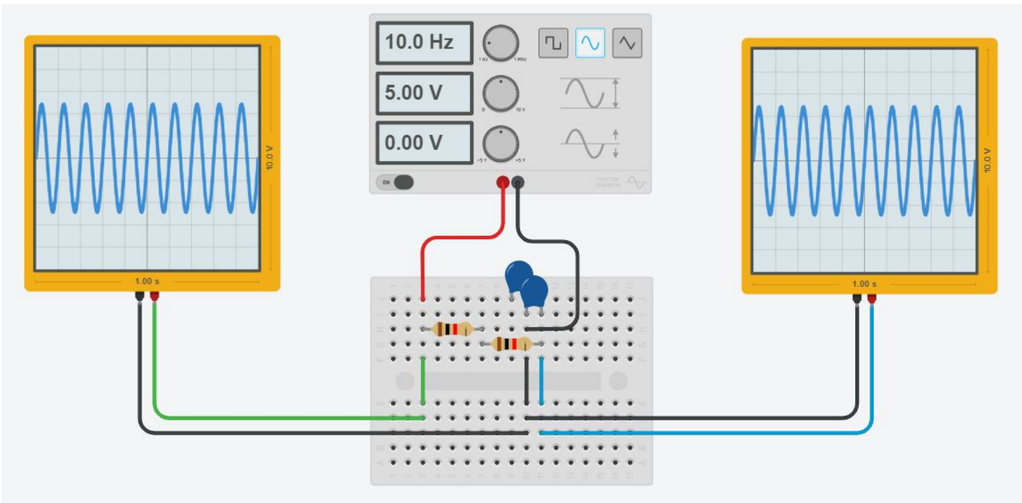


Figure 3: **Schematics of the circuit built in the Breadboard.**

1.2 Assignment 2

a +b) The parameters t_start and t_stop were adjusted as instructed in the exercise sheet and a signal with frequencies f_start = 55 Hz and f_stop = 22 kHz was generated.

c) Figure 4 shows the Bode plot of the calculated and measured Transfer Function and phase for the first and second-order Low-pass filter for the generated signal. We can see that the transfer function of the second-order low-pass filter is steeper than the transfer function of the first order low-pass filter.

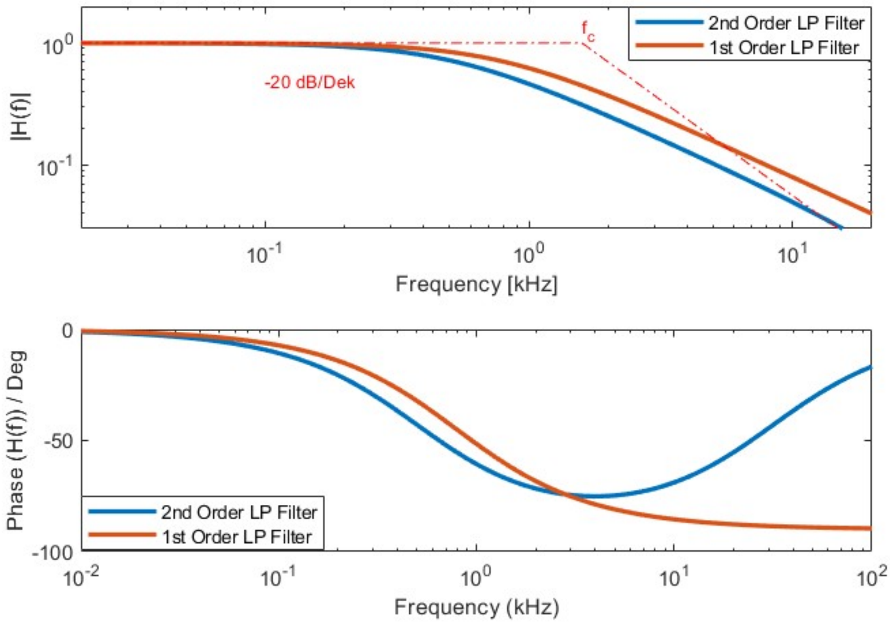


Figure 4: **Calculated Transfer Function and phase of the first and second order Low-pass Filter for the generated signal.**

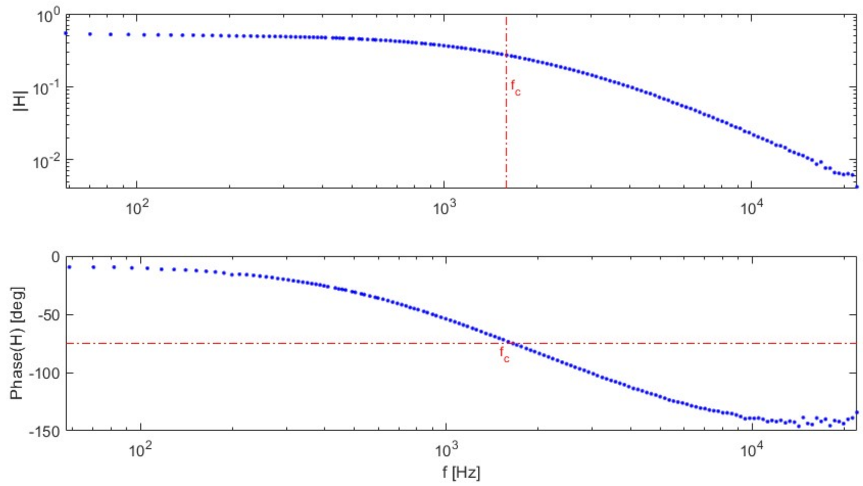
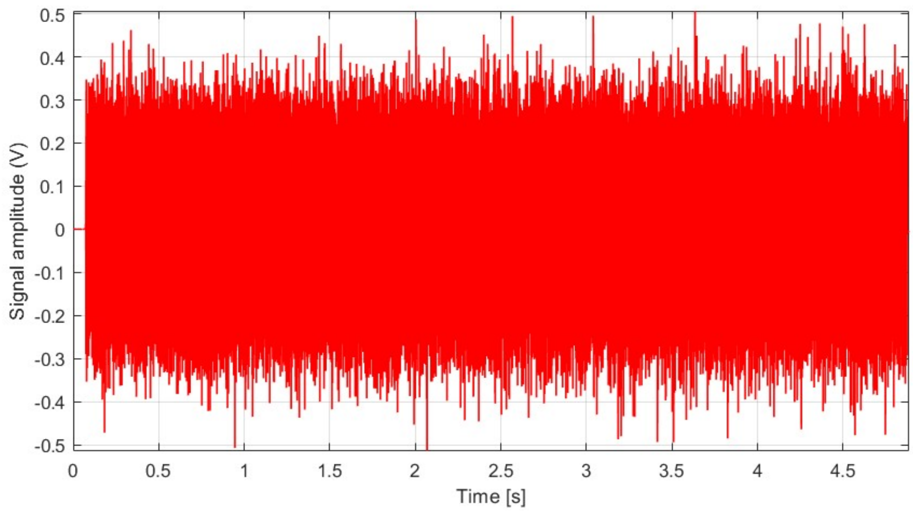


Figure 5: **Measured Transfer Function and phase of the second order Low-pass Filter for the generated signal.**

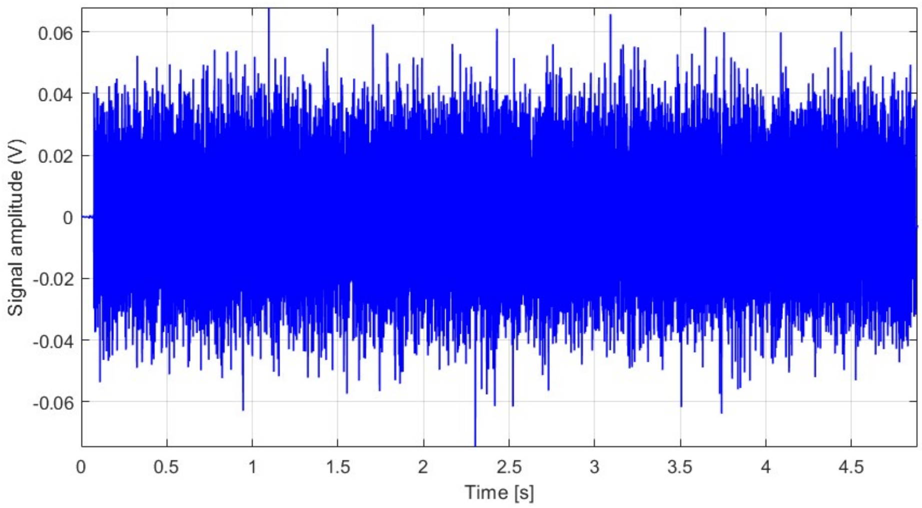
1.3 Assignment 3

a) A white noise signal with an amplitude of 0.4 V was generated and ramped with a Hanning window. The measured input signal to the filter circuit and output of the signal from the filter circuit. The results are illustrated in created (figure 6)

b)The mean and the standard deviation of the input signal was -2.44×10^{-5} V and 0.1124 V respectively. On the other hand, The mean and the standard deviation of the output signal were -2.169×10^{-5} V and 0.0149 V respectively. As shown in the figures, the amplitude of the output signal is damped relative to the input signal. This is due to the effect of the low-pass filter and the presence of the high-frequency components in the signal. In addition, the mean and standard deviation values of the output signals have smaller values than the values of the mean and standard deviation of the input signal. We do not observe a significant difference in the mean values between the input and the output signals.



(a)

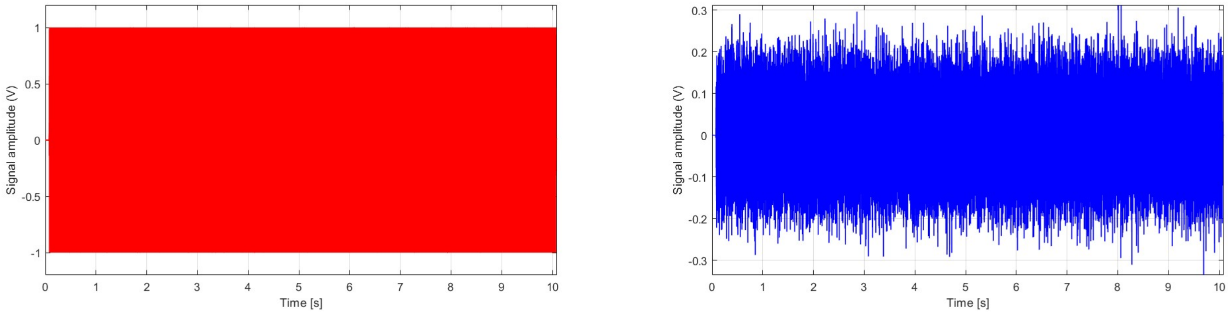


(b)

Figure 6: **The measured input signal to the filter circuit and output of the signal from the filter circuit.** **(a)** Measured input signal to the filter. **(b)** Measured Output signal from the filter circuit.

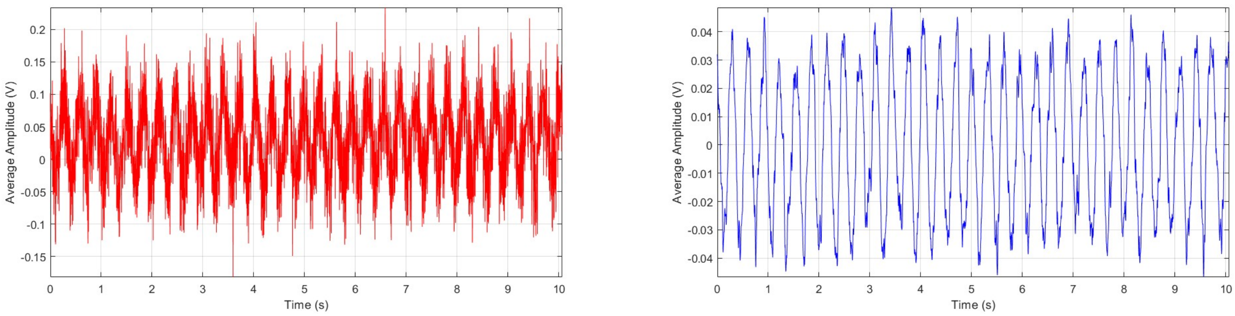
1.4 Assignment 4

a) A sinusoidal signal with an amplitude of 0.05 V buried in noise with a peak amplitude of 0.4 V. The results are illustrated in created (figure 7).
b) the recorded signals into blocks with a sample size of 2^{12} (4096 samples). The results are illustrated in created (figure 8).
c)The mean and the standard deviation of the input signal were 0.0291 Vand 0.0623 V, respectively. On the other hand, The mean and the standard deviation of the output signal were -3.44×10^{-5} V and 0.0254 V respectively. As shown in the figures, the amplitude of the output signal is damped relative to the input signal. We can also see that the output signal is less noisy than the input signal. This is due to the effect of the low-pass filter and the presence of the high-frequency components in the signal. In addition, the mean and standard deviation values of the output signals have smaller values than the values of the mean and standard deviation of the input signal, and we can see a vast difference between the values of the input and output signals.
d) The main difference between the averaged signal and the non-averaged signal is that the mean and the standard deviation of the input and output signal do not show a huge difference for the non-averaged signal. On the other hand, the mean and standard deviation of the averaged output signal decreases drastically relative to the averaged input signal. This is because averaging the signal increases the strength of the signal relative to the noise (signal-to-noise ratio). This is also why the input and output of the averaged signal look much less noisy than the input signal.



(a) (b)

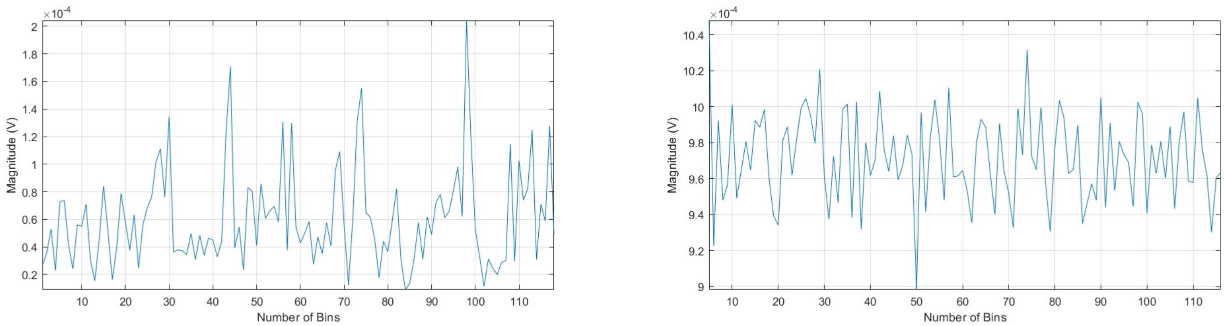
Figure 7: **Input and Output non-averaged signal amplitude in time domain (a)** Input signal in time domain. **(b)** Output signal in time domain.



(a) (b)

Figure 8: **Input and Output averaged signal amplitude in time domain (a)** Input signal in time domain. **(b)** Output signal in time domain.

e) The FFT for each block was computed to obtain the output signal in the frequency domain. Firstly, the average of the complex spectra and the magnitudes were plotted. Secondly, the magnitude of the spectra for each block was computed, and their average was plotted. The results are illustrated in created (figure 9) The first plot shows the magnitudes of the averaged complex spectra across all blocks. On the other hand, the second plot shows the averaged magnitudes of the spectra for each block individually and then takes the average across all blocks. Hence, the first plot gives a more overall view of the dominant frequency components present in the signal. In contrast, the second plot provides more detailed information about the frequency content of individual blocks and how it varies over time.



(a) (b)

Figure 9: **Average magnitude of complex spectra domain and Average magnitudes of spectra of each block (a)** Average magnitude of complex spectra. **(b)** Average magnitudes of spectra of each block

f) The recorded signal was then divided into blocks with sample sizes of 2^9, 2^{10}, 2^{11}. The results are illustrated in figure (sizes(figure 10) The results show that when the sample size increases with a factor of 2, the magnitude of the spectra for each block is reduced by half. This is because as the sample size of the block increases, the noise in the signal decreases, and the filter is enhanced.

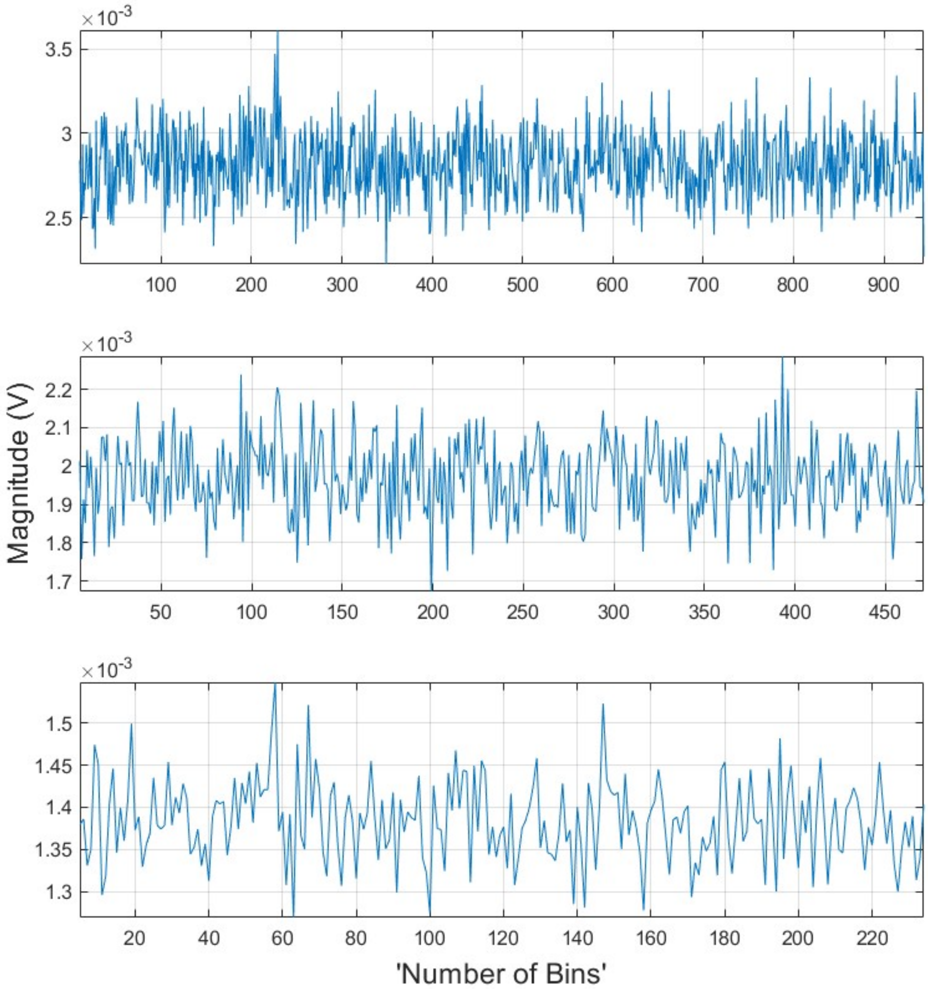


Figure 10: **average of the magnitude of the spectra for every block size**