Technical University of Munich

Neuroprosthetics Exercise 3 Report

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1. SOLVER IMPLEMENTATION

As discussed in the previous assignment, if the equations given contain a function and its derivative, they are referred to as differential equations. Using finite time steps, we can find the numerical approximations to the solutions of differential equations with the help of numerical methods often referred to as solvers. The methods we will be using in this assignment are Forward (Explicit Euler), Heun (2. Order), and Exponential Euler (1. Order).

A. Forward (Explicit) Euler method:

By taking the slope at a point and multiplying it by the change in time, and adding this value to the value of the initial point, we can compute the next point. Equation **(1)** below shows the formula of the Forward Euler method.

$$u_{n+1} = u_n + \mathrm{f}(t_n, u_n)\Delta t \tag{1}$$

B. Heun (2. Order):

Heun's method computes the position of the next point using the average of the slope of an initial point and an additional support point. Equations **(2)** and **(3)** below show the slopes of the initial and additional points, respectively. Equation **(4)** below shows the formula of the Heun (2. Order) method:

$$k_1 = \mathrm{f}(t_{n,}u_n) \tag{2}$$

$$k_2 = \mathrm{f}(t_{n,} + \Delta t, u_n + \Delta t k_1) \tag{3}$$

$$u_{n+1} = u_n + \frac{\Delta t}{2}\,(k_1 + k_2) \tag{4}$$

C. Exponential Euler (1. Order)

The exponential Euler can be used for equations similar to Equation **(5)**. Equation **(6)** below shows the formula of the exponential Euler method:

$$\frac{du}{dt} = A(t)u(t) + B(u, t) \tag{5}$$

$$u_{n+1} = u_n e^{A(t_n)\Delta t} + \frac{B(t_n)}{A(t_n)} e^{A(t_n)\Delta t} - 1 \tag{6}$$

2. SOLVE FUNCTIONS

The second part of the task is to solve equation **(7)** below with the differential equation solvers where V (t = −4s) = V0 = −3 V (initial conditions).

$$\frac{dV}{dt} = 1 - V - t \tag{7}$$

Figure 1 below demonstrates the approximations of the given differential equation with different solver methods and different step sizes.

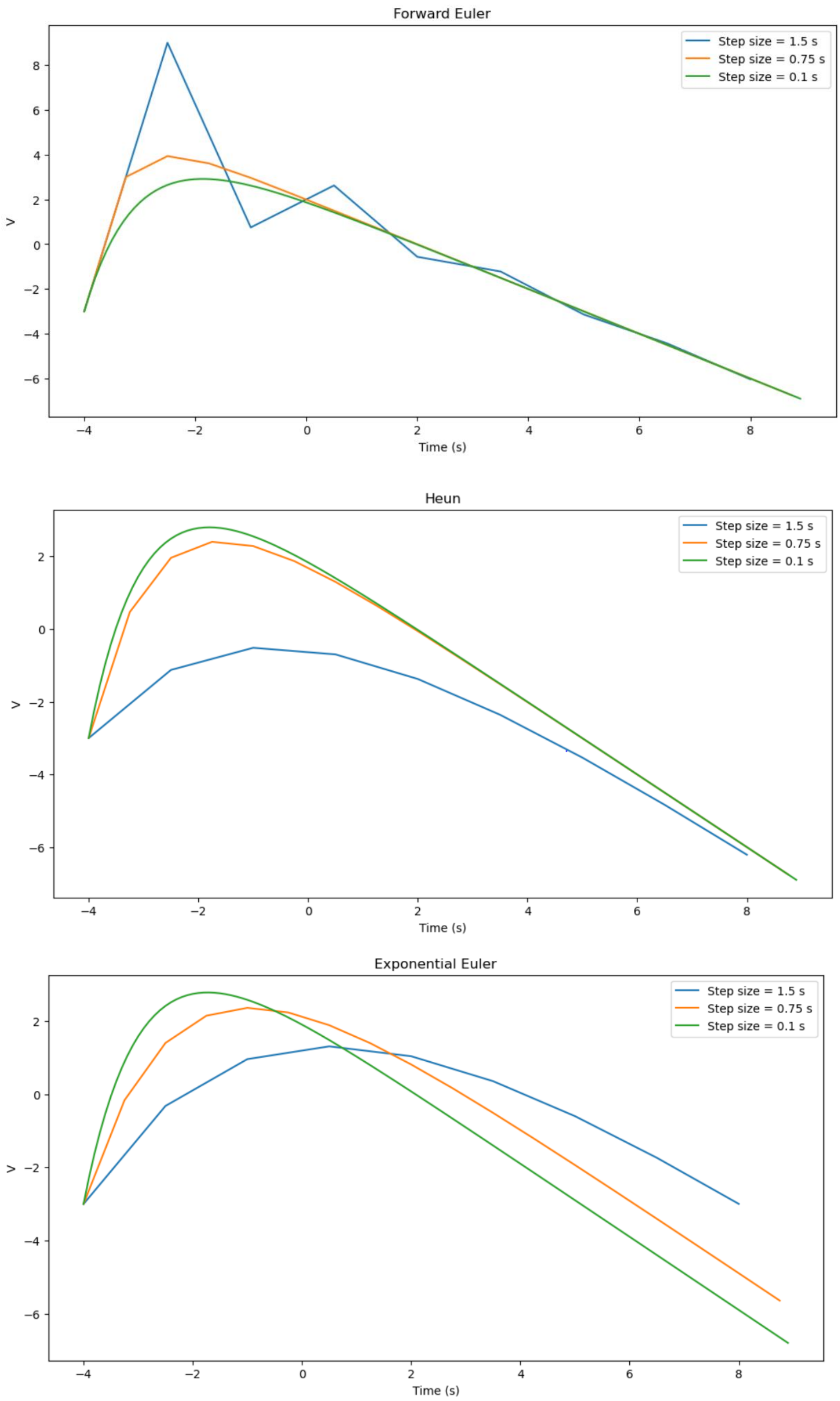

Figure 1. Approximations of the given differential equation 7 with different solver methods and step sizes. Top: Forward-Euler-Method. Middle: Heun Method. Bottom: Exponential-Euler Method

A. Forward (Explicit) Euler: Simple and easy to implement but may be less accurate for stiff problems. As mentioned in the exercise introduction video, when we halve the step size in Explicit Euler method, the error is divided by 2.
   Heun (2nd Order): Improved accuracy over Forward Euler by using a predictor-corrector approach. As mentioned in the exercise introduction video, when we half the step size in the Heun method, the error is divided by 4.
   Exponential Euler: Particularly useful for problems with exponential growth or decay.
B. Smaller step sizes generally lead to more accurate results but require more computational resources. Larger step sizes may result in faster computations but less accurate solutions, especially for rapidly changing functions.
C. The tradeoff lies in balancing computational efficiency with solution accuracy. Very small step sizes increase accuracy but also computational cost. Choosing an infinitesimal step size may not be practical due to computational limitations, and it may introduce numerical instability. In practice, it's essential to choose a step size that is small enough to capture the dynamics of the system accurately but large enough to maintain computational efficiency.

3. The Leaky Integrate and Fire Neuron

The passive membrane corresponds to an RC-Circuit and is represented by a linear differential equation. It cannot account for spike firing, and to generate spikes we need to introduce the notion of a threshold. In the third part of the exercise, we should implement a Leaky Integrate and Fire Neuron Model according to equation **(8)**.

$$V_{n+1} = \begin{cases} V_n + \dfrac{\Delta t}{C_m}\Big(-g_{leak}(V_n - V_{rest}) + I_{input}(t_n)\Big) & V_n < V_{thr} \\ V_{spike} & V_{thresh} \leq V_n < V_{spike} \\ V_{rest} & V_{spike} \leq V_n \end{cases} \tag{8}$$

The parameters of the equation are the following:

- Vn : cell membrane voltage at a timestep n (corresponding to a time t)
- Vn+1 : cell membrane voltage at a timestep n + 1 (corresponding to a time t + Δt)
- Cm = 1 μF : cell membrane capacitance
- gleak = 100 μS : cell membrane leak conductivity
- Vrest = −60 mV : cell membrane resting voltage
- Vthr = −20 mV : cell membrane spiking threshold voltage
- Vspike = 20 mV : spike voltage
- I_{input1}: rectified 50Hz sine with 10 μA amplitude
- I_{input2}: rectified 50Hz sine with 30 μA amplitude

Figures 2 and **3** illustrate the input current sinusoidal signals with 10 μA and 30 μA amplitudes respectively. **Figures 4** and **5** showcase the cell membrane voltage of a LIF-Model using the two different current inputs. As we can see from the results, the cell membrane voltage increase from V_0 which is equal to the V_{rest} (-60 mV) with input current increase until it reaches the V_{thresh} (Threshold Voltage=-20mV). This gradual increase in membrane potential is referred to as the integrating property of the LIF model. As soon as the membrane voltage reaches the V_{thresh}, it spikes (fires an action potential) to reach 20 mV which is the spike voltage in our equation. This is referred to as the firing property of the LIF model. When the cell membrane voltage becomes equal to the spike voltage, the membrane voltage resets to the V_{rest}. This is referred to as the leaky property of the LIF model. The LIF fires regularly with continuous input current. With the increase of current, the approach to the threshold is faster. Therefore, the time intervals between two spikes will be smaller as the current increases from 10 μA to 30 μA. That is why we can observe more spikes with 30 μA input current.

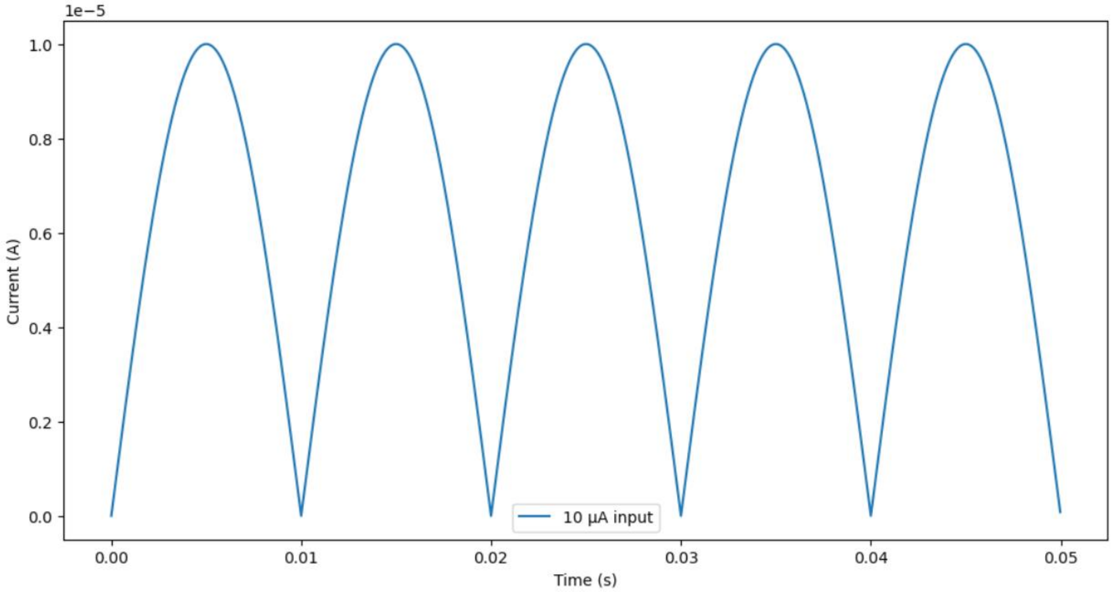

Figure 2. Rectified sine current input for the LIF model with an amplitude of 10 μA

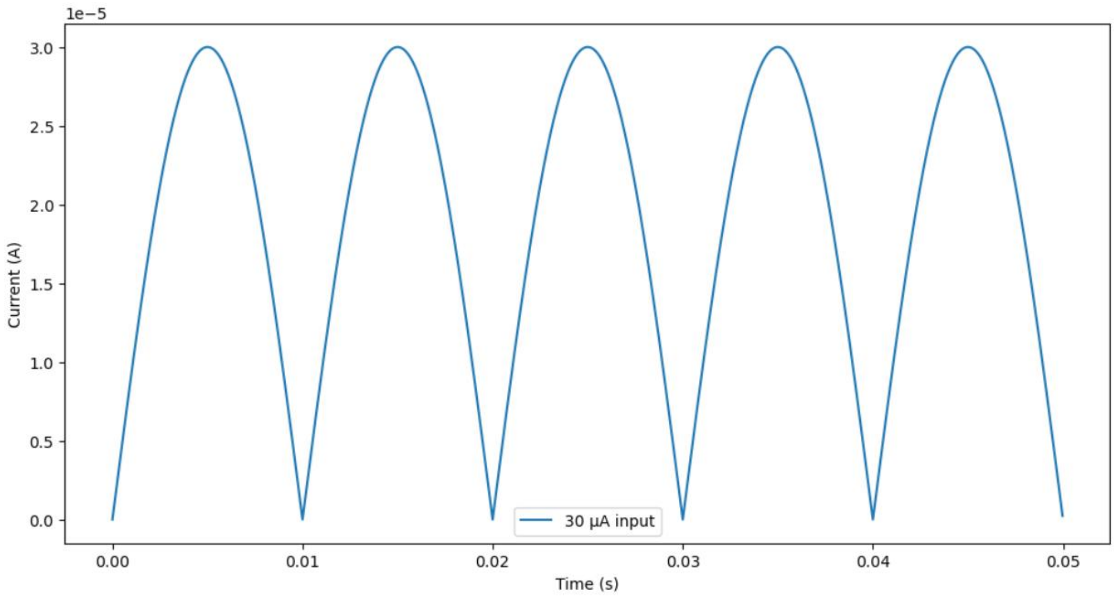

Figure 3. Rectified sine current input for the LIF model with an amplitude of 10 μA

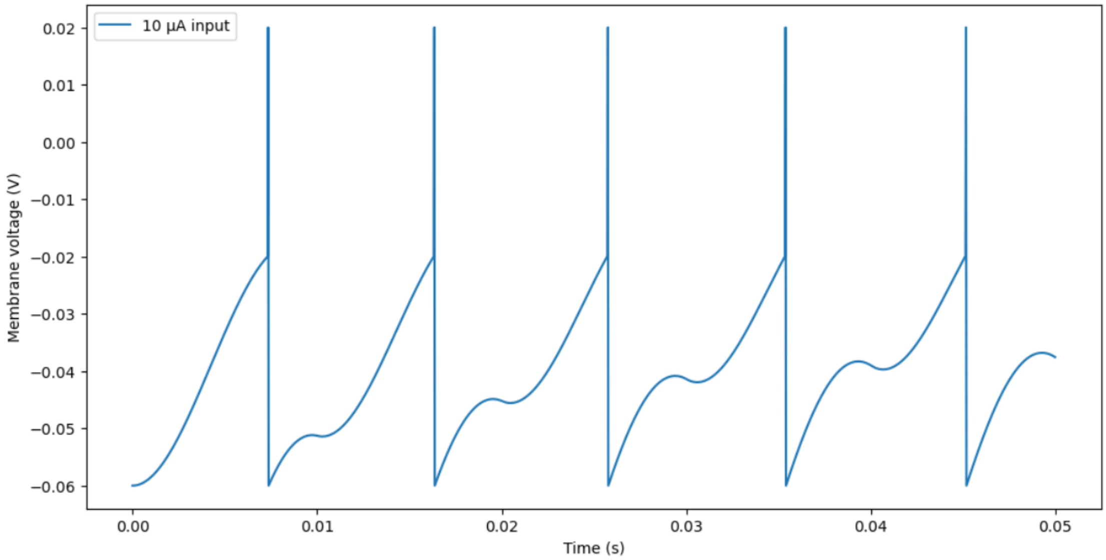

Figure 4. Cell membrane voltage of a LIF-Model with 10 μA input current

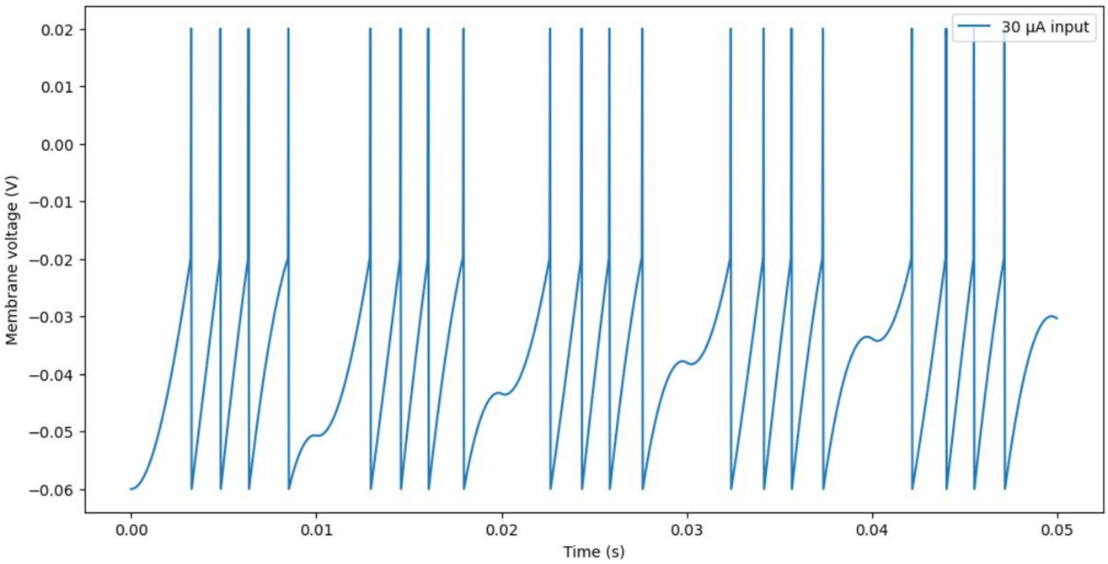

Figure 5. Cell membrane voltage of a LIF-Model with 30 μA input current