Technical University of Munich

Neuroprosthetics Exercise 2 Report

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1. SLOPE FIELDS

The equations given contain a function and its derivative, therefore, they are referred to as differential equations. A derivative of a function gives you slopes at different points of a plane. For example, equation **(1)** below gives the slope at each point in the (x,y) plane. In the exercise, we are given equations **(2)** and **(3)** below. The equations show the rate of change of Voltage (volts) over time (seconds) with respect to voltage and time. To obtain an isocline we set the derivative or slope to a specific value. For equation 1 we are given the set of slope values [-3 V/s, -1 V/s and 1 V/s] and for equation 2 we are given the set of slope values [-2 V/s, 0 V/s and 2 V/s]. **Figures 1** and **2** illustrate the plotted slope fields of equations **(2)** and **(3)** with their isoclines respectively.

$$y' = f(x, y) \tag{1}$$

$$\frac{dV}{dt} = -10 - V - t \tag{2}$$

$$\frac{dV}{dt} = \cos(t) - \frac{1}{2}V + 20 \tag{3}$$

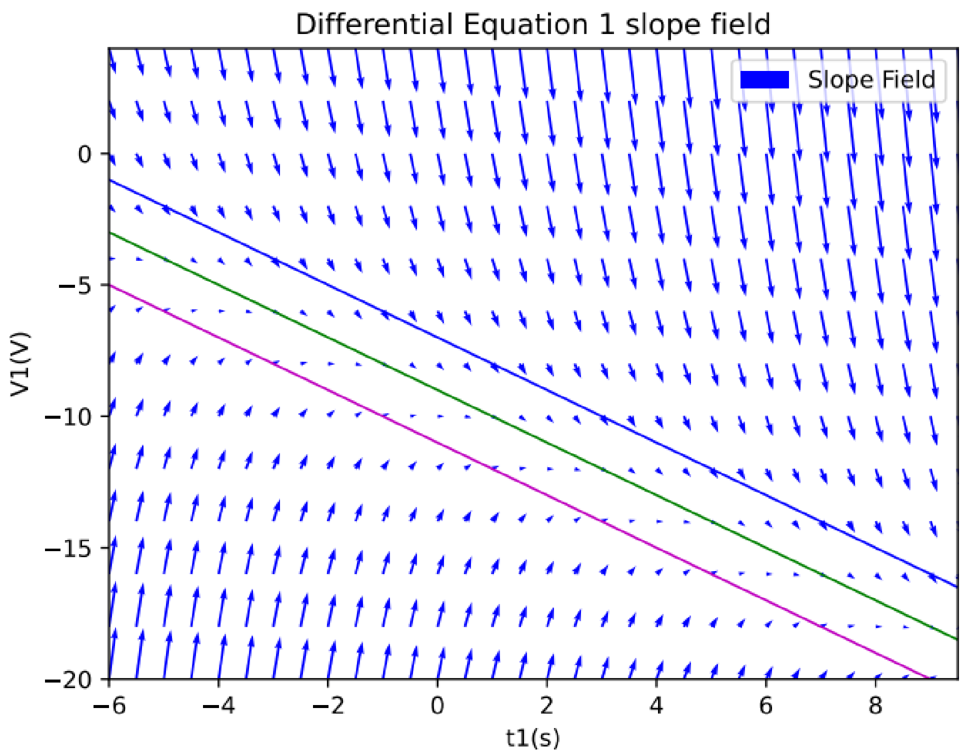

Figure 1. Slope Field and Isoclines of equation 1

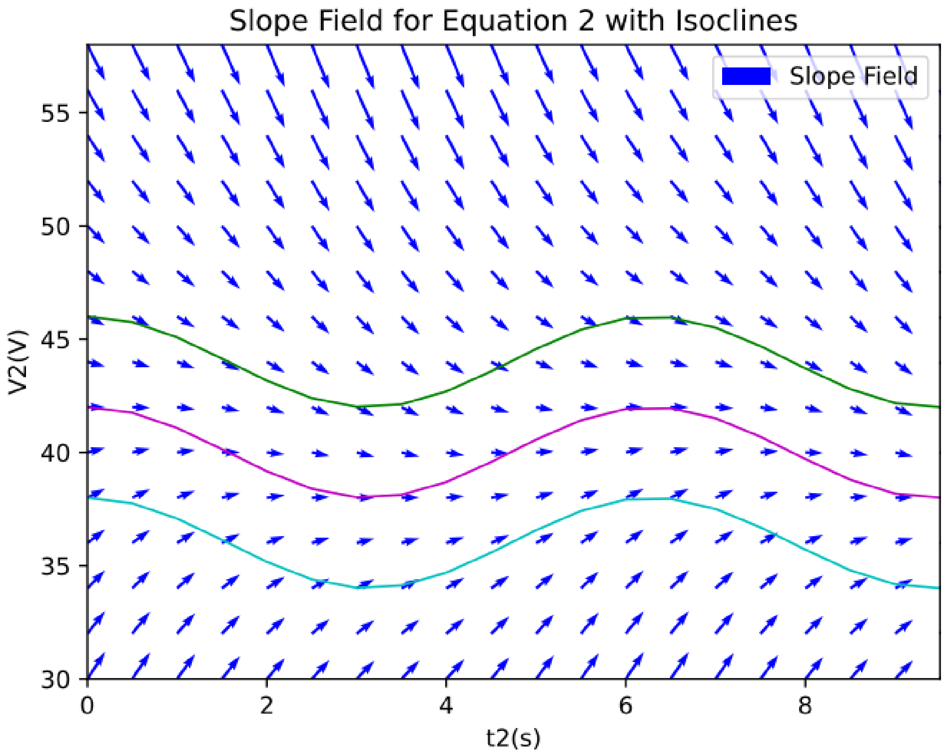

Figure 2. Slope Field and Isoclines of equation 2

2. Simple Cell Model

As per discussed in the lecture, a simple circuit of a cell can be represented as an R-C circuit. The circuit given in the exercise consists of a current source (I_{ex}), and a resistor (R_l) and a capacitor (C_m) connected together in **parallel.** If two electrical components are connected together in parallel then the total current is divided over the two electrical components. Hence, we obtain equations **(4).** After substituting the value of (I_{ex}) given in equation **(5)** we obtain equation **(6).** After making the slope (dV/dt) the subject and simplifying the equation we obtain equation **(7).**

$$I_{ex} = C_m \frac{dV}{dt} + \frac{V}{R_l} \tag{4}$$

$$I_{ex} = I_{max}.Sin(t) \tag{5}$$

$$I_{max}.Sin(t) = C_m \frac{dV}{dt} + \frac{V}{R_l} \tag{6}$$

$$\frac{dV}{dt} = \frac{1}{C_m}(I_{max}.\sin(t) - \frac{V}{R_l}) \tag{7}$$

2.1. Plotted Slope Fields

a) **Figures 3** show the plotted slope fields for equation **(7)** given the following parameters respectively:

- R_{l1} = 1 Ohm; C_{m1} = 2 F; I_{max1} = 0 A
- R_{l2} = 1 Ohm; C_{m2} = 2 F; I_{max2} = 10 A

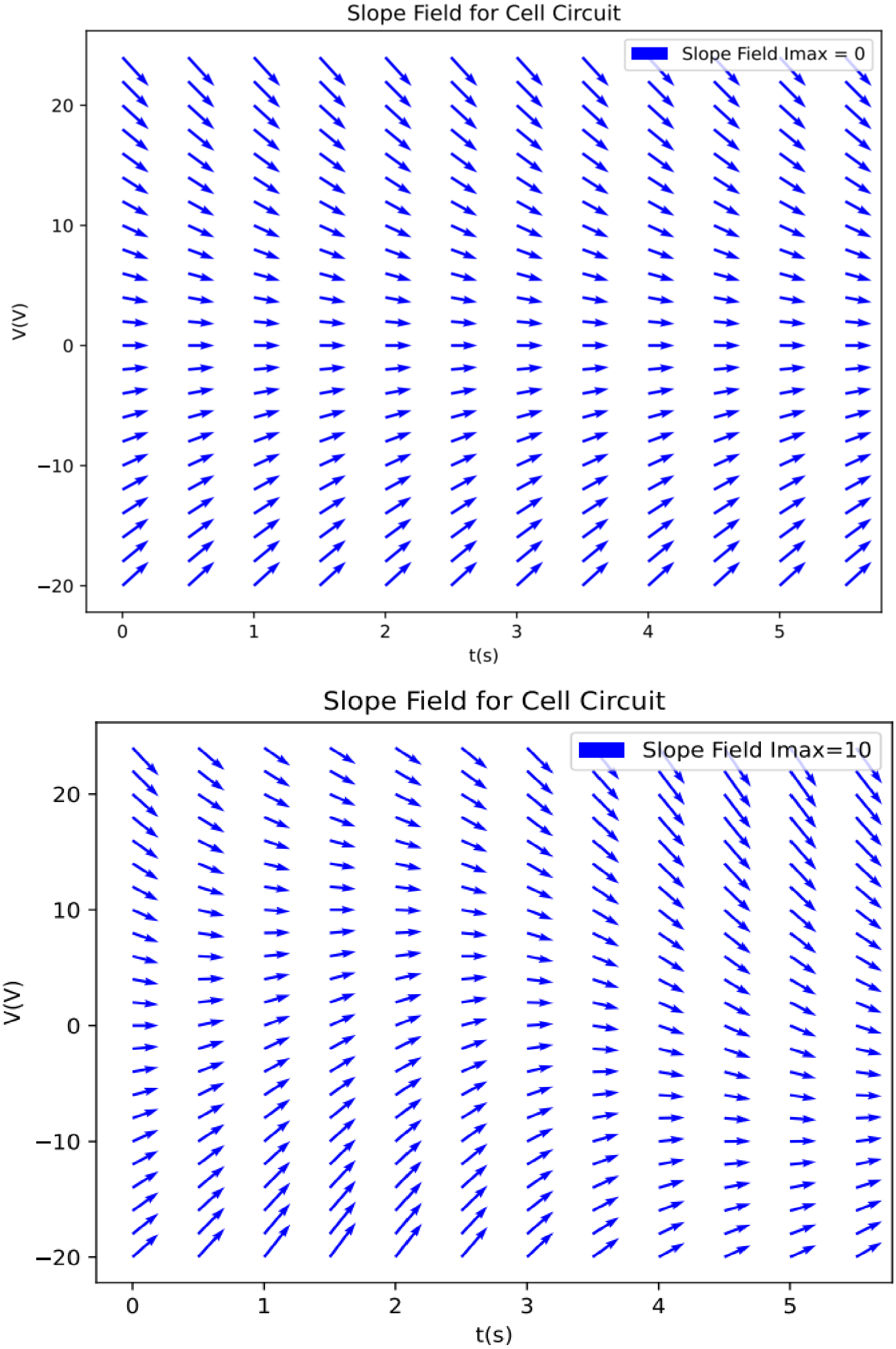

Figure 3. Slope fields for the equation derived from the cell equivalent circuit. Top: Rl1 = 1 Ohm; Cm1 = 2 F; Imax1 = 0 A. Bottom: Rl2 = 1 Ohm; Cm2 = 2 F; Imax2 = 10 A

b) Now after we add a constant term D = 5 A to the current source, we obtain equation **(8)** for the total current. Then we substitute the value of I_{ex} with the value given in equation **(5)** and obtain equation **(9)**. After making the slope (dV/dt) the subject and simplifying the equation we obtain equation **(10)**. **Figures 4** and **6** show the plotted slope fields for equation **(7)** given the following parameters respectively.

- R_{l1} = 1 Ohm; C_{m1} = 2 F; I_{max1} = 0 A
- R_{l2} = 1 Ohm; C_{m2} = 2 F; I_{max2} = 10 A

$$I_{ex} + D = C_m \frac{dV}{dt} + \frac{V}{R_l} \tag{8}$$

$$I_{max}.\sin(t) + D = C_m \frac{dV}{dt} + \frac{V}{R_l} \tag{9}$$

$$\frac{dV}{dt} = \frac{1}{C_m}(I_{max}.\sin(t) + D - \frac{V}{R_l}) \tag{10}$$

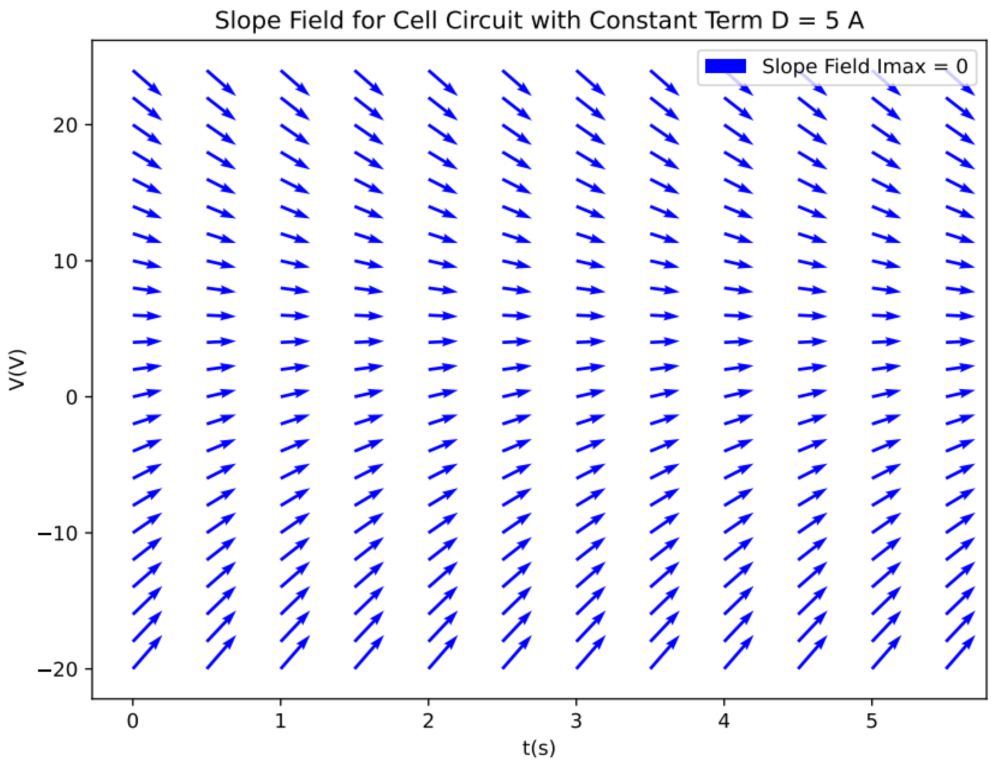

Figure 4. Slope field for the equation derived from the cell equivalent circuit with constant term D = 5A. Top: Rl1 = 1 Ohm; Cm1 = 2 F; Imax1 = 0 A.

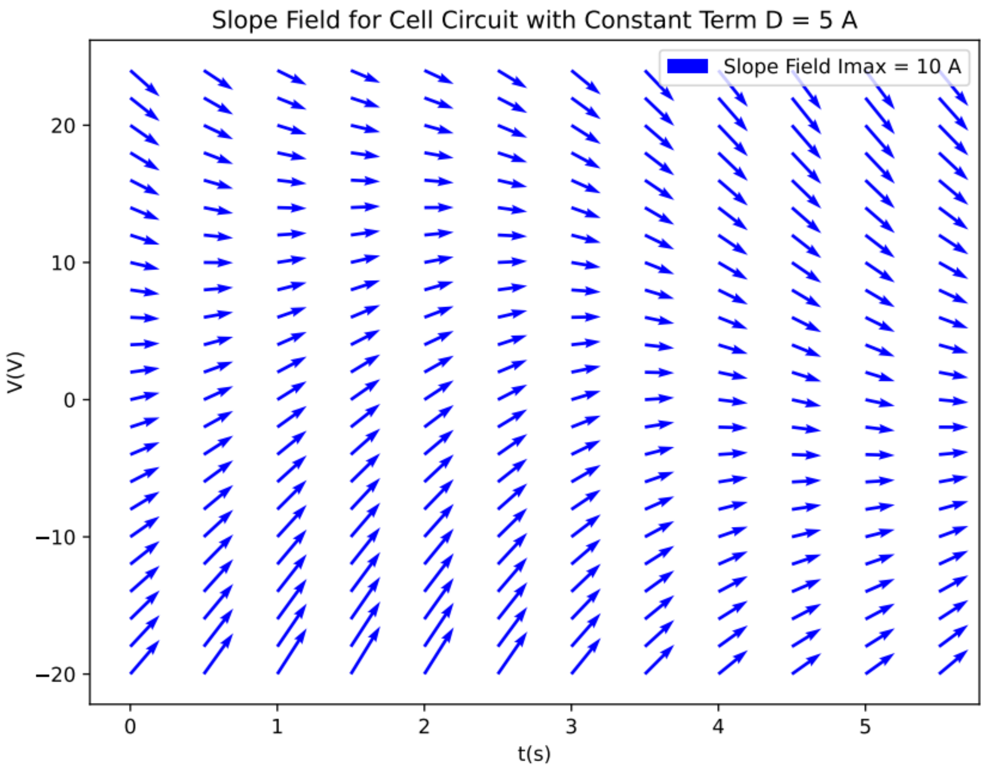

Figure 5. Slope field for the equation derived from the cell equivalent circuit with constant term D = 5A. Top: Rl1 = 1 Ohm; Cm1 = 2 F; Imax2 = 10 A.