Analog integrated Circuits.

Lec17: Noise fundamentals.

(1) Noise in time domain.

- Noise is unwanted random Signal.
- We Cannot Predic (model) its instantaneous Value
- But we Can Predic (model) noise statistical dis.

(2) Noise statistical distribution.

- Noise has normal (gaussian) distribution.
- the mean = 0 $\longrightarrow$ ∴ avg noise Voltage in time domain = 0
- the Variance (σ^2) is the mean-Square Value
- the standard diviation (σ) is the r.m.s Value.
- usually: Peak to Peak instantaneous noise Voltage is within $\pm 3\sigma$

(3) Noise Power and noise Voltage

* the avg Power of a Periodic Sig. $\rightarrow$

$$P_{avg} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} P(t)\,dt = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{V^2(t)}{R_L}\,dt$$

$$= \frac{1}{R_L}\cdot\underbrace{\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \overbrace{V_n^2(t)}^{\text{Square}}\,dt}_{\text{mean}} = \frac{\overbrace{\overline{V^2}}^{\text{mean-Square / Variance}}}{R_L} = \frac{\sigma^2}{R_L}$$

* The avg Power of a non-Periodic Sig. $\longrightarrow$

$$P_{avg} = \frac{1}{R_L}\cdot\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} V_n^2(t)\,dt = \frac{\overline{V_n^2}}{R_L} = \frac{\sigma_n^2}{R_L}$$

* the mean-Square noise Voltage $\longrightarrow$

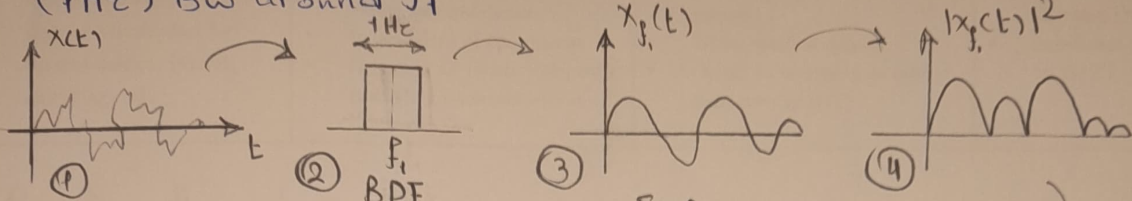
$$\overline{V_n^2} = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} V_n^2(t)\,dt = \sigma_n^2 \propto P_{avg}$$

* the root-mean-Square of noise Voltage $\rightarrow$

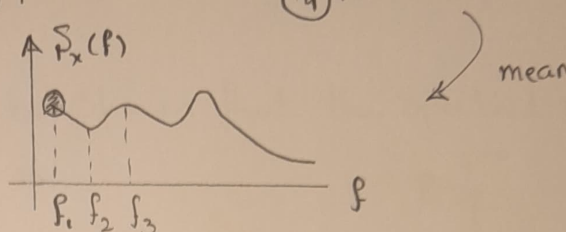
$$V_{n\,rms} = \sqrt{\overline{V_n^2}} = \sigma_n$$

4 Noise in frequency domain.

- a Signal $x(t)$ has Power spectral density (PSD) = $S_x(f)$
  - ↳ how much Power is Carried arround each frequency.
- PSD of a noise Signal $x(t)$ at freq f_1 is the avg Power (σ^2) in (1 Hz) BW around f_1



repeat 1 to 4 for each freq (f_2, f_3 ---) for range of intrest



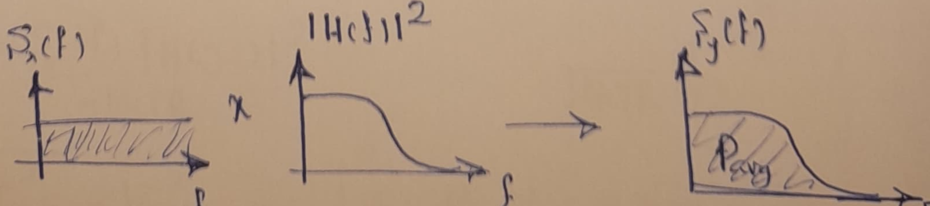
- $S_x(f)$ measured in W/Hz or V^2/Hz
- Voltage noise density: $V_n(f) = \sqrt{S_x(f)} \longrightarrow V/\sqrt{Hz}$
- Power at freq (e.g., f_2) exactly Can not be measured (we take 1 Hz BW)

5 White noise and noise Shaping.

- White noise: noise PSD has the Same Value at all freq.
- The noise spectrum is shaped by the system transfer func.
- Avg Power (output noise Power) (σ^2) is the area under the output PSD Curve

$$P_{avg} = \int_{-\infty}^{\infty} S_y(f)\,df$$

$$S_x(f) \longrightarrow \boxed{H(f)} \longrightarrow S_y(f)$$



is wide BW good BW

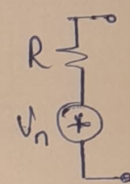
- Wide BW means more noise Power is integrated
- Wide BW is not good BW → amp BW should just fit the Sig

6 Types of noise.

External Noise	Internal Noise		
	Inherent noise due to the fundamental physical properties of the circuit components. Can be reduced but cannot be eliminated.		
	Thermal noise	**Shot noise**	**Flicker noise**
• A.k.a. interference noise or man-made noise • Unwanted interaction between the outside world • Ex. EM interference noise and power supply noise • Can be eliminated by careful design, layout, shielding, etc.	• Due to thermal excitation of charge carriers • White spectral density • Independent of DC current • Occurs in all resistive elements (including semiconductors)	• Due to non-smooth DC current (flow of individual carriers) • White spectral density • Occurs in pn-junctions (and consequently BJT)	• Due to traps in semiconductors affecting DC current flow • Significant noise source in MOSFET

7 Resistor thermal noise

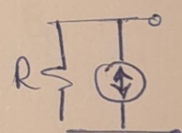
- From thermodynamics, it can be shown that the spectral density of a resistor thermal noise is



$$\overline{v_n^2}(f) = S_v(f) = 4\underbrace{k}_{1.38 * 10^{-23}\ J/k}TR$$

- For $R = 1k\Omega \longrightarrow v_n(f) \simeq 4 \frac{nV}{\sqrt{Hz}}$ @ $T = 300°k = 27°C$

$v_n(f) \simeq \sqrt{\frac{R}{1k}} * 4 \frac{nV}{\sqrt{Hz}}$

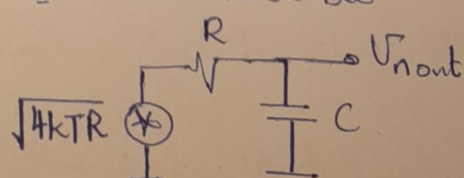


- the norton equivalent $\overline{i_n^2}(f) = \frac{4kT}{R}$
- For $R = 1k\Omega \longrightarrow I_n(f) = 4 \frac{PA}{\sqrt{Hz}}$

$i_n(f) \simeq \sqrt{\frac{1k}{R}} \cdot 4 \frac{PA}{\sqrt{Hz}}$

8 Noise in RC Circuits.

- in RC circuits R generates noise & C limits noise BW



$$S_{v_{nout}}(f) = \underbrace{S_v(f)}_{4kTR}|H(j\omega)|^2$$

$$\therefore \overline{v_{nout}^2} = v^2_{nout\,rms} = \int_{-\infty}^{\infty} S_{v_{nout}}(f)\,df = \boxed{\frac{kT}{C}}$$ # $\neq f(R)$ why?

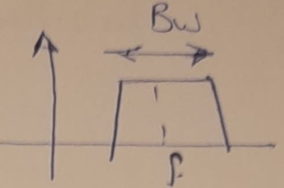
when $R\uparrow$, $4kTR\uparrow$, $BW\downarrow$ (↳ of (H)), noise shaped S_v = Const ↲

- for 1pF Capacitance $\longrightarrow V_{n\,rms} = 64\,\mu V_{rms}$

$V_{n\,rms} \simeq \sqrt{\frac{1p}{C}} * 64\mu V_{rms} = 8$

- for Specific BW application (BP)

noise Limited by R

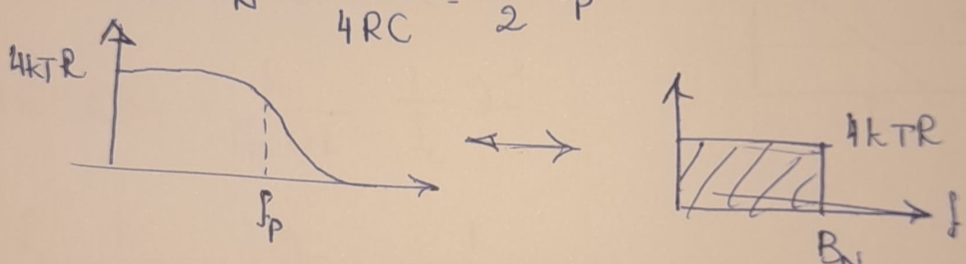


(9) Equivalent noise BW

- Define an equivalent noise BW (B_N) such that the area under a brickWall response is the Same area under the actual spectral density Curve

$$V_{n\,rms}^2 = \int_{-\infty}^{\infty} S_{v_n}(f)\,df = 4kTR * B_N = \frac{kT}{C}$$

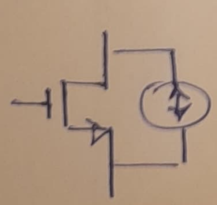
$$\therefore B_N = \frac{1}{4RC} = \frac{\pi}{2} f_p$$



(10) MOSFET thermal noise

- MOS has thermal noise due to the resistive nature of the Channel

- it Can be shown that the noise Current spectral density is given by $I_n^2(f) = 4kT\gamma g_m$



↳ Similar to a resistor with $R = \frac{1}{\gamma g_m}$

↳ γ: Mosfet thermal noise Coefficient

$\gamma \approx \frac{2}{3}$ for Long Channel, $\gamma \approx 1$ for short Channel

- the noise Current Can be referred to the gate Voltage

- the relation between I_n and V_n is g_m (not G_m)

$$V_n^2(f) = \frac{4kT\gamma}{g_m}$$ $\longrightarrow$ Valid at Zero gate Current (Low/medium) freq

(11) flicker noise

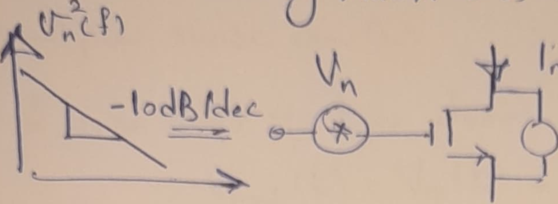
- mainly due to dangling bonds at the oxide/silicon interface
- it can be shown that the noise voltage spectral density is given by.

flicker noise coefficient

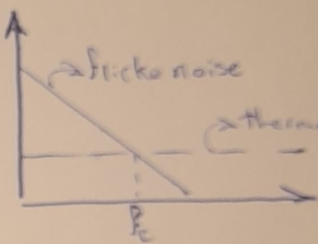
$$V_n^2(f) = \frac{K}{C_{ox}WL}\cdot\frac{1}{f}$$

scale down hurts flicker noise

- PMOS has usually much less flicker noise



$I_n = f(G_m)$ if it was I_D
or
$i_n = f(g_m)$ if it current source between S/D

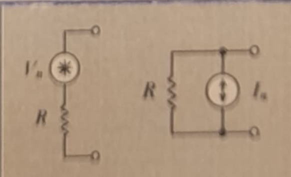
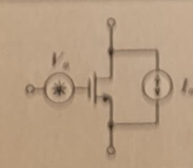
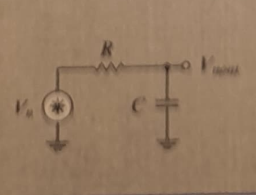


- flicker noise Corner (f_c)

$$4kT\gamma g_m = \frac{k}{C_{ox}WL}\cdot\frac{1}{f_c}\, g_m^2$$

$$f_c = \frac{k}{C_{ox}WL}\cdot g_m\cdot\frac{1}{4kT\gamma}$$

- f_c tells which type of noise (thermal / flicker) is dominant for a given signal band

Resistor thermal noise		$V_n(f) = \sqrt{4kTR} = \sqrt{\frac{R}{1k}}\times 4\frac{nV}{\sqrt{Hz}}$ $I_n(f) = \sqrt{\frac{4kT}{R}} = \sqrt{\frac{1k}{R}}\times 4\frac{pA}{\sqrt{Hz}}$
MOSFET thermal and flicker noise		$I_n^2(f) = 4kT\gamma g_m$ $V_n^2(f) = \frac{K}{C_{ox}WL}\frac{1}{f}$
RMS noise		$V^2_{noutrms} = 4kTR\times B_N = \frac{kT}{C}$ $B_N = \frac{1}{4RC} = \frac{\pi}{2}f_p$ $V_{noutrms} \approx \sqrt{\frac{1p}{C}}\times 64\,\mu Vrms$

(12) Signal to noise ratio.

- is the ratio between Signal Power and noise Power

$$\# \text{ SNR} = \frac{P_{sig}}{P_{noise}} = \frac{V^2_{sig\,rms}}{V^2_{n\,rms}}$$

- SNR is usually expressed in dB

$$\# \text{ SNR} = 10 \log \frac{P_{sig}}{P_{noise}} = 20 \log \frac{V_{sig\,rms}}{V_{n\,rms}}$$

(13) multiple noise sources

1. Noise adds in time domain

$$V_{nout}(t) = V_{n1}(t) + V_{n2}(t)$$

2. remember: $V_n(t)$ is a random Variable
   ↳ We can not add rms Values

$$V_{nout\,rms} \neq V_{n1\,rms} + V_{n2\,rms}$$

3. if $V_{1n}(t)$, and $V_{2n}(t)$ are uncorrelated (independent) random Variables)

$$V^2_{nout\,rms} = V^2_{n1\,rms} + V^2_{n2\,rms}$$

4. the Largest noise Contributor dominates

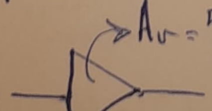
$$3^2 + 1^2 \simeq 3^2$$

(14) output referred noise

- do not Compare output noises for Compare two designs
- instead Compare input refferred as output noise depends on design gain

$$\hookrightarrow \frac{V_{nout\,rms}}{A_v}$$

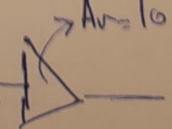
- Example

$20\,mV_{rms}$  $A_v = 50$

$V_{out\,rms} = 100\,\mu V_{rms}$

$V_{sig\,out} = 1\,V_{rms}$

$SNR = 10^4$

$20\,mV_{rms}$  $A_v = 10$

$V_{nout\,rms} = 50\,\mu V_{rms}$

$V_{sig\,out} = 200\,mV_{rms}$

$SNR = 4 \times 10^3$

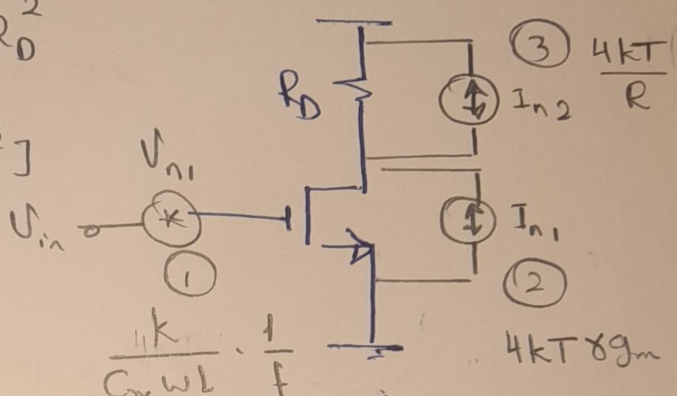
(15) Noise analysis example (CS with resistive load)

1. Deactivate the input signal
2. identify noise sources
   - ↳ Resistor: thermal
   - ↳ Mosfet: Thermal + flicker
3. find the noise spectral density at output using superposition

① $\overline{V^2_{nout}(f)}' = \frac{k}{C_{ox}WL} \cdot \frac{1}{f} \cdot g_m^2 R_D^2$

② $\overline{V^2_{nout}(f)}'' = 4kT\gamma g_m [R_D^2 \parallel r_o^2]$
$= 4kT\gamma g_m R_D^2$

③ $\overline{V^2_{nout}(f)}''' = \frac{4kT}{R_D} \cdot R_D^2$



↳ noise denisties can not be added unless they all has same shapping (they all depends on the same node = same Pole)

$$\therefore \overline{V^2_{nout}(f)} = \left[4kT\gamma g_m + \frac{k}{C_{ox}WL} \cdot \frac{1}{f} \cdot g_m^2 + \frac{4kT}{R_D}\right] R_D^2$$

⟶ for a fair comparison

$$\overline{V^2_{nin}(f)} = \frac{\overline{V^2_{nout}(f)}}{A_v^2} = \frac{4kT\gamma}{g_m} + \frac{k}{C_{ox}WL} \cdot \frac{1}{f} + \frac{4kT}{g_m^2 R_D}$$

⟶ assume thermal noise is dominant

$$\overline{V^2_{nout}(f)} \simeq 4kTg_m \left(\gamma + \frac{1}{g_m R_D}\right) R_D^2$$

⟶ assume BW is limited by C_L

$$\therefore V^2_{nout\,rms} = \overline{V^2_{nout}} = \overline{V^2_{nout}(f)} \cdot \frac{1}{4 R_D C_L} \simeq kT(1+\gamma g_m R_D)\frac{1}{C_L}$$

assume $\partial = (1+\gamma g_m R_D) = (1+\gamma|A_v|)$

$$\therefore V^2_{nout\,rms} \simeq \frac{kT\partial}{C_L}$$

- assume input signal is sinusoide with amplitud = V_p

$$SNR = \frac{V_{out\,rms}^2}{V_{nout\,rms}^2} \approx \left(\frac{V_P}{\sqrt{2}} \cdot g_m R_D\right)^2 \cdot \frac{C_L}{kT\partial}$$

$$= \frac{V_P^2}{2} \; \frac{g_m^2 R_D^2 C_L}{kT\partial} = \frac{2 V_p^2 V_{R_D}^2 C_L}{V^{*2} kT\partial}$$

- assume $V_{R_D} = \frac{V_{DD}}{2}$

V_{DD} improve SNR

$$SNR \approx \frac{V_P^2 V_{DD}^2 C_L}{2V^{*2} kT\partial}$$

- assume rms output max amplitude = $K_0 V_{DD}$

$$\therefore SNR = \frac{(K_0 V_{DD})^2 C_L}{kT\partial} = \frac{(K_0 V_{DD})^2 C_L}{kT(1+\gamma|A_v|)}$$

- assume speed spec is fixed (Noise Vs Power)

$$SNR \approx \frac{V_p^2 V_{DD}^2 C_L}{2V^{*2} kT\partial}$$

$$GBW = \frac{g_m}{C_L}$$

↳ to improve SNR by 6 dB (equivalent 1B) in a system limited by thermal nois

$\therefore C_L \times 4$

$\therefore g_m \times 4$

$\therefore P_{cons} \times 4$ → assuming V^* is constant

- assume P_{cons} is fixed

↳ to improve by 6 dB

$\therefore C_L \times 4$

$\therefore GBW \times \frac{1}{4}$ → decreasing V^* may help But $f_T \downarrow$, lower speed