

# Analog Integrated Systems Design

## Lecture 10 Digital-to-Analog Conversion (2)

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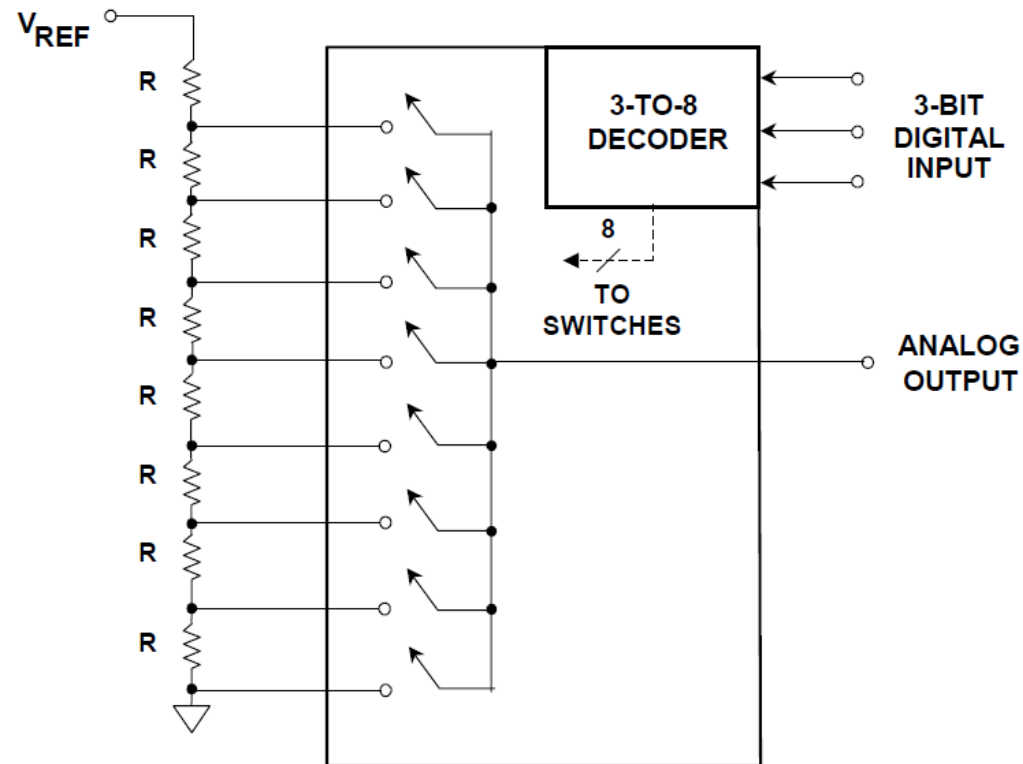
# DAC Architectures

	Unary	Binary
Voltage	Resistor string <i>Flash ADC</i>	R-2R <i>Low-performance DAC</i>
Current	Current matrix <i>High bandwidth DAC</i>	Current splitting
Charge/capacitor	Capacitor bank <i>Low power DAC</i>	Capacitor bank
Time	PWM, $\Sigma\Delta$ mod <i>Low bandwidth DAC</i>	Limited by distortion

In italic the main application area is indicated

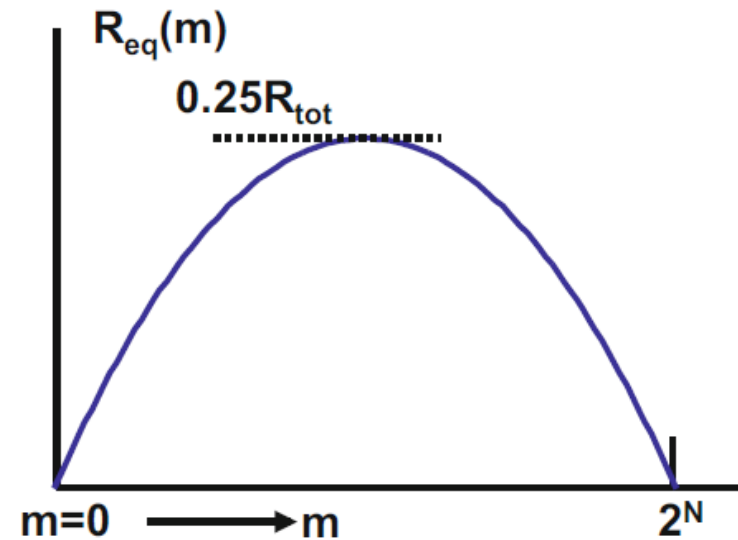
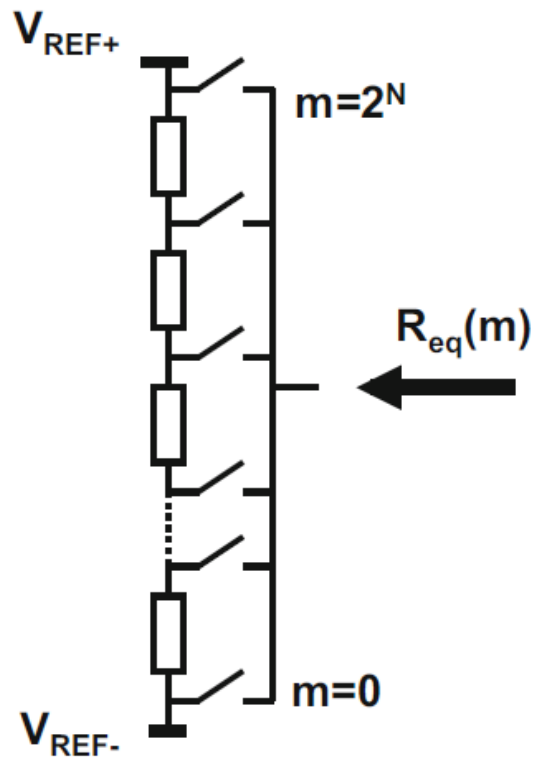
# Resistor String (Ladder) DAC

- Simply a voltage divider using  $2^N$  identical unit resistors
- No. of switches =  $2^N$  (**every** unit element needs a switch)
  - This is an example of a **unary** (thermometer) DAC
- Monotonicity is guaranteed
- The decoder is a bit complex



# Resistor String (Ladder) DAC

- The resistor ladder impedance varies according to the tap position
  - Min at ends and max at the middle
  - The ladder output voltage must be buffered



# Resistor String (Ladder) DAC

❑ Transitions will cause spurious charges that disturbs the tap voltages of the string

- A rough estimate for the time constant that assumes middle tap disturbance

$$\tau \approx \left( \frac{R_{tot}}{2} // \frac{R_{tot}}{2} \right) \times \frac{C_{tot}}{2} = \frac{R_{tot} C_{tot}}{8}$$

- $R_{tot}$  is total resistance and  $C_{tot}$  is total parasitic capacitance
- We use  $C_{tot}/2$  because not all parasitic caps are fully charged
- Exact solution yields:  $\tau = \frac{R_{tot} C_{tot}}{\pi^2}$
- $\tau$  must be significantly smaller than the DAC sample rate

# How Many Time Constants Are Enough?

- Assume linear settling of first order system

$$V(t) = V_i + (V_f - V_i)(1 - e^{-\frac{t}{\tau}})$$
$$T = \tau \ln \left( \frac{V_f - V_i}{V_f - V(T)} \right) = \tau \ln \left( \frac{\Delta V}{\epsilon} \right)$$

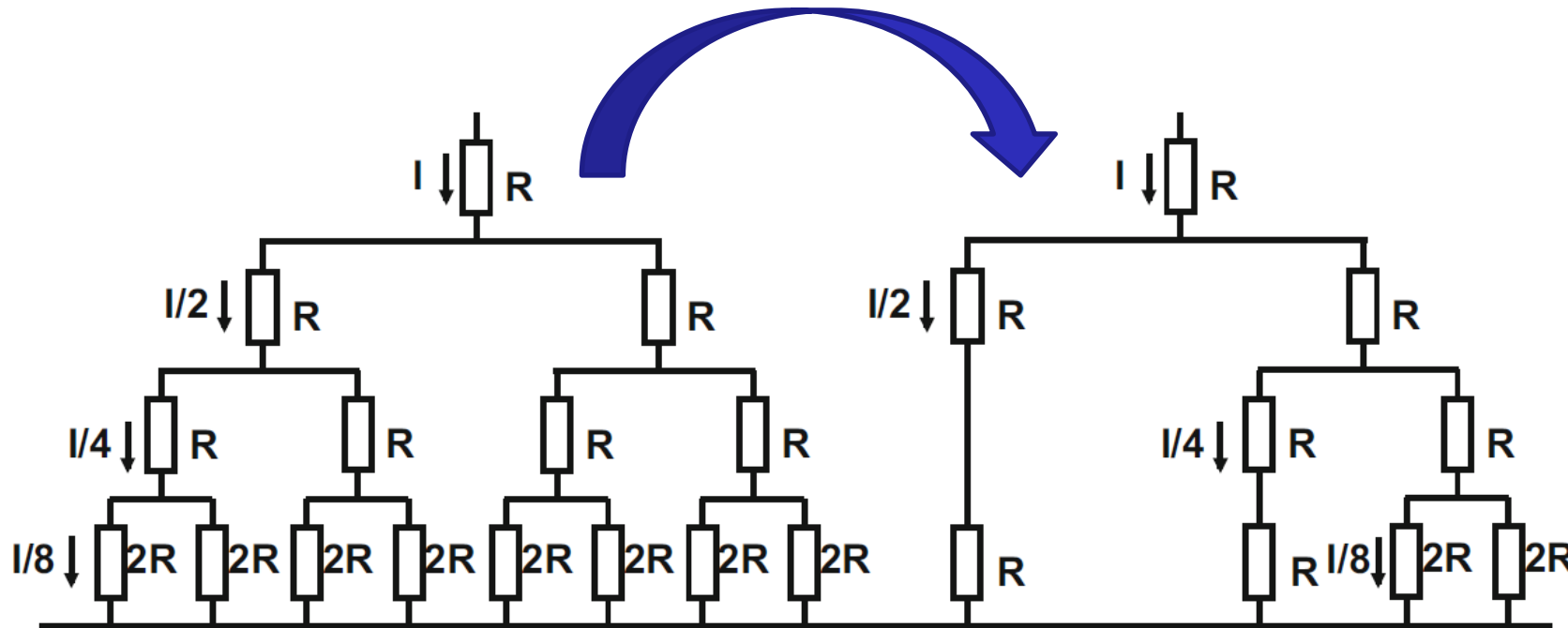
- If  $\Delta V = FS$  and  $\epsilon < 1LSB = \frac{FS}{2^N}$

$$T = \tau \ln(2^N) = 0.69N\tau$$

Resolution (N-bit)	No. of time constants
4	2.8
8	5.5
12	8.3
16	11.1
20	13.9
24	16.6

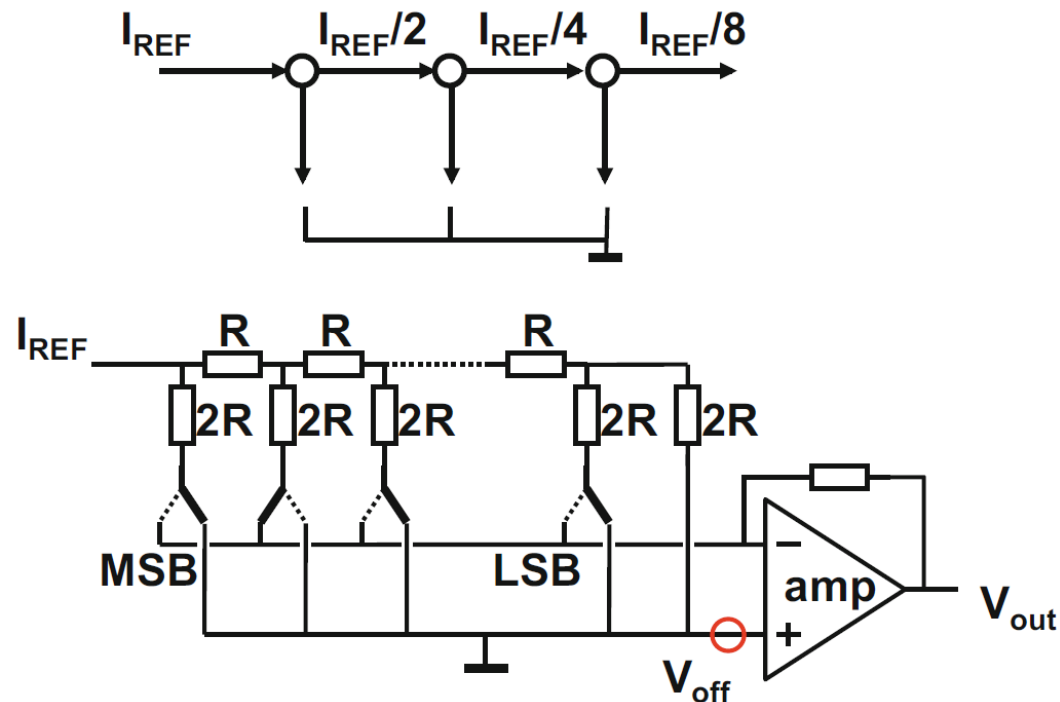
# R-2R Ladder DAC

- ❑ Current splits equally at each level  $\rightarrow$  Binary weighted currents
- ❑ At each node, looking downward the equivalent impedance is  $R$ 
  - Each branch can be replaced by a resistance  $R$  with no effect on current splitting
- ❑ Current splits equally at each node between the “ $R$ ” branch and the “ $2R$ ” branch



# R-2R Ladder DAC

- ❑ Performance limited to 8-10-bit at low/medium speed
- ❑ Limitations
  - Buffer offset (virtual ground is not 0V)
  - Switches resistance (must be  $\ll R$ , or is made part of the  $2R$ )
  - Buffer bandwidth





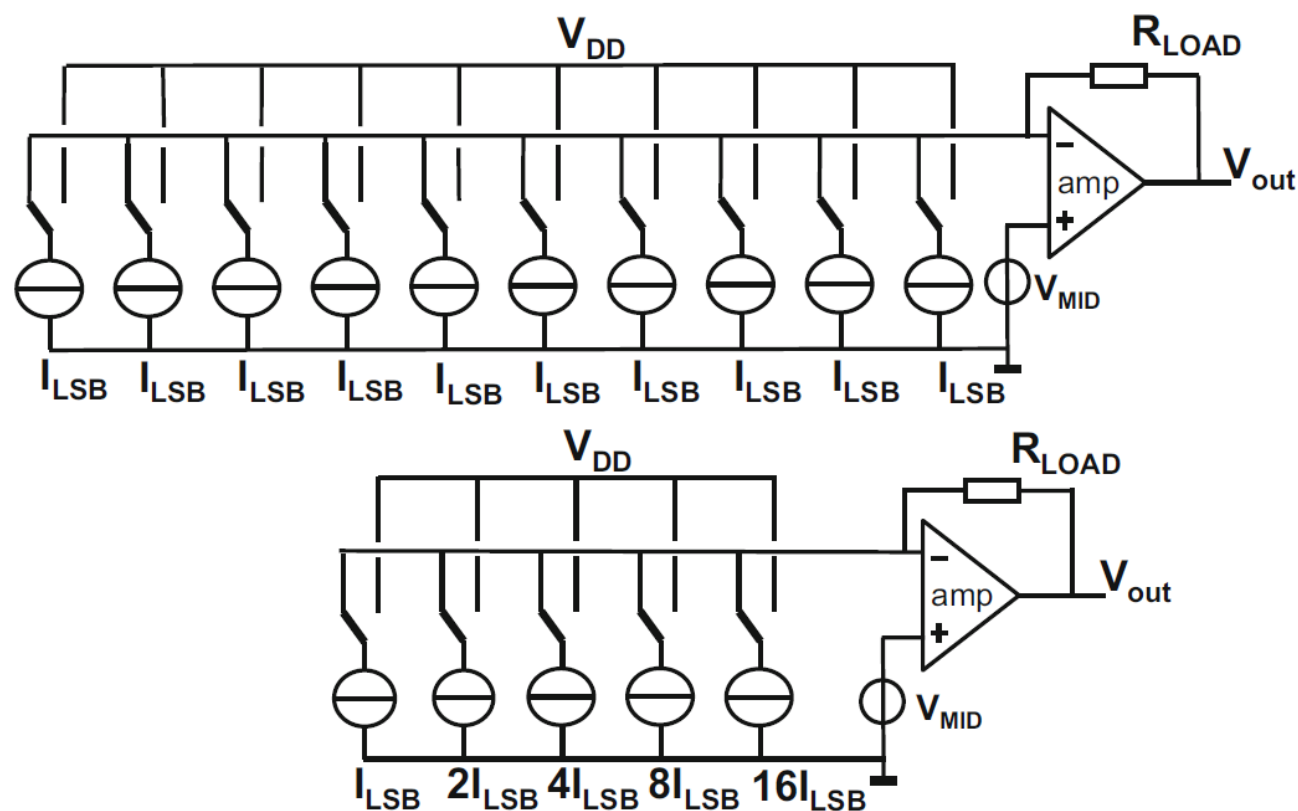
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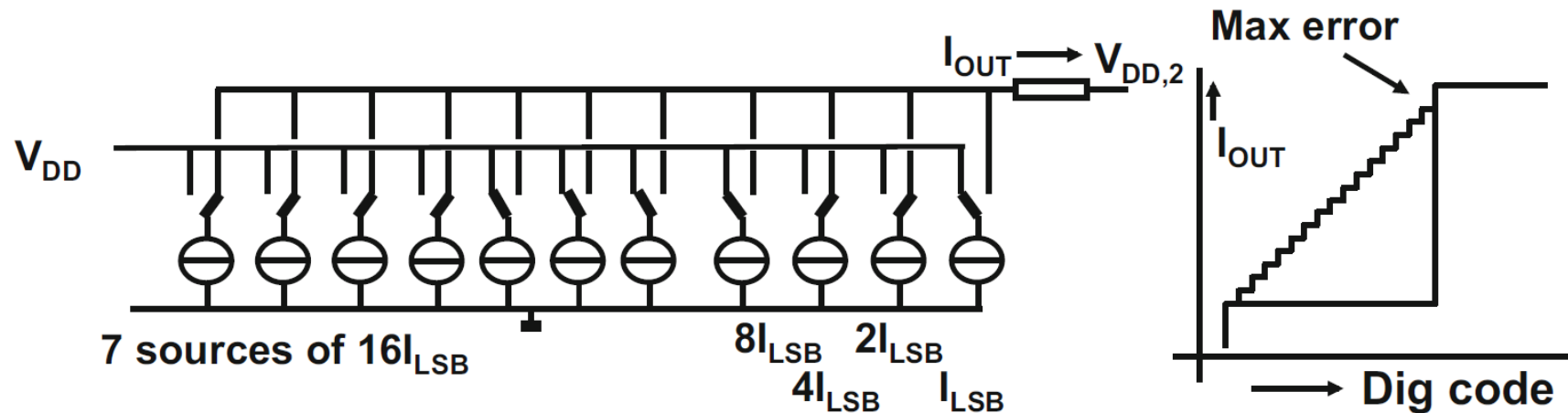
# Buffered Current Domain DAC

- ❑ Currents sources designed with relatively large area for good matching
- ❑ Current always active: steered to output or dumped to rail
- ❑ Op-amp keeps voltage across CS fixed, but limits speed



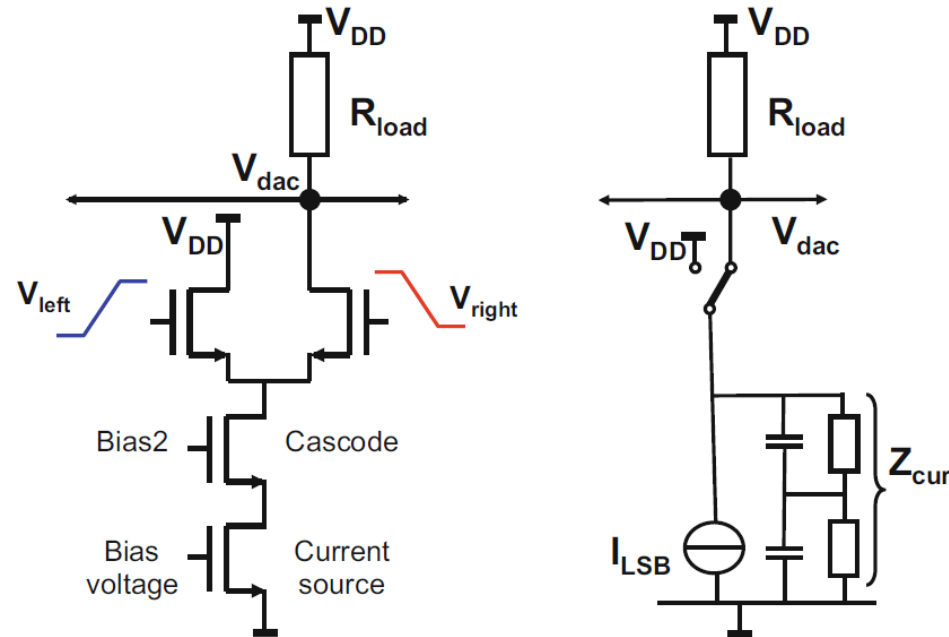
# Current Steering DAC

- ❑ No bandwidth-limiting buffer → High-speed architecture
  - Voltage across CS varies: Current modulation due to finite CS output impedance is a major problem
- ❑ Directly supplies current into 50-75 Ohm load
- ❑ Usually a segmented architecture is used
- ❑ The most popular high performance DAC architecture



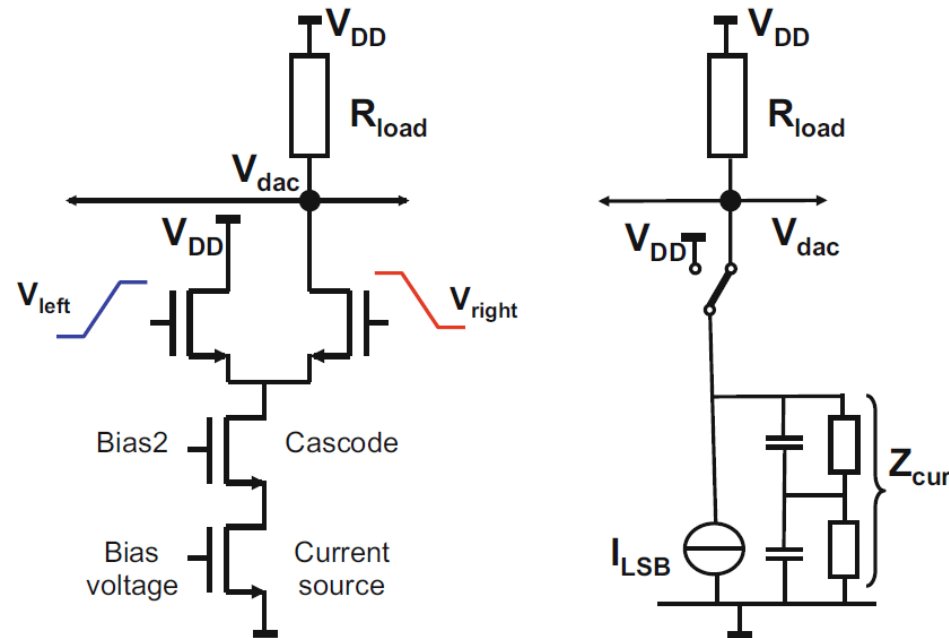
# Current Steering Unit Cell

- ❑ Current is steered (not switched off). Why?
  - Avoid discharging the caps
    - Takes extra delay to build charge back (lower speed and more distortion)
  - Need small differential voltage to steer the current using the diff pair
- ❑ Max current consumption independent of digital input



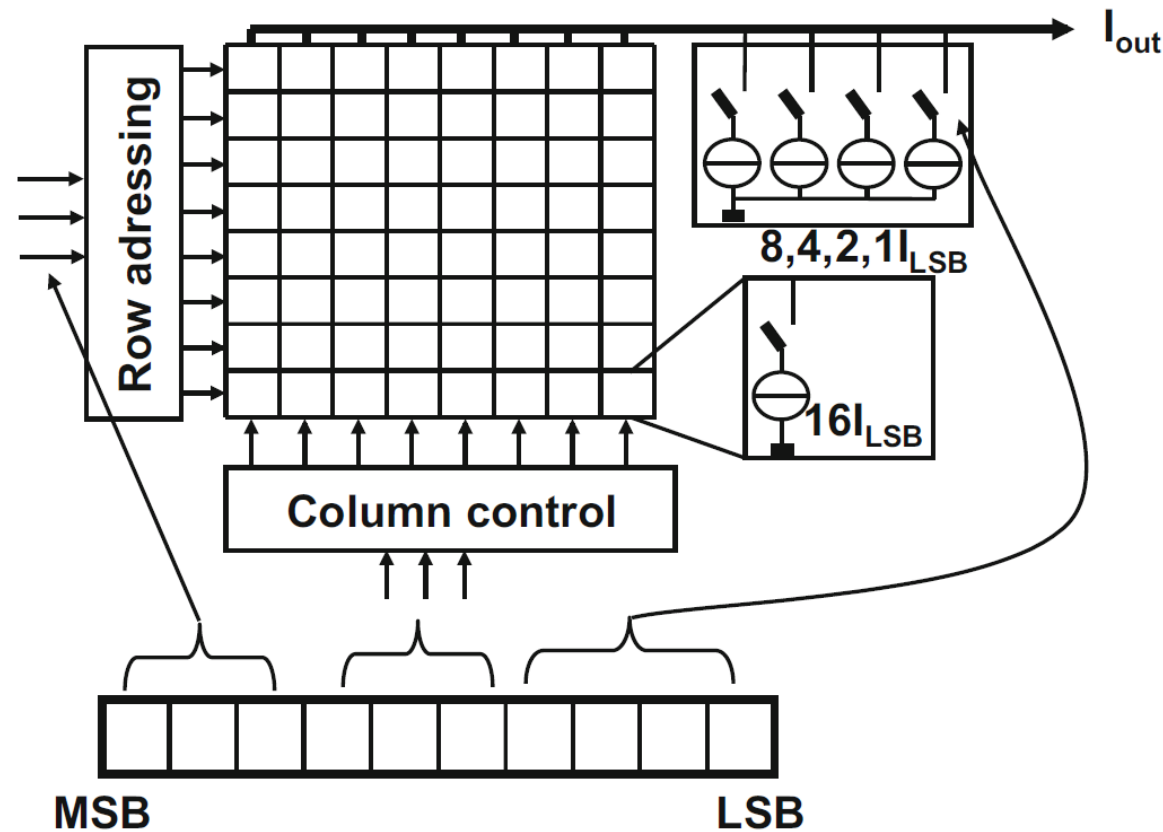
# Current Steering Unit Cell

- ❑ Current is modulated by  $V_{dac}$  due to finite  $Z_{cur}$ 
  - Current source cascoded to improve accuracy
- ❑ Non-linear distortion mitigated by making  $Z_{cur}/2^N \gg R_{load}$
- ❑  $2^N$  current cells typically decomposed into a  $(2^{N/2} \times 2^{N/2})$  matrix



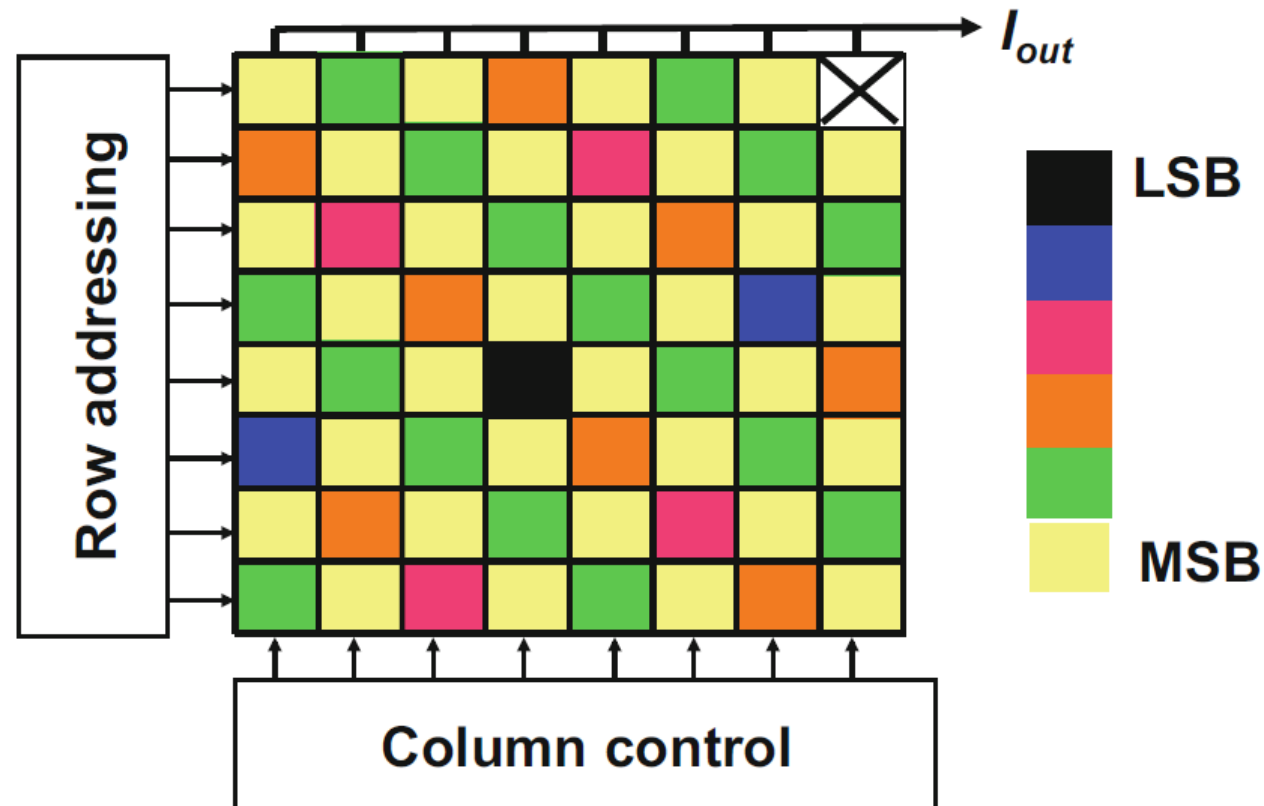
# Matrix Decoded Current DAC

- 10-bit digital-to-analog converter:
  - Six MSBs are implemented as 64 unary current sources in a matrix configuration
    - Gradient effects mitigated
  - Four LSBs are designed in a binary series



# Matrix Decoded Current DAC

- ❑ Matrix decoding can be used as well for binary arrays
  - Common-centroid layout mitigates process gradients
  - Addresses first-order (linear) gradients only



# DAC Architectures

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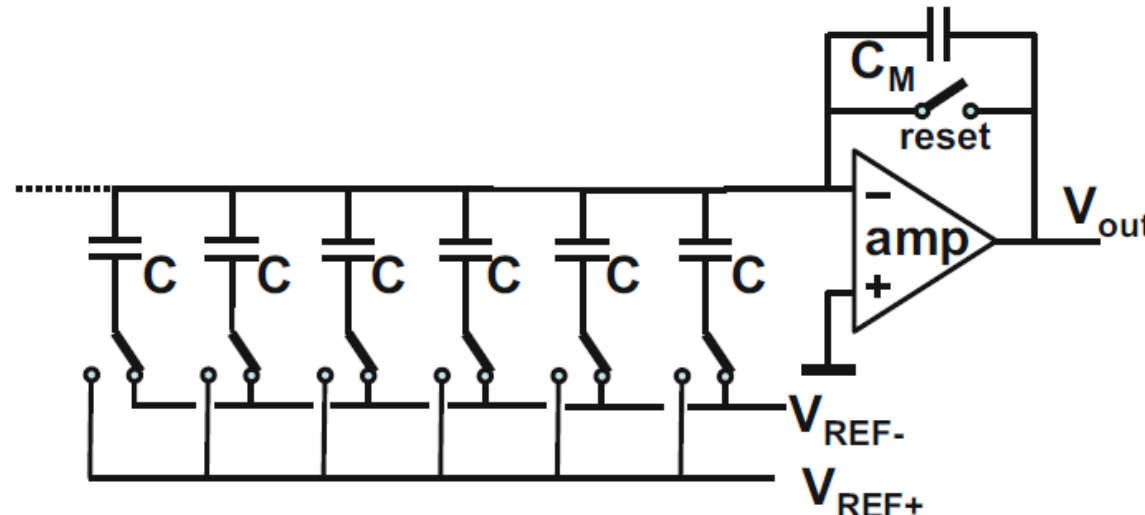
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# SC Charge Domain DAC (1)

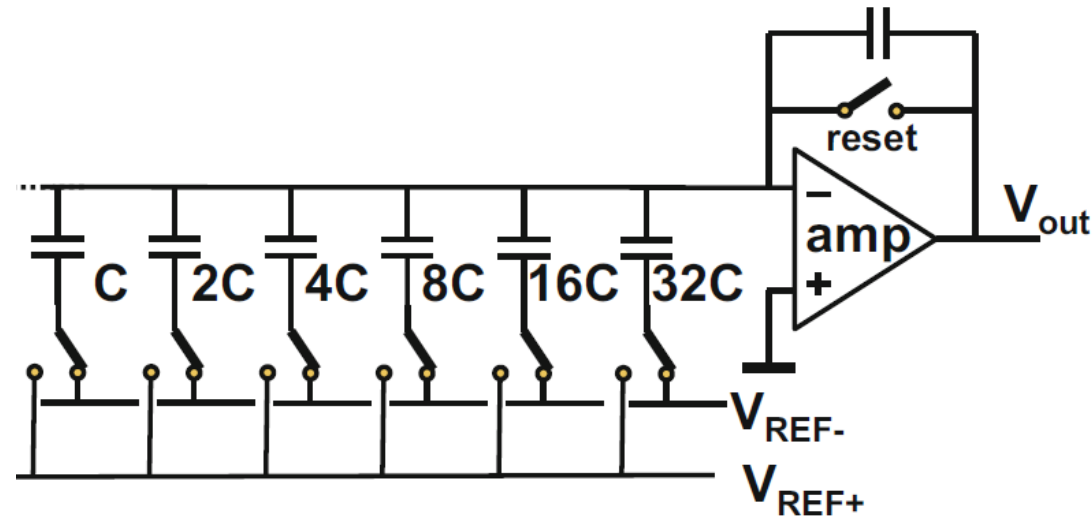
- ❑ Parasitic (stray) insensitive architecture
  - Bottom plate connected to ref voltage (bottom plate sampling)
  - Top plate connected to virtual ground
- ❑ Assume m unit capacitors are switched from  $V_{REF+}$  to  $V_{REF-}$

$$V_{out} = -\frac{mC(V_{REF-} - V_{REF+})}{C_M}$$



# SC Charge Domain DAC (2)

- ❑ Key advantage: No static power (except in OTA)
  - Excellent energy efficiency
- ❑ A.k.a. charge redistribution DAC, capacitive DAC
- ❑ Charge transfer (redistribution) is insensitive to jitter
- ❑ Capacitor array is usually binary or segmented
  - Unary capacitor array wiring and switches are impractical
- ❑ INL and DNL limited by capacitor array mismatch



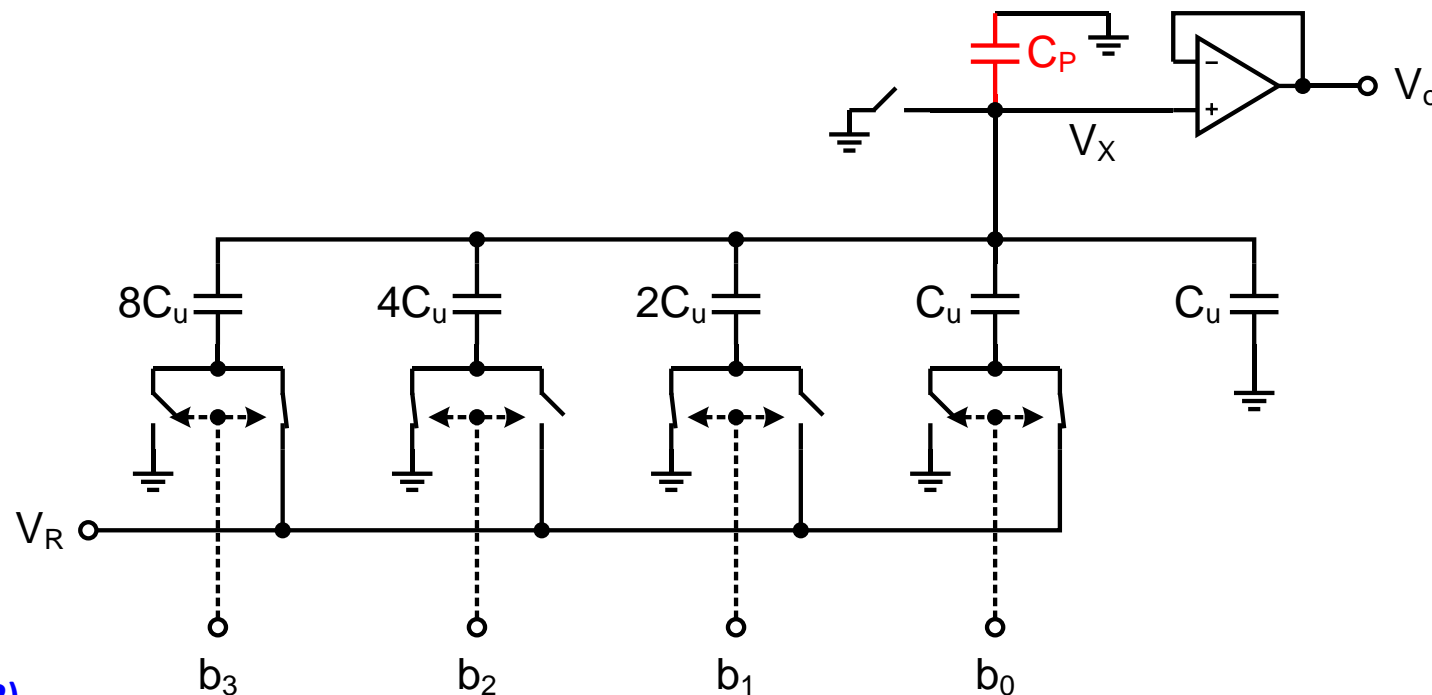
# SC Charge Domain DAC (3)

❑ Stray sensitive architecture

❑ Assume  $m$  unit capacitors are switched from 0 to  $V_R$

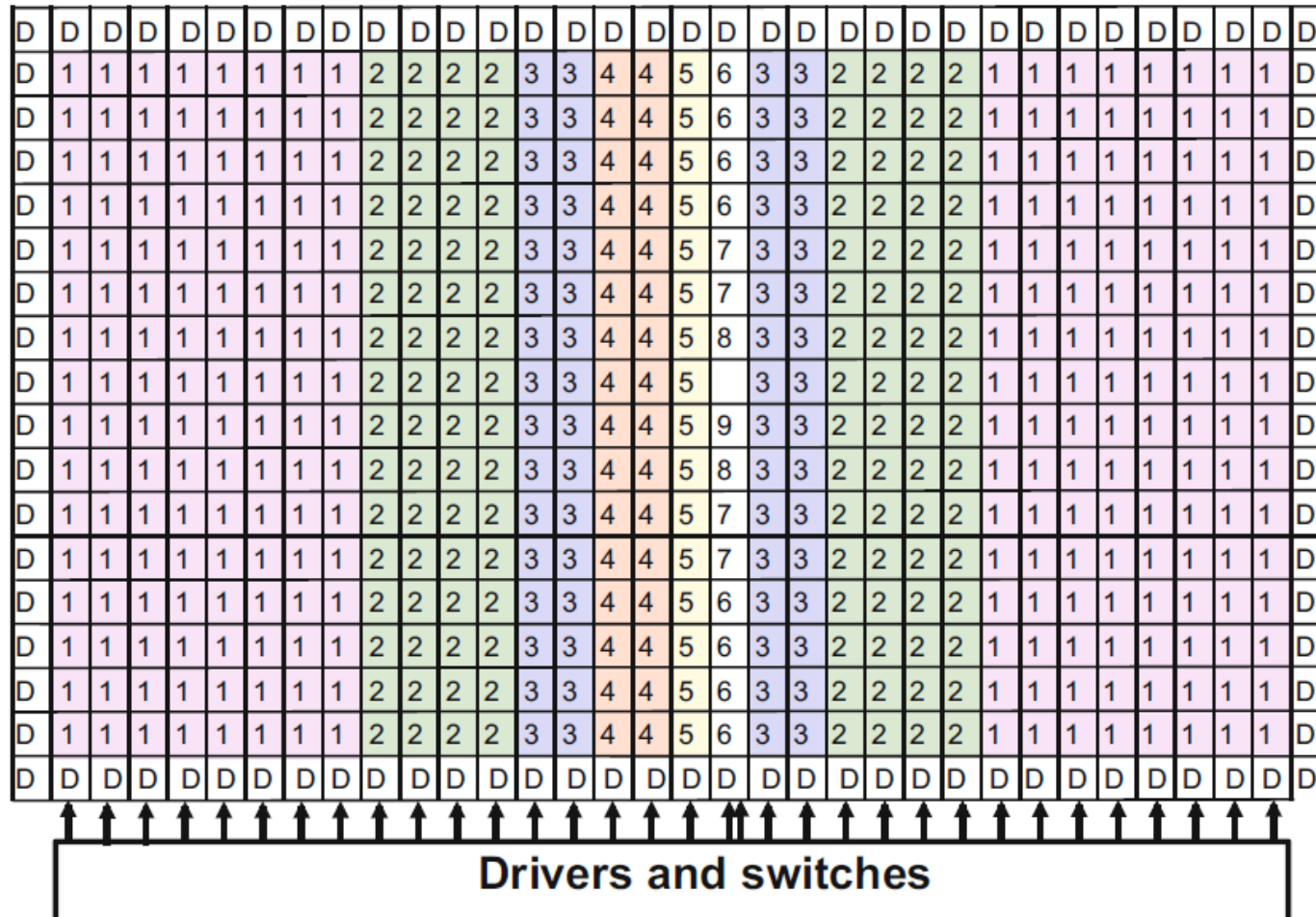
$$V_o = \frac{mC_u}{C_p + nC_u} \cdot V_R = \frac{C_u \cdot \sum_{j=0}^{N-1} 2^j b_j}{C_p + 2^N C_u} \cdot V_R$$

❑  $C_p \rightarrow$  gain error (nonlinearity if  $C_p$  is nonlinear, e.g.,  $C_{in}$  of OTA or CMP)



# Capacitor Array

- ❑ Capacitors can have very good matching properties
  - Main limitation is the exponential growth of the cap array area

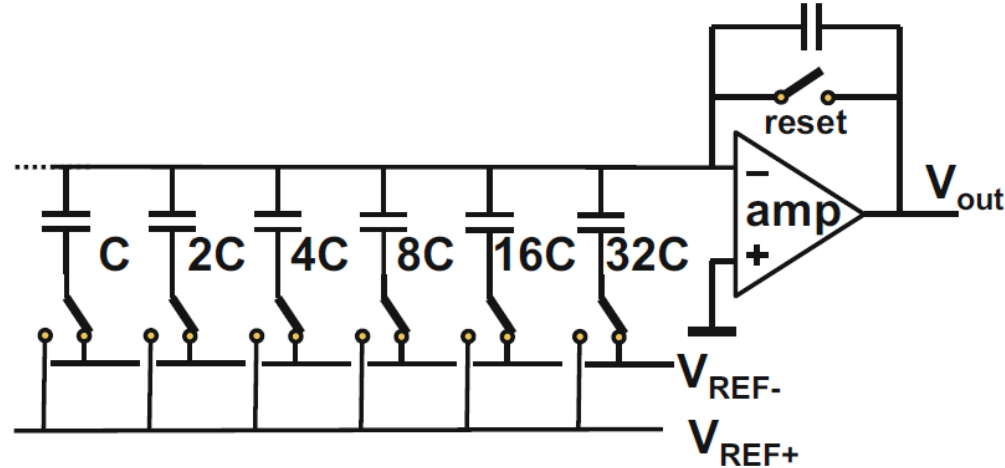


# Capacitor Array

- ❑ Unit capacitors can be few (or sub) femtofarad(s)
  - H. Omran *et al.*, "Matching Properties of Femtofarad and Sub-Femtofarad MOM Capacitors," in *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 63, no. 6, pp. 763-772, June 2016.
  - H. Omran *et al.*, "Direct Mismatch Characterization of Femtofarad Capacitors," in *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 63, no. 2, pp. 151-155, Feb. 2016.
  - P. J. A. Harpe *et al.*, "A 26  $\mu$ W 8 bit 10 MS/s Asynchronous SAR ADC for Low Energy Radios," in *IEEE Journal of Solid-State Circuits*, vol. 46, no. 7, pp. 1585-1595, July 2011.

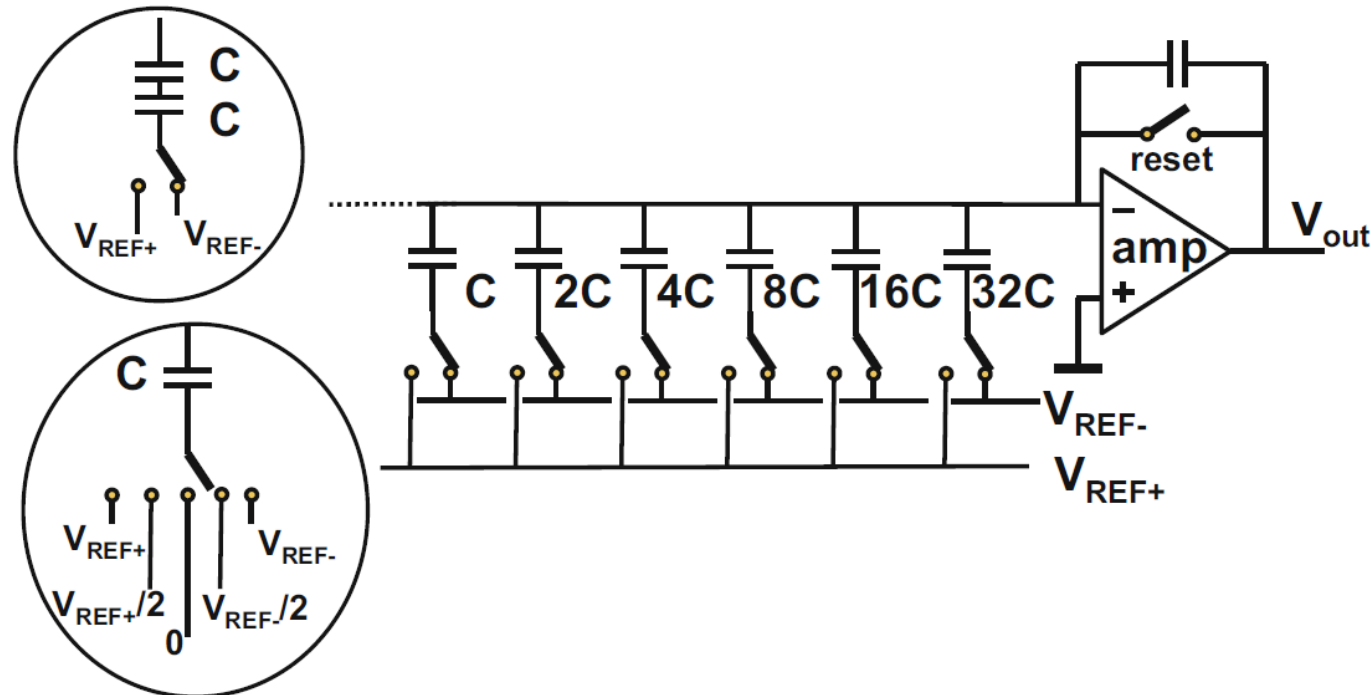
# Capacitor Array

- ❑ Unit capacitors can be few (or sub) femtofarad(s)
- ❑ Alternatives to using very small unit capacitors:
  1. Using capacitors in series (parasitic sensitive)
  2. Using intermediate bottom plate voltages generated by a resistor string DAC (extra power consumption)



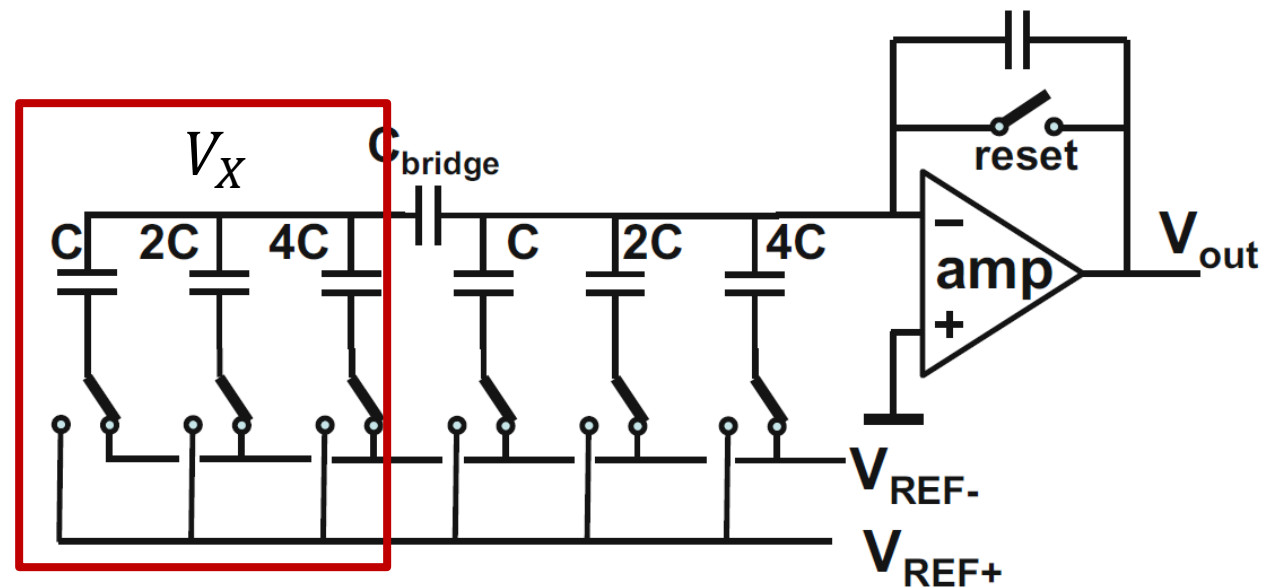
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# Capacitor Array

- ❑ Alternatives to using very small unit capacitors:
  3. Using bridging capacitor (parasitic sensitive)
    - The array to the left of  $C_{bridge} = C$  replaces the string DAC
    - The voltage on the left side plate of  $C_{bridge}$  is set by a capacitive divider

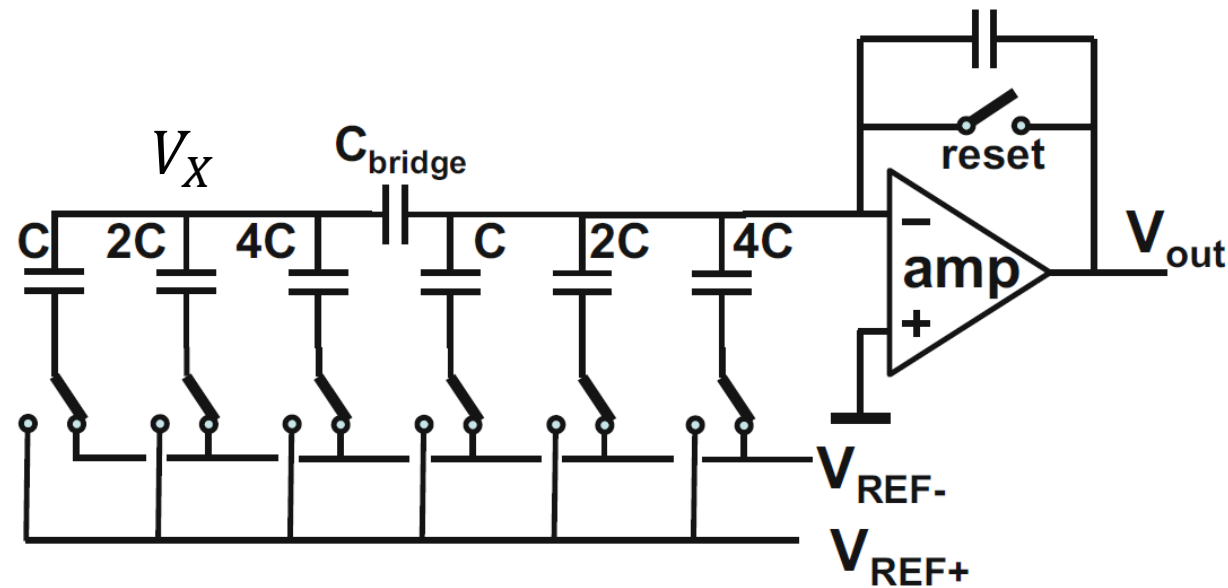




# Bridge Capacitor Example

□ Let  $V_{REF+} = 1V$  and  $V_{REF-} = 0$

- $XXX001 \rightarrow V_X = \frac{C}{C_b + 7C} = \frac{1}{8}$
- $XXX111 \rightarrow V_X = \frac{7C}{C_b + 7C} = \frac{7}{8}$



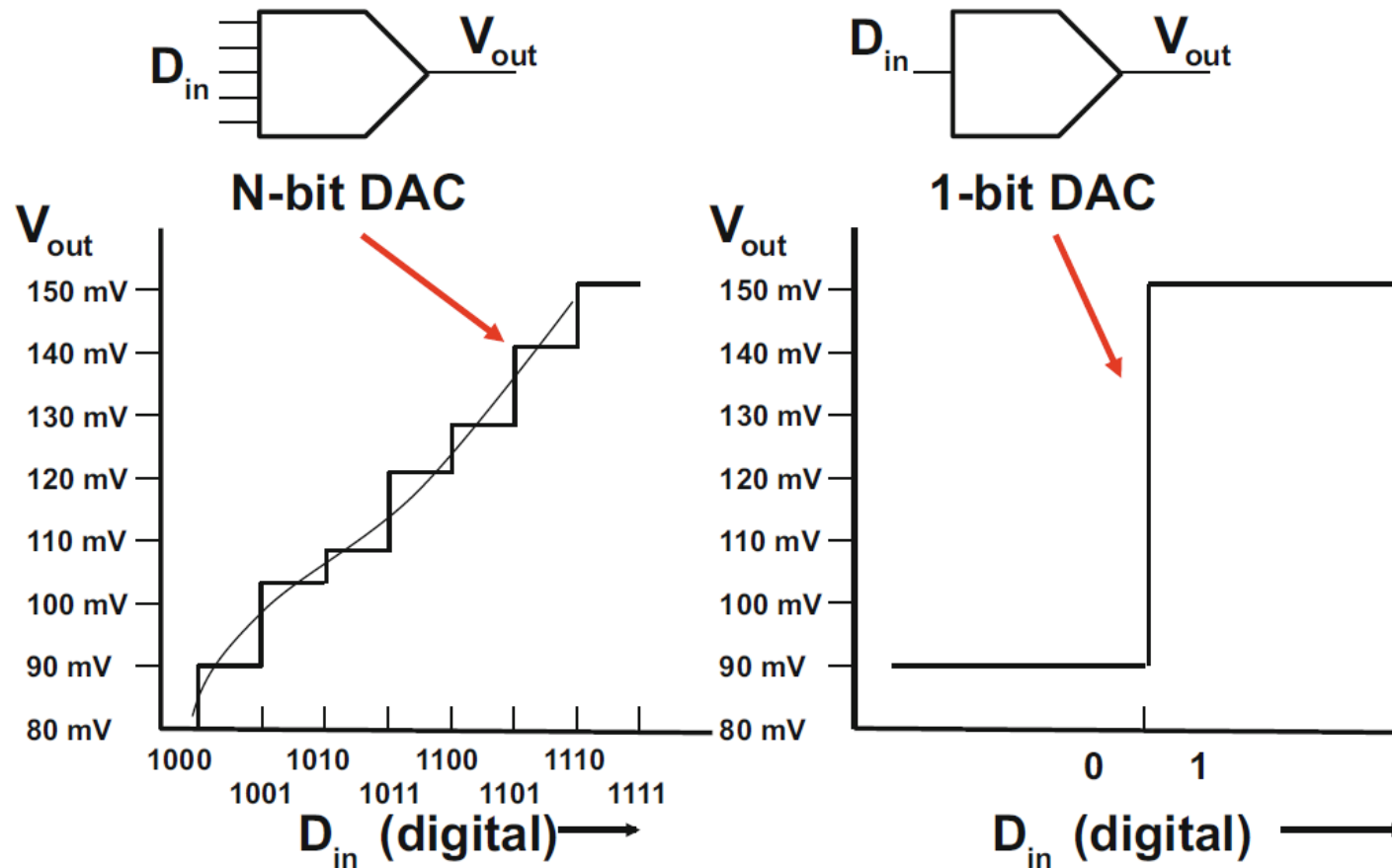
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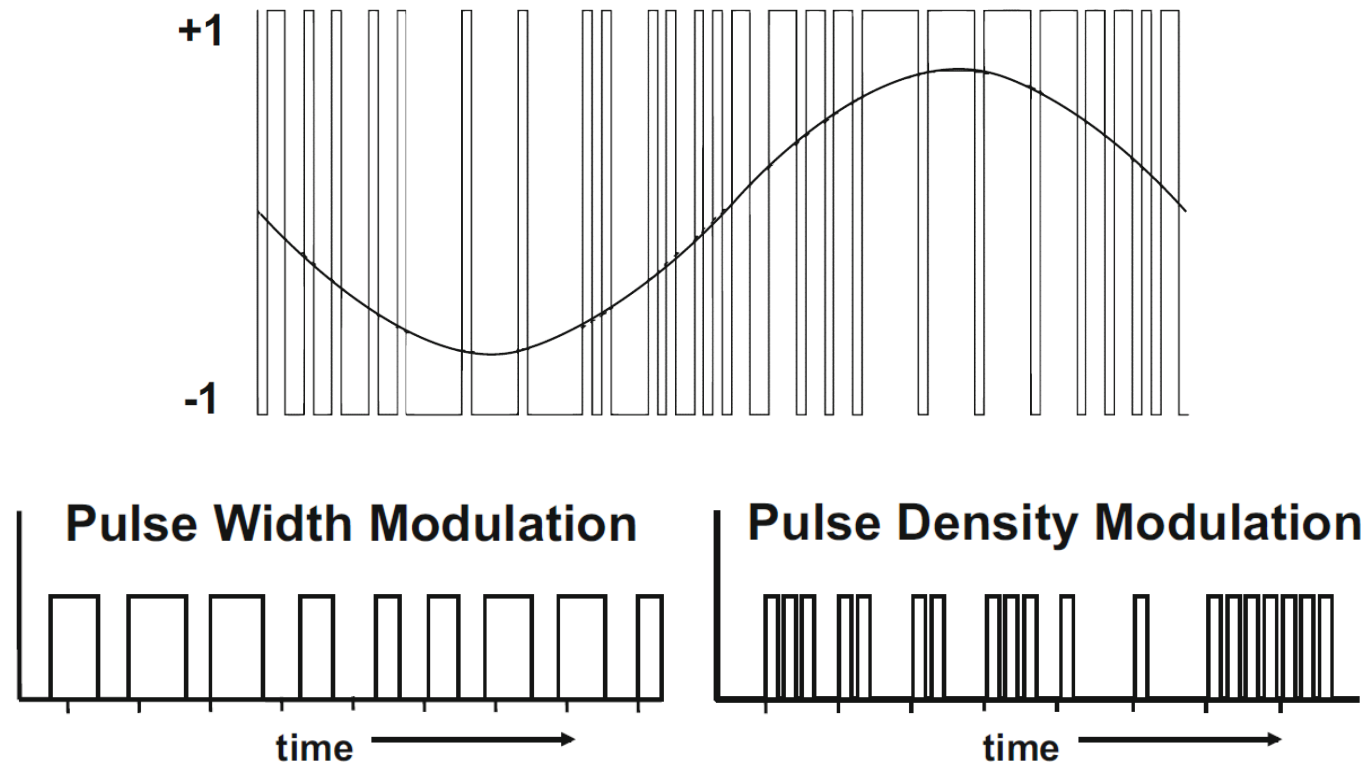
# Single-Bit DAC

- ❑ One-bit DAC has two levels only → It cannot have linearity errors!
  - No matched components are required!



# Single-Bit DAC

- ❑ Use large oversampling ratio (OSR):  $f_s \gg f_{Nyquist}$
- ❑ Pulse width modulation (PWM) or pulse density modulation (PDM)
- ❑ PDM avoids distortion due to asymmetries in falling and rising edges
- ❑ The signal is reconstructed by a LPF

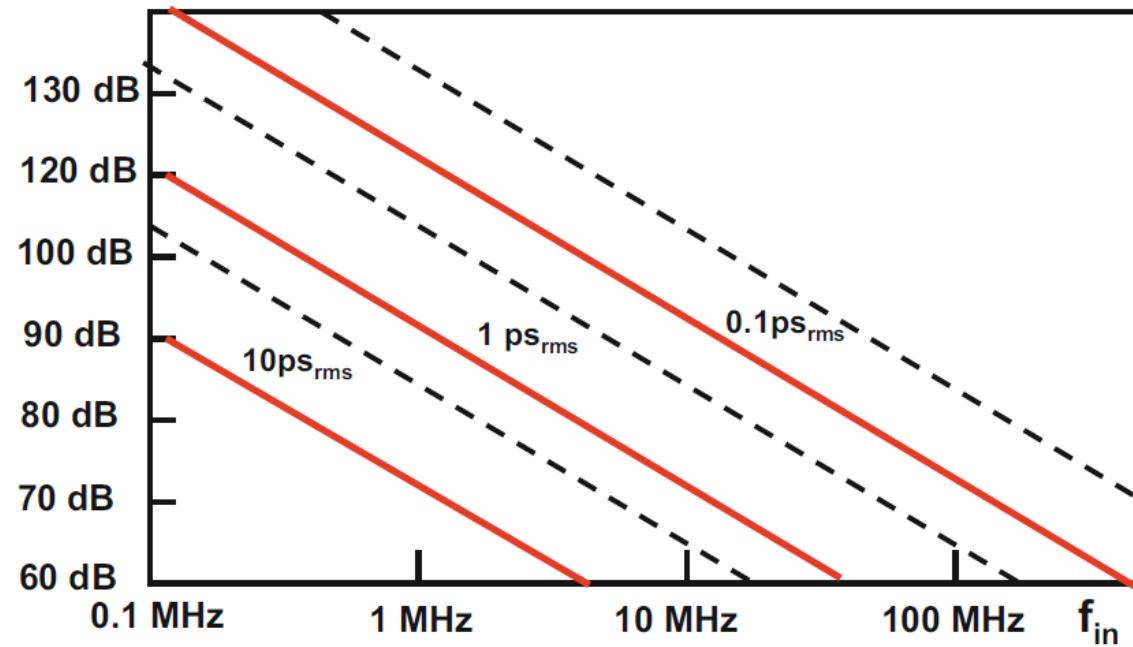


# Jitter-Limited SNR

- Assume single-bit DAC output from  $-A$  to  $A$  with activity factor  $= \alpha$

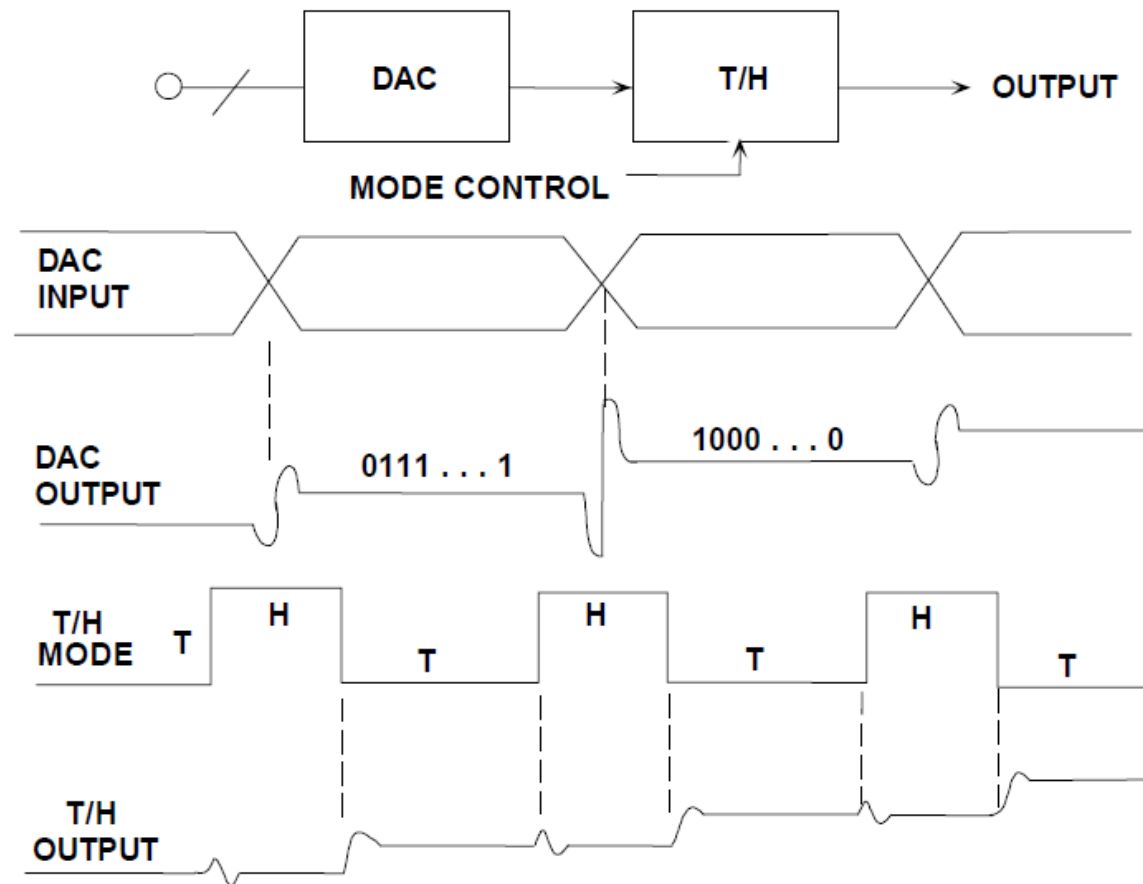
$$SNR = \frac{A^2/2}{\left(2A \times \sigma_{jit} \times \frac{1}{T_s} \times \alpha\right)^2} = \frac{1}{\left(2\sqrt{2}\alpha f_s \sigma_{jit}\right)^2} = \frac{1}{\left(4\sqrt{2}\alpha f_{sig} \sigma_{jit} \times OSR\right)^2}$$

- Solid: single-bit DAC with  $OSR = 5$  and dotted: sampled DAC
  - Single-bit DAC suffers much more from jitter ( $\sigma_{jit}$  is multiplied by pk-to-pk swing)



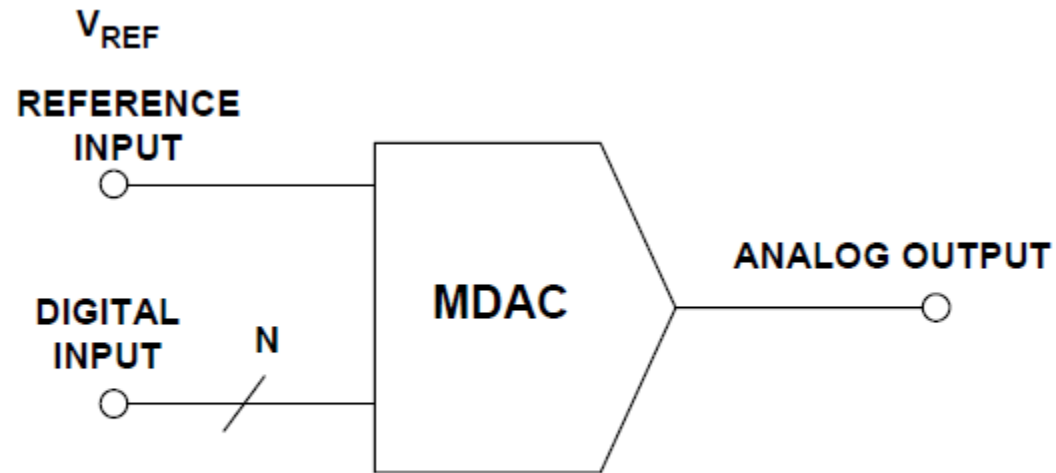
# Deglitching using a Track & Hold

- ❑ DAC switching glitches are isolated from the output
- ❑ T/H transients are code independent and occurs at the clock frequency (can be filtered out)



# Multiplying DAC

- ❑ DAC output is the produce of  $V_{REF}$  and a digital code
- ❑ If  $V_{REF}$  can be a varying signal → multiplying DAC (MDAC)
- ❑ Four-quadrant multiplying DAC: A bipolar DAC with a bipolar  $V_{REF}$



$$\text{ANALOG OUPUT} = V_{REF} \times \text{DIGITAL INPUT} \times \text{CONSTANT}$$

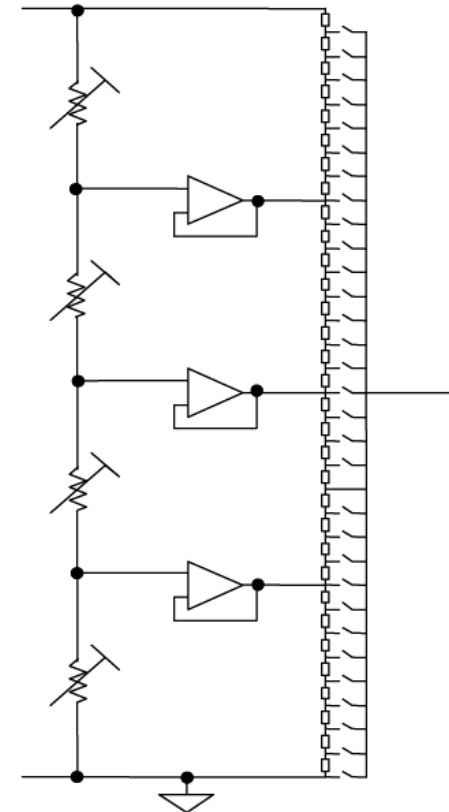
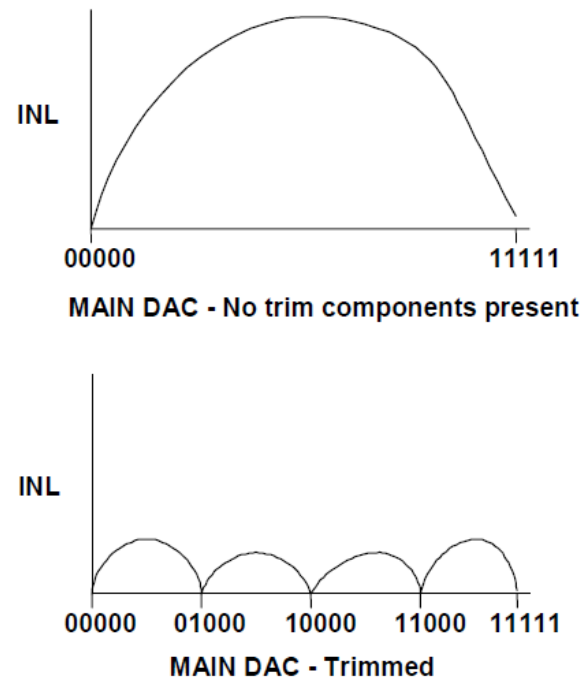
# Improving DAC Accuracy

- ❑ Good design and layout can largely eliminate systematic errors
  - But random errors will limit DAC resolution to  $\sim 12$ -bit
- ❑ Methods to further improve the accuracy
  - Trimming
  - Calibration
  - Dynamic element matching (DEM)
  - ...



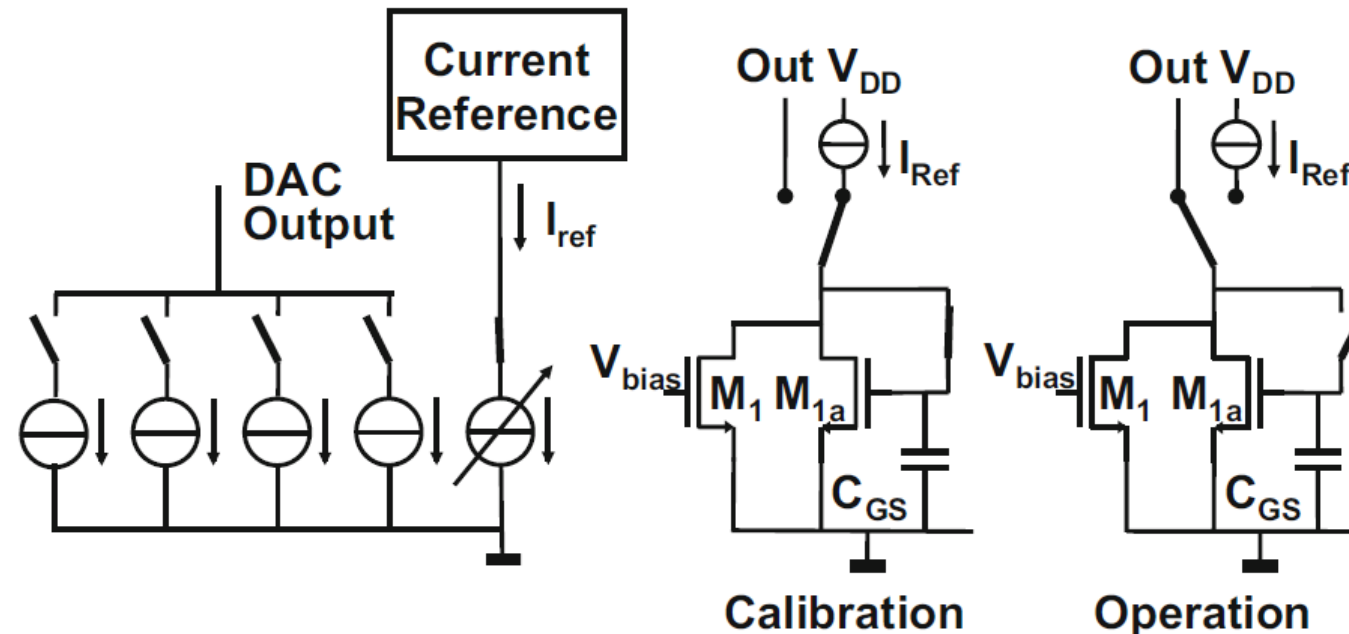
# Trimming

- A 2<sup>nd</sup> resistor string is connected in parallel with the main string
  - The resistors are made physically large enough to laser trim
  - Connected by buffer amplifiers to the  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{3}{4}$  points
  - INL is reduced by a factor of four



# Current Calibration

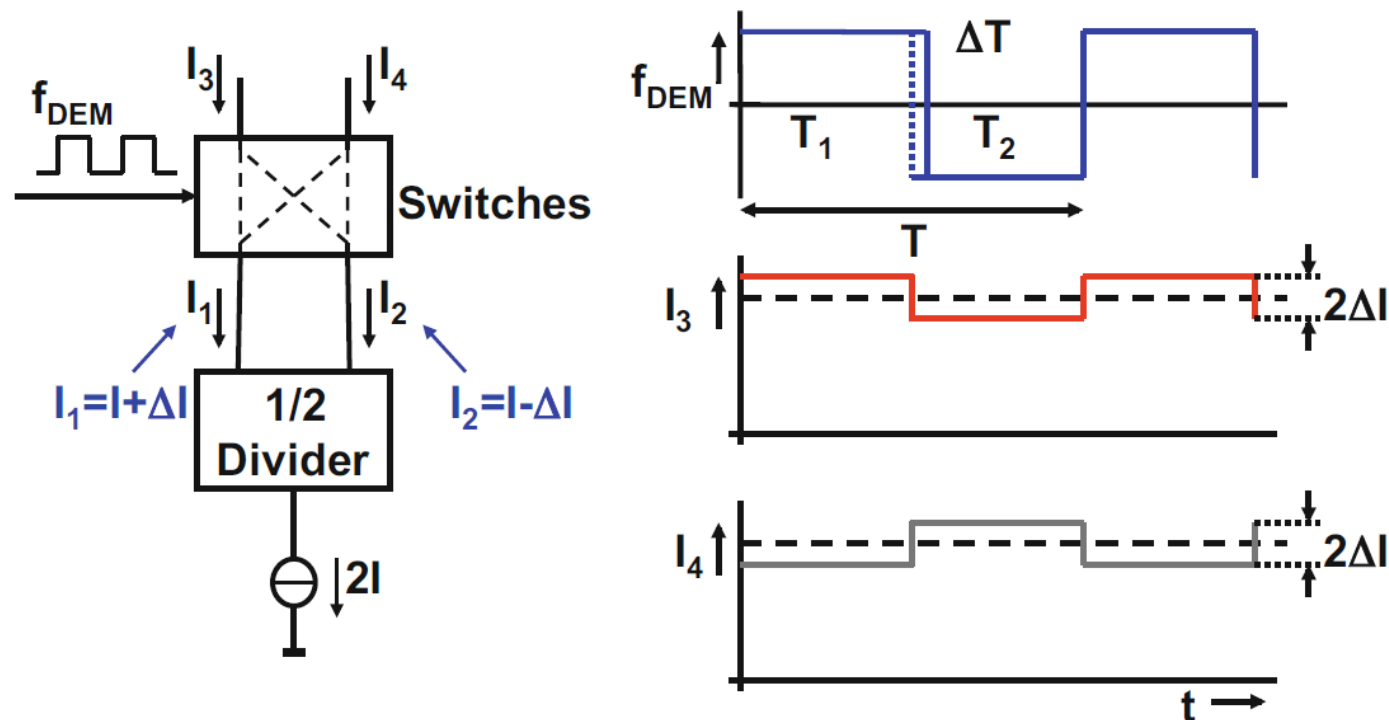
- ❑ M1 supplies the majority of current as a traditional current source
- ❑ M1a is used to calibrate and to mitigate the inherent mismatch
- ❑ The calibration mechanism rotates through the array tuning all current sources
- ❑ The main problem in this arrangement is the sampling of noise in the calibration cycle



# Dynamic Element Matching (DEM)

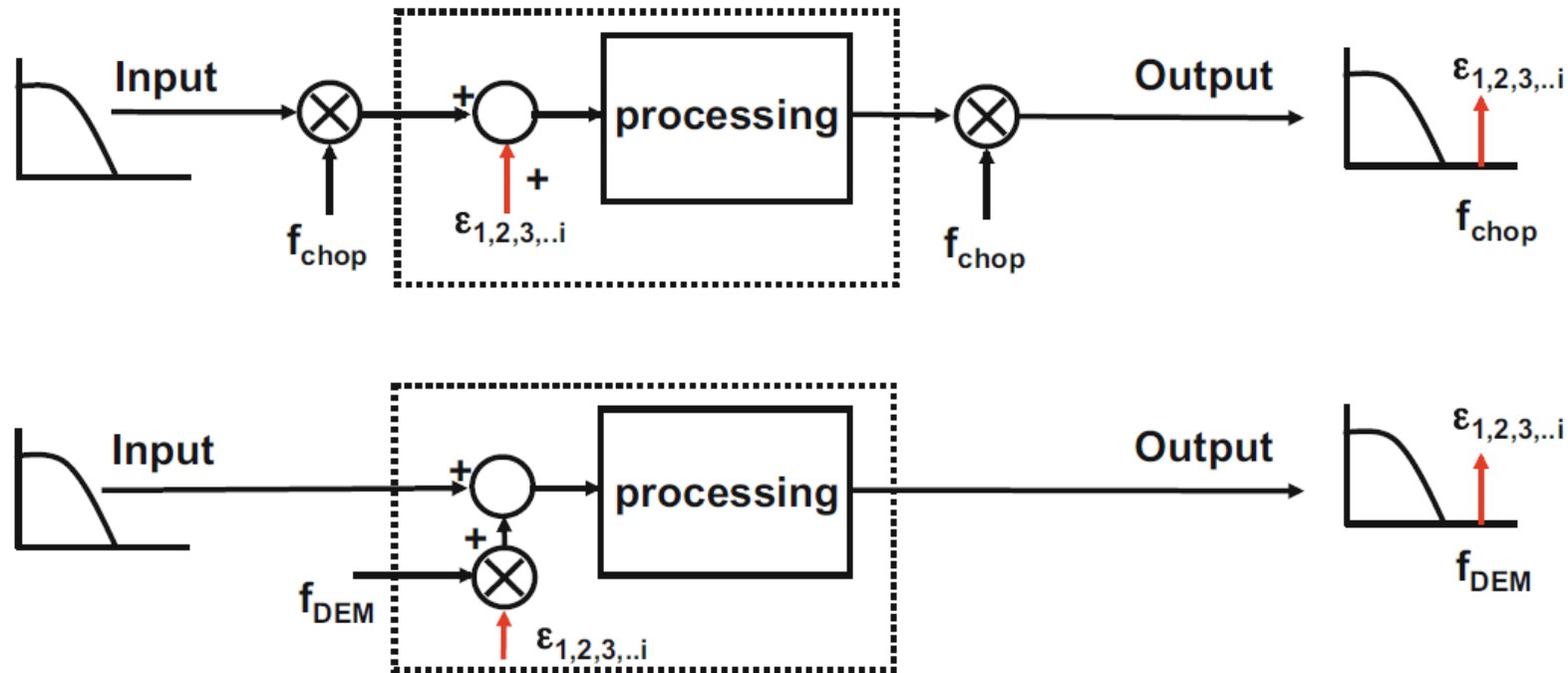
- Unit currents are swapped  $\rightarrow$  Average error tends to zero

$$\frac{I_3 - I_4}{I_3 + I_4} = \frac{I_1 - I_2}{I_1 + I_2} \times \frac{t_1 - t_2}{t_1 + t_2}$$



# Chopping vs DEM

- ❑ A DEM with a fixed swapping frequency generates an error spectrum with tones at the multiples of the swapping frequency
- ❑ Some designs use a pseudorandom swapping sequence to spread out the error energy over the spectrum (mismatch scrambling)
- ❑ Mismatch shaping can be also applied



# References

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- ❑ M. Pelgrom, Analog-to-Digital Conversion, Springer, 3<sup>rd</sup> ed., 2017.
- ❑ W. Kester, The Data Conversion Handbook, ADI, Newnes, 2005.
- ❑ B. Boser and H. Khorramabadi, EECS 247 (previously EECS 240), Berkeley.
- ❑ B. Murmann, EE 315, Stanford.
- ❑ Y. Chiu, EECT 7327, UTD.

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**Thank you!**