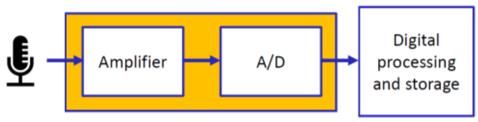
# Analog Systems Design

## 1. Introduction

1. Analog Vs Digital
   - Analog : Continuous in Time and Amplitude
   - Digital : Discrete in Time and Amplitude

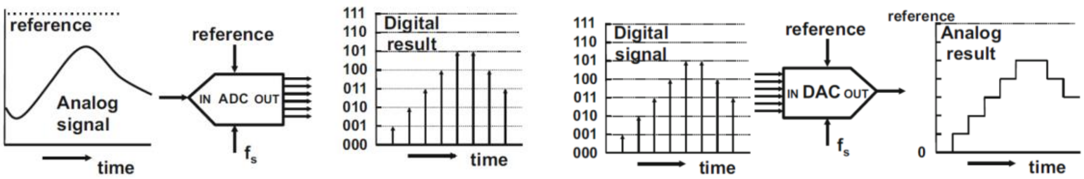
2. Why Digital ?
   - Less sensitive to noise
   - Easier to store (Digital Memories)
   - Easier to process (Digital Signal Processing DSP)
   - Amenable to automated design and testing
   - Direct beneficiary of Moor's law

3. Why Analog ?
   - All the physical signals in the world around us are analog
   - We always need an analog interface circuit to connect between our physical world and our digital electronics




4. ADC Functions
   - Convert analog signal (Continuous in time and amplitude) to a digital signal (Discrete in time and amplitude) by
     - Sampling : Discretization of analog signal in time domain
     - Quantization : Discretization of analog signals in amplitude domain
     - Linking to a reference ($V_{FS}$)

5. DAC Functions
   - Convert digital signal (Discrete in time and amplitude) to an analog signal (Continuous in time and amplitude) by
     - Amplitude Restoration : Convert digital levels to voltage amplitude
     - Holding : Holding voltage amplitude to convert signal form DT to CT
     - Linking to a reference ($V_{FS}$)



# 2. Sampling

1. Sampling introduction

- Sampling is time discretization
    - Converts a continuous time signal to a discrete time signal
    - The result is a sequence of samples
- The Sampling instants are defined by a clock signal controls an electronic switch e.g. MOS
- The sampled signal is stored as a voltage on a capacitor
- The circuits is called sample and hold S/H circuit

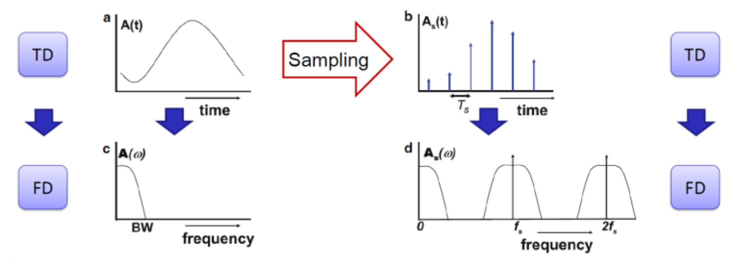
2. Time and Frequency domains

- $TD_{step} = \Delta t = 1/f_s = 1/\text{FD period}$
- $FD_{step} = \Delta f = 1/T_o = 1/\text{TD period}$

| Time domain | | Technique | Frequency domain | | Where in the chain? |
|---|---|---|---|---|---|
| CT/DT | Periodic | ↔ | C/D | Periodic | |
| CT | Yes | CT Fourier series (CTFS) | Discrete | No | - |
| CT | No | CT Fourier transform (CTFT) | Continuous | No | Before S/H |
| DT | Yes | DT Fourier series (DTFS) → FFT | Discrete | Yes | After ADC |
| DT | No | DT Fourier transform (DTFT) | Continuous | Yes | After S/H |

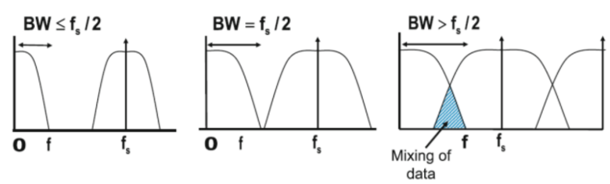
3. Discrete and periodicity

- Sampling causes "images" in the frequency domain
    - The sampled signal is folded around fs and its multiples
    - The part from 0 to fs/2 is the only part that has a physical meaning




4. Aliasing and Nyquist criterion

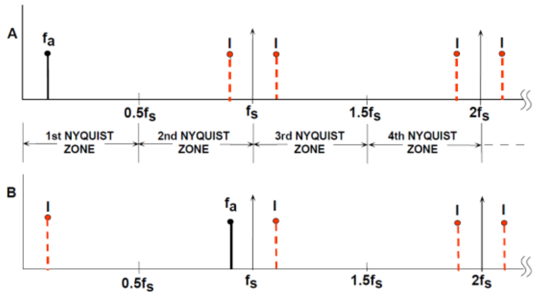
- Aliasing is an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled
- Nyquist criterion $f_s \geq f_{nyq} = 2\,BW$



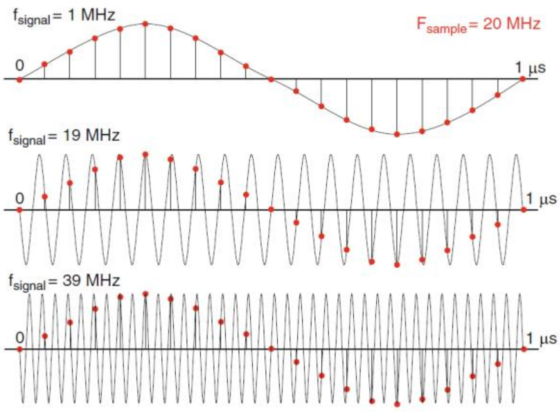
# Analog Systems Design

## 2. Sampling

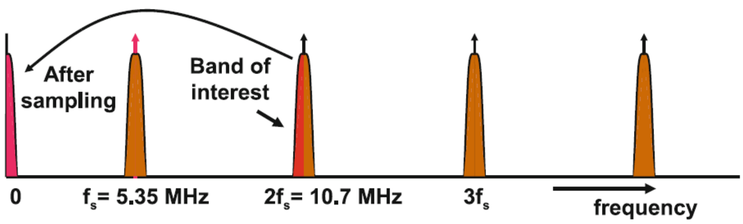
1. Aliasing and subsampling in Frequency / Time domain
   - The sampled signal does not have to be a base-band signal
     - Images appears at $|\pm k f_s \pm f_a|$ where $k = 0,1,2,\ldots$
   - If $f_s < 2f_{max}$ under sampling (band-pass sampling) is happening
   - The signal with $f_{max} = f_a$ in fig. B is down converted to a base-band signal as $f_s < 2f_{max}$



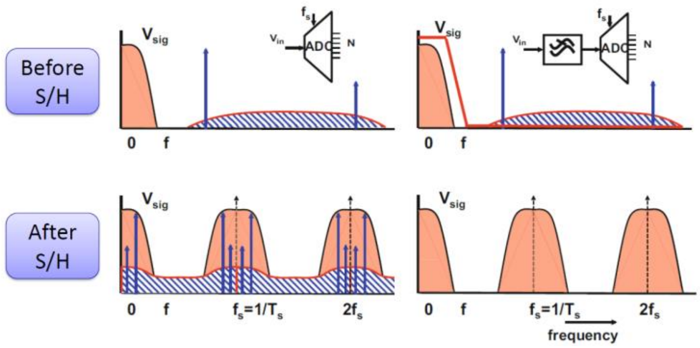

   - The subsampling can be used to demodulate (down convert) an RF signal
   - Samples of a 1 MHz signal is identical to a 19 MHz signal and 39 MHz Signal as they are sampled with $f_s = 20$ MHz where at 19 and 39 MHz signals $f_s < 2f_{max}$




   - Example: FM signal of 100 kHz bandwidth at 10.7 MHz, The signal is down converted by a …….. MHz sampling clock
     - As the signal appears at fs and its multiples and its image at fsignal = 2fs = 10.7 MHz, so fs = 5.35 MHz



2. Anti-Aliasing Filter (AAF)
   - To ensure the Nyquist criterion, we must ensure that the signal is band-limited signal, the problem is that, there is a duality between the time and band limited signals, as any time limited signal is band unlimited and vice versa
   - All real signals are time limited, so we need these signals to be limited at frequency domain to ensure the Nyquist criterion and avoid aliasing
   - This can be achieved by Anti-Aliasing Filters (AAF)
   - AAF can be
     - Active filters or Passive filter
     - Continues time or Discrete time
     - Low pass filters (for base band signals) or Band pass filters (for band pass filters)




   - Example: Signal band: 33 MHz to 39 MHz, Which sampling rate to choose to subsample the signal?

     a) 78 MHz : fs = 2 fmax so not a valid option
     b) 39 MHz : fs < 2 fmax so, it is a valid option
     c) 19.5 MHz : fs < 2 fmax so, it is a valid option
     d) 13 MHz : fs < 2 fmax so, it is a valid option

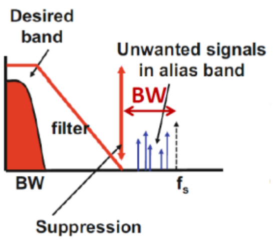
     Option b ) has easier AAF requirements as it has a large relaxation region but its higher sampling rate means higher power consumption and higher memory
     Option d ) needs a very complex AAF but with lower power consumption

     I choose option b) as the technology scaling tends to move all the complexity to digital domain

# Analog Systems Design

## 2. Sampling

1. Alias Band Suppression: What is the suppression requirements of AAF?
   - Working at the limit of Nyquist criterion requires an ideal filter that does not exist
   - Signal in the alias band will alias in the desired signal band after sampling so, it must be suppressed by AAF

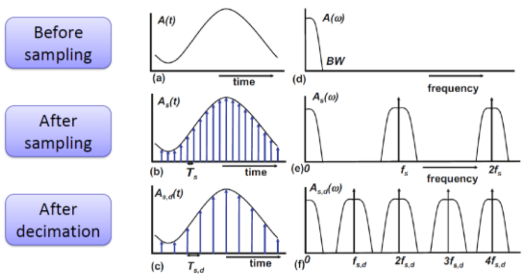



   - Each pole gives a roll off 20 dB/decade so if $f_s/BW = 2$ for a 4th order filter the suppression will equal?

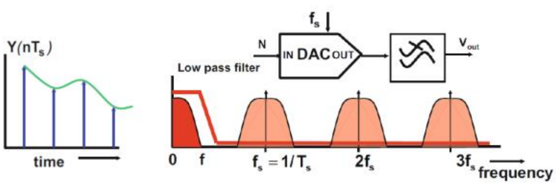
$$Aliasing\ band = fs - BW\ to\ f_s = f_s/2 = BW$$
$$@\ BW\ each\ pole\ gives\ 3\ dB \rightarrow for\ a\ 4^{th} order\ filter\ the\ supression = 4 * 3 = 12\ dB$$

2. Oversampling
   - Oversampling relaxes the requirements of baseband anti-aliasing filter
   - Higher relaxation zone means less pole numbers, simpler design, lower power consumption

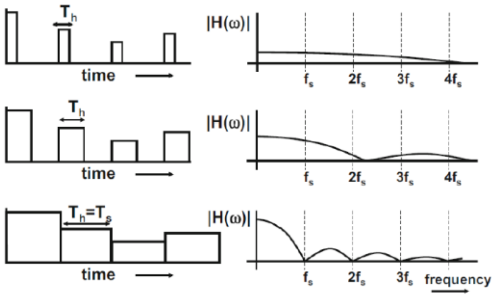
3. Decimation
   - Decimation is the process of reducing the sample rate of a sampled signal
   - Unless the original signal is already filtered and oversampled, digital filtering is necessary
   - We use decimation instead of using low sample rate from the beginning because oversampling makes filtering easy



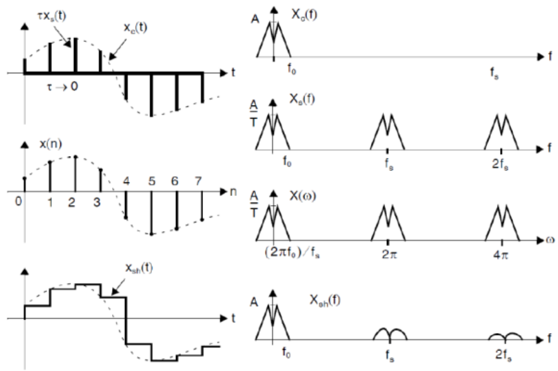

4. Reconstruction filter
   - As ADC needs an AAF also the DAC needs a reconstruction filter (smoothing filter) which:
     - In TD: interpolate / restore / reconstruct the original signal
     - In FD: reconstruction filter suppress the images



5. Zero Order Hold ( ZOH )
   - Keeps the value of the signal at the sample moment
   - The Fourier transform of ZOH is a sinc function $sinc(x) = \sin(x)/x$ where the nulls of $sinc(x)$ at the inverse of the fold time




   - The ZOH performs inherent reconstruction (filtering out images)



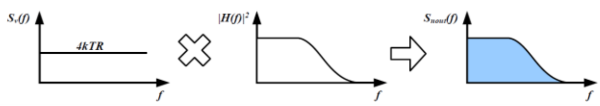

   - Passband droop: ZOH suppresses images but introduce amplitude distortion
     - The passband distortion may be compensated by inverse sinc response in digital or analog domain

6. Noise in RC circuit
   - Resistors generate white thermal noise but the BW is always limited by the capacitor

$$S_{nout}(f) = S_v(f)\left|\frac{V_{nout}(j\omega)}{V_n(j\omega)}\right|^2$$

$$\overline{V_{nout}^2} = V_{noutrms}^2 = \int_{-\infty}^{\infty} S_{nout}(f)\,df$$

$$\boxed{\overline{V_{nout}^2} = \frac{kT}{C}}$$




   - RMS noise is independent of R (why?)
     $R\uparrow, S_v(f) = 4kTR\uparrow$, but the $BW = 1/RC \downarrow \rightarrow$ area under the curve $S_{nout}(f)$ remains constant

$$V_{nrms} = \sqrt{\frac{1p}{C}} * 64\ uV_{rms}$$

   - Equivalent noise bandwidth: define an equivalent noise BW ($B_N$) such that the area under a brick-well response is the same area under the actual spectral density curve
   - For a first order system: $B_N = 1/4RC = \pi/2\,f_{pole}$

7. Sampling noise
   - The sampling capacitor determines noise power, SNR, and the No. of ADC bits
   - C ↑, noise ↓, SNR ↑, but BW (speed) ↓ @ the same power consumption

| $C_{hold}$ | $V_{nrms} = \sqrt{\frac{kT}{C}}$ at $T = 300\ K$ | SNR (assume $V_{sigrms} = 1\ Vrms$) | No. of bits (see next lecture) |
|---|---|---|---|
| 100 fF | 203 $\mu$Vrms | 74 dB | 12-bit |
| 1 pF | 64 $\mu$Vrms | 84 dB | 13.7-bit |
| 10 pF | 20.3 $\mu$Vrms | 94 dB | 15.4-bit |

8. Noise folding
   - As sampling folds the signal it is also folds the noise

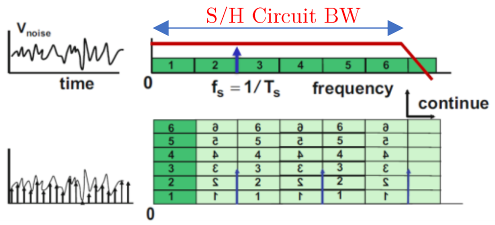
Before sampling: $P_n = kT/C = S_n(f) \times B_N$

After sampling $P_n$ is unchanged: $P_n = kT/C = S_{n,sampled}(f) \cdot \frac{f_s}{2}$

$$S_{n,sampled}(f) = \frac{kT}{C} \times \frac{2}{f_s} = S_n(f) \times B_N \times \frac{2}{f_s}$$

$$S_{n,sampled}(f) = S_n(f) \times \frac{2B_N}{f_s} = S_n(f) \times \frac{\pi BW}{f_s}$$

Noise power is unchanged, but noise density increases (noise folding).




   - BW of the S/H circuit cannot be small because if

$$BW_{SH} \downarrow, \tau = RC \uparrow, \text{S/H Cap charges slowly} \rightarrow \text{slow S/H response}$$