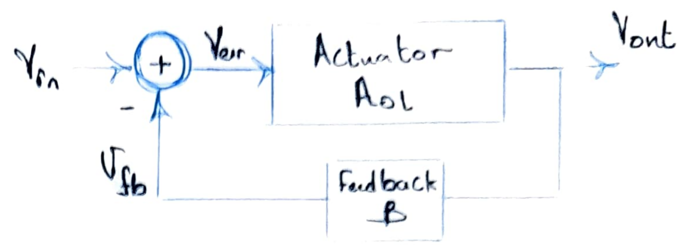
Analog Integrated Circuits.

lec 15: Negative feedback System.

① General feedback system.



∴ 1. $V_{out} = V_{err} A_{OL}$

2. $V_{err} = V_{in} - V_{fb}$

3. $V_{fb} = V_{out} \beta$

∴ $V_{out} = (V_{in} - V_{out}\beta) A_{OL}$

∴ $V_{out}(1 + \beta A_{OL}) = V_{in} A_{OL} \rightarrow \dfrac{V_{out}}{V_{in}} = A_{CL} = \dfrac{A_{OL}}{1 + \beta A_{OL}}$

4. $\beta A_{OL} = LG \gg 1 \rightarrow A_{CL} \approx \dfrac{1}{\beta}$

∴ A_{CL} is independent of A_{OL}

5. $V_{err} = V_{in} - V_{fb} = V_{in} - \beta V_{out} = V_{in} - \beta A_{OL} V_{err}$

∴ $V_{err} = \dfrac{V_{in}}{1 + \beta A_{OL}} = \dfrac{V_{in}}{1 + LG}$

∴ if $LG \gg 1 \rightarrow V_{err} \simeq 0$

∴ Negative feedback loop works to minimize the error sig.

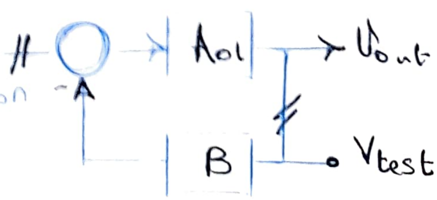
② Calculation of Loop Gain

1. deactivate the input → 2. Break the loop → 3. Apply V_{test}

then calculate the gain around the loop

$V_{out} = -\beta A_{OL} V_{test} \rightarrow -\dfrac{V_{out}}{V_{test}} = \beta A_{OL}$

③ Why Negative feedback



Negative feedback gives
→ Gain Desensitization
→ BW extension
→ Modification of I/O impedances.

3.1 Gain Desensitization.

1. $A_{CL} = \dfrac{A_{OL}}{1+\beta A_{OL}} \rightarrow A_{CL} = \dfrac{1}{\beta}\left(\dfrac{1}{\frac{1}{\beta A_{OL}}+1}\right) = \dfrac{1}{\beta}\left(1-\dfrac{1}{\beta A_{OL}}\right) = \dfrac{1}{\beta}\left(1-\dfrac{1}{LG}\right)$

where if $x \approx 0 \rightarrow \dfrac{1}{1+x} = 1-x$

2. static gain error

$$\epsilon_s = \frac{|A_{CL,\,ideal} - A_{CL\,actual}|}{A_{CL\,ideal}} \approx \frac{1}{\beta A_{OL}} = \frac{1}{LG}$$

- A_{OL} Varies due to PVT, Load, and input signal Var.
- A_{CL} almost independent of A_{OL} (if $LG \gg 1$)

$\therefore$ A_{CL} is independent of → PVT, → Load, → input Level

- Altough A_{CL} depends on β but β is insensitive to Variations as it's a ratio between matched Component

3.2 BW extension.

- Assume A_{OL} is a 1st order System

$$A_{OL}(S) = \frac{A_{OL_0}}{1+\frac{S}{\omega_{P_{OL}}}}$$

- The Closed Loop TF is also a first order System

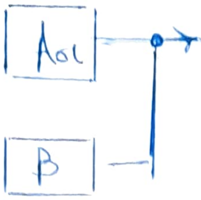
$$A_{CL} = \frac{A_{OL}(S)}{1+\beta A_{OL}(S)} = \frac{\frac{A_{OL_0}}{1+\beta A_{OL_0}}}{1+\frac{S}{(1+\beta A_{OL_0})\,\omega_{P_{OL}}}} = \frac{A_{CL_0}}{1+\frac{S}{\omega_{P_{CL}}}}$$

$\therefore \omega_{P_{CL}} = (1+LG_0)\,\omega_{P_{OL}}$ ☹

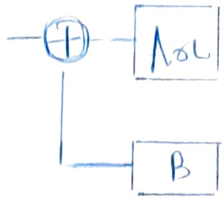
- The DC gain reduced by $(1+LG_0)$
- The BW extend by $(1+LG_0)$ ☺

→ GBW remains Const

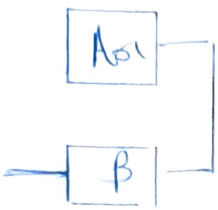
3.3 Modification of I/O impedances



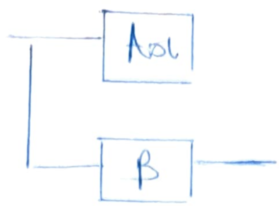
Shunt (Voltage) Sensing



Series (Voltage) mixing



Series (Current) Sensing



Shunt (Current) mixing

- Shunt Sensing / mixing → Reduce R by (1+LG)
- Series Sensing / mixing → Increase R by (1+LG)

1. The Price we pay to buy feedback useful properties is
   - → Gain reduction
   - → The risk of instability.

(4) Stability of feedback system

1. $$A_{CL}(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{A_{OL}(s)}{1+\beta A_{OL}(s)}$$

If $\angle \beta A_{OL}(s) = -180$ → the negative feedback turns to Positive feedback (oscillators)

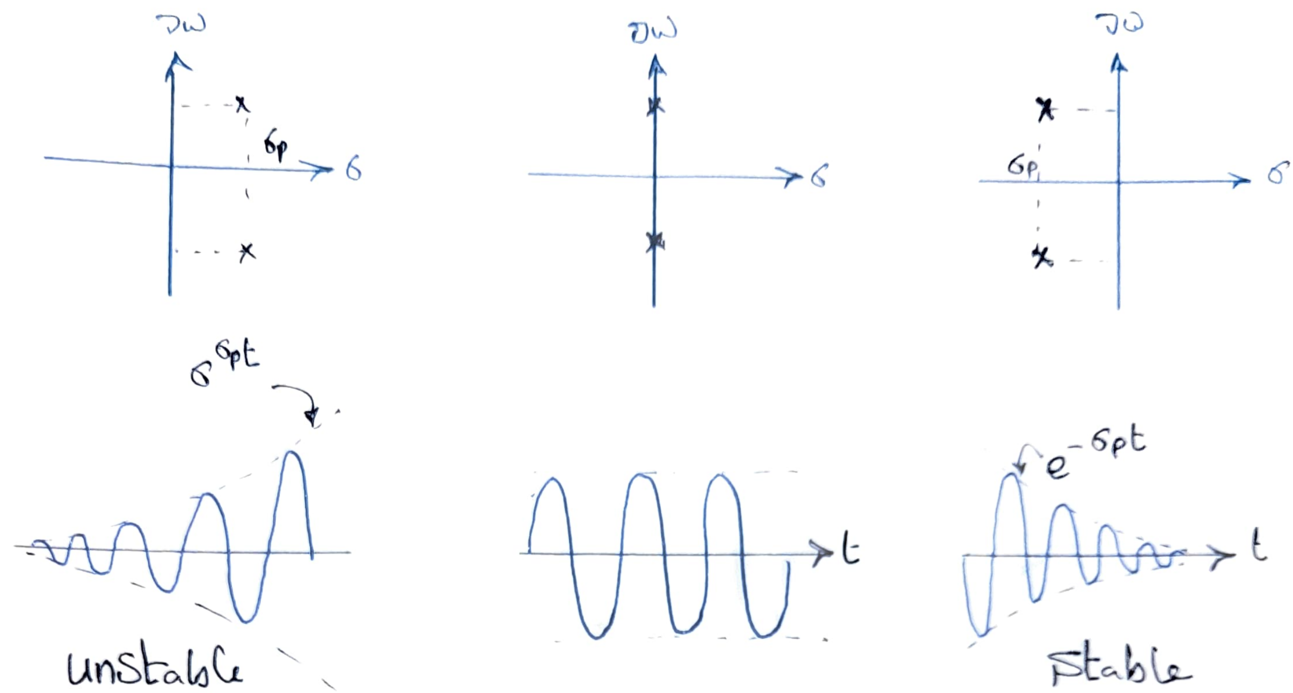
2. Barkhausen's oscillation Criteria

$$|\beta A_{OL}| = 1 \quad \text{and} \quad \angle \beta A_{OL}(s) = -180$$

(5) Stable Vs Unstable System : Bode plot

1. from oscilation Criteria :

We want : $|LG| = 1$ and the $\angle LG < -180^\circ$



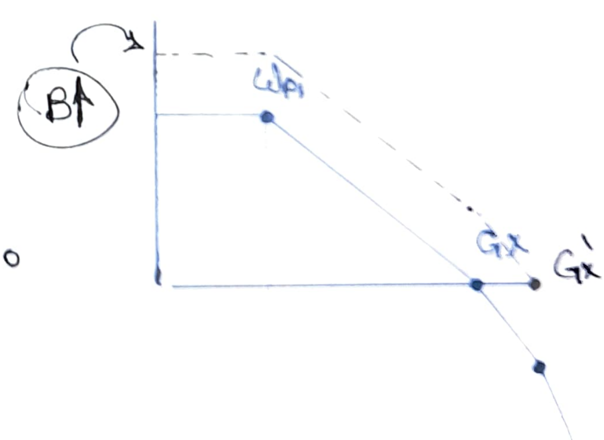
(Laplace domain) $\dfrac{1}{S-a} \longrightarrow e^{at}$ (time domain)

2. Gain Crossover (G_x)

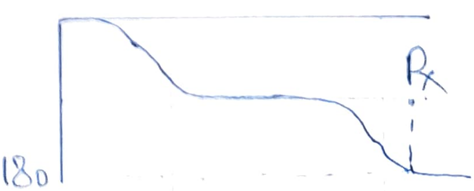
is the freq which the $|LG| = 1$

3. Phase Crossover (P_x)

is the freq which the $\angle LG = -180$



$$\boxed{\begin{array}{c}\text{For a stable System} \\ G_x < P_x\end{array}}$$



(6) The effect of B on Stability.

1. assume B is freq independent

$\therefore \angle B A_{ol}(s)$ is independent of B $\rightarrow P_x$ is $\neq f(B)$

2. $B\uparrow, |LG|\uparrow, G_x\uparrow \rightarrow$ Bad for stability ☹

3 Worst Case stability @ $\beta = 1 \rightarrow \beta A_{OL} = A_{OL}$

∴ Unity gain feedback → buffer

∴ Smallest A_{CL}

4 β only affects Gx of 1G and does not affect Poles/Zeros

(7) Two-Pole System.

1 If poles are always @ LHP

∴ System will be unconditionaly stable but it may suffer from peaking and ringing

$$A_{CL} = \frac{A_{OL}}{1+\beta A_{OL}} = \frac{A_{CL_0}}{1+\frac{1}{Q}\frac{s}{\omega_0}+\frac{s^2}{\omega_0^2}} = \frac{A_{CL_0}}{1+2\eta\frac{s}{\omega_0}+\frac{s^2}{\omega_0^2}}$$

Quality factor — damping factor

η
- > 1 ($Q < 0.5$) → over damped system
- $= 1$ ($Q = 0.5$) → Critical damped
- < 1 ($Q > 0.5$) → under damped → over shot in step resp.

2 Bode plot

- If Phase shift always < 180°

∴ System will be unconditionaly stable

- For under damped system ($\eta < 0.707$)

↳ Peaking in freq response → Ringing in step response.

- $\beta\uparrow$, $Q\uparrow$ → overdamped goes to under damped

- PM ↓, Peaking ↑, Ringing ↑

(8) Phase margin and Gain margin.

$$PM = 180^\circ - |\angle BA_{OL}(G_x)| = 180^\circ - \tan^{-1}\left(\frac{\omega_u}{\omega_{p_1}}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p_2}}\right)$$

$$GM = 0 - 20\log|BA_{OL}(P_x)|$$

If $\omega_{p_2} \gg \omega_{p_1}$

$\therefore PM = 90^\circ - \tan^{-1}\left(\frac{\omega_u}{\omega_{p_2}}\right) \rightarrow$

PM > 0 → stable System
But Small PM means We have:
Freq domain Peaking → noise amplification
time domain ringing → Poor settling T

If $\omega_{p_2} = \omega_u$

$\therefore PM = 45^\circ \longrightarrow$ typically inadequate (Peaking | ringing)

Thus We need $\omega_{p_2} > \omega_u$

$$\boxed{\therefore\ \omega_{p_1} \ll \omega_u < \omega_{p_2}}$$

ω_{p_2} → define max GBW or max CL BW

ω_{p_1} → define OL BW

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max CL BW without Peaking occurs at $\eta = Q = 0.707$

Actual ω_u slightly < ω_{u1}
$\therefore$ UGF < GBW

ω_{p_2}/ω_{u1}	Q	ω_u/ω_{u1}	PM	notes.
1	1	0.786	51.8°	
# 2	0.707	0.91	65.5	max flat But $\eta < 1$ ∴ under damped ∴ over shoot exist in transient response
⋮	⋮	⋮		
# 4	0.5	0.972	76.3	Critical damped → fastest settling without overshoot
⋮				
∞	–	1	90°	1st order System