Analog Integrated Circuits.

Chapter 6: Frequency Responce.

① Poles and Zeros.

- Any System Transfer function $H(s) = \frac{N(s)}{D(s)}$
- Zeros: roots of numerator.

  Poles: roots of denominator

- Freq Response: $s \to j\omega$

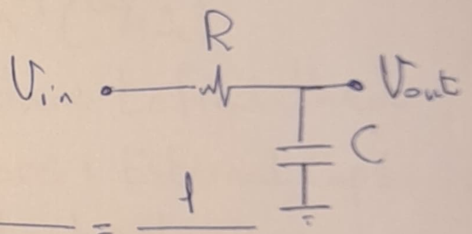
$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = |H(j\omega)| e^{j\phi}$$

$*$ Mag of $a + jb = r = \sqrt{a^2 + b^2}$

$*$ Phase of $(a + jb) = \phi = \tan^{-1} \frac{b}{a}$

- Example 1

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$



$$= \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + s\tau}$$

$$\rightarrow H(j\omega) = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

↳ $\tau = RC$ = time Constant

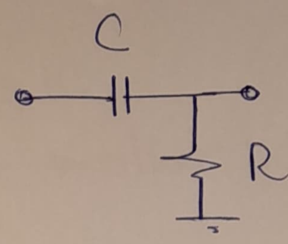
↳ $\omega_c = \frac{1}{\tau} = \frac{1}{RC}$: Cut-off / Corner freq.

↳ Poles: $S_p = -1/\tau = -\omega_c$, no zeros.

↳ $$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

$$P(H(j\omega)) = \underbrace{-}_{\text{denominator}} \tan^{-1} \frac{\omega}{\omega_c}$$

Example 2 :



$$H(s) = \frac{R}{R + 1/sC} = \frac{sRC}{1 + sRC}$$

$$= \frac{s\underbrace{RC}_{\tau}}{1 + s\tau}$$

$$\rightarrow H(j\omega) = \frac{j\omega / \omega_c}{1 + j\omega / \omega_c}$$

↳ Poles : $S_p = -\frac{1}{\tau} = -\omega_c$

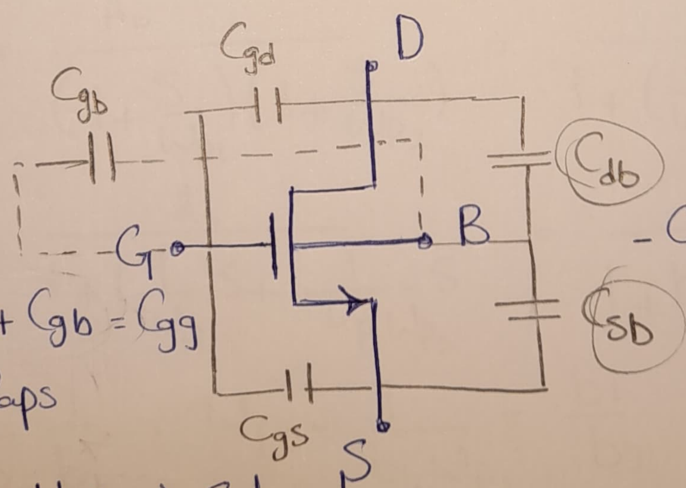
Zeros : $S_z = 0$

$$\hookrightarrow |H(j\omega)| = \frac{\omega/\omega_c}{\sqrt{1 + (\omega/\omega_c)^2}}$$

$$\angle(H(j\omega)) = 90^\circ - \tan^{-1}(\omega/\omega_c)$$

(2) MOSFET Capacitors and where to find them

- Coupling and Bypass Capacitors : External Caps.
  * Not Common in CMOS technology.
  * Acts as a HPF
- Internal Capacitors



- C_{sb} and C_{db} are Parasitics

- $C_{gs} + C_{gd} + C_{gb} = C_{gg}$ are Gate Caps

* for MOS in Sat.
  - ↳ $C_{gb} = 0$: due to Channel formation
  - ↳ $C_{gs} \gg C_{gd}$: due to Channel Pinched off at drain, So Most of Channel goes to S
  - ↳ $C_{sb} > C_{db}$

Not Valid for $\lambda \neq 0$

③ SCTC and OCTC techniques. → Page 3

* SCTC

↳ for (LFR): not Common in Analog IC

↳ only Consider one Cap @ a time → other SC

↳ $\omega_{L,3dB} = \omega_{L1} + \omega_{L2} + \cdots$

\# Highest Pole dominants (LIN dominants)

* OCTC

↳ for High frequency range → more Common.

↳ only Consider one Cap @ a time → other O.C

↳ $\omega_{H,3dB} = \omega_{H1} \parallel \omega_{H2} \parallel \cdots$

\# Lowest Pole dominants (HIN dominants)

* Both gives good aprox if only one pole dominants and Poles are real.

④ Dominant pole Approx.

- Assume: Poles are real and $\omega_{P_1} \ll \omega_{P_2}$

$$A_v(S) = \frac{A_o}{\left(1+\frac{S}{\omega_{P1}}\right)\left(1+\frac{S}{\omega_{P2}}\right)} = \frac{A_o}{1+\left(\frac{1}{\omega_{P_1}}+\cancel{\frac{1}{\omega_{P2}}}\right)S+\frac{S^2}{\omega_{P_1}\omega_{P2}}}$$

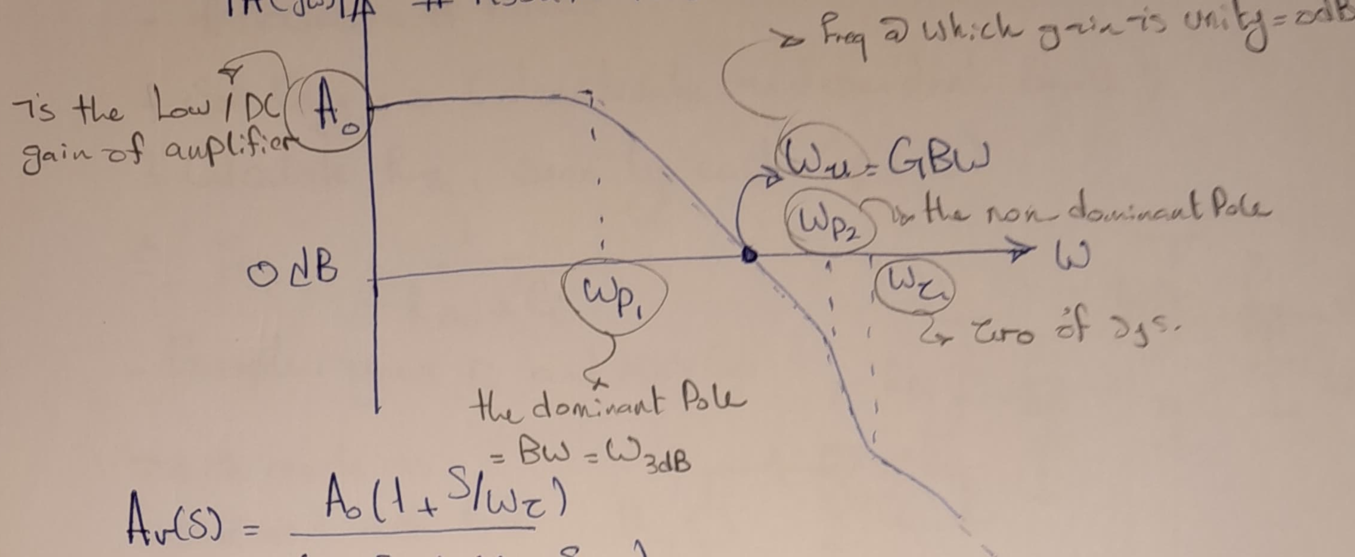
$$\simeq \frac{A_o}{1+\left(\frac{1}{\omega_{P_1}}\right)S+\frac{1}{\omega_{P_1}\omega_{P_2}}S^2} = \frac{A_o}{1+b_1 S+b_2 S^2}$$

$$\therefore \omega_{P_1} = \frac{1}{b_1}, \quad \omega_{P2} \simeq \frac{1}{b_2\omega_{P1}} = \frac{b_1}{b_2}$$

- gives good aprox for both dominant and non-dominant.

⑤ IC Amplifiers Freq response.

$|A(j\omega)|$ # Assume first order Sys.



$$A_v(S) = \frac{A_o(1 + S/\omega_z)}{(1+\frac{S}{\omega_{p1}})(1+\frac{S}{\omega_{p2}})}$$

- $GBW = A_v * BW = A_o\,\omega_{p1}$
- We usually design amplifiers such that ω_{p2} and $\omega_z > \omega_u$

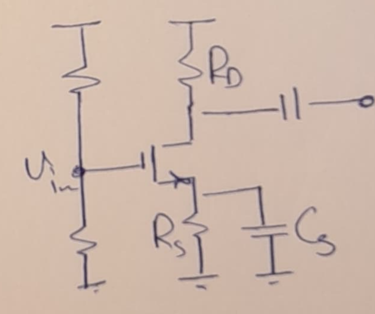
⑥ Calculating Poles by inspection.

↳ 1: Set $V_{sig} = 0$, deactivate independent Source.
2: Calculate thevenin resistance. Seen by each Cap
3: $S_{p,i} = -\frac{1}{R_{th\,i}C_i}$

Example

@ C_s: $R_{th} = R_s \,||\, \frac{1}{g_m}$

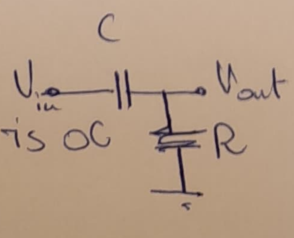
$$S_p = \frac{-1}{(R_s \,||\, \frac{1}{g_m})\,C_s}$$



⑦ Calculating Zeros by inspection

↳ 1: Find S that makes $V_{out} = 0$ for each Cap

Example 1:



$V_{out} = 0$ @ C is OC

$\therefore \frac{1}{SC} = \infty$

$\therefore S_z = 0$

Example 2:
For C_s in Circuit above

$V_{out} = 0$ @ Z_s is OC $= \infty$

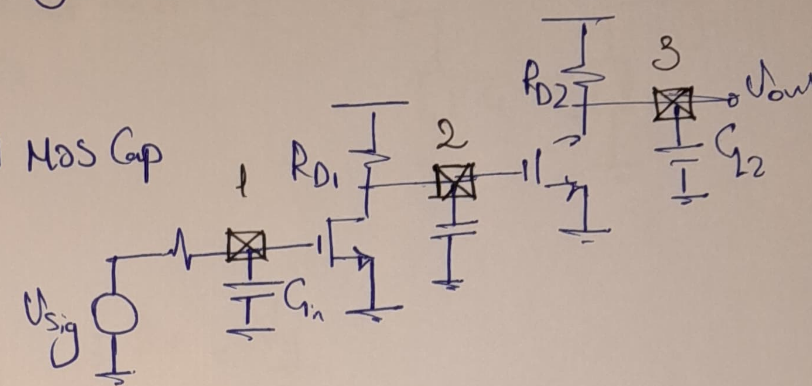
$\therefore Z_s = \frac{R_s}{1 + SR_sC_s} \longrightarrow S_z = \frac{-1}{R_sC_s}$

8) Associating Poles with nodes

- # nodes = # Poles

1. Set $V_{sig} = 0$ (deactivate independent Source)
2. Calculate $R_{Th,i}$ seen by each Cap (C_i)
3. $S_{Pi} = -\dfrac{1}{R_{thi} * C_i}$

- Example: ignore r_o and MOS Cap



∵ each node is associated with pole

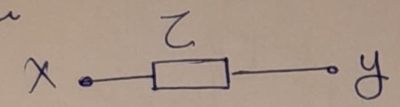
∴ HIN dominates

$$\therefore H(s) = \frac{(g_{m1} R_{D1})(g_{m2} R_{D2})}{(1 + sR_{sig} C_{in})(1 + sR_D C_{L1})(1 + sR_{D2} C_{L2})}$$

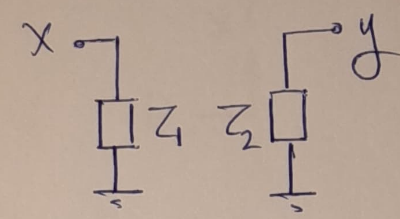
(9) Miller Effect

↳ Miller theorem

if we have a floating impedance Z between two point x, y



it can be converted to Z_1 and Z_2 not floating components



where.

$$Z_1 = \frac{Z}{1 - A_v}$$

$$Z_2 = \frac{Z}{1 - A_v^{-1}} \quad ; \quad A_v = \frac{V_y}{V_x}$$

Proof: if the two circuits equivalent → $I_Z = I_{Z_1}$

$$\frac{V_x - V_y}{Z} = \frac{V_x}{Z_1} \rightarrow Z_1 = \frac{Z V_x}{V_x - V_y} = \frac{Z}{1 - \frac{V_y}{V_x}}$$

Similarly

$$Z_2 = \frac{Z}{1 - \frac{V_x}{V_y}}$$

- this decomposition of a "floating" impedance Z into two "grounded" impedances proves useful in analysis and design
- if the impedance Z form the only path between x and y, then the convertion is often invalid
- Miller's theorm proves useful in cases where the impedance Z appears in parallel with the main signal

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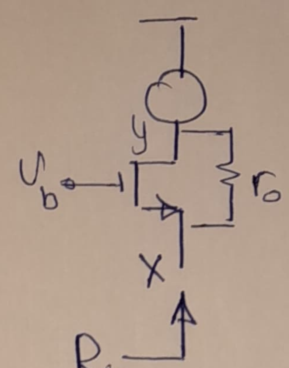
Example 3. Calc. R_{in}

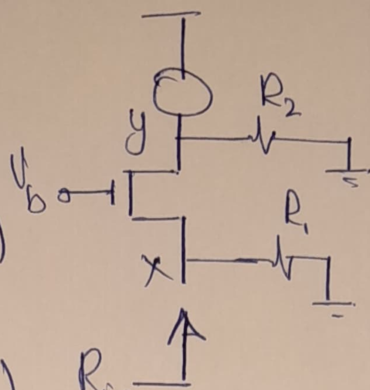


From direct shortcuts

$\therefore R_{in} = R_{LFS} = \dfrac{1}{g_m + g_{mb}} \left(1 + \dfrac{R_{Drain}}{r_o}\right)$

$R_{Drain} = \infty \longrightarrow R_{in} = \infty$

using miller's.



$R_{in} = R_{LFS} \parallel R_1$

$$R_{LFS} = \frac{1}{g_m + g_{mb}} \left(1 + \frac{R_2}{r_o}\right)$$

$$= \frac{1}{g_m + g_{mb}} \left(1 + \frac{r_o / r_o}{1 - \frac{1}{A_v}}\right)$$

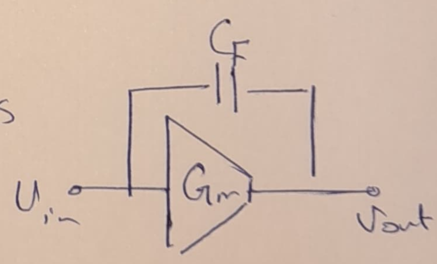
$$= \frac{1}{g_m + g_{mb}} \left(1 + \frac{1}{1 - 1/A_v}\right) = \frac{1}{g_m + g_{mb}}$$

$$\therefore R_1 = \frac{r_o}{1 - A_v} = \frac{r_o}{1 - (g_m + g_{mb}) r_o} = \frac{r_o}{-(g_m + g_{mb}) r_o} = \frac{-1}{g_m + g_{mb}}$$

$\therefore R_{in} = \infty$

the feed forward zero

- Miller's theorm eliminate zeros
- it may predict additional poles
- it does not correctly compute the output impedance



$V_{out} = 0 \longrightarrow i_{out} = 0$

$V_{in} s C_F = - G_m V_{in}$

$\therefore S_z = - G_m / C_F$