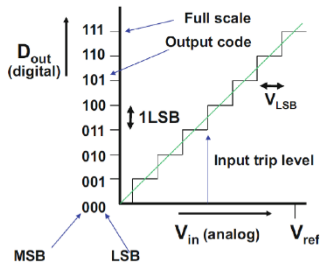
Analog Systems Design

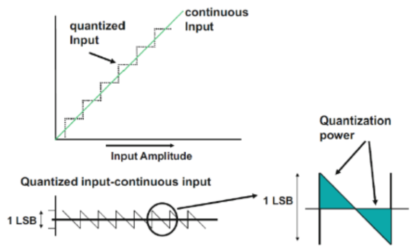
3. Quantization

1. Quantization
   - is the discretization of the analog signal in voltage/amplitude domain while sampling is the discretization in time domain
   - There is a limited (finite) number of valid amplitude levels
   - Quantization process is a lossy one where quantization error is the difference between the actual analog signal amplitude and the corresponding quantization level
   - Quantization error peak to peak value < V_{LSB} where V_{LSB} is the quantization step (difference between each quantization level)
   - If signal amplitude is
     - Rounded to the nearest quantization level : $-0.5\ V_{LSB} < error < 0.5\ V_{LSB}$
     - Floored : $0 < error < V_{LSB}$
     - Ceiled : $-V_{LSB} < error < 0$
   - Quantization error is also referred to as noise, as it for a large number of bits quantization error can be approximated as a white noise

2. Binary representation
   - $B_s = \sum_{i=0}^{i=N-1} b_i 2^i = b_0 2^0 + b_1 2^1 + \cdots + b_{N-1} 2^{N-1}$
     - N: word width, resolution, No. of bits
     - No. of levels, steps = 2^N
     - b_0 is the least significant bit LSB
     - b_{N-1} is the most significant bit MSB
     - $V_{LSB} = \Delta = V_{FS} / 2^N$
     - Full scale of digital domain = 111…1 = $2^N - 1$
     - Full scale of analog domain = $V_{FS} - \Delta$



3. Quantization error (Noise)
   - For low resolution ADC the quantization error will show as distortion components (harmonics of the input)
   - For N > 6 : the quantization error can be approximated as
     - Uniformly distributed PDF (from -0.5 LSB to 0.5 LSB)
     - White noise in frequency domain (from 0 to fs/2)



   - Quantization noise (deterministic approach)
     - Assume linear ramp input: quantization error is a sawtooth wave, $V_{Q,rms} = V_{LSB}/\sqrt{12}$
   - Quantization noise (stochastic approach)

- Assume uniformly distributed random error, $V_{Q,rms} = V_{LSB}/\sqrt{12}$

$$V_{Q(rms)} = \left[\int_{-\infty}^{\infty} x^2 f_e(x)\, dx\right]^{1/2} = \left[\frac{1}{V_{LSB}}\left(\int_{-V_{LSB}/2}^{V_{LSB}/2} x^2\, dx\right)\right]^{1/2} = \frac{V_{LSB}}{\sqrt{12}}$$

4. Signal to quantization noise ratio

$$SQNR = 10\log\left(\frac{Signal\ Power}{Quantization\ Power}\right) = 20\log\left(\frac{V_{sigrms}}{V_{Qnrms}}\right)$$

$$Signal\ Power = \frac{\left(\frac{2^N V_{LSB}}{2}\right)^2}{2} = \frac{2^{2N} V_{LSB}^2}{8}$$ Assume sinusoidal input signal from 0 to V_{FS}

$$Quantization\ Power = \frac{V_{LSB}^2}{12}$$

$$SQNR = 10\log\left(\frac{Signal\ Power}{Quantization\ Power}\right) = 10\log\left(\frac{3}{2}2^{2N}\right)$$

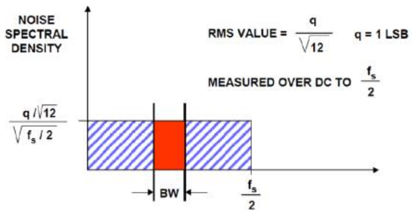
$$\boldsymbol{SQNR = 6.02 \times N + 1.76\ \ [dB]}$$

- Although the results comes from to different assumption (linear ramp input for the Quantization power and sinusoidal input for the signal power) but it gives good results especially for large N
- Quantization noise dominates up to around N = 14 bit
- Each bit adds 6 dB to SNR.
- dBFS means decibel relative to the full scale which is a measurement for amplitude levels in digital systems
- 0 dBFS is assigned to be the maximum possible digital level

5. Processing gain (oversampling gain)
- For large No. of bits, error is considered as a random signal and will be shown as a random distributed noise form 0 to fs/2
- If we use fs much larger than signal BW, noise will be spread on a large range and after the quantization we can filtering it in digital domain to select the BW which means that the quantization noise will also be filtered, lower quantization noise, better SQNR
- Using large fs called oversampling and the improvement in the SQNR us called processing gain

$$P_{Qn-total} = \frac{V_{LSB}^2}{12} = S_Q(f) \times \frac{f_s}{2} \ \rightarrow\ S_Q(f) = \frac{V_{LSB}^2}{12} \times \frac{2}{f_s}$$

$$P_{Qn-red} = S_Q(f) \times BW = \frac{V_{LSB}^2}{12} \times \frac{BW}{f_s/2}$$



$$SQNR = 10\log\left(\frac{Signal\ Power}{Quantization\ Power \times \frac{BW}{f_s/2}}\right)$$

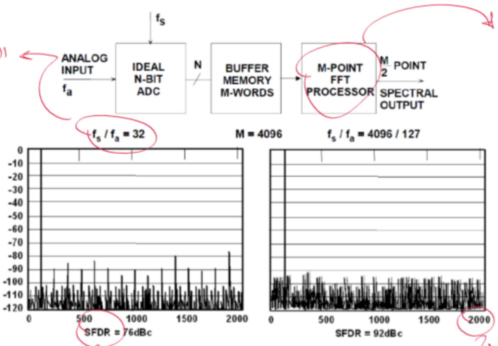
$$SQNR = 6.02 \times N + 1.76 + 10\log\left(\frac{f_s/2}{BW}\right)$$

6. Quantization Noise Spectrum
- In most practical applications, the input to the ADC is a band of frequencies + noise
  - The quantization noise tends to be random white noise.

  - Uniformly distributed from 0to $fs/2$

- In ADC testing/simulation, a pure single tone sine wave is used.
  - The sampling frequency may be correlated to the test tone.
  - The quantization noise power may also become correlated to the input signal.
  - Quantization noise may appear as harmonic distortion.
- This is a testing artifact.
  - It should be avoided so that the true ADC distortion is measured, rather than the correlated quantization noise.

ال ratio بين ال f_s/f_a
لو correlated فدا
هيؤدي لاني اجيب كل مرة
ال Samples في نفس
الاماكن وبالتالي ال
error مش هيكون uniform
فلازم اتأكد إنه الحالة
دي ماتحصلش



distance from signal fs and the highst harmonic

FFT on Matlab gives the same No. of the entered points (M) from 0 → fs, but we just needs from 0 → fs/2 so we use M/2 points and ignore the rest.

≈ M/2

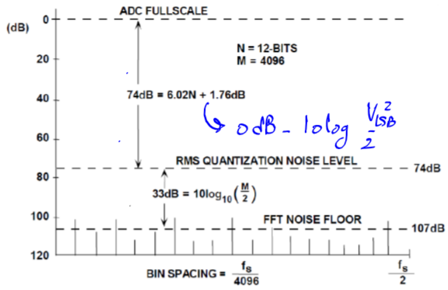
7. FFT noise floor
   - How to interpret the FFT plot?

Continous in frequency domain from 0 → $\frac{f_s}{2}$

$$\text{Quantization Power} = \frac{V_{LSB}^2}{12} = S_Q \times \frac{f_s}{2} = S_{Q,FFT} \times \frac{M}{2} \rightarrow S_{Q,FFT} = \frac{V_{LSB}^2}{12} / \left(\frac{M}{2}\right)$$

$$\text{FFT Noise Floor} = 10 \log S_{Q,FFT} = 10 \log \frac{V_{LSB}^2}{12} - 10 \log \frac{M}{2}$$

discrete with $\frac{M}{2}$ points



↳ 0dB - 10 log $\frac{V_{LSB}^2}{12}$

Noise Power is Constant

increasing No. of Points (M)
↳ will decrease the share of Power for single point
↳ this will make Noise floor below the Quantization Power by 10 log $\frac{M}{2}$

increasing (M)↑, FFT noise floor ↓, harmonics will be more obvious, Simulation time ↑

8. Dithering
   - The addition of a random signal allows to determine the value of a DC signal at greater accuracy than the quantization process allows.
   - Additional signal processing like averaging can then lead to resolution improvement for low frequency signals.