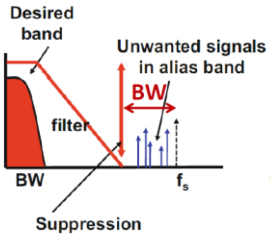
Analog Systems Design

2. Sampling

1. Alias Band Suppression: What is the suppression requirements of AAF?
   - Working at the limit of Nyquist criterion requires an ideal filter that does not exist
   - Signal in the alias band will alias in the desired signal band after sampling so, it must be suppressed by AAF

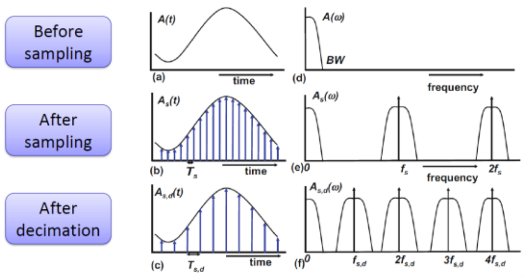


   - Each pole gives a roll off 20 dB/decade so if $f_s/BW = 2$ for a 4th order filter the suppression will equal?

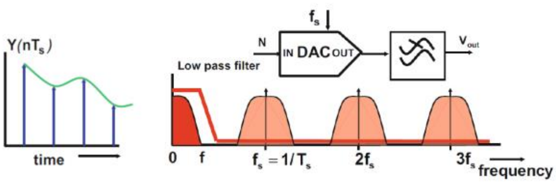
$$Aliasing\ band = fs - BW\ to\ f_s = f_s/2 = BW$$
$$@\ BW\ each\ pole\ gives\ 3\ dB \rightarrow for\ a\ 4^{th} order\ filter\ the\ supression = 4 * 3 = 12\ dB$$

2. Oversampling
   - Oversampling relaxes the requirements of baseband anti-aliasing filter
   - Higher relaxation zone means less pole numbers, simpler design, lower power consumption

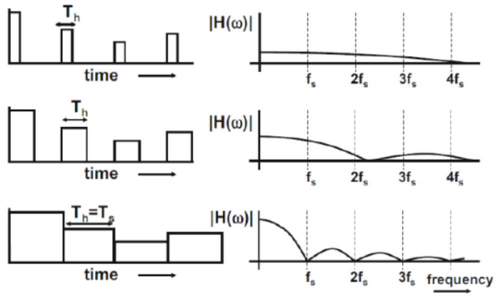
3. Decimation
   - Decimation is the process of reducing the sample rate of a sampled signal
   - Unless the original signal is already filtered and oversampled, digital filtering is necessary
   - We use decimation instead of using low sample rate from the beginning because oversampling makes filtering easy



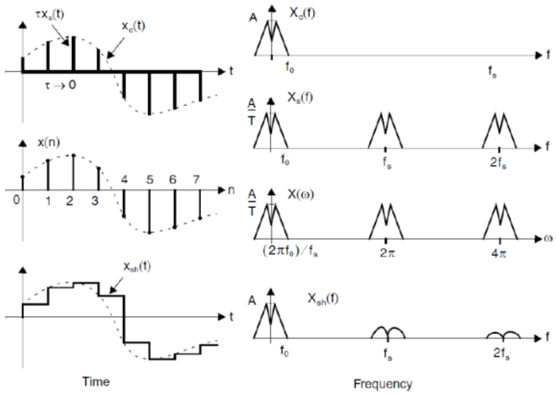
4. Reconstruction filter
   - As ADC needs an AAF also the DAC needs a reconstruction filter (smoothing filter) which:
     - In TD: interpolate / restore / reconstruct the original signal
     - In FD: reconstruction filter suppress the images



5. Zero Order Hold (ZOH)
   - Keeps the value of the signal at the sample moment
   - The Fourier transform of ZOH is a sinc function $sinc(x) = \sin(x)/x$ where the nulls of $sinc(x)$ at the inverse of the fold time



   - The ZOH performs inherent reconstruction (filtering out images)



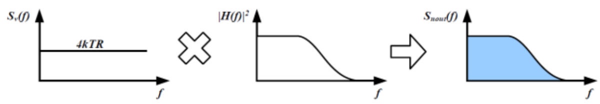
   - Passband droop: ZOH suppresses images but introduce amplitude distortion
     - The passband distortion may be compensated by inverse sinc response in digital or analog domain

6. Noise in RC circuit
   - Resistors generate white thermal noise but the BW is always limited by the capacitor

$$S_{nout}(f) = S_v(f)\left|\frac{V_{nout}(j\omega)}{V_n(j\omega)}\right|^2$$

$$\overline{V_{nout}^2} = V_{noutrms}^2 = \int_{-\infty}^{\infty} S_{nout}(f)\,df$$

$$\boxed{\overline{V_{nout}^2} = \frac{kT}{C}}$$



   - RMS noise is independent of R (why?)
     $R\uparrow, S_v(f) = 4kTR\uparrow$, but the $BW = 1/RC \downarrow \rightarrow$ area under the curve $S_{nout}(f)$ remains constant

$$V_{nrms} = \sqrt{\frac{1p}{C}} * 64\ uV_{rms}$$

   - Equivalent noise bandwidth: define an equivalent noise BW (B_N) such that the area under a brick-well response is the same area under the actual spectral density curve
   - For a first order system: $B_N = 1/4RC = \pi/2\, f_{pole}$

7. Sampling noise
   - The sampling capacitor determines noise power, SNR, and the No. of ADC bits
   - C ↑, noise ↓, SNR ↑, but BW (speed) ↓ @ the same power consumption

C_{hold}	$V_{nrms} = \sqrt{\frac{kT}{C}}$ at $T = 300\ K$	SNR (assume $V_{sigrms} = 1\ Vrms$)	No. of bits (see next lecture)
100 fF	203 μVrms	74 dB	12-bit
1 pF	64 μVrms	84 dB	13.7-bit
10 pF	20.3 μVrms	94 dB	15.4-bit

8. Noise folding
   - As sampling folds the signal it is also folds the noise

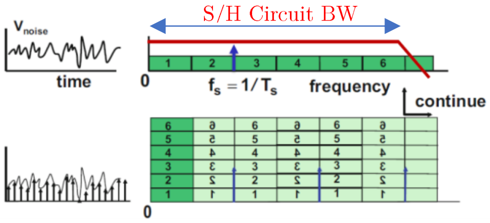
Before sampling: $P_n = kT/C = S_n(f) \times B_N$

After sampling P_n is unchanged: $P_n = kT/C = S_{n,sampled}(f) \cdot \frac{f_s}{2}$

$$S_{n,sampled}(f) = \frac{kT}{C} \times \frac{2}{f_s} = S_n(f) \times B_N \times \frac{2}{f_s}$$

$$S_{n,sampled}(f) = S_n(f) \times \frac{2B_N}{f_s} = S_n(f) \times \frac{\pi BW}{f_s}$$

Noise power is unchanged, but noise density increases (noise folding).



   - BW of the S/H circuit cannot be small because if

$\mathrm{BW_{SH}} \downarrow, \tau = \mathrm{RC} \uparrow$, S/H Cap charges slowely → slow S/H response