Difference Method (FDM) and Jacobi Iteration

Do We Use Finite Difference Method (FDM)?

Yes, the project absolutely uses the **Finite Difference Method (FDM)** as the core technique to discretize **Laplace's Equation**, which describes the voltage distribution in an electrical grid. After applying FDM, we solve the resulting system of equations using the **Jacobi Iteration Method**. Here's a clear breakdown :

1. Laplace's Equation in 2D

The problem involves solving **Laplace's Equation**, which is a partial differential equation (PDE) that models the steady-state voltage distribution in a 2D grid without charges:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

This equation describes how voltage V varies across a grid, such as a circuit board or electrical network.

- solving Laplace's Equation analytically is complex

2. Applying Finite Difference Method (FDM)

Since solving Laplace's Equation analytically is complex for arbitrary boundary conditions, we use **FDM** to approximate the partial derivatives. Specifically, we use the **Central Difference Approximation** to discretize the second derivatives:

$$\frac{\partial^2 V}{\partial x^2} \approx \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta x)^2}$$

$$\frac{\partial^2 V}{\partial y^2} \approx \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta y)^2}$$

Assuming a uniform grid where $\Delta x = \Delta y$, we substitute these into Laplace's Equation and simplify (often normalizing $\Delta x = 1$ for simplicity in coding):

$$V_{i,j}^{(new)} = \frac{1}{4}\left(V_{i+1,j}^{(old)} + V_{i-1,j}^{(old)} + V_{i,j+1}^{(old)} + V_{i,j-1}^{(old)}\right)$$

- This formula is used to **update the voltage at each interior point** during every iteration.
- This equation relates the voltage at each grid point to its four neighbors, forming a system of linear equations. This is a direct result of applying **FDM**.

3. Solving with Jacobi Iteration

The system of equations generated by FDM is large and sparse, so we solve it iteratively using the **Jacobi Iteration Method**. In each iteration, we update the voltage $V_{i,j}$ at every interior grid point based on the average of its neighbors, as shown in the equation above. This process continues until the solution converges (i.e., changes between iterations become negligible). or by using a pre-defined max_number_of_iterations.

Correct Order of Steps

- **FDM**: Transforms the continuous PDE (Laplace's Equation) into a discrete system of algebraic equations.
- **Jacobi Iteration**: Solves this system iteratively to find the voltage at each grid point.

Summary

our project uses the **Finite Difference Method** to approximate Laplace's Equation, converting it into a set of algebraic equations that describe the voltage at each point in a 2D grid. We then apply the **Jacobi Iteration Method** to iteratively solve these equations.