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Ladies or Tigers?

Many of you are familiar with Frank Stockton's story "The Lady or the Tiger?," in which the prisoner must choose between two rooms, one of which contains a lady and the other a tiger. If he chooses the former, he marries the lady; if he chooses the latter, he (probably) gets eaten by the tiger.

The king of a certain land had also read the story, and it gave him an idea. "Just the perfect way to try my prisoners!" he said one day to his minister. "Only, I won't leave it to chance; I'll have signs on the doors of the rooms, and in each case I'll tell the prisoner certain facts about the signs. If the prisoner is clever and can reason logically, he'll save his life—and win a nice bride to boot!"

"Excellent idea!" said the minister.

THE TRIALS OF THE FIRST DAY

On the first day, there were three trials. In all three, the king explained to the prisoner that each of the two rooms contained either a lady or a tiger, but it *could* be that there were

tigers in both rooms, or ladies in both rooms, or then again, maybe one room contained a lady and the other room a tiger.

1 ◆ The First Trial

"Suppose both rooms contain tigers," asked the prisoner. "What do I do then?"

"That's your hard luck!" replied the king.

"Suppose both rooms contain ladies?" asked the prisoner.

"Then, obviously, that's your good luck," replied the king. "Surely you could have guessed the answer to that!"

"Well, suppose one room contains a lady and the other a tiger, what happens then?" asked the prisoner.

"In that case, it makes quite a difference which room you choose, doesn't it?"

"How do I know which room to choose?" asked the prisoner.

The king pointed to the signs on the doors of the rooms:

I	II
IN THIS ROOM THERE IS A LADY, AND IN THE OTHER ROOM THERE IS A TIGER	IN ONE OF THESE ROOMS THERE IS A LADY, AND IN ONE OF THESE ROOMS THERE IS A TIGER

"Is it true, what the signs say?" asked the prisoner.

"One of them is true," replied the king, "but the other one is false."

If you were the prisoner, which door would you open (assuming, of course, that you preferred the lady to the tiger)?

2 ◆ The Second Trial

And so, the first prisoner saved his life and made off with the lady. The signs on the doors were then changed, and new occupants for the rooms were selected accordingly. This time the signs read as follows:

I	II
AT LEAST ONE OF THESE ROOMS CONTAINS A LADY	A TIGER IS IN THE OTHER ROOM

"Are the statements on the signs true?" asked the second prisoner.

"They are either both true or both false," replied the king. Which room should the prisoner pick?

3 ◆ The Third Trial

In this trial, the king explained that, again, the signs were either both true or both false. Here are the signs:

I	II
EITHER A TIGER IS IN THIS ROOM OR A LADY IS IN THE OTHER ROOM	A LADY IS IN THE OTHER ROOM

Does the first room contain a lady or a tiger? What about the other room?

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THE SECOND DAY.

"Yesterday was a fiasco," said the king to his minister. "All three prisoners solved their puzzles! Well, we have five trials coming up today, and I think I'll make them a little tougher."

"Excellent idea!" said the minister.

Well, in each of the trials of this day, the king explained that in the lefthand room (Room I), if a lady is in it, then the sign on the door is true, but if a tiger is in it, the sign is false. In the righthand room (Room II), the situation is the opposite: a lady in the room means the sign on the door is false, and a tiger in the room means the sign is true. Again, it is possible that both rooms contain ladies or both rooms contain tigers, or that one room contains a lady and the other a tiger.

4 ◆ The Fourth Trial

After the king explained the above rules to the prisoner, he pointed to the two signs:

I	II
BOTH ROOMS CONTAIN LADIES	BOTH ROOMS CONTAIN LADIES

Which room should the prisoner pick?

5 ◆ The Fifth Trial

The same rules apply, and here are the signs:

I	II
AT LEAST ONE ROOM CONTAINS A LADY	THE OTHER ROOM CONTAINS A LADY

6 ◆ The Sixth Trial

The king was particularly fond of this puzzle, and the next one too. Here are the signs:

I	II
IT MAKES NO DIFFERENCE WHICH ROOM YOU PICK	THERE IS A LADY IN THE OTHER ROOM

What should the prisoner do?

7 ◆ The Seventh Trial

Here are the signs:

I	II
IT DOES MAKE A DIFFERENCE WHICH ROOM YOU PICK	YOU ARE BETTER OFF CHOOSING THE OTHER ROOM

What should the prisoner do?

8 ♦ The Eighth Trial

"There are no signs above the doors!" exclaimed the prisoner.

"Quite true," said the king. "The signs were just made, and I haven't had time to put them up yet."

"Then how do you expect me to choose?" demanded the prisoner.

"Well, here are the signs," replied the king.

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┌─────────────────┐
│   THIS ROOM     │
│   CONTAINS      │
│   A TIGER       │
└─────────────────┘
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┌─────────────────┐
│   BOTH ROOMS    │
│   CONTAIN       │
│   TIGERS        │
└─────────────────┘
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"That's all well and good," said the prisoner anxiously, "but which sign goes on which door?"

The king thought awhile. "I needn't tell you," he said. "You can solve this problem without that information.

"Only remember, of course," he added, "that a lady in the lefthand room means the sign which should be on that door is true and a tiger in it means the sign should be false, and that the reverse is true for the righthand room."

What is the solution?

THE THIRD DAY

"Confound it!" said the king. "Again all the prisoners won! I think tomorrow I'll have *three* rooms instead of two; I'll put a

lady in one room and a tiger in each of the other two rooms. Then we'll see how the prisoners fare!"

"Excellent idea!" replied the minister.

"Your conversation, though flattering, is just a bit on the repetitious side!" exclaimed the king.

"Excellently put!" replied the minister.

9 ◆ The Ninth Trial

Well, on the third day, the king did as planned. He offered three rooms to choose from, and he explained to the prisoner that one room contained a lady and the other two contained tigers. Here are the three signs:

I	II	III
A TIGER IS IN THIS ROOM	A LADY IS IN THIS ROOM	A TIGER IS IN ROOM II

The king explained that at most one of the three signs was true. Which room contains the lady?

10 ◆ The Tenth Trial

Again there was only one lady and two tigers. The king explained to the prisoner that the sign on the door of the room containing the lady was true, and that at least one of the other two signs was false.

Here are the signs:

I	II	III
A TIGER IS IN ROOM II	A TIGER IS IN THIS ROOM	A TIGER IS IN ROOM I

What should the prisoner do?

11 ◆ First, Second, and Third Choice

In this more whimsical trial, the king explained to the prisoner that one of the three rooms contained a lady, another a tiger, and the third room was empty. The sign on the door of the room containing the lady was true, the sign on the door of the room with the tiger was false, and the sign on the door of the empty room could be either true or false. Here are the signs:

I	II	III
ROOM III IS EMPTY	THE TIGER IS IN ROOM I	THIS ROOM IS EMPTY

Now, the prisoner happened to know the lady in question and wished to marry her. Therefore, although the empty room was preferable to the one with the tiger, his first choice was the room with the lady.

Which room contains the lady, and which room contains the tiger? If you can answer these two questions, you should have little difficulty in also determining which room is empty.

THE FOURTH DAY

"Horrible!" said the king. "It seems I can't make my puzzles hard enough to trap these fellows! Well, we have only one more trial to go, but this time I'll *really* give the prisoner a run for his money!"

12 ✦ A Logical Labyrinth

Well, the king was as good as his word. Instead of having three rooms for the prisoner to choose from, he gave him nine! As he explained, only one room contained a lady; each of the other eight either contained a tiger or was empty. And, the king added, the sign on the door of the room containing the lady is true; the signs on doors of all rooms containing tigers are false; and the signs on doors of empty rooms can be either true or false.

Here are the signs:

I	II	III
THE LADY IS IN AN ODD-NUMBERED ROOM	THIS ROOM IS EMPTY	EITHER SIGN V IS RIGHT OR SIGN VII IS WRONG

IV	V	VI
SIGN I IS WRONG	EITHER SIGN II OR SIGN IV IS RIGHT	SIGN III IS WRONG

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VII	VIII	IX
THE LADY IS NOT IN ROOM I	THIS ROOM CONTAINS A TIGER AND ROOM IX IS EMPTY	THIS ROOM CONTAINS A TIGER AND VI IS WRONG

The prisoner studied the situation for a long while.

"The problem is unsolvable!" he exclaimed angrily. "That's not fair!"

"I know," laughed the king.

"Very funny!" replied the prisoner. "Come on, now, at least give me a decent clue: is Room Eight empty or not?"

The king was decent enough to tell him whether Room VIII was empty or not, and the prisoner was then able to deduce where the lady was.

Which room contained the lady?

♦ SOLUTIONS ♦

1 ♦ We are given that one of the two signs is true and the other false. Could it be that the first is true and the second false? Certainly not, because if the first sign is true, then the second sign must also be true—that is, if there is a lady in Room I and a tiger in Room II, then it is certainly the case that one of the rooms contains a lady and the other a tiger. Since it is not the case that the first sign is true and the second one false, then it must be that the second sign is true and the first one false. Since the second sign is true, then there really is a lady in one room and a tiger in the other. Since the

first sign is false, then it must be that the tiger is in Room I and the lady in Room II. So the prisoner should choose Room II.

2 ◆ If Sign II is false, then Room I contains a lady; hence at least one room contains a lady, which makes Sign I true. Therefore, it is impossible that both signs are false. This means that both signs are true (since we are given that they are either both true or both false). Therefore, a tiger is in Room I and a lady in Room II, so again the prisoner should choose Room II.

3 ◆ The king must have been in a generous mood this time, because both rooms contain ladies! We prove this as follows:

Sign I says in effect that *at least* one of the following alternatives holds: there is a tiger in Room I; there is a lady in Room II. (The sign does not preclude the possibility that both alternatives hold.)

Now, if Sign II is false, then a tiger is in Room I, which makes the first sign true (because the first alternative is then true). But we are given that it is not the case that one of the signs is true and the other one false. Therefore, since Sign II is true, both signs must be true. Since Sign II is true, there is a lady in Room I. This means that the first alternative of Sign I is false, but since at least one of the alternatives is true, then it must be the second one. So there is a lady in Room II also.

4 ◆ Since the signs say the same thing, they are both true or both false. Suppose they are true; then both rooms contain ladies. This would mean in particular that Room II contains a lady. But we have been told that if Room II contains a lady, the sign is false. This is a contradiction, so the signs are not

true; they are both false. Therefore, Room I contains a tiger and Room II contains a lady.

5 ◆ If the first room contains a tiger, we get a contradiction. Because if it does contain a tiger, then the first sign is false, which would mean that neither room contains a lady; both rooms would contain tigers. But we have been told that a tiger in the second room indicates that the second sign is true, which would mean that the other room contains a lady, contrary to the assumption that the first room contains a tiger. So it is impossible for the first room to contain a tiger; it must contain a lady. Therefore, what the second sign says is true, and the second room contains a tiger. So the first room contains a lady and the second room contains a tiger.

6 ◆ The first sign says, in effect, that either both rooms contain ladies or both contain tigers—that is the only way it could make no difference which room is picked.

Suppose the first room contains a lady. Then the first sign is true, which means the second room also contains a lady. Suppose, on the other hand, the first room contains a tiger. Then the first sign is false, which means that the two occupants are not the same, so again the second occupant is a lady. This proves that Room II must contain a lady regardless of what is in Room I. Since Room II contains a lady, Sign II is false and Room I must contain a tiger.

7 ◆ The first sign says in effect that the two occupants are different (one a lady and the other a tiger), but doesn't say which room contains which. If Room I's occupant is a lady, the sign is true; hence Room II must contain a tiger. If, on the other hand, Room I's occupant is a tiger, then the first sign is false, which means that the two occupants are not really dif-

ferent, so Room II must also contain a tiger. Therefore, Room II definitely contains a tiger. This means that the second sign is true, so a lady must be in the first room.

8 ◆ Suppose the top sign, THIS ROOM CONTAINS A TIGER, were on the door of Room I. If a lady is in the room, the sign is false, violating the conditions given by the king. If a tiger is in the room, the sign is true, which again violates the king's conditions. So that sign can't be on the first door; it must be on the second. This means the other sign is to be put on the first door.

The sign belonging on the first door thus reads: BOTH ROOMS CONTAIN TIGERS. So the first room can't contain a lady, or the sign would be true, which would mean that both rooms contain tigers—an obvious contradiction. Therefore, the first room contains a tiger. From this it follows that the sign is false, so the second room must contain a lady.

9 ◆ Signs II and III contradict each other, so at least one of them is true. Since at most one of the three signs is true, then the first one must be false, so the lady is in Room I.

10 ◆ Since the sign of the room containing the lady is true, then the lady certainly can't be in Room II. If she is in Room III, then all three signs must be true, which is contrary to the given condition that at least one sign is false. Therefore, the lady is in Room I (and sign II is true and sign III is false).

11 ◆ Since the sign on the door of the room containing the lady is true, then the lady cannot be in Room III.

Suppose she is in Room II. Then sign II would be true; hence the tiger would be in Room I and Room III would be

empty. This would mean that the sign on the door of the tiger's room would be true, which is not possible. Therefore, the lady is in Room I; Room III must then be empty, and the tiger is in Room II.

12 ◆ If the king had told the prisoner that Room VIII was empty, it would have been impossible for the prisoner to have found the lady. Since the prisoner did deduce where the lady was, the king must have told him that Room VIII was not empty, and the prisoner reasoned as follows:

"It is impossible for the lady to be in Room Eight, for if she were, Sign Eight would be true, but the sign says a tiger is in the room, which would be a contradiction. Therefore, Room Eight does not contain the lady. Also, Room Eight is not empty; therefore, Room Eight must contain a tiger. Since it contains a tiger, the sign is false. Now, if Room Nine is empty, then Sign Eight would be true; therefore, Room Nine cannot be empty.

"So, Room Nine is also not empty. It cannot contain the lady, or the sign would be true, which would mean that the room contains a tiger; this means Sign Nine is false. If Sign Six is really wrong, then Sign Nine would be true, which is not possible. Therefore, Sign Six is right.

"Since Sign Six is right, then Sign Three is wrong. The only way Sign Three can be wrong is that Sign Five is Wrong and Sign Seven is right. Since Sign Five is wrong, then Sign Two and Sign Four are both wrong. Since Sign Four is wrong, then Sign One must be right.

"Now I know which signs are right and which signs are wrong—namely:

"1 – Right	4 – Wrong	7 – Right
2 – Wrong	5 – Wrong	8 – Wrong
3 – Wrong	6 – Right	9 – Wrong

"I now know that the lady is in either Room One, Room Six, or Room Seven, since the others all have false signs. Since Sign One is right, the lady can't be in Room Six. Since Sign Seven is right, the lady can't be in Room One. Therefore, the lady is in Room Seven."