Data Compression

Lossy Compression
Scalar Quantization

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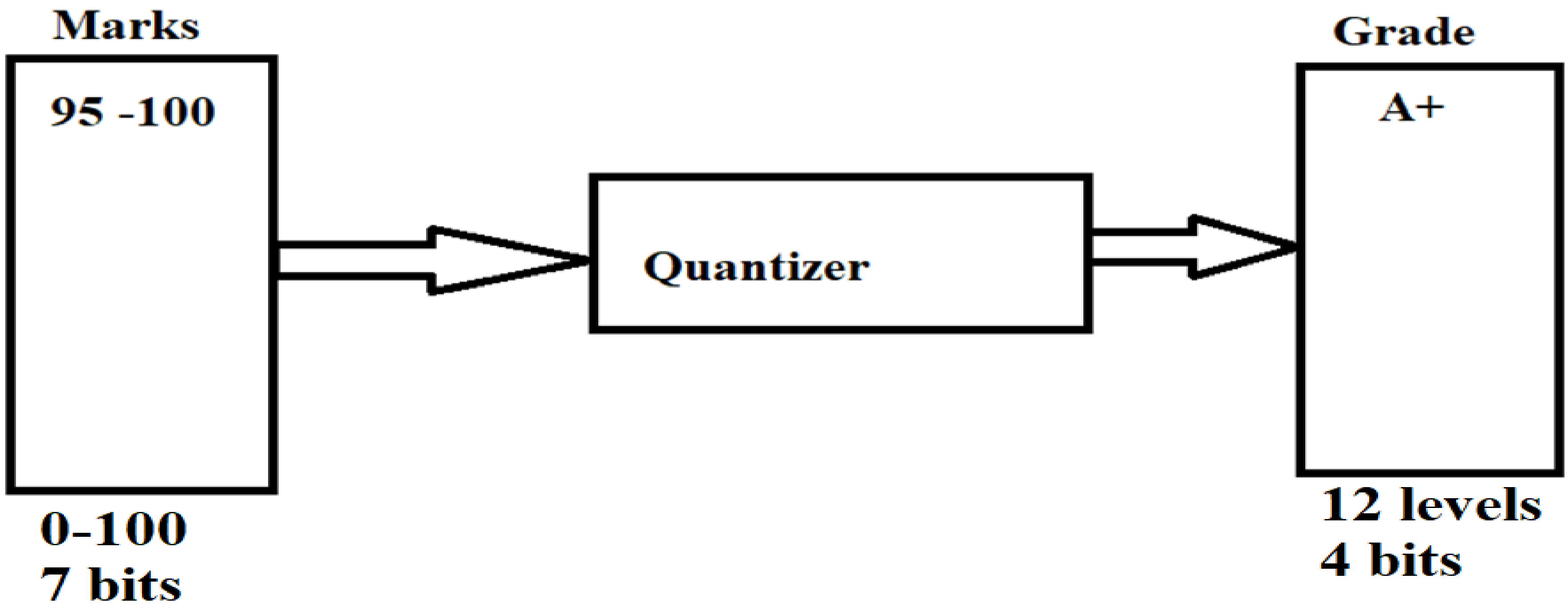
Quantization:

A process of representing a large, possibly infinite, set of values with a much smaller set.



- Reconstruct: (95-100) → A+ → 97
- Quantization is one of the simplest and most general idea in lossy compression.

<u>**Quantizer:**</u>

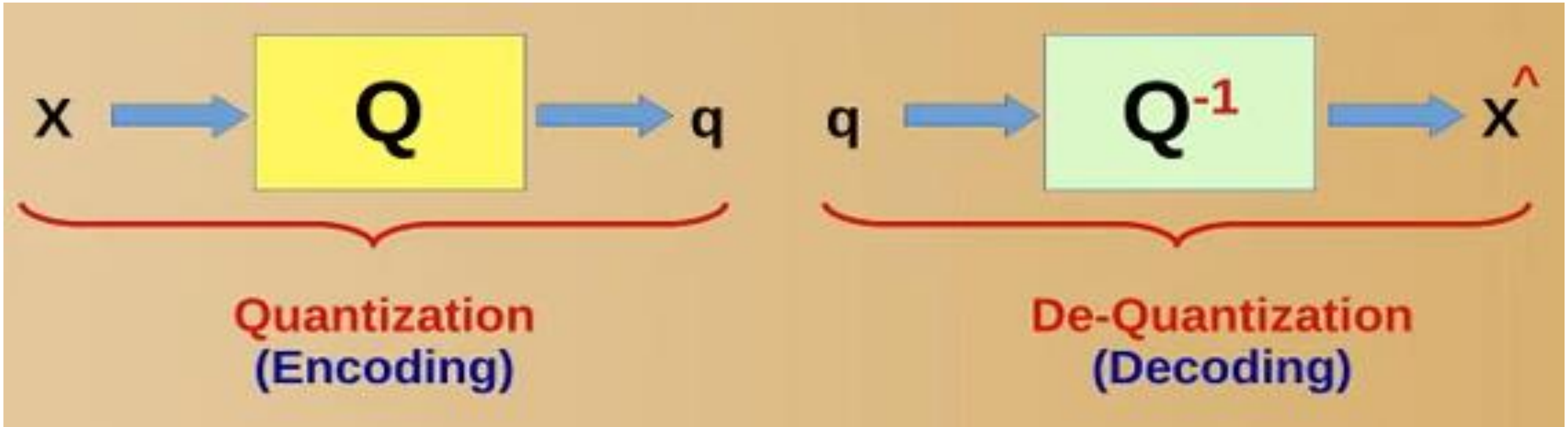
There are 2 types:

1) Scalar Quantization
   - Uniform Quantization.
   - Non-uniform (Optimal) Quantization.
2) Vector Quantization

1- Uniform Scalar Quantizer:

- A uniform scalar quantizer partitions the domain of input values into equally spaced intervals.
- Each interval is represented by a distinct codeword (Q).
- The output or reconstruction value (Q^{-1}) corresponding to each interval is taken to be the midpoint of the interval.
- The length of each interval is referred to as the step size.

Quantization and Dequantization:



- x: input value
- Q: codeword for x (Encoded value of x)
- $\hat{x}$: Output value (Reconstructed values of x)

Uniform Quantizer (Encoding):

- Input Output mapping.

Example:

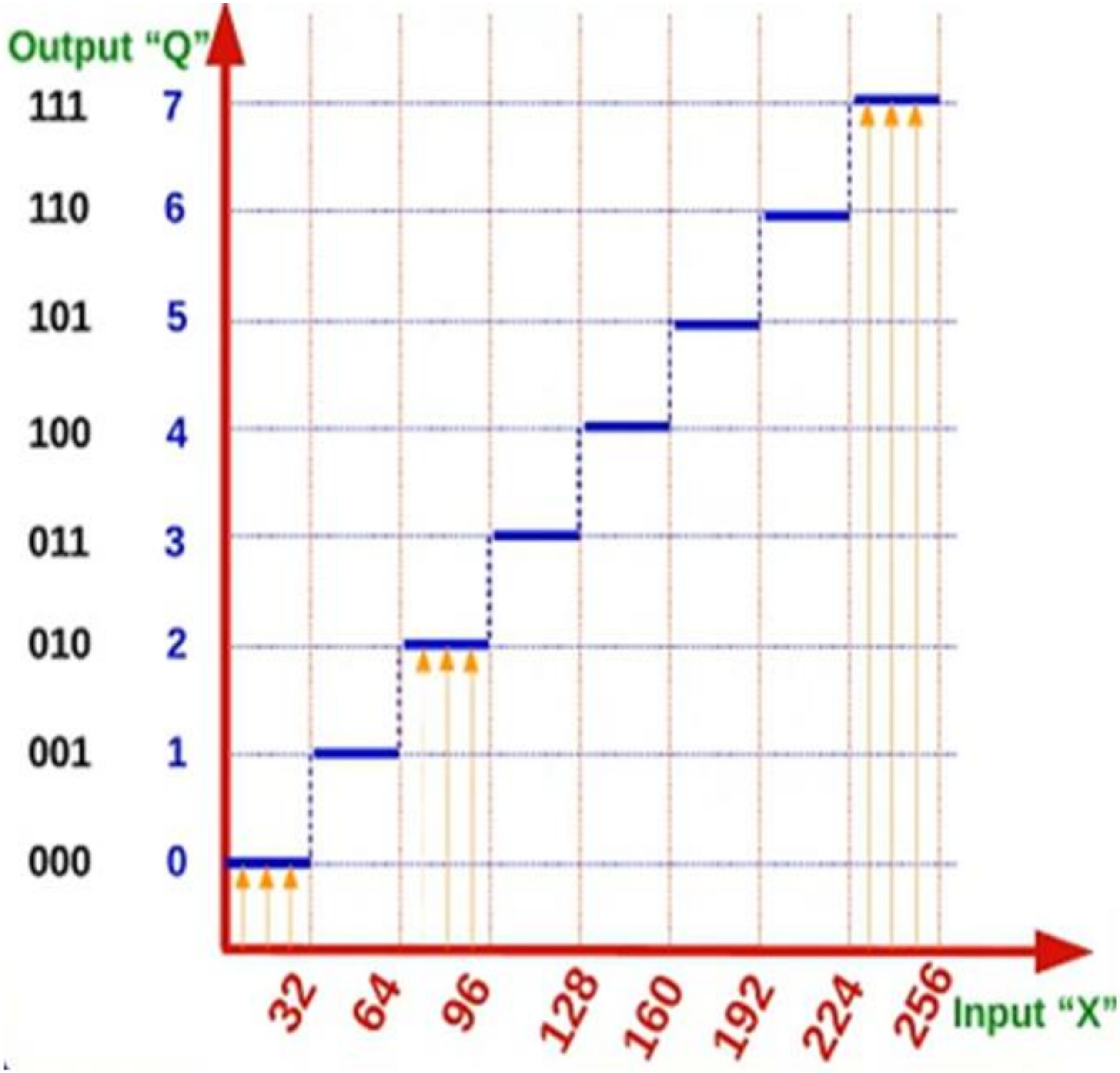
- Pixel (0 -255)
- Full scale = 255 (0-255)
- Step = 32
- No of steps (levels) = 256/32

  = 8 steps (3 bits)
- Compression ration $= \frac{\log_2 255}{\log_2 7} = \frac{8}{3}$

Range	Q
0 – 31	0
32 – 63	1
64 – 95	2
96 – 127	3
128 – 159	4
160 – 191	5
192 – 223	6
224 - 255	7

Uniform Quantizer (Encoding):

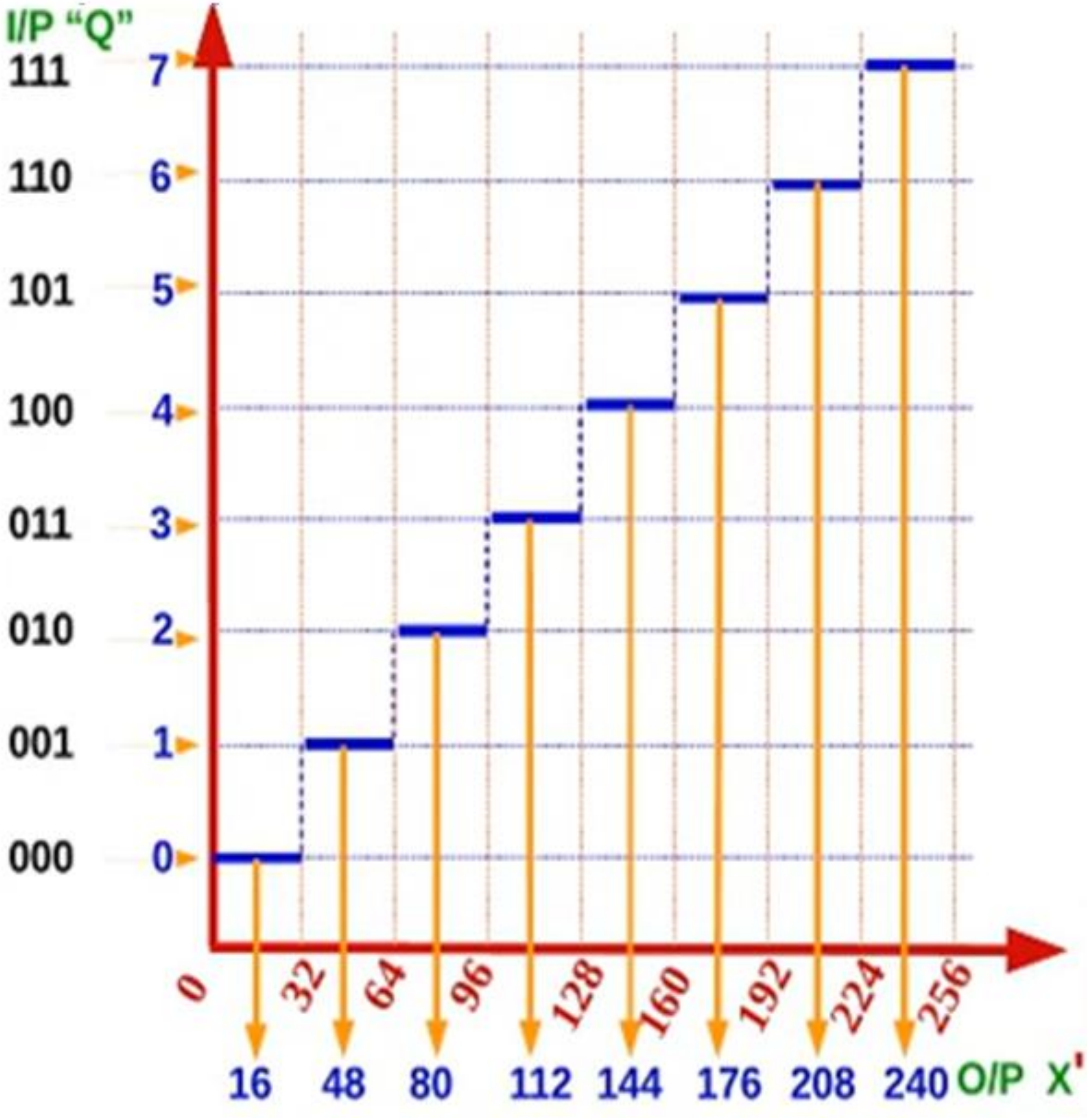
Example:



Range	Q
0 – 31	0
32 – 63	1
64 – 95	2
96 – 127	3
128 – 159	4
160 – 191	5
192 – 223	6
224 - 255	7

Dequantization (Decoder):

- Max Error $= \frac{1}{2} step = 16$



Range	Q	Q^{-1}
0 – 31	0	16
32 – 63	1	48
64 – 95	2	80
96 – 127	3	112
128 – 159	4	144
160 – 191	5	176
192 – 223	6	208
224 - 255	7	240

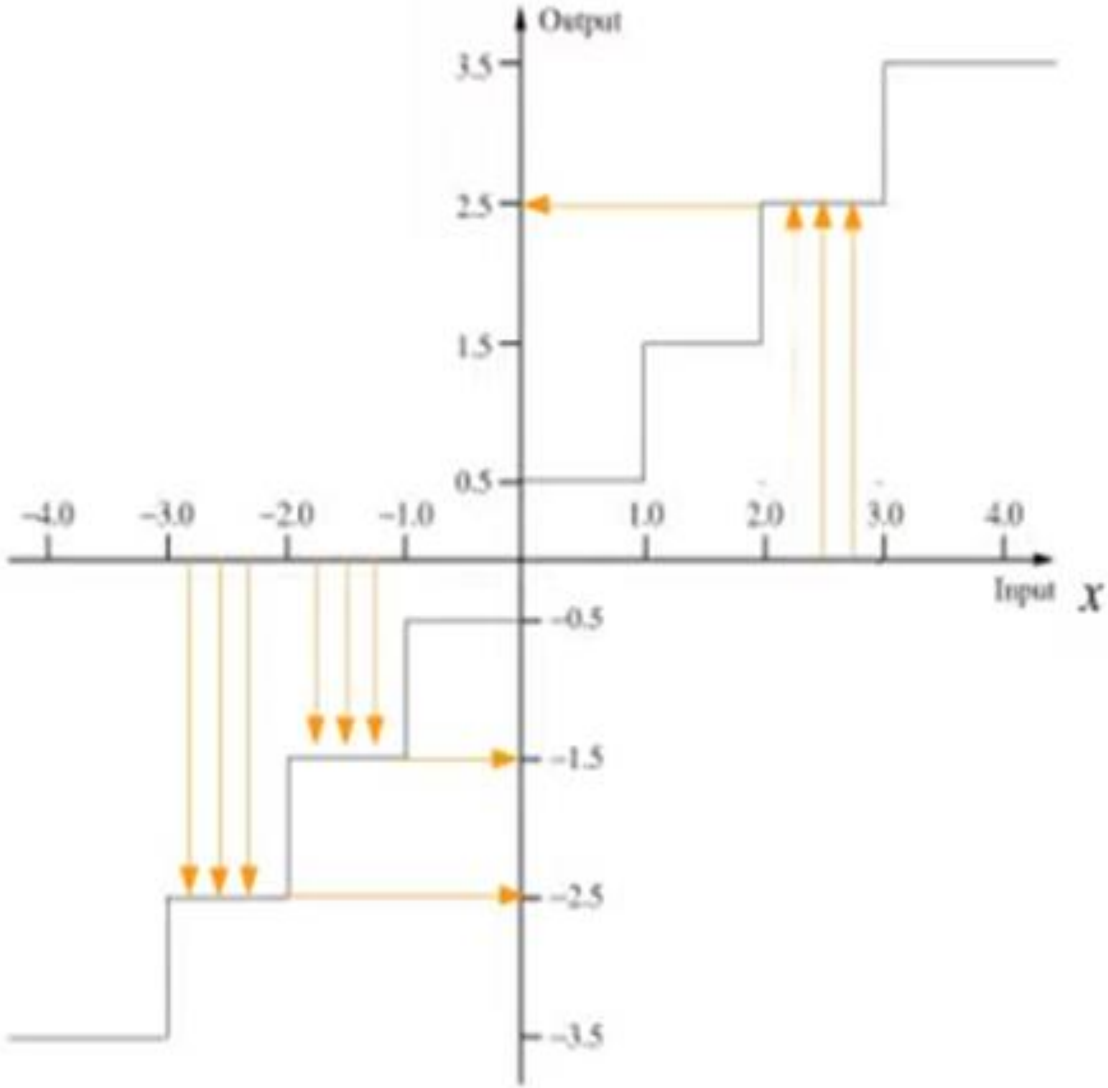
First $Q^{-1} = \frac{1}{2} step$
Second $Q^{-1} = First + step$
Third $Q^{-1} = second + step$

Design of uniform Quantizer for a given number of steps:

- Full scale
- Number of steps.
- Step size.
- Max error
- Compression ratio.

Scalar Quantizer with positive and negative input values:

Example: Quantization for temperature:

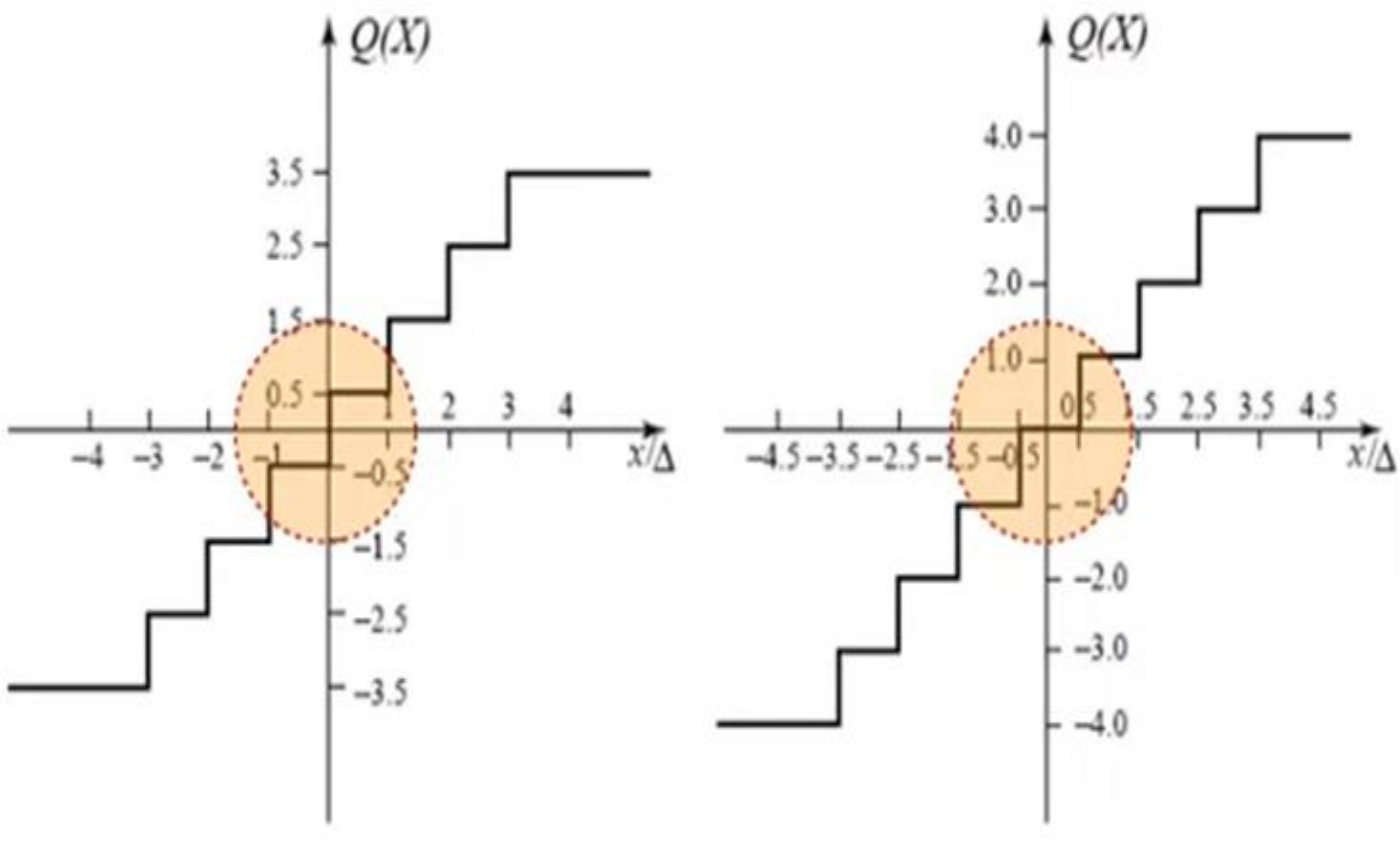


Q	Range	Q^{-1}
000	[-4….-3[	-3.5
001	[-3….-2[	-2.5
010	[-2…-1[	-1.5
011	[-1….0[	-0.5
100	[0…1[	0.5
101	[1….2[	1.5
110	[2….3[	2.5
111	[3….4[	3.5

Types of Uniform Scalar Quantizers:

a) Midrise quantizers have even number of output levels.

b) Midtread quantizers have odd number of output levels including zero as one of them.



(a) (b)

Effect on increasing number of bits (Number of levels) on Quantization Error (Decompressed Image Quality)

Notes:

We measure the quality of quantizer by MSE (Mean Square Error)

Example:

Compress the following data using 2 bits uniform quantizer with step = 32, full scale = 127

6, 15, 17, 60, 100, 90, 66, 59, 18, 3, 5, 16, 14, 67, 63, 2, 98, 92.

Calculate MSE.

Solution:

Range	Q	Q^{-1}
0 – 31	0	16
32 – 63	1	48
64 – 95	2	80
96 - 127	3	112

Solution cont.

$$MSE = \frac{E^2}{18}$$

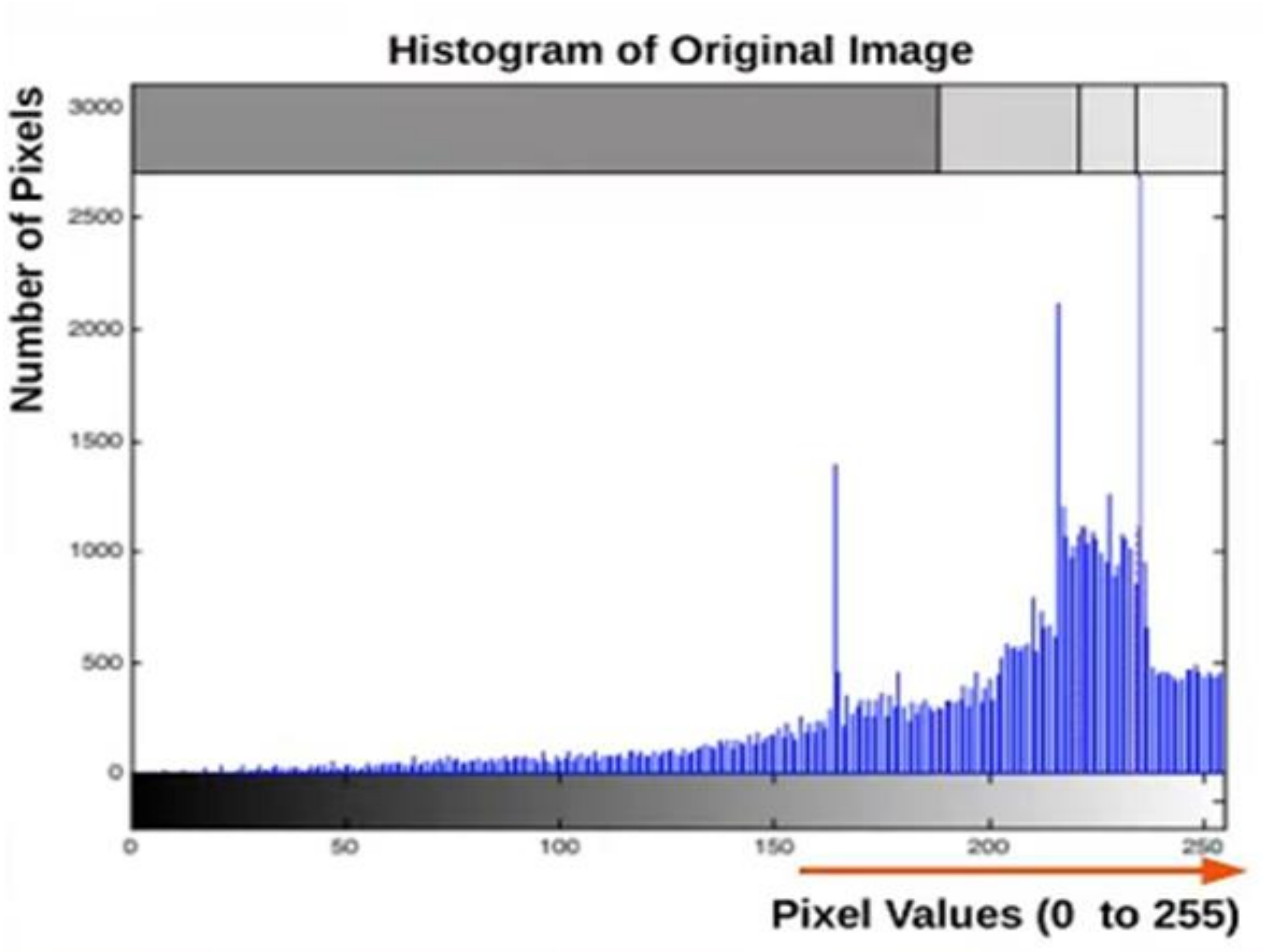
$$MSE = \frac{\mathbf{2035}}{18} = \mathbf{113}$$

Max Error = 15 < $\frac{1}{2}$ $\boldsymbol{step}$

Range	Q	Q^{-1}
0 – 31	0	16
32 – 63	1	48
64 – 95	2	80
96 - 127	3	112

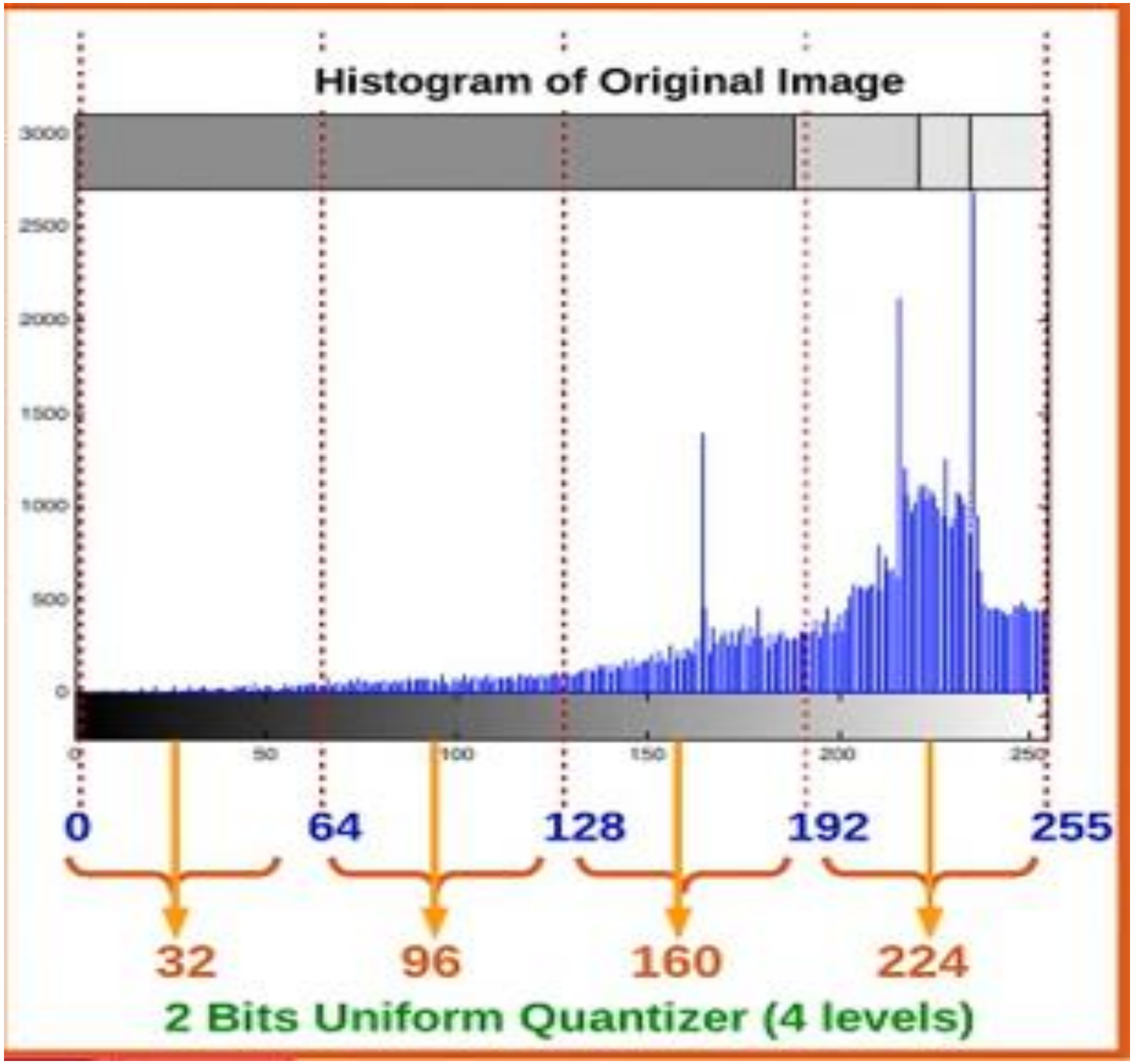
Data	Q	Q^{-1}	$Error$	$Error^2$
6	0	16	10	100
15	0	16	1	1
17	0	16	1	1
60	1	48	12	144
100	3	112	12	144
90	2	80	10	100
66	2	80	14	196
59	1	48	11	121
18	0	16	2	4
3	0	16	13	169
5	0	16	11	121
16	0	16	0	0
14	0	16	2	4
67	2	80	13	169
63	1	48	15	225
2	0	16	14	196
98	3	112	14	196
92	2	80	12	144

Is uniform quantizer the best?



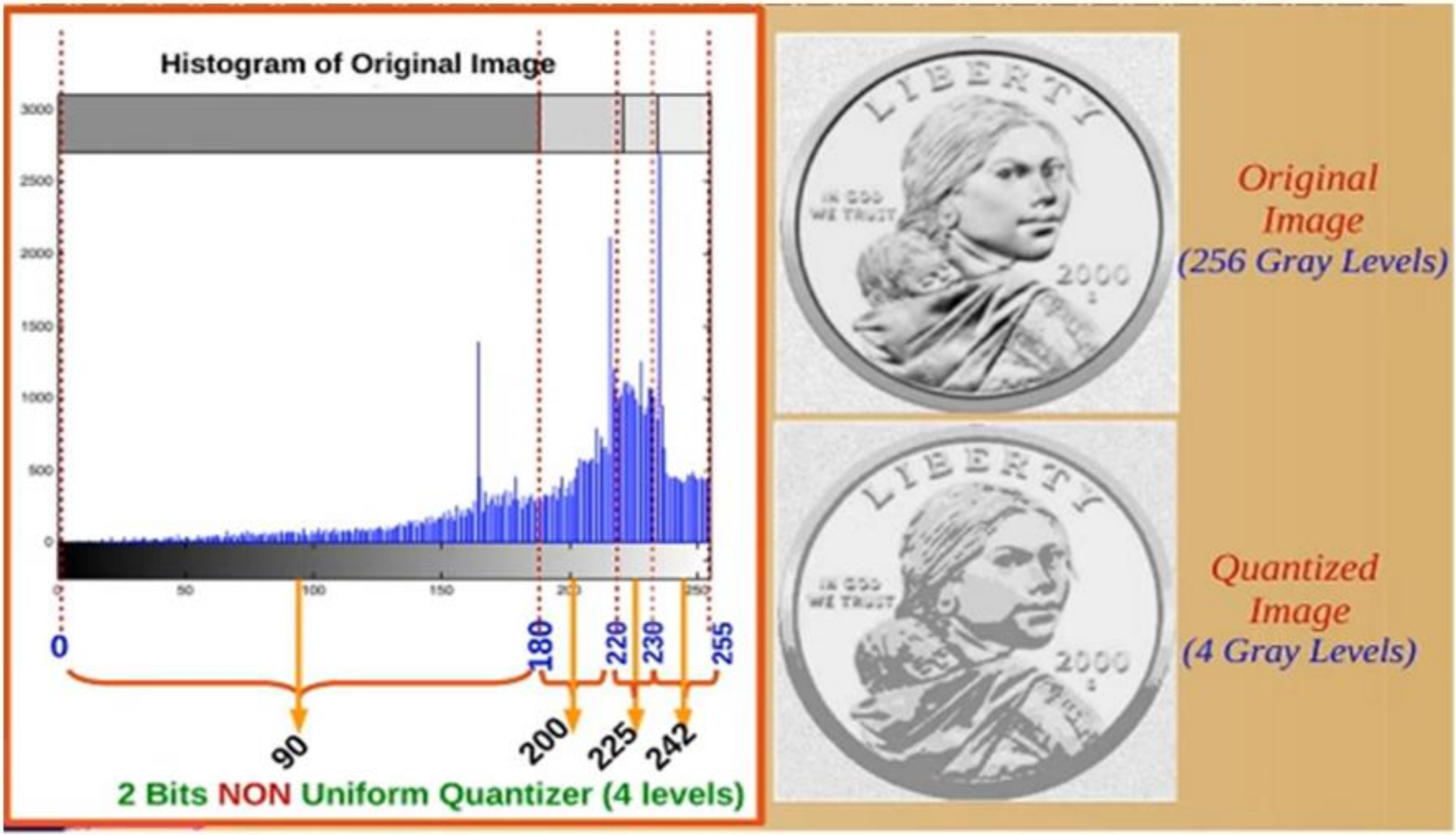
Is uniform quantizer the best?

2 Bits Uniform quantizer:



Is uniform quantizer the best?

2 Bits Non-Uniform quantizer:



Comparison Between Uniform and Non -Uniform Scalar Quantizers:

- **Both Compressed image are of same size.**
- **Compression ration =** $\frac{8}{2} = \frac{4}{1}$ $\boldsymbol{for\ both\ images}$

Example:

Compress the following data using the following 2 bits Non-uniform quantizer.

6, 15, 17, 60, 100, 90, 66, 59, 18, 3, 5, 16, 14, 67, 63, 2, 98, 92.

Calculate MSE.

Range	Q	Q^{-1}
0 – 10	0	4
11 – 39	1	16
40 – 79	2	63
80 - 127	3	95

Solution:

$$MSE = \frac{E^2}{18}$$

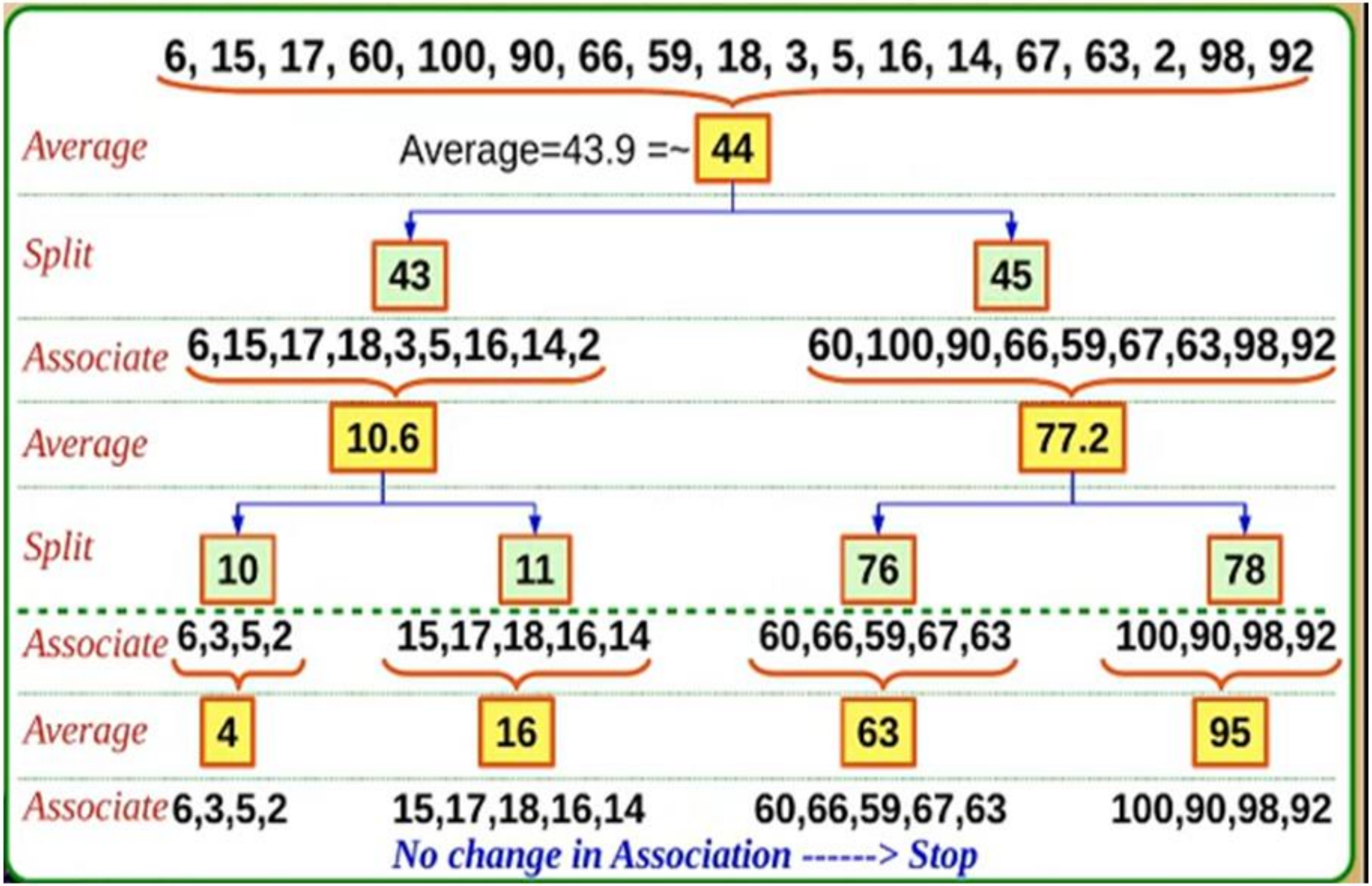
$$MSE = \frac{\mathbf{138}}{18} = \mathbf{7.66}$$

Max Error = 5

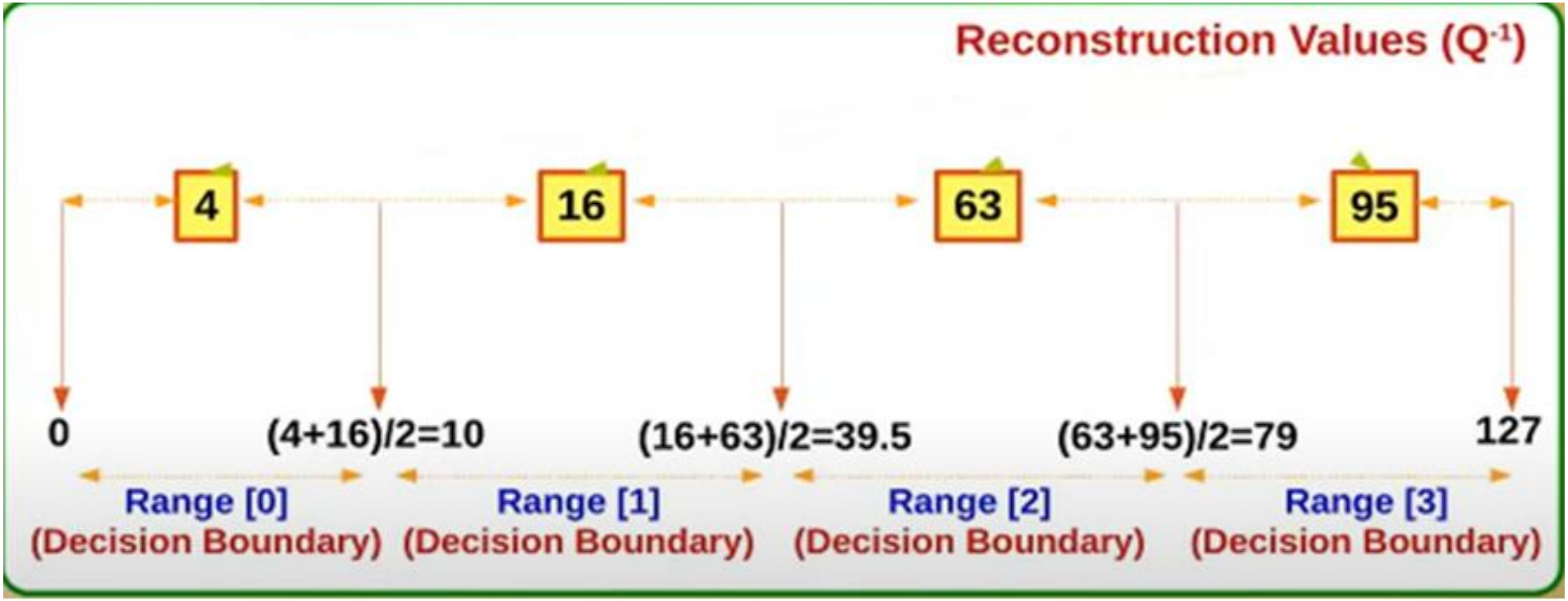
Range	Q	Q^{-1}
0 – 10	**0**	**4**
11 – 39	**1**	**16**
40 – 79	**2**	**63**
80 - 127	**3**	**95**

Data	Q	Q^{-1}	$Error$	$Error^2$
6	0	4	2	4
15	1	16	1	1
17	1	16	1	1
60	2	63	3	9
100	3	95	5	25
90	3	95	5	25
66	2	63	3	9
59	2	63	4	16
18	1	16	2	4
3	0	4	1	1
5	0	4	1	1
16	1	16	0	0
14	1	16	2	4
67	2	63	4	16
63	2	63	0	0
2	0	4	2	4
98	3	95	3	9
92	3	95	3	9

Design of Non-Uniform Quantizer Using LBG Algorithm with Splitting:



Design of Non-Uniform Quantizer Using LBG Algorithm with Splitting:



Range	Q	Q^{-1}
[0….10[	0	4
[10… 39.5[	1	16
[39.5….79[	2	63
[79…..127]	3	95