

a)

Pseudocode for Prim's Algorithm

1. Input:

- A graph $G(V,E)$ is represented as an adjacency matrix or adjacency list, where V is the set of vertices and E is the set of edges.
- A starting vertex r .

2. Output:

- The MST represented as a set of edges.

Algorithm:

PRIM(G, r):

1. Initialize MST as an empty set.
 2. Initialize a min-priority queue Q with all vertices and their key values (infinity for all except r , which is 0).
 3. While Q is not empty:
 - a. Extract the vertex u with the smallest key from Q .
 - b. Add u to the MST.
 - c. For each vertex v adjacent to u :
 - i. If v is in Q and the weight of edge (u, v) is less than the current key of v :
 - Update v 's key to the weight of edge (u, v) .
 - Update v 's parent to u .
 4. Return the MST.
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b)

Time Complexity

1. Using an adjacency matrix:

- Extracting the minimum vertex takes $O(V^2)$.
- Time complexity: $O(V^2)$.

2. Using an adjacency list with a min-heap:

- Extracting the minimum vertex takes $O(\log V)$.

- Updating adjacent vertices takes $O(\log V)$ per edge.
- Time complexity: $O(E \log V)$.