Electronic Systems

Active Filters

Lecture 3

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The Active Filters Contents:

1. Introduction to Filters.
2. Low Pass Filter.
3. High Pass Filter.
4. Band Pass Filter.
5. **Butterworth Filter.**
6. Chebyshev Filter.
7. Bessel Filter.
8. KHN Biquad filter.
9. Multiple Feedback Filters.
10. State Variable Filters.

Filter Transfer Function Approximations

1. **Butterworth Filter**
2. **Chebyshev Filter**
3. **Bessel Filter**

1. Butterworth Filter

- The **Butterworth filter** is a type of signal processing filter designed to have a frequency response that is as flat as possible in the passband. It is also referred to as a **maximally flat magnitude filter**.

- Butterworth had a reputation for solving very complex mathematical problems thought to be 'impossible'. At the time, filter design required a considerable amount of designer experience due to limitations of the theory then in use. The filter was not in common use for over 30 years after its publication.

- Butterworth stated that:

"An ideal electrical filter should not only completely reject the unwanted frequencies but should also have uniform sensitivity for the wanted frequencies".

1. Butterworth Filter

- Such an ideal filter cannot be achieved, but Butterworth showed that successively closer approximations were obtained with increasing numbers of filter elements of the right values. At the time, filters generated substantial ripple in the passband, and the choice of component values was highly interactive.

- Butterworth solved the equations for two-pole and four-pole filters, showing how the latter could be cascaded when separated by vacuum tube amplifiers and so enabling the construction of higher-order filters despite inductor losses.

1. Butterworth Filter

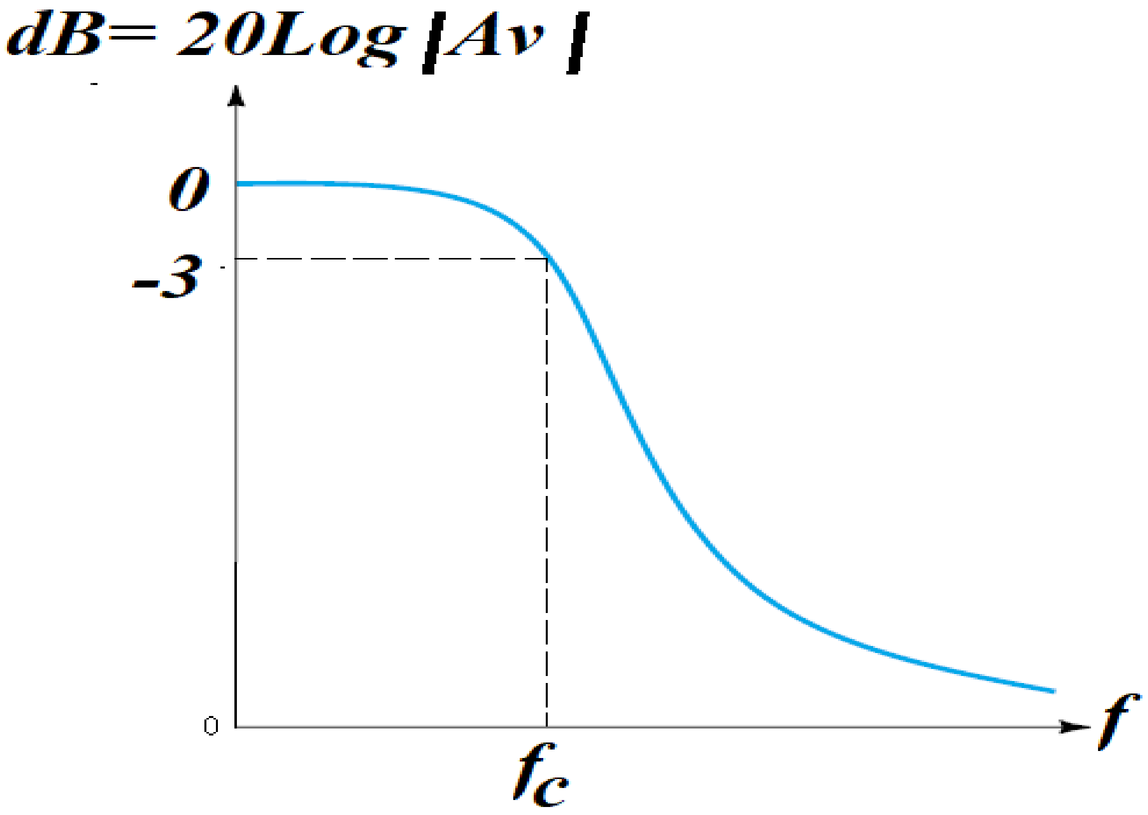
- In applications that use filters to shape the frequency spectrum of a signal such as in communications or control systems, the shape or width of the roll-off also called the "transition band". For simple first-order filters this transition band maybe too long or too wide, so active filters designed with more than one "order" are required. These types of filters are commonly known as "High-order" or "n^{th}-order" filters.

- Then, for a filter that has an n^{th} number order, it will have a subsequent roll-off rate of 20n dB/decade. So a first-order filter has a roll-off rate of 20dB/decade, a second-order filter has a roll-off rate of 40dB/decade, and a fourth-order filter has a roll-off rate of 80dB/decade, etc.

- High-order filters, such as third, fourth, and fifth-order are usually formed by cascading together single first-order and second-order filters.

1.Butterworth Filter

- The most commonly used filter.
- Provide the maximum Flatness during the pass band.
- Low Roll-off after *fc* .



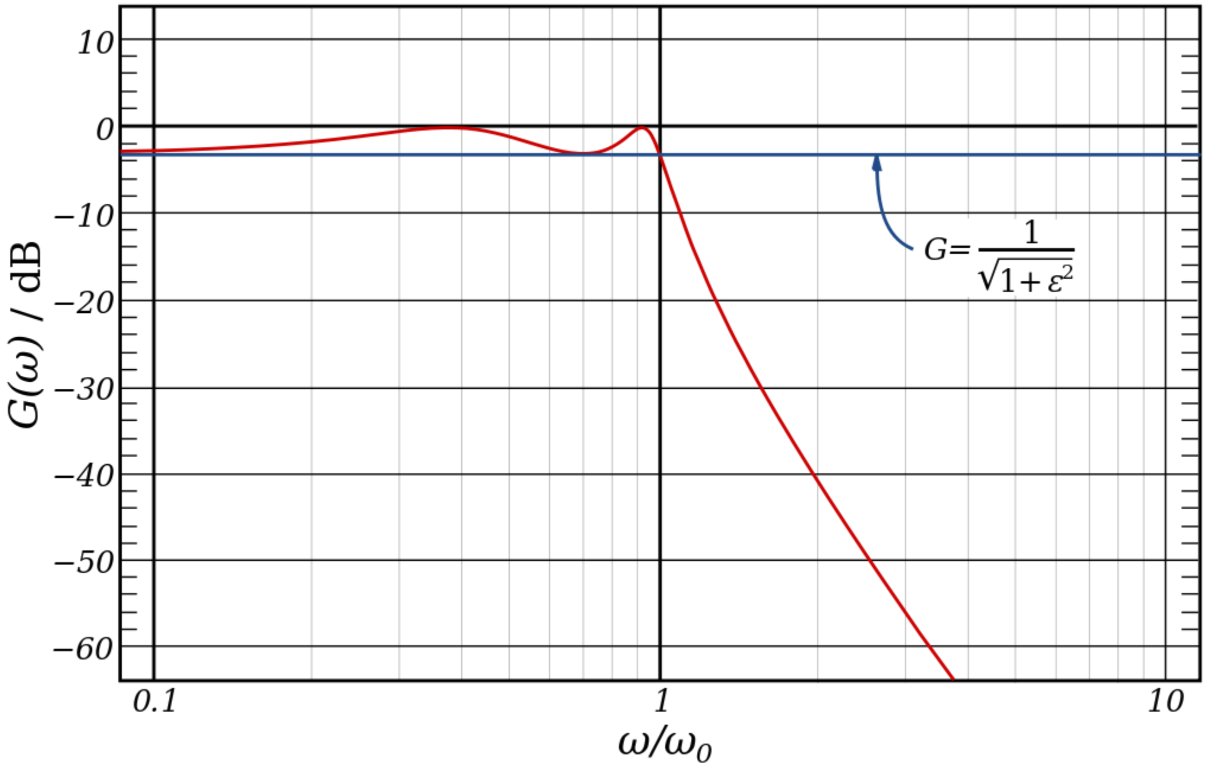
2. Chebyshev Filter

- **Chebyshev filters** are analog or digital filters that have a steeper roll-off than Butterworth filters, and have either passband ripple (type I) or stopband ripple (type II).

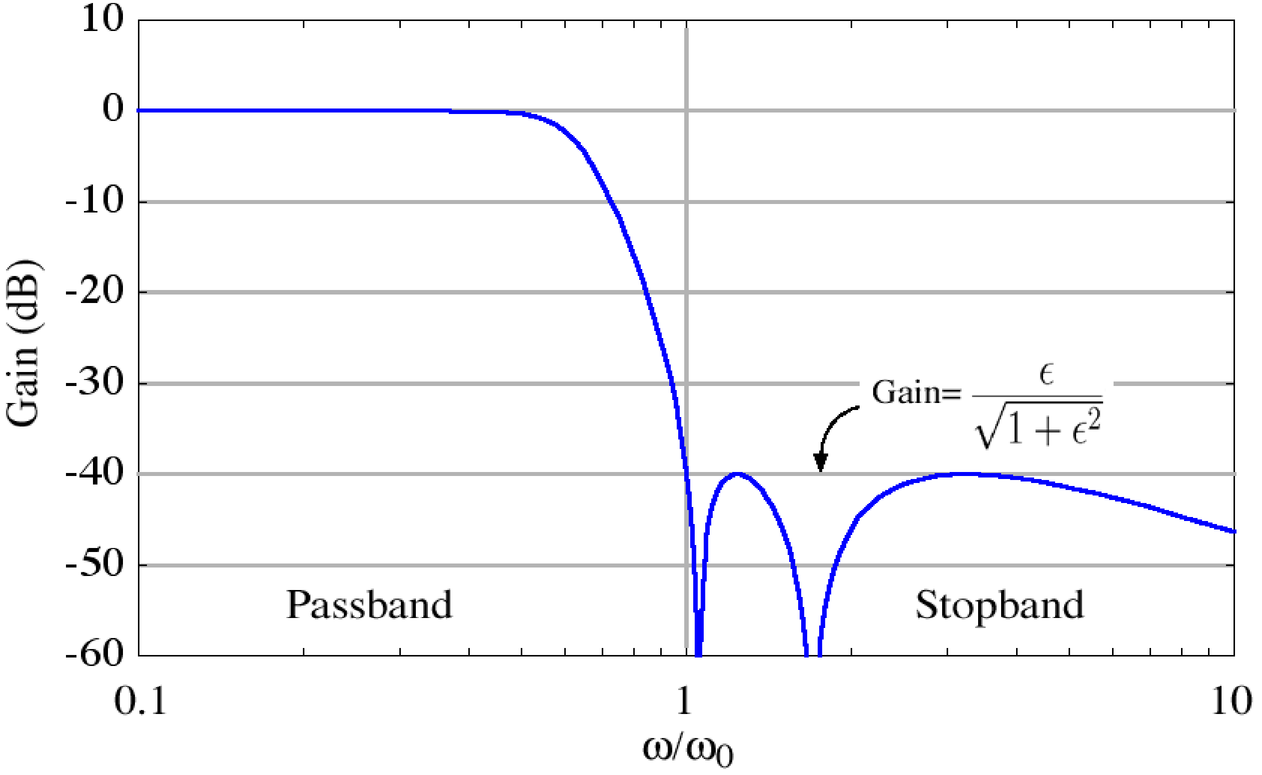
- Chebyshev filters have the property that they minimize the error between the idealized and the actual filter characteristic over the range of the filter, but with ripples in the passband.

- This type of filter is named after Pafnuty Chebyshev because its mathematical characteristics are derived from Chebyshev polynomials. Type I Chebyshev filters are usually referred to as "Chebyshev filters", while type II filters are usually called "inverse Chebyshev filters".

2. Chebyshev Filter



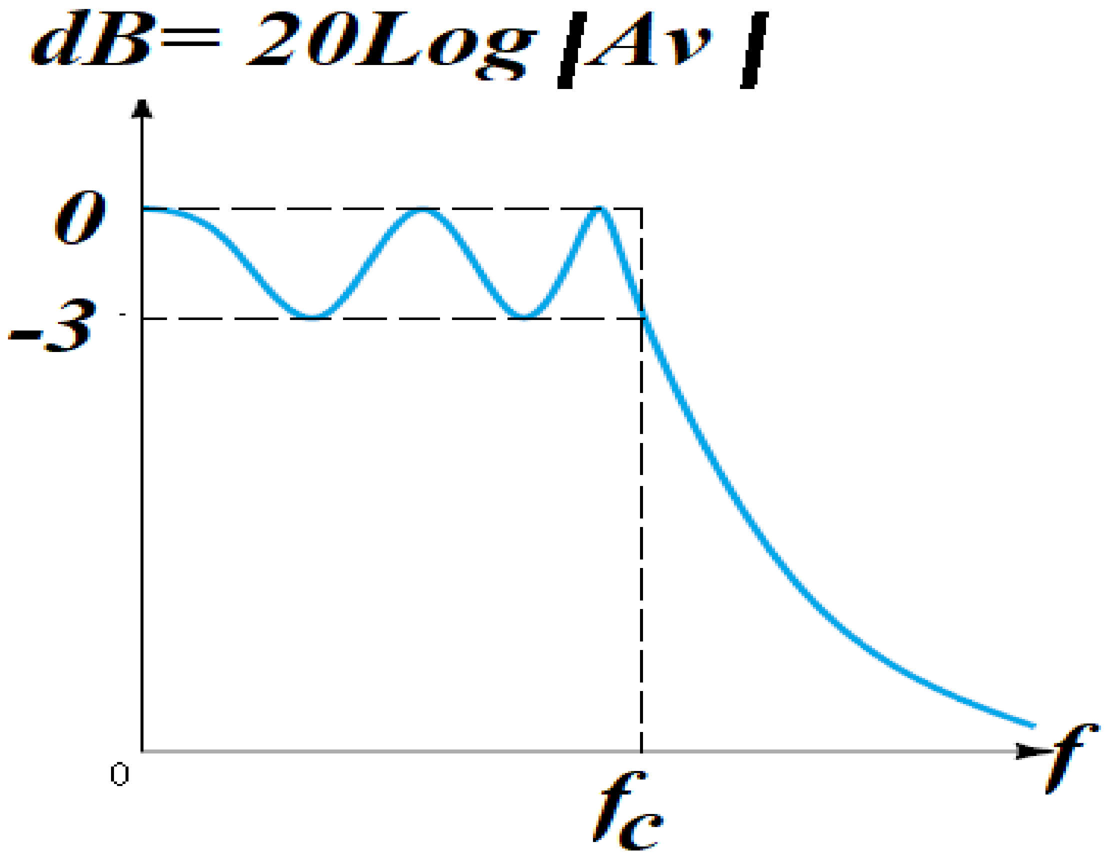
(Type I)



(Type II)

2. Chebyshev Filter

- ❑ **Provide a ripples during the pass band.**
- ❑ **High Roll-off after *fc* .**



3. Bessel Filter

- In electronics and signal processing, a **Bessel filter** is a type of analog linear filter with a maximally flat group/phase delay (maximally linear phase response), which preserves the wave shape of filtered signals in the passband. Bessel filters are often used in audio crossover systems.

- The filter's name is a reference to German mathematician Friedrich Bessel, who developed the mathematical theory on which the filter is based. The filters are also called **Bessel–Thomson filters** in recognition of W. E. Thomson, who worked out how to apply Bessel functions to filter design in.

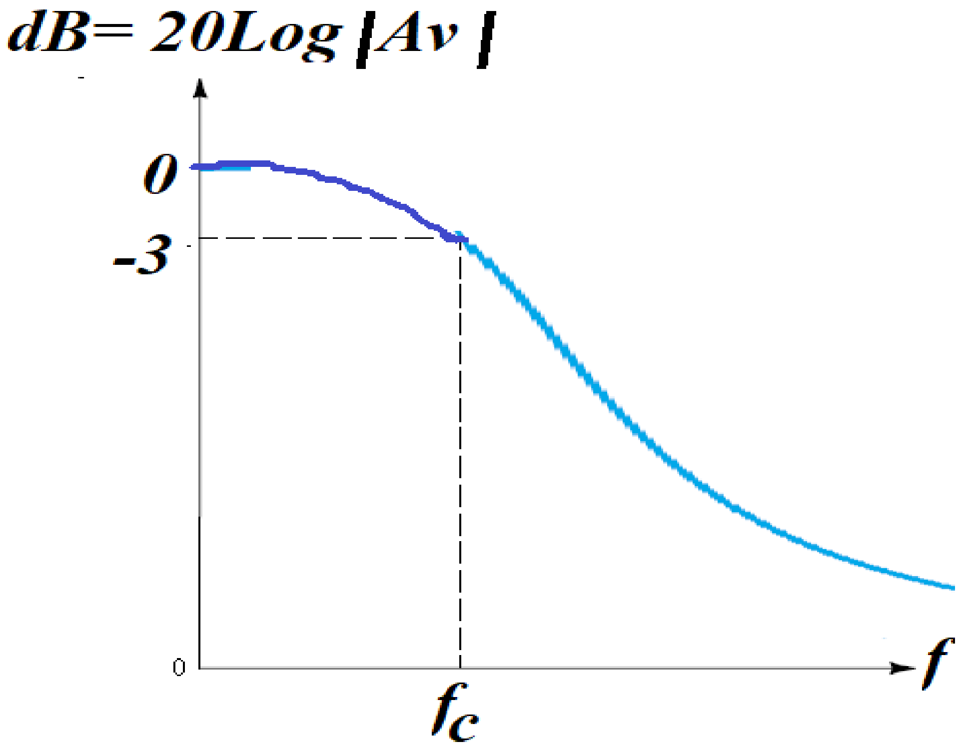
- The Bessel filter is very similar to the Gaussian filter, and tends towards the same shape as filter order increases.

3. Bessel Filter

- **Less Flat area during the pass band.**
- **Less sharpness than the last two types.**

But:

It has a linear phase response over wide frequency range.



Compare between Butterworth,Chebyshev and Bessel Filters

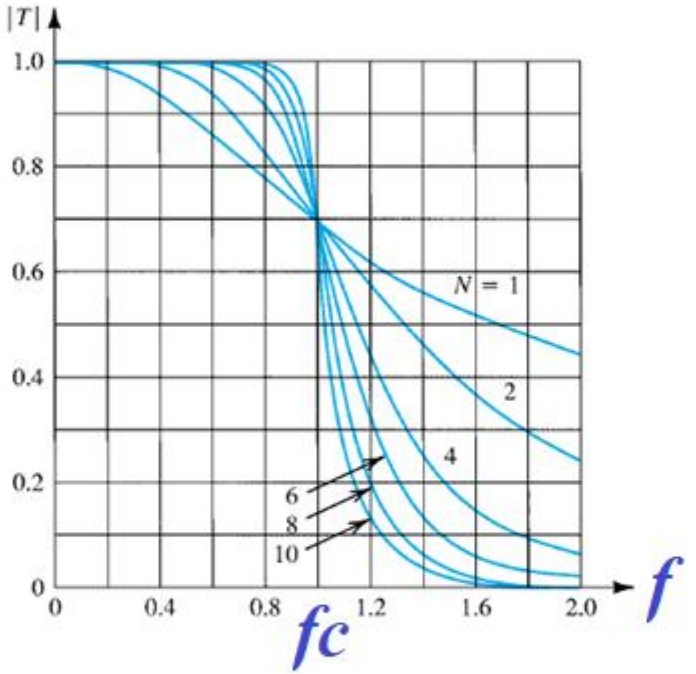
	Advantages	Disadvantages
Butterworth	Provide the maximum Flatness during the pass band	Low Roll-off after *fc*
Chebyshev	High Roll-off after fc	Provide a ripples during the pass band
Bessel	It has a linear phase response over wide frequency range	- Less Flat area during the pass band - Less sharpness than the last two types

1. Butterworth Filter

- The filter transfer function is:

$$A_v(S) = \frac{A_o}{B_n(S)} = \frac{A_m}{B_n(S)}$$

- Where: $A_o = A_m$ is the maximum gain and $B_n(S)$ is the Butterworth polynomial.



- **Normalized Butterworth polynomial Table:**

Filter Order (n)	$B_n(S)$
1	S+1
2	$S^2 + 1.414\,S + 1$
3	$(S+1).(S^2 + S + 1)$
4	$(S^2 + 0.765\,S + 1).(S^2 + 1.848\,S + 1)$
5	$(S+1).(S^2 + 0.618\,S + 1).(S^2 + 1.618\,S + 1)$

1. Butterworth Filter

n	Factors of Butterworth Polynomials $B_n(s)$
1	$(s+1)$
2	$(s^2+1.414214s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765367s+1)(s^2+1.847759s+1)$
5	$(s+1)(s^2+0.618034s+1)(s^2+1.618034s+1)$
6	$(s^2+0.517638s+1)(s^2+1.414214s+1)(s^2+1.931852s+1)$
7	$(s+1)(s^2+0.445042s+1)(s^2+1.246980s+1)(s^2+1.801938s+1)$
8	$(s^2+0.390181s+1)(s^2+1.111140s+1)(s^2+1.662939s+1)(s^2+1.961571s+1)$
9	$(s+1)(s^2+0.347296s+1)(s^2+s+1)(s^2+1.532089s+1)(s^2+1.879385s+1)$
10	$(s^2+0.312869s+1)(s^2+0.907981s+1)(s^2+1.414214s+1)(s^2+1.782013s+1)(s^2+1.975377s+1)$

1.Butterworth Low Bass Filter (LBF)

(a) First Order LPF:

Transfer Function

$$A_V(S) = \frac{A_m}{1 + \frac{S}{W_c}}$$

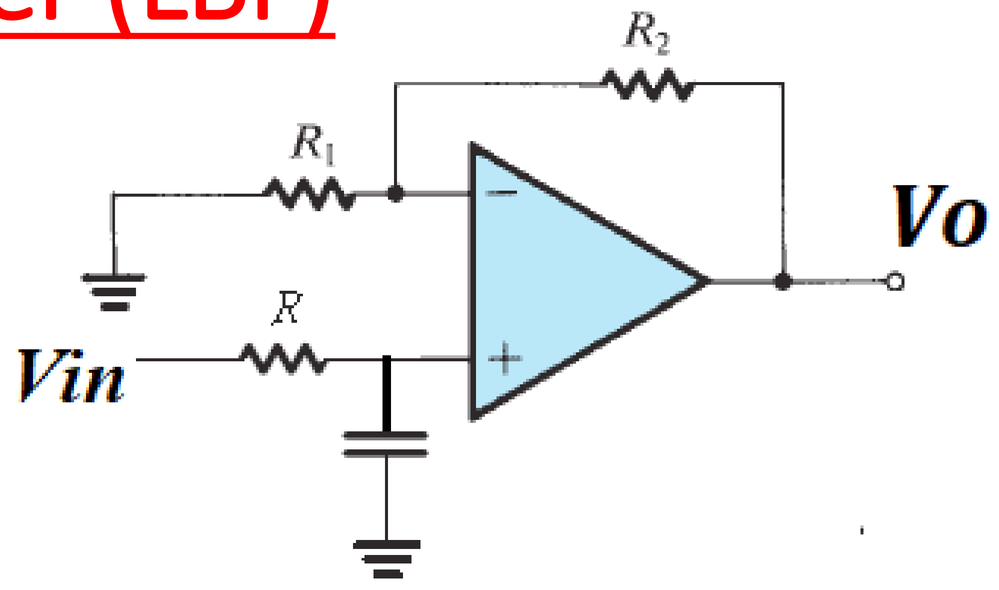
A_m *The maximum gain*

$W_c = 2\pi f_c$

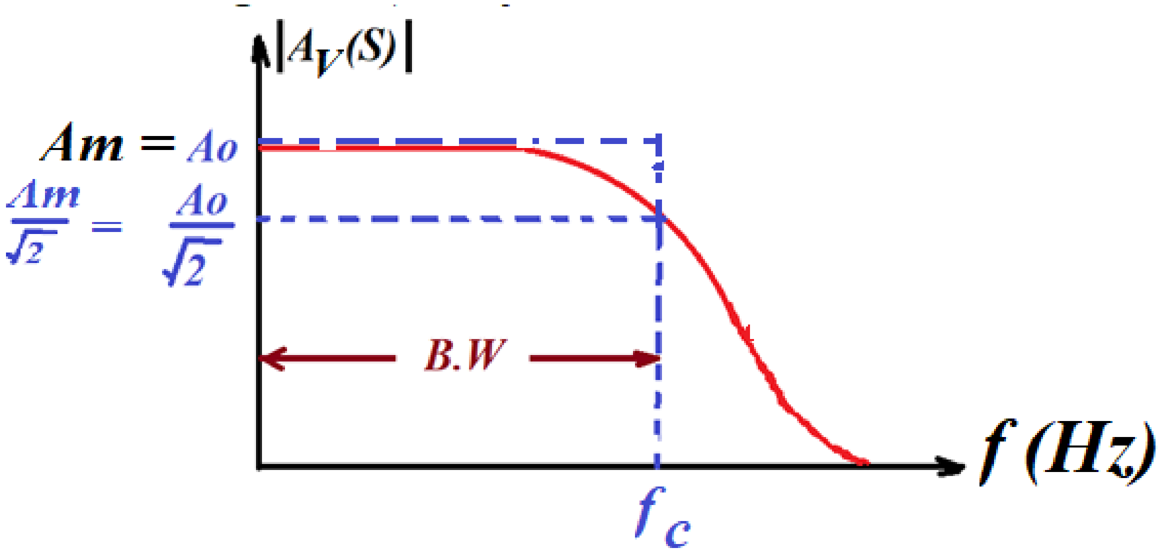
fc *The cut-off frequency*

Design Rules:

- ❑ *Maximum gain* $A_m = 1 + \frac{R_2}{R_1}$
- ❑ *Cut-Off Frequency* $f_c = \frac{1}{2\pi R C}$

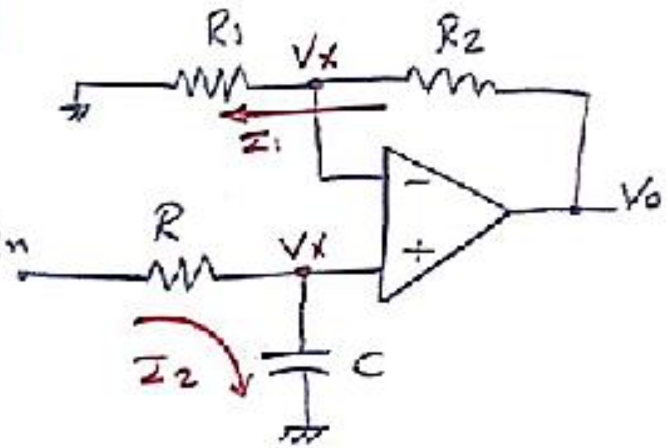


Circuit



1.Butterworth Low Bass Filter (LBF)

Proof:



$$I_1 = \frac{V_x - 0}{R_1} = \frac{V_o - 0}{R_1 + R_2}$$

$$V_o = \frac{R_1 + R_2}{R_1} V_x$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right) \cdot V_x$$

$$V_x = \frac{1}{1 + R_2/R_1} V_o$$

$$* \; V_x = I_2 \cdot \frac{1}{sC}$$

$$V_x = \left[\frac{V_{in}}{R + \frac{1}{sC}}\right] \cdot \frac{1}{sC}$$

$$V_x = \frac{V_{in}}{sRC + 1}$$

↓

$$\frac{V_o}{1 + \frac{R_2}{R_1}} = \frac{V_{in}}{sRC + 1}$$

$$A_v = \frac{V_o}{V_{in}} = \frac{1 + \frac{R_2}{R_1}}{1 + sRC}$$

$$A_v = \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 + \frac{s}{1/RC}}$$

$$A_v = \frac{A_m}{1 + \frac{s}{\omega_c}}$$

$$\therefore \; A_m = 1 + \frac{R_2}{R_1}$$

→ max. gain

∴ cut-off frequency

$$\omega_c = \frac{1}{RC}$$

$$\therefore \; f_c = \frac{1}{2\pi RC}$$

1.Butterworth Low Bass Filter (LBF)

Example 1

Design a First order Butterworth LPF with a cut-off frequency of 10 KHz and a DC gain of 10.

Solution

$$\boldsymbol{A_m = 10 = 1 + \frac{R_2}{R_1}}$$

$\frac{R_2}{R_1} = 9$

Let R1 =1KΩ

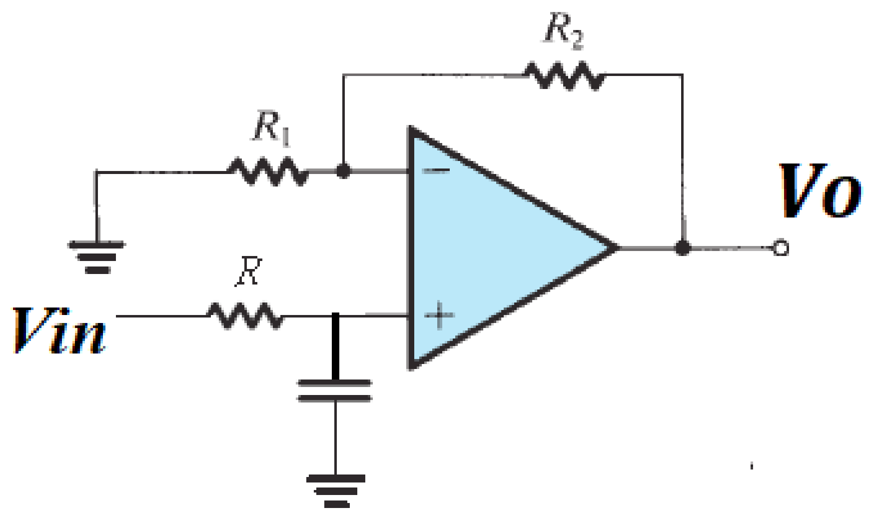
R2 =9KΩ

Fc =10KHz =$\frac{1}{2\pi Rc}$

Choose C=0.01 μF

$$10^4 = \frac{1}{2\pi R(0.01x10^{-6})}$$

R=1.591KΩ≅ 1.6KΩ



Circuit

1. Butterworth Second Order (LBF)

Transfer Function:

$$A_V(S) = \frac{V_o}{Vin} = \frac{A_m}{(\frac{S}{Wc})^2 + 2K\left(\frac{S}{Wc}\right) + 1}$$

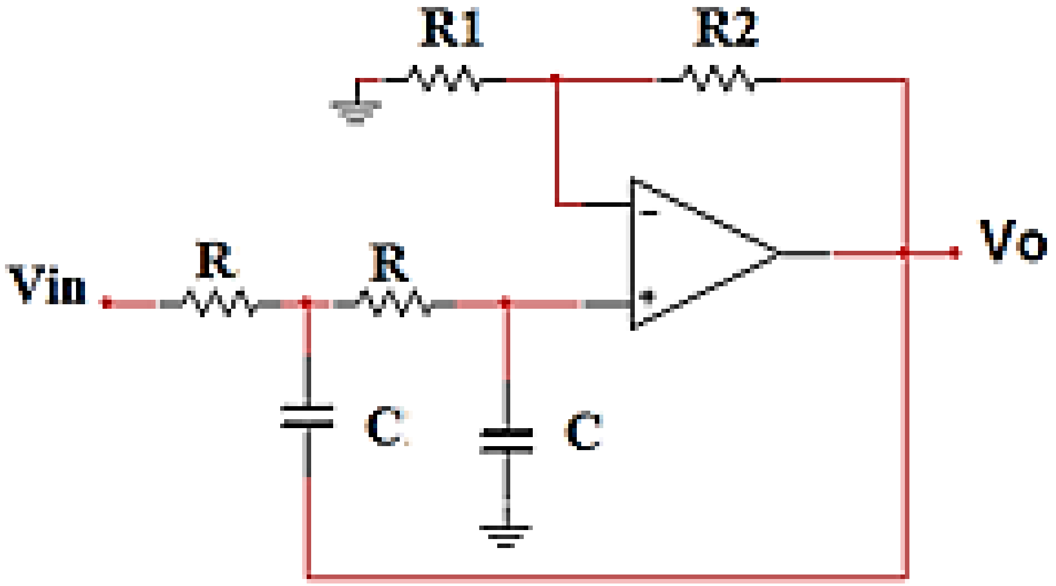
A_m ***The maximum gain***

$W_c = 2\pi fc$

fc ***The cut-off frequency***

K ***The damping ratio***

$(2K)$ ***The coefficient of (S) in the table***



Circuit

Design Rules:

- ***Maximum gain*** $A_m = 1 + \frac{R_2}{R_1}$
- ***Cut-Off Frequency*** $f_c = \frac{1}{2\pi R C}$
- ***Damping Ratio*** $2K =$ ***Coefficient of (S) in the polynomial*** $B_n(S)$.

1. Butterworth Second Order (LBF)

Proof:

$$* \; I = \frac{V_1 - 0}{R_1} = \frac{V_o - 0}{R_1 + R_2}$$

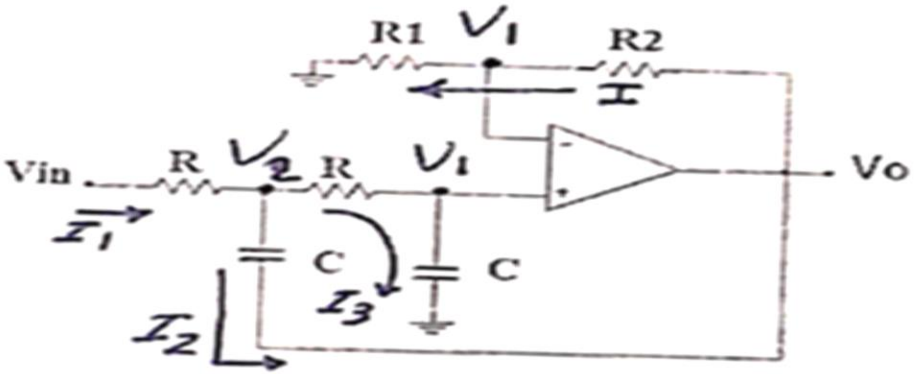
$$\therefore V_o = \frac{R_1 + R_2}{R_1} V_1$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right) . V_1$$

$$V_o = A_m V_1$$

$$\therefore \boxed{V_1 = \frac{V_o}{A_m}} \quad \boxed{1}$$

where $\boxed{A_m = A_o = 1 + \frac{R_2}{R_1}}$ $\boxed{2}$

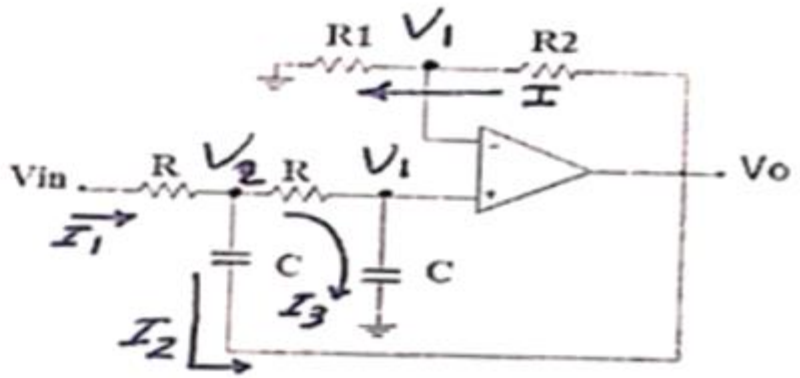


$$* \; V_1 = I_3 \frac{1}{sC} = \left[\frac{V_2}{R + \frac{1}{sC}}\right] . \frac{1}{sC} = \frac{V_2}{sCR + 1}$$

$$\therefore V_2 = (sCR + 1) . V_1 \quad \text{sub. From } \boxed{1}$$

$$\therefore \boxed{V_2 = \frac{sCR + 1}{A_m} V_o} \quad \boxed{3}$$

1. Butterworth Second Order (LBF)



$$* \ I_1 = I_2 + I_3$$

$$\frac{V_{in} - V_2}{R} = \frac{V_2 - V_o}{1/SC} + \frac{V_2}{R + \frac{1}{SC}}$$

$$\frac{V_{in} - V_2}{R} = SC(V_2 - V_o) + \frac{SC}{SCR+1} V_2 \qquad \times R$$

$$V_{in} - V_2 = SCR(V_2 - V_o) + \frac{SCR}{SCR+1} V_2$$

$$V_{in} = V_2 \left[(1 + SCR) + \frac{SCR}{SCR+1}\right] - SCR\,V_o$$

$$\downarrow$$

$$V_{in} = \left(\frac{SCR+1}{Am}\right) V_o \left[(SCR+1) + \frac{SCR}{SCR+1}\right] - SCR\,V_o$$

$$V_{in} = V_o \left\{ \frac{(SCR+1)^2}{Am} + SCR/Am - SCR \right\}$$

$$V_{in} = V_o \left\{ \frac{(SCR+1)^2 + SCR - SCR\,Am}{Am} \right\}$$

$$\therefore \frac{V_o}{V_{in}} = Av(s) = \frac{Am}{s^2C^2R^2 + (3 - Am)\,SCR + 1}$$

$$Av(s) = \frac{Am}{\left(\frac{S}{1/RC}\right)^2 + (3 - Am)\frac{S}{1/RC} + 1}$$

$$Av(s) = \frac{Am}{\left(\frac{S}{\omega_c}\right)^2 + 2k\frac{S}{\omega_c} + 1}$$

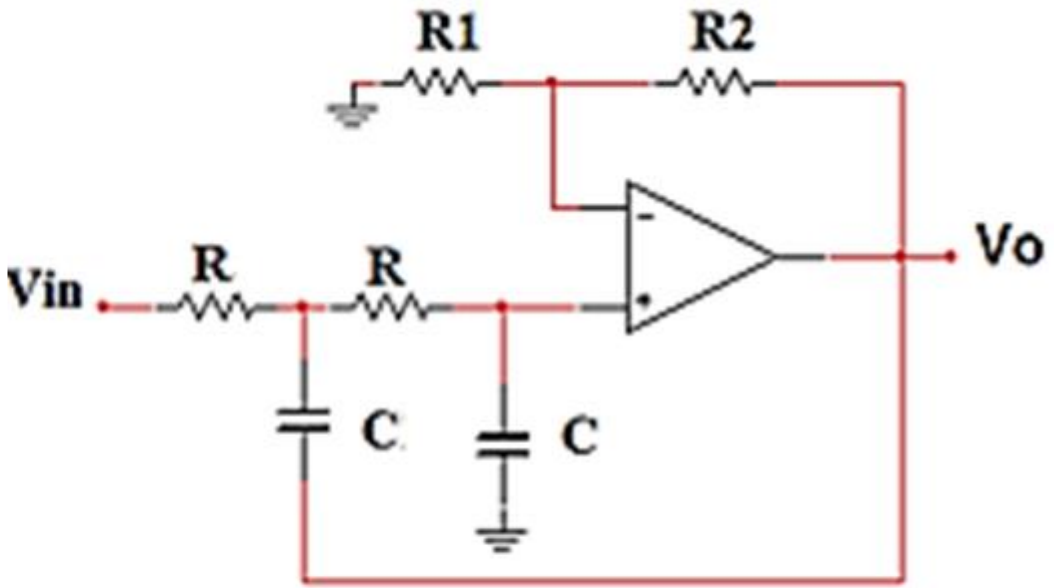
- ❑ *Maximum gain* $\quad A_m = 1 + \frac{R_2}{R_1}$
- ❑ *Cut-Off Frequency* $\quad w_c = \frac{1}{R\,C}$
- ❑ *Then,* $\quad f_c = \frac{1}{2\pi\,R\,C}$

1. Butterworth Second Order (LBF)

Example 2

Design a Second order Butterworth LPF with a cut-off frequency of 10 KHz.

Given n=2, $Bn(s)$= $S^2 + 1.414S + 1$



Solution

$$\boldsymbol{A_m} = 3 - 2\mathrm{k} = 3 - 1.414$$

$$\boldsymbol{A_m} = 1.586 = \boldsymbol{1} + \frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}$$

$\frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}$ = 0.586

Let R1 =10KΩ

R2 =5.86KΩ

Fc = $10^4 = \frac{1}{2\pi Rc}$

Choose C=0.01 μF

$$10^4 = \frac{1}{2\pi R(0.01x10^{-6})}$$

R=1.591KΩ≅ 1.6KΩ

1.Butterworth Higher Order (LBF)

Example 3

Design a 5^{th} order Butterworth Low-Pass-Filter (LPF) with a cut-off frequency of 10 KHz. The Butterworth polynomial for n = 5 is $(S+1)\,(S^2+0.618S+1)\,(S^2+1.618S+1)$.

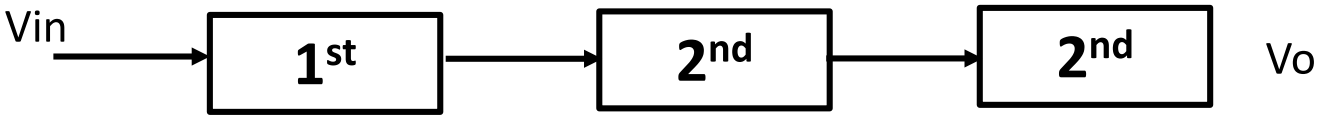
The overall DC gain of the filter should be 10.

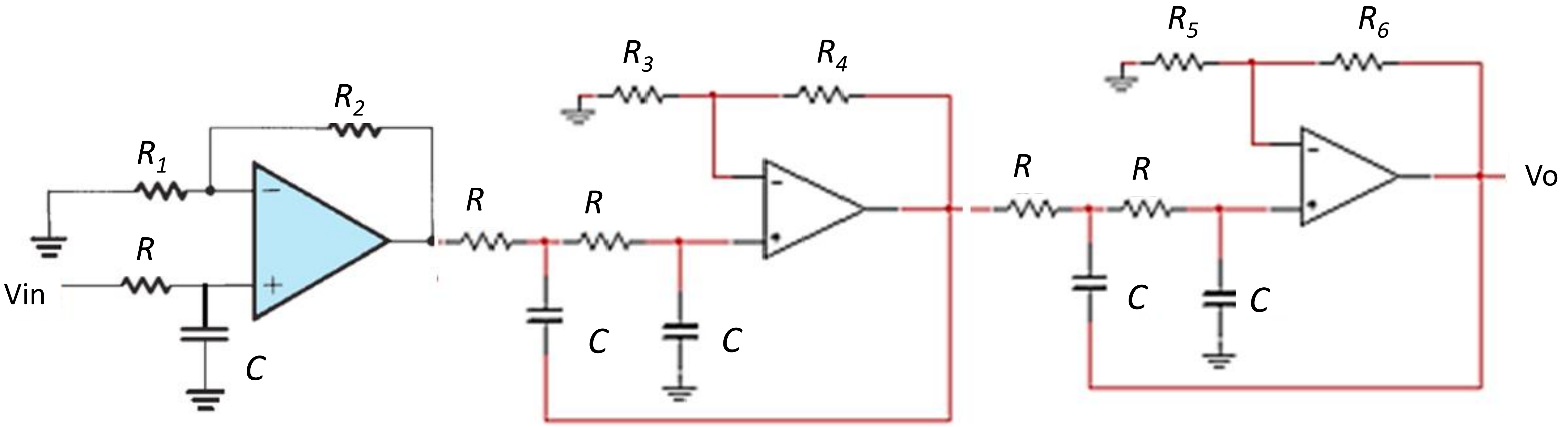
(a) Draw the Circuit- diagram for the filter.

(b) Calculate the values of the circuit components.

1. Butterworth Higher Order (LBF)

(a) Circuit Diagram





1. Butterworth Higher Order (LBF)

(b) Circuit Components:

$$* \ f_o = 10^4 = \frac{1}{2\pi RC}$$

choose $\boxed{C = 0.01\,\mu F}$, Then, $\boxed{R = 1591.55\,\Omega \cong 1.59\,K\Omega}$

$$B_n = (s+1)(s^2+0.618s+1)(s^2+1.618s+1)$$

$$* \ 2k_2 = 0.618 \rightarrow A_{o2} = 3-2k = 3-0.618$$

$$A_{o2} = 2.382 = 1+\frac{R_4}{R_3} \rightarrow \boxed{\frac{R_4}{R_3} = 1.382}$$

1. Butterworth Higher Order (LBF)

Let $\rightarrow \boxed{R_3 = 10\,K\Omega \checkmark \rightarrow \therefore R_4 = 13.82\,K\Omega}$

* $2K_3 = 1.618 \checkmark \rightarrow A_{o3} = 3 - 2k = 3 - 1.618 = 1.382$

$A_{o3} = 1.382 = 1 + \frac{R_6}{R_5} \checkmark \rightarrow \therefore \boxed{\frac{R_6}{R_5} = 0.382}$

Let $\boxed{R_5 = 10\,K\Omega \checkmark \rightarrow \therefore R_6 = 3.82\,K\Omega}$

* Total D.C gain $A_o = A_{o1} A_{o2} A_{o3}$

$10 = A_{o1} \times 2.382 \times 1.382$

$\therefore A_{o1} \cong 3 = 1 + \frac{R_2}{R_1}$

$\therefore \boxed{\frac{R_2}{R_1} \cong 2}$ Let $\boxed{\begin{array}{l} R_1 = 1\,K\Omega \\ \therefore R_3 = 2\,K\Omega \end{array}}$

2. Butterworth High Bass Filter (HBF)

(a) First Order HPF:

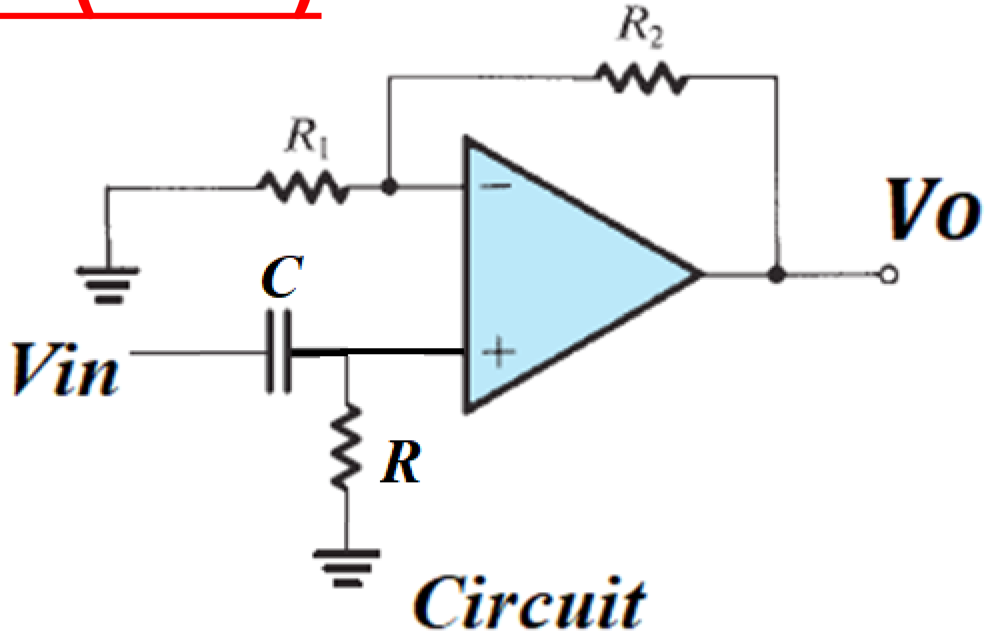
Transfer Function

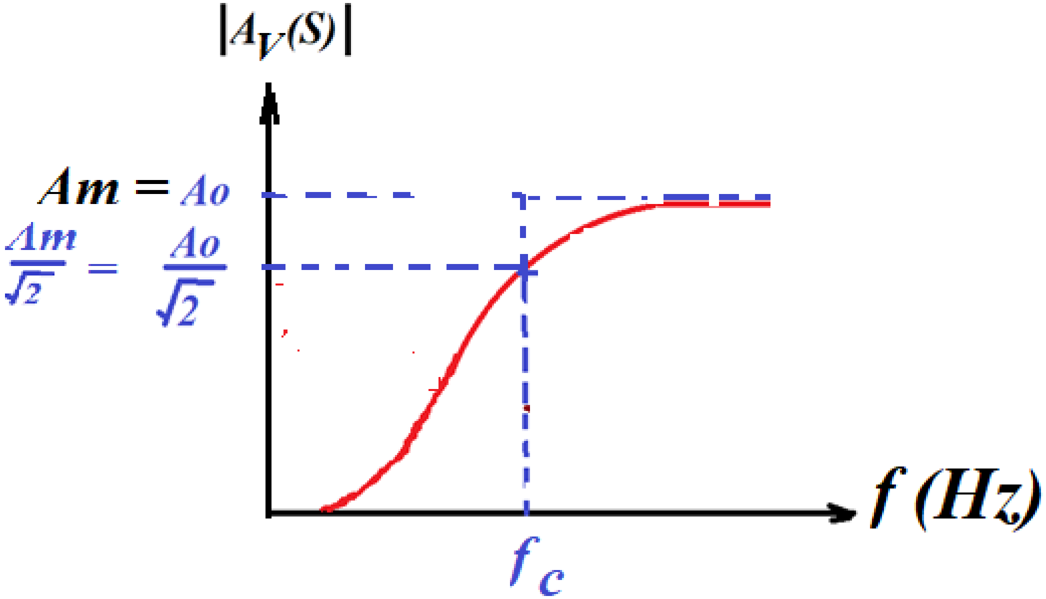
$$A_V(S) = \frac{A_m}{1 + \frac{W_c}{S}}$$

A_m *The maximum gain*

$W_c = 2\pi fc$

fc *The cut-off frequency*





Design Rules:

- ❑ *Maximum gain* $A_m = 1 + \frac{R_2}{R_1}$
- ❑ *Cut-Off Frequency* $f_c = \frac{1}{2\pi\, R\, C}$

2. Butterworth High Bass Filter (HBF)

(b) Second Order HPF:

$\boldsymbol{Transfer\ Function}$:

$$A_V(S) = \frac{V_o}{Vin} = \frac{A_m}{(\frac{Wc}{S})^2 + 2K\left(\frac{Wc}{S}\right) + 1}$$

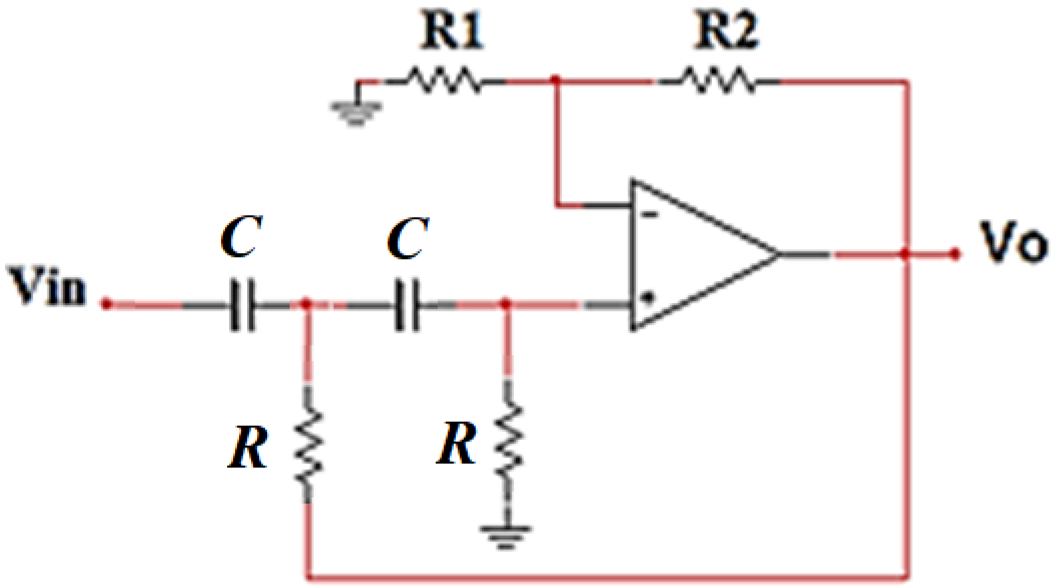
A_m *The maximum gain*

$W_c = 2\pi fc$

fc *The cut-off frequency*

K *The damping ratio*

$(2K)$ *The coefficient of (S) in the table*



Circuit

Design Rules:

- ❑ Maximum gain $A_m = 1 + \frac{R_2}{R_1}$
- ❑ *Cut-Off Frequency* $f_c = \frac{1}{2\pi R C}$
- ❑ *Damping Ratio* $2K =$ *Coefficient of (S) in the polynomial* $B_n(S)$.

2. Butterworth Second Order (HBF)

Example 4

Design a Second order Butterworth HPF with a cut-off frequency of 10 KHz.

Given n=2, $Bn(s)= S^2 + 1.414S + 1$

Solution

$2\mathrm{k} = 1.414$

$\boldsymbol{A_m} = 3 - 2\mathrm{k} = 3 - 1.414$

$\boldsymbol{A_m} = 1.586 = \mathbf{1} + \frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}$

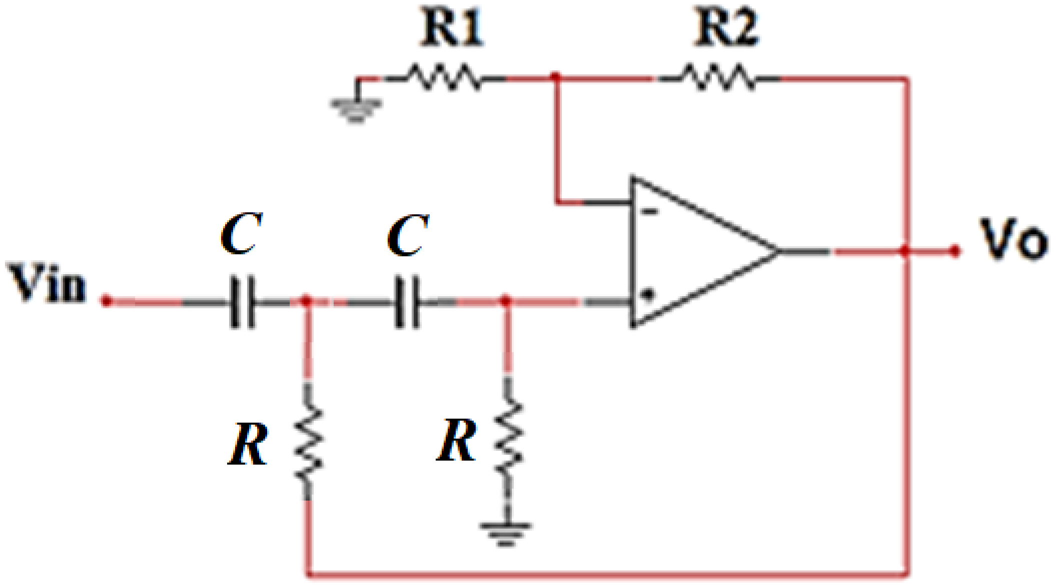
$\frac{\boldsymbol{R_2}}{\boldsymbol{R_1}} = 0.586$ Let R_1 =10KΩ

R_2 =5.86KΩ

$F_c = 10^4 = \frac{1}{2\pi Rc}$ Choose C=0.01 μF

$$10^4 = \frac{1}{2\pi R(0.01x10^{-6})}$$

R=1.591KΩ≅ 1.6KΩ

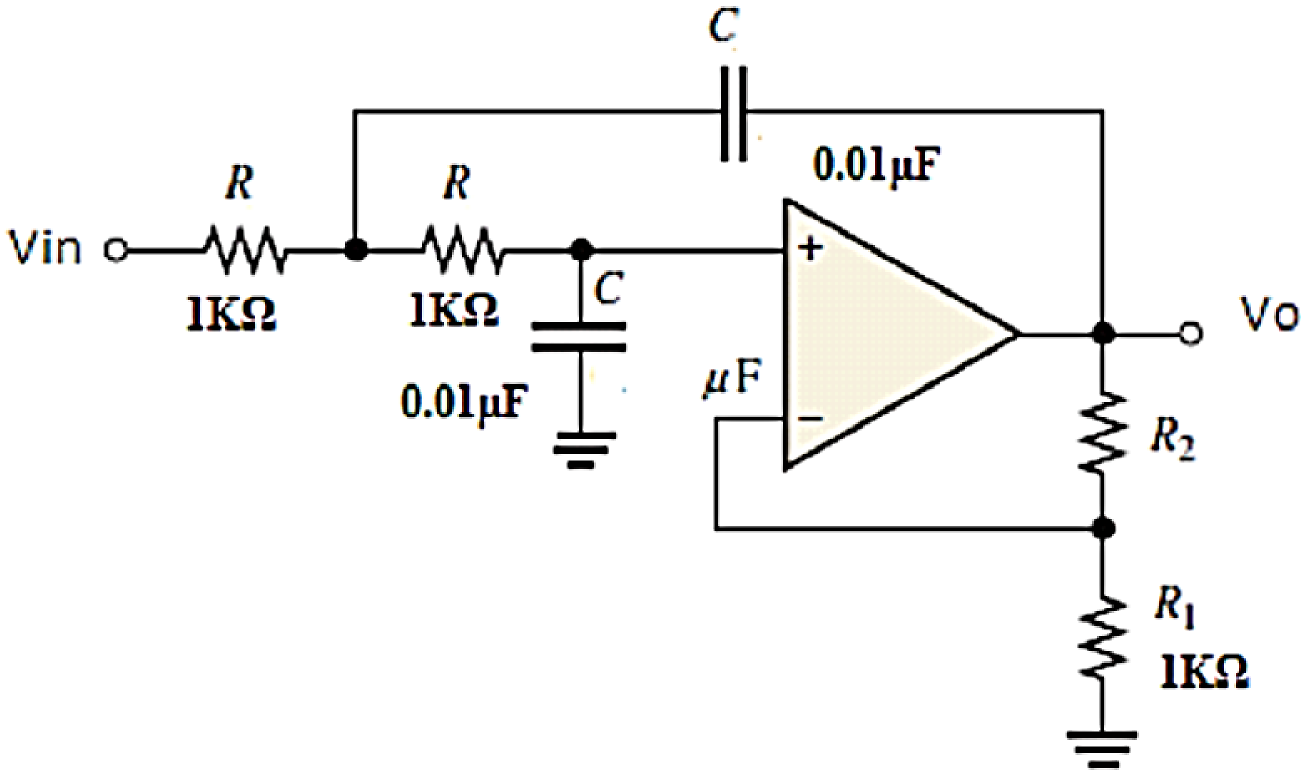


Circuit

Example 5

Example:

The circuit shown in Figure represents an Op-Amp Sallen-Key Active Filter.



(a) Identify the order and type of that filter?

(b) Drive an expression for the filter transfer function H(s).

(C) Calculate the cutoff frequency of the filter and the value of R_2 for a DC gain of 10.

Example 5

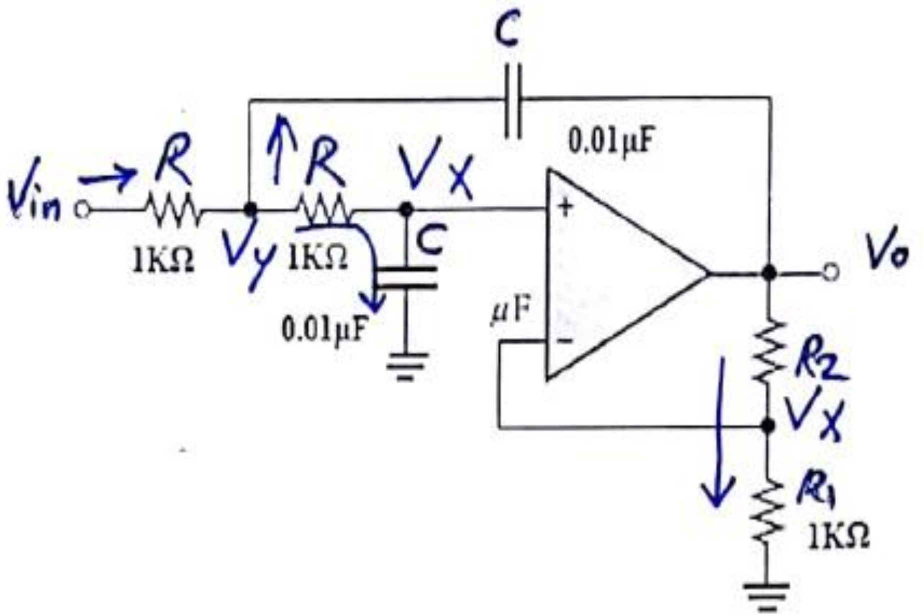
(a) Identify the order and type of that filter?

* Second-order , Low-Pass- Filter (LPF)

(b) Drive an expression for the filter transfer function H(s).

$$* \; V_X = \frac{V_o \cdot R_1}{R_1 + R_2} = \frac{V_o}{1 + \frac{R_2}{R_1}} = \frac{V_o}{A_o}$$

$$\therefore \boxed{V_X = \frac{V_o}{A_o}} \; \boxed{1} \;, \text{ where } \boxed{A_o = 1 + \frac{R_2}{R_1}}$$



$$* \; V_X = \frac{V_Y \cdot \frac{1}{SC}}{R + \frac{1}{SC}} = \frac{V_Y}{SCR + 1}$$

$$\therefore V_Y = (SCR+1) \cdot V_X = (SCR+1) \frac{V_o}{A_o}$$

$$\boxed{V_Y = \frac{SCR+1}{A_o} \cdot V_o} \; \boxed{2}$$

Example 5

* Node of V_Y

$$\frac{V_{in}-V_Y}{R} = \frac{V_Y - V_o}{\left(\frac{1}{sC}\right)} + \frac{V_Y}{R+\frac{1}{sC}} \qquad \times R$$

$$V_{in}-V_Y = sCR(V_Y - V_o) + \frac{R}{R+\frac{1}{sC}} V_Y$$

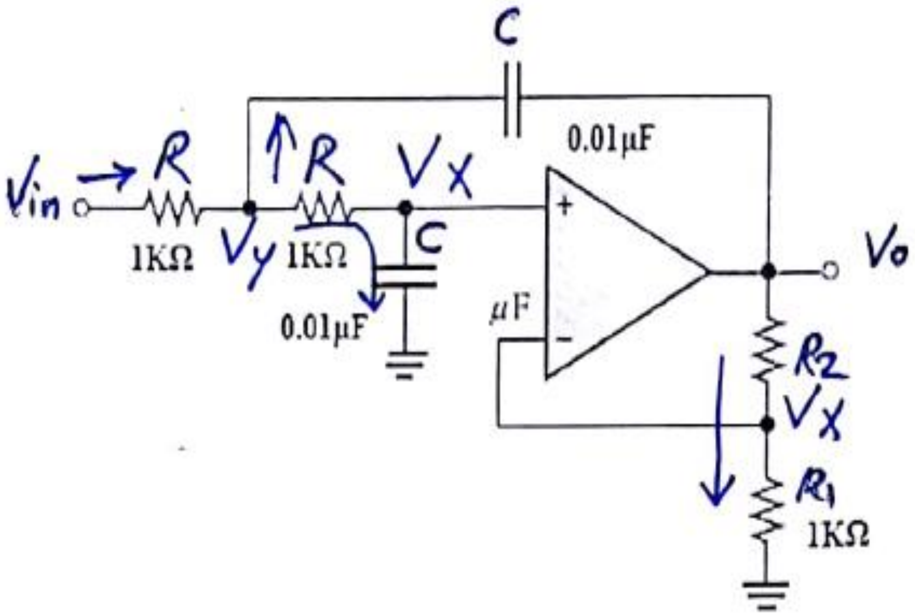
$$V_{in} = V_Y\left[sCR+1 + \frac{R}{R+\frac{1}{sC}}\right] - sCR\,V_o$$

$$V_{in} = \frac{sCR+1}{A_o} V_o\left[(sCR+1) + \frac{sCR}{sCR+1}\right] - sCR\,V_o$$

$$V_{in} = V_o\left\{\frac{sCR+1}{A_o}\left[(sCR+1) + \frac{sCR}{sCR+1}\right] - sCR\right\}$$

$$V_{in} = V_o\left[\frac{(sCR+1)^2}{A_o} + \frac{sCR}{A_o} - sCR\right]$$

$$V_{in} = V_o\left[\frac{(sCR+1)^2 + sCR - sCRA_o}{A_o}\right]$$

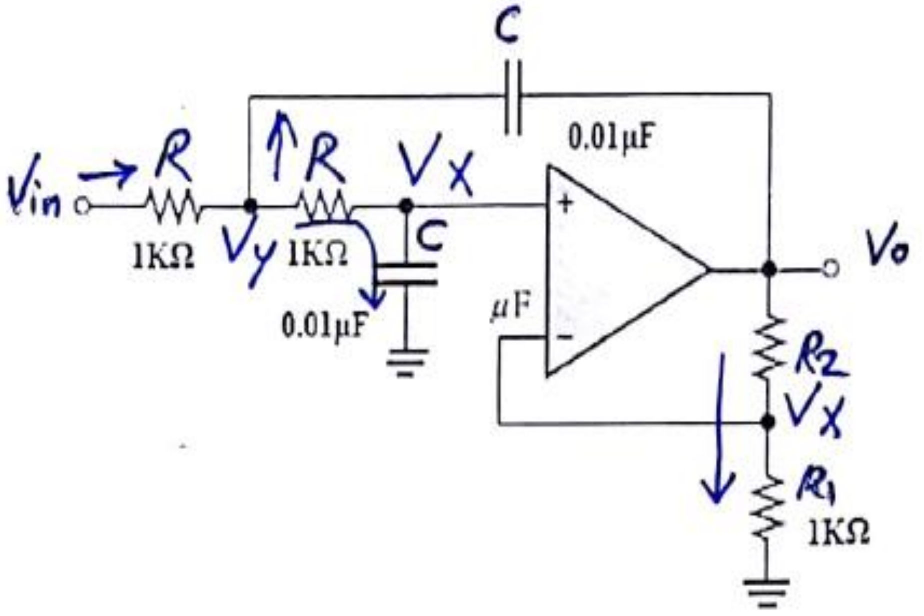


Example 5

$$\therefore H(s) = \frac{V_o}{V_{in}} = \frac{A_o}{s^2C^2R^2 + 1 + 2sCR + sCR - sCRA_o}$$

$$H(s) = \frac{A_o}{(sCR)^2 + (3 - A_o)sCR + 1}$$

$$H(s) = \frac{A_o}{\left(\frac{s}{1/RC}\right)^2 + (3 - A_o)\left(\frac{s}{1/RC}\right) + 1}$$



(c) Calculate the cutoff frequency of the filter and the value of R_2 for a DC gain of 10.

$$* \ \omega_o = \frac{1}{RC} \rightarrow f_o = \frac{1}{2\pi RC}$$

$$\therefore f_o = \frac{1}{2\pi(10^3) \times (0.01 \times 10^{-6})} \rightarrow f_o \cong 15.92 \text{ KHz}$$

$$* \ \text{D.C gain} = A_o = 1 + \frac{R_2}{R_1} = 10$$

$$\therefore \frac{R_2}{R_1} = 9 \quad , \text{Given } R_1 = 1K\Omega$$

$$\therefore \boxed{R_2 = 9 \ K\Omega}$$