Electronic Systems

Active Filters

Lecture 4

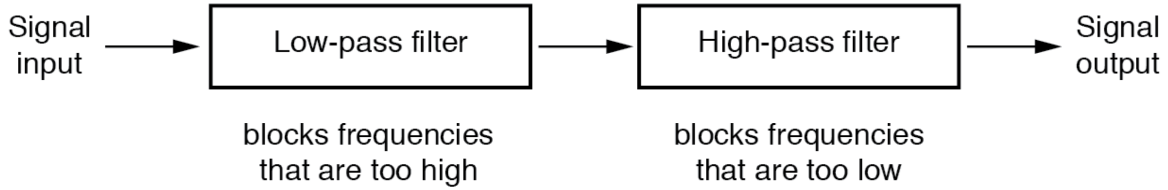
Dr. Roaa Mubarak

The Active Filters Contents:

1. Introduction to Filters.
2. Low Pass Filter.
3. High Pass Filter.
4. Band Pass Filter.
5. **Butterworth Filter.**
6. Chebyshev Filter.
7. Bessel Filter.
8. KHN Biquad filter.
9. Multiple Feedback Filters.
10. State Variable Filters.

3. Butterworth Band Pass Filter (BBF)

- f_L is the lower Cut-off Frequency
- f_H is the Higher Cut-off Frequency
- B.W is the Band-Width
- $B.W = f_H - f_L$
- Condition: $f_H\ (f_{c(LPF)}) > f_L\ (f_{c(HPF)})$

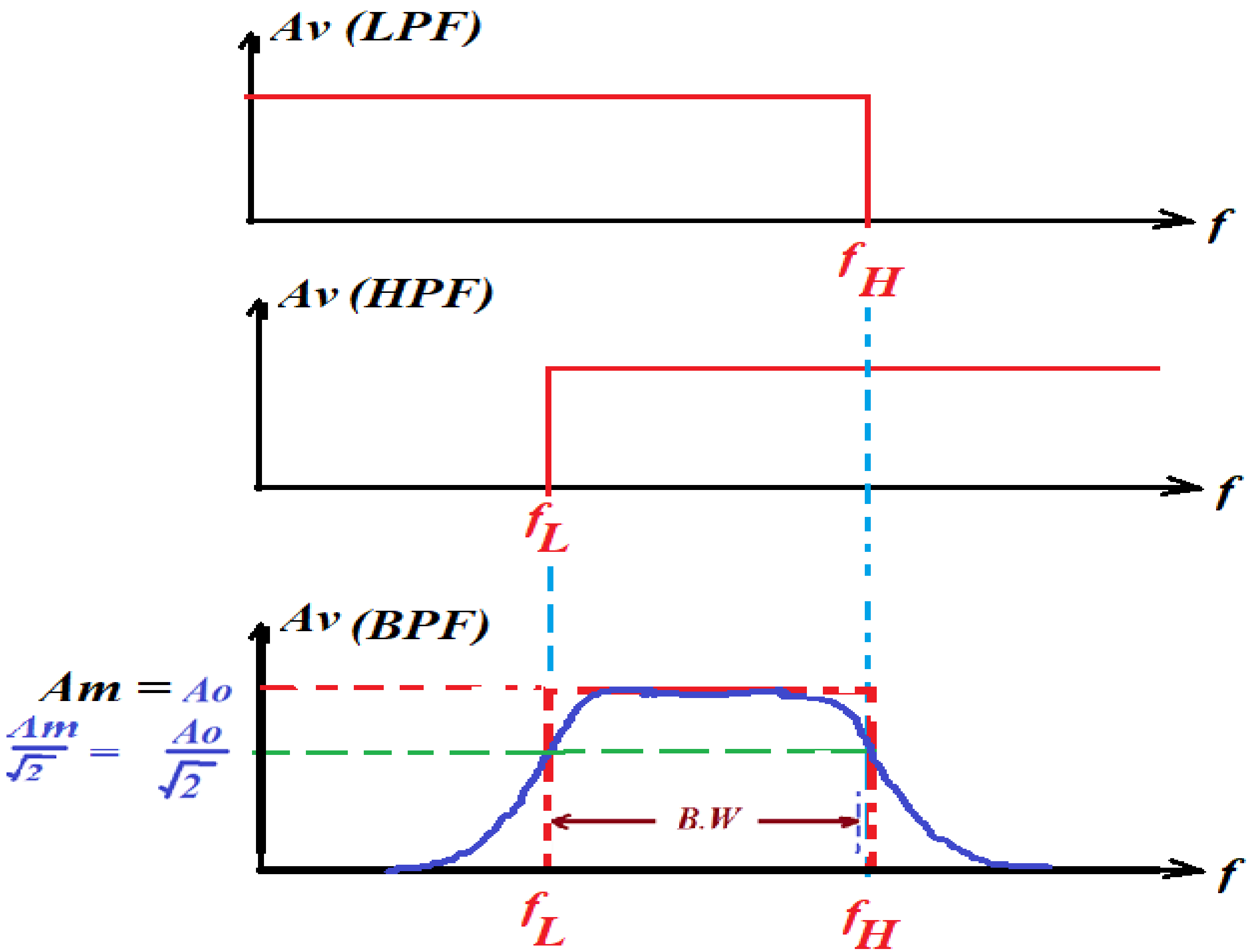


The Center Frequency (Tuned Freq.)

$$\boxed{f_o = \sqrt{f_L \cdot f_H}}$$

The Quality Factor
(How Sharp is the response)

$$\boxed{Q = \frac{f_o}{B.W}}$$



3. Butterworth Band Pass Filter (BBF)

Example 1:

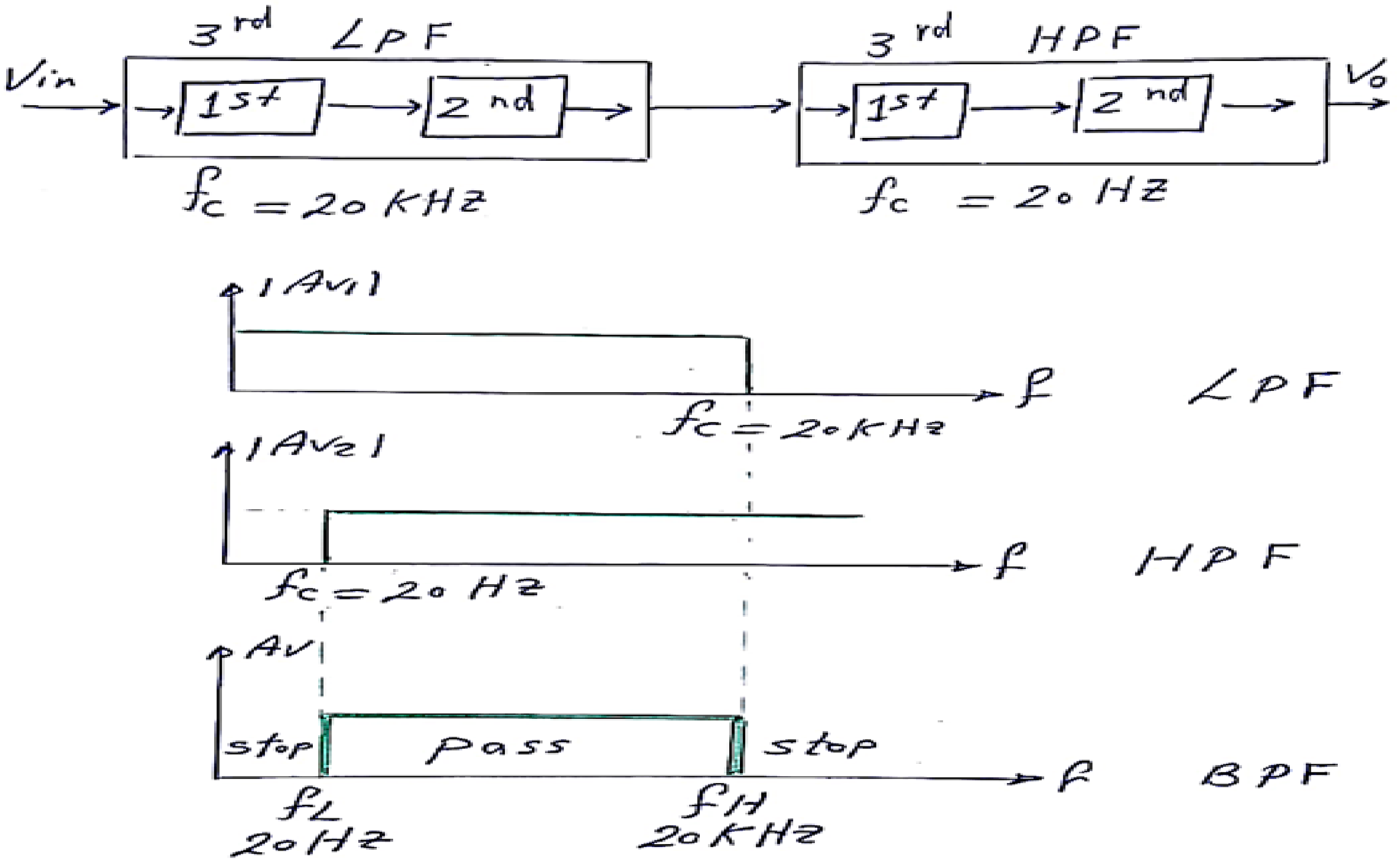
Design a 3^{rd} order Butterworth Band-Pass-Filter (BPF) for the audio frequency band from 20 Hz to 20 KHz. The Butterworth polynomial for n = 3 is (S+1) (S^2+ S +1).

(a) Draw the Block- diagram for the filter.
(b) Draw the Circuit- diagram for the filter.
(c) Calculate the values of the circuit components.

3. Butterworth Band Pass Filter (BBF)

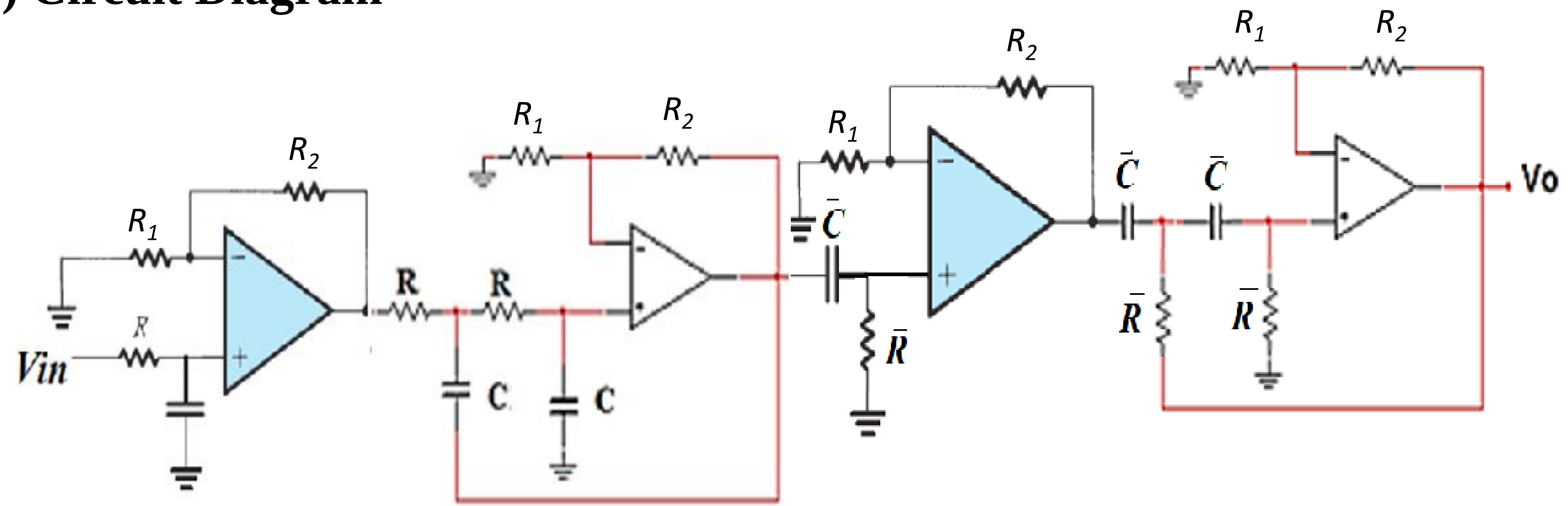
Solution:

(a) Block Diagram



3. Butterworth Band Pass Filter (BBF)

(b) Circuit Diagram



3rd LBF

3rd HBF

3. Butterworth Band Pass Filter (BBF)

(c) Circuit Components Calculations

I For LPF :-

$* \; f_o = f_c = \frac{1}{2\pi RC} \rightarrow 20\times10^3 = \frac{1}{2\pi \; R \; C}$

choose $\boxed{C = 0.01\mu F} \rightarrow \therefore \boxed{R = 795.77\Omega}$

* For HPF :-

$* \; f_o = f_c = \frac{1}{2\pi \bar{R} \bar{C}} \rightarrow 20 = \frac{1}{2\pi \; \bar{R} \; \bar{C}}$

choose $\boxed{\bar{C} = 0.01\mu F} \rightarrow \therefore \boxed{\bar{R} = 795.775 \, K\Omega}$

For $n = 3 \rightarrow$ polynomial is $(s+1)(s^2+s+1)$

$\boxed{2K = 1} \quad \therefore A_o = 3 - 2K = 3 - 1 = 2$

$\boxed{A_o = 2} = 1 + \frac{R_2}{R_1} \rightarrow \therefore \frac{R_2}{R_1} = 1$

Let $\boxed{\begin{matrix} R_1 = 10\,K\Omega \\ \therefore R_2 = 10\,K\Omega \end{matrix}}$

3. Butterworth Band Pass Filter (BBF)

Example2:

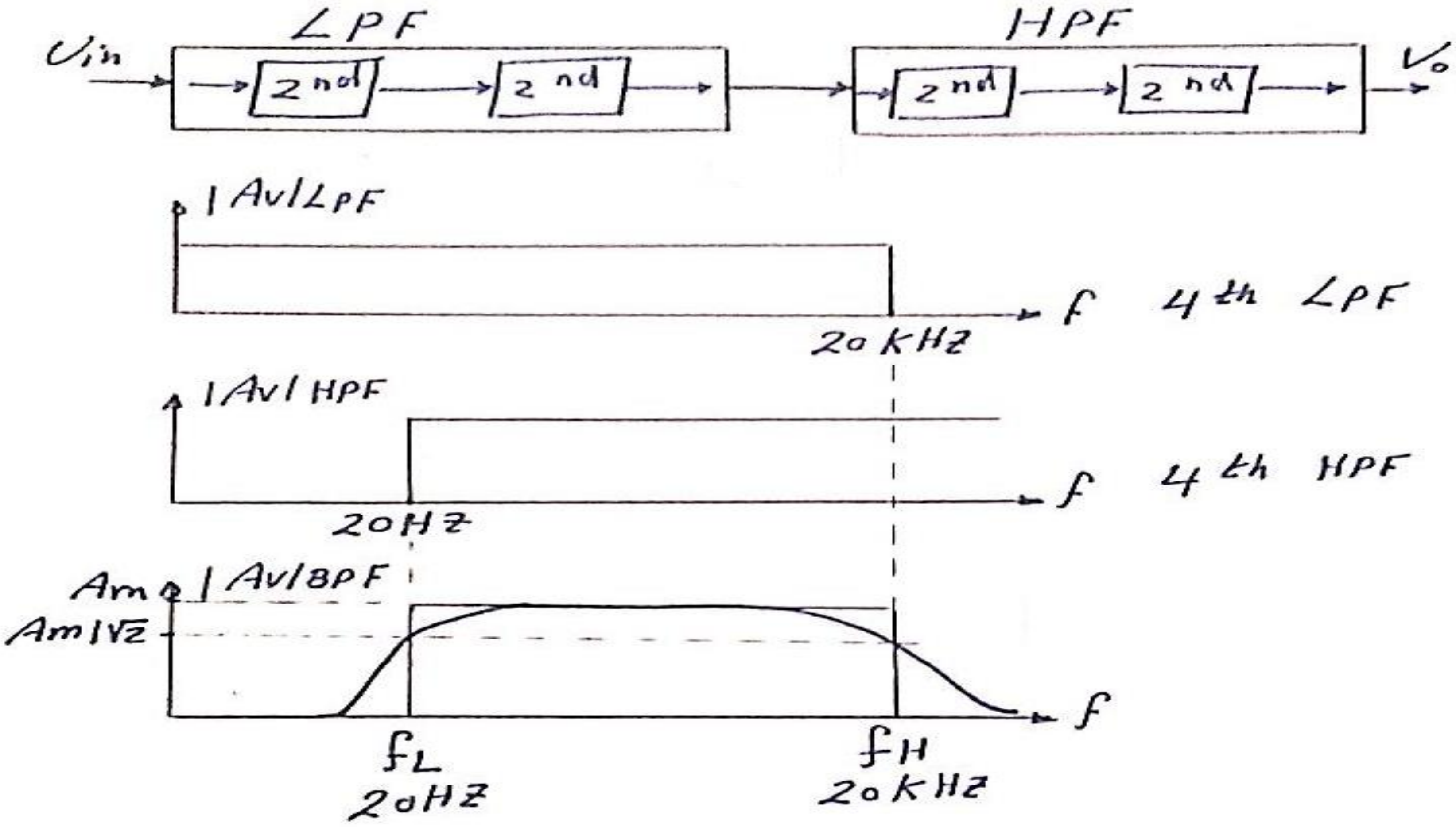
Design a 4^{th} order Butterworth Band-Pass-Filter (BPF) for the audio frequency band from 20 Hz to 20 KHz. The Butterworth polynomial for n = 4 is $(S^2+0.765S+1)\,(S^2+1.848S+1)$

(a) Draw the Block- diagram for the filter.
(b) Draw the Circuit- diagram for the filter.
(c) Calculate the values of the circuit components.

3. Butterworth Band Pass Filter (BBF)

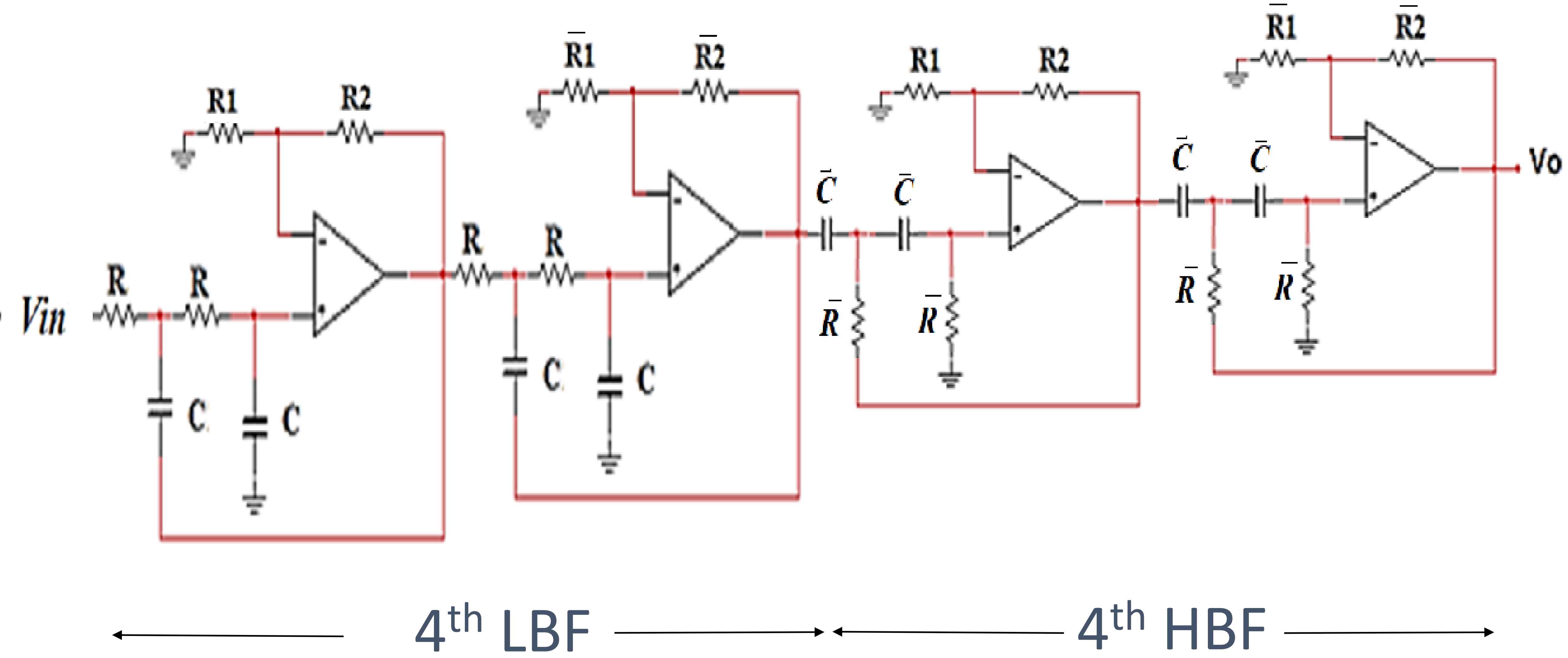
Solution:

(a) Block diagram



3. Butterworth Band Pass Filter (BBF)

(b) Circuit- diagram



3. Butterworth Band Pass Filter (BBF)

(b) Circuit Components Calculations:

For LPF :-

$$f_o = \frac{1}{2\pi R C} = 20 \times 10^3 \ Hz$$

choose $\boxed{C = 0.01 \mu F}$ $\checkmark \rightarrow \therefore \boxed{R = 795.775 \Omega}$

For HPF :-

$$f_o = \frac{1}{2\pi R' C'} = 20 \ Hz$$

choose $\boxed{C = 0.01 \mu F}$ $\checkmark \rightarrow \therefore \boxed{R = 795.775 k\Omega}$

$$B_n(s) = (s^2 + 0.765 s + 1)(s^2 + 1.848 s + 1)$$

$2K_1 = 0.765$

$\therefore A_{m1} = 3 - 2K_1 = 2.235$

$A_{m1} = 1 + \frac{R_2}{R_1} = 2.235$

$\frac{R_2}{R_1} = 1.235$

let $\checkmark \rightarrow$ $\boxed{R_1 = 10 \ k\Omega}$

$\therefore$ $\boxed{R_2 = 12.35 k\Omega}$

$2K_2 = 1.848$

$\therefore A_{m2} = 3 - 2K_2 = 1.152$

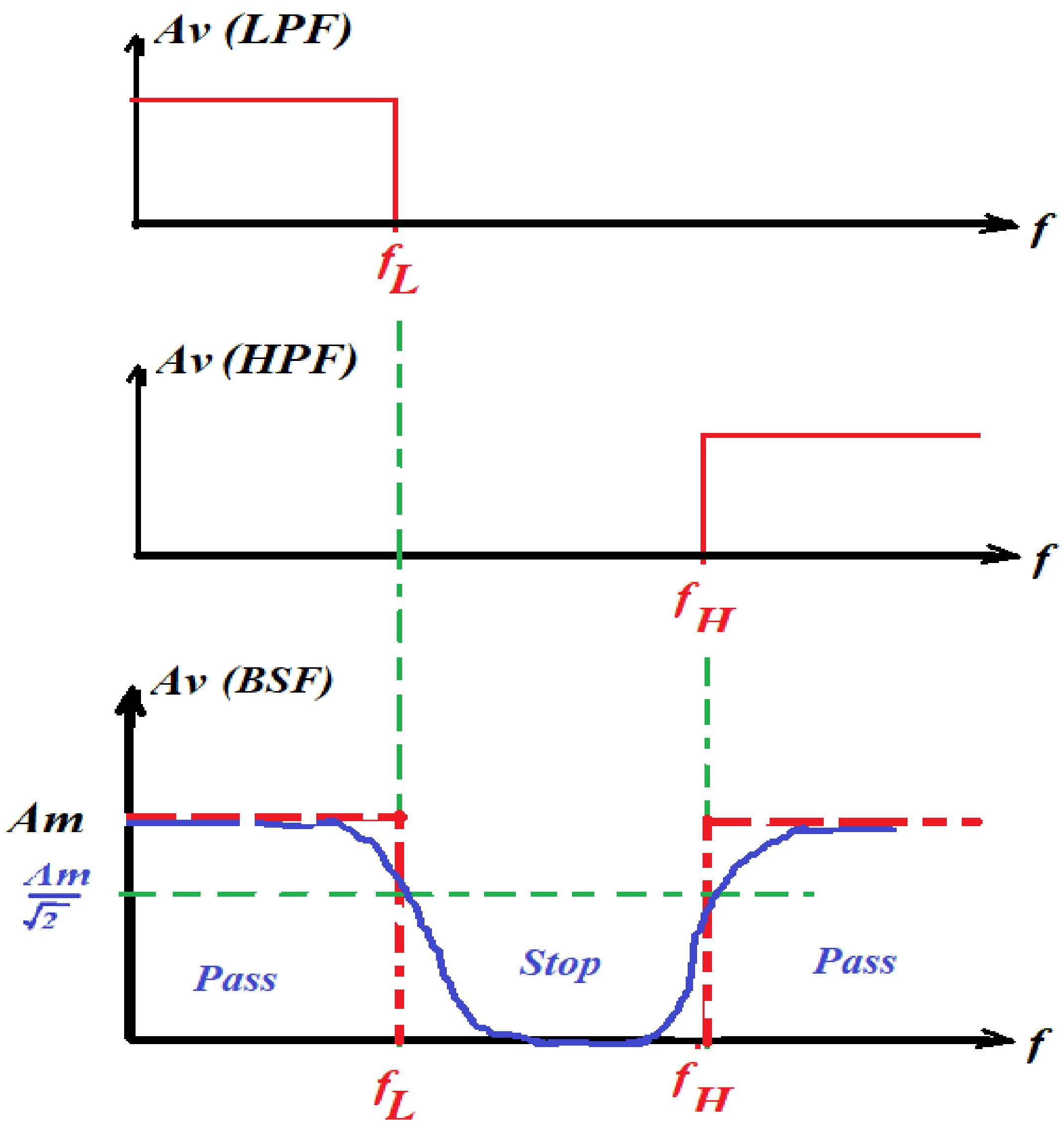
$A_{m2} = 1 + \frac{R_2'}{R_1'} = 1.152$

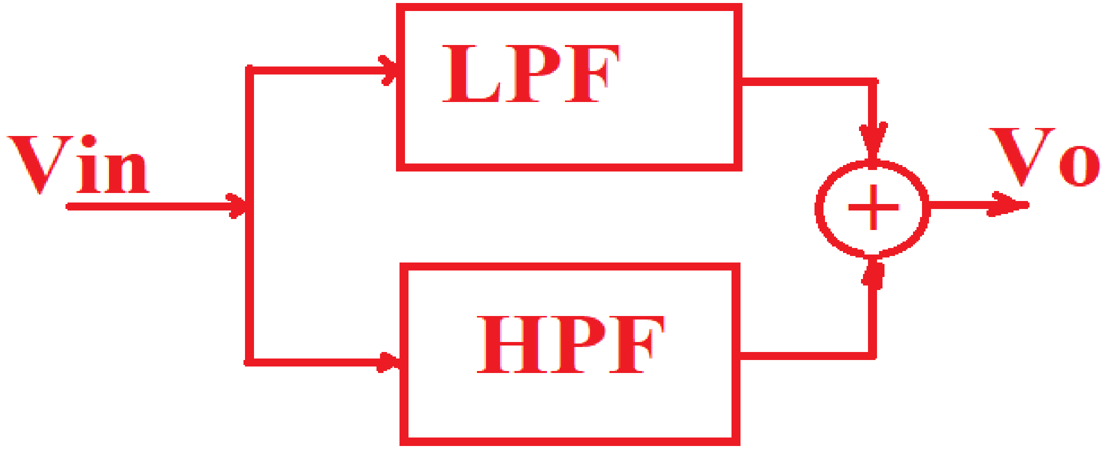
$\frac{R_2'}{R_1'} = 0.152$

let $\checkmark \rightarrow$ $\boxed{R_1' = 10 \ k\Omega}$

$\therefore$ $\boxed{R_2' = 1.52 \ k\Omega}$

4. Butterworth Band Stop Filter (BSF) (BRF)





- **Condition:** $f_L\ (f_{c(LPF)}) < f_H\ (f_{c(HPF)})$

4. Butterworth Band Stop Filter (BSF) (BRF)

Example3:

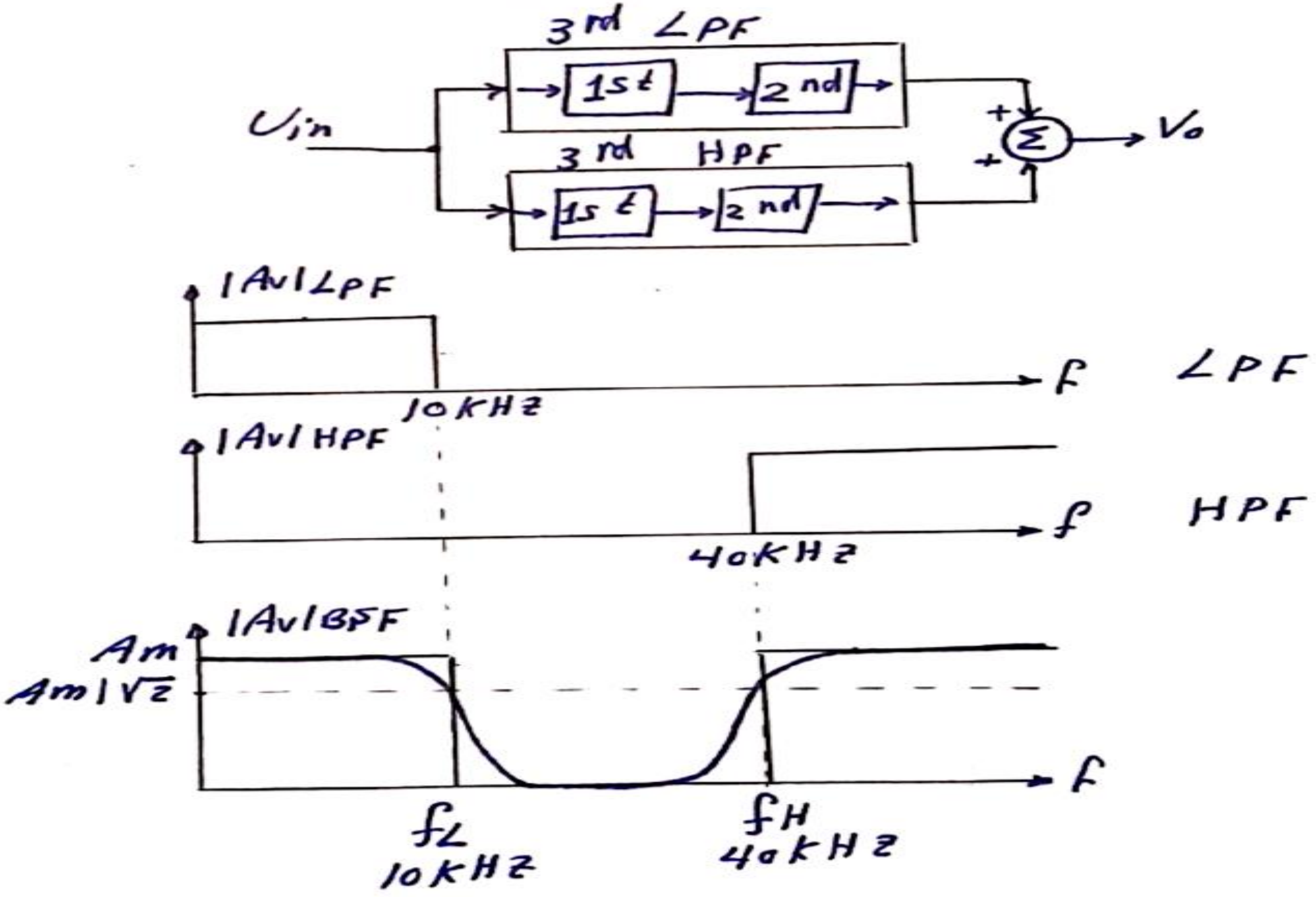
3rd order Butterworth BSF to reject the frequency band from 10 KHz to 40 KHz. The Butterworth polynomial for n = 3 is (S +1) (S2 + S + 1).

1. Draw the block diagram for the filter.
2. Draw the circuit diagram.
3. Calculate all circuit components values.

4.Butterworth Band Stop Filter (BSF) (BRF)

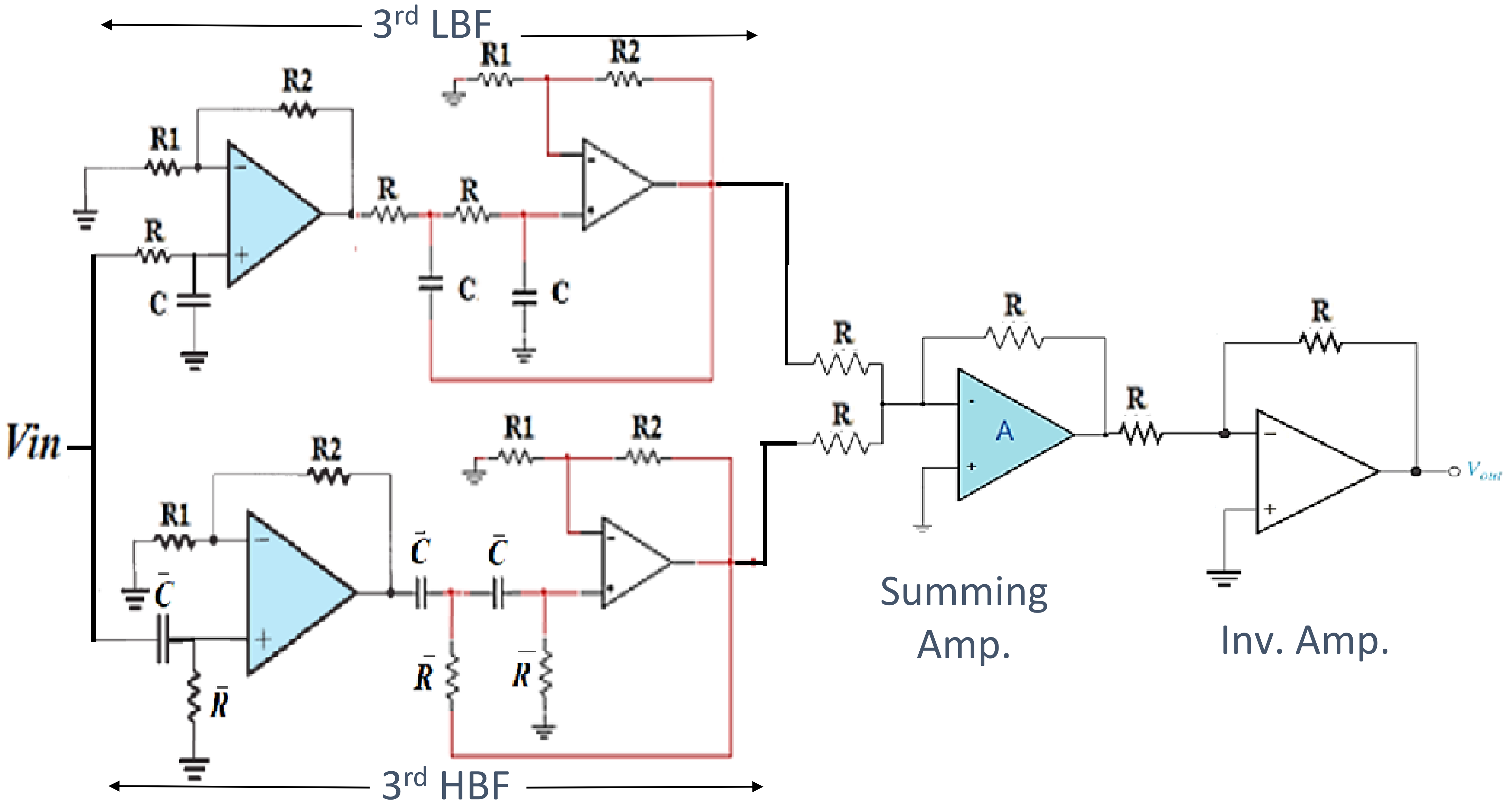
Solution:

(a) Block Diagram:



4.Butterworth Band Stop Filter (BSF) (BRF)

(b) Circuit- diagram



4.Butterworth Band Stop Filter (BSF) (BRF)

(C) Components Calculations

For LPF:-

$$f_o = \frac{1}{2\pi RC} = 10^4 \ Hz$$

choose $\boxed{C = 0.01 \mu F}$ $\rightarrow \therefore$ $\boxed{R = 1591.55 \Omega \cong 1.59 k\Omega}$

For HPF:-

$$f_o = \frac{1}{2\pi R' C'} = 40 \times 10^3 \ Hz$$

choose $\boxed{C' = 0.01 \mu F}$ $\rightarrow \therefore$ $\boxed{R' = 397.89 \Omega \cong 398 \Omega}$

$$B_n(s) = (s+1)(s^2+s+1)$$

* For the second-order

$$A_m = 3 - 2\zeta = 3 - 1 = 2 = 1 + \frac{R_2}{R_1}$$

$$\therefore \frac{R_2}{R_1} = 1$$

Let $\boxed{R_1 = 10 k\Omega \rightarrow \therefore R_2 = 10 k\Omega}$

Assuming the gain of the first-order part is equal to that of the second-order part.

4. Butterworth Band Stop Filter (BSF) (BRF)

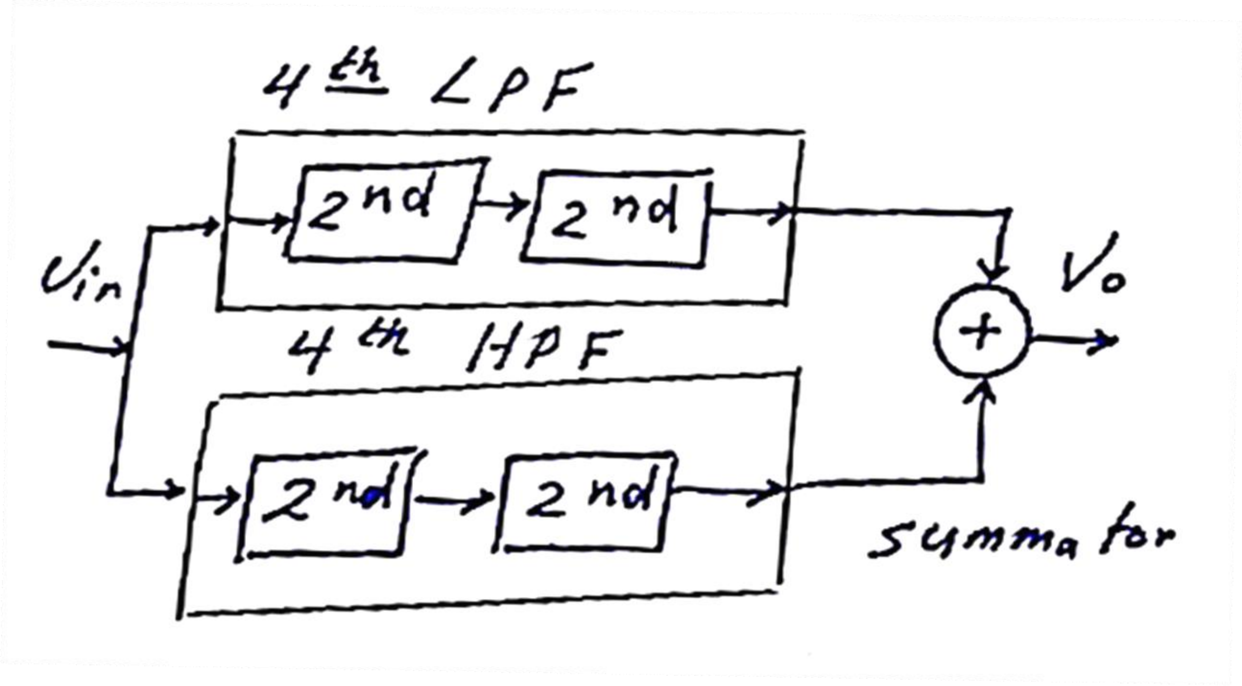
Example 4:

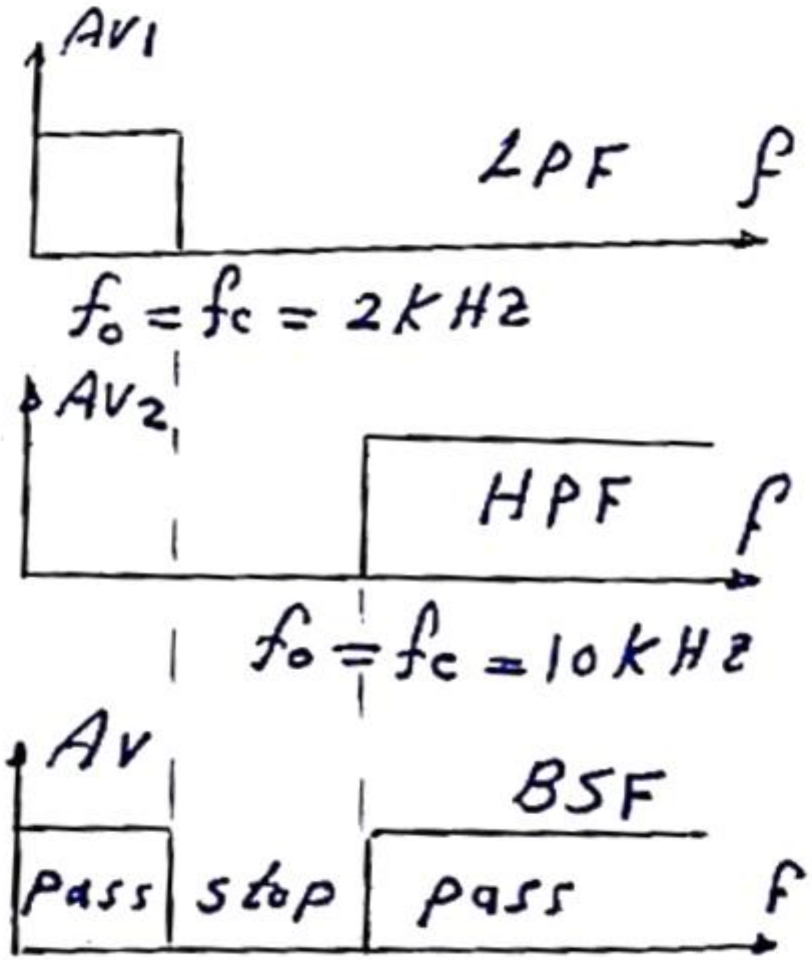
Design a 4th order Butterworth BSF to reject the frequency band from 2 KHz to 10 KHz. The Butterworth polynomial for n = 4 is $(S^2+1.848\ S+1)\ (S^2+0.765\ S+1)$.

1. **Draw the block diagram for the filter.**
2. **Draw the circuit diagram.**
3. **Calculate all circuit components values.**

4.Butterworth Band Stop Filter (BSF) (BRF)

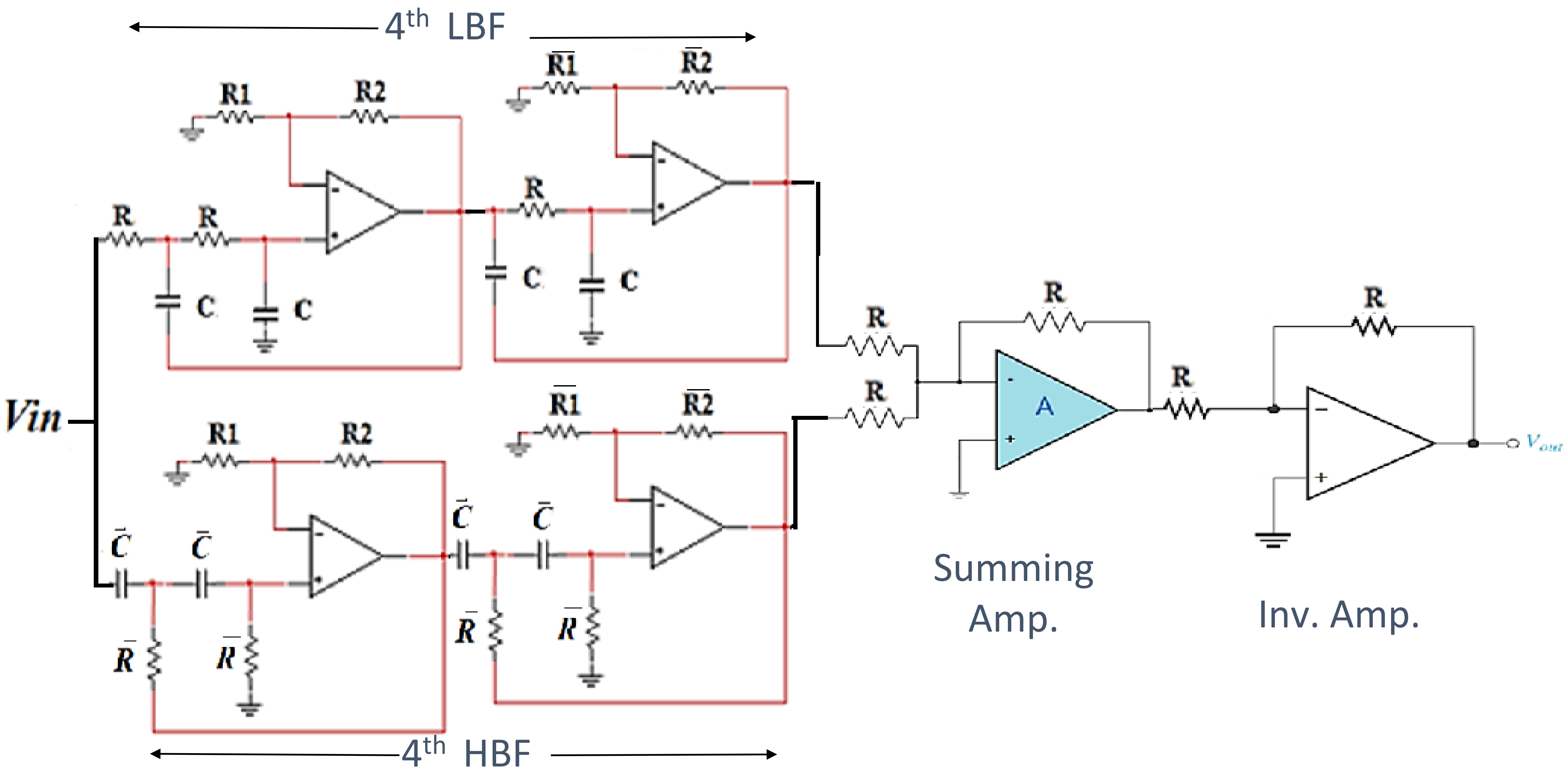
(a) Block diagram





4.Butterworth Band Stop Filter (BSF) (BRF)

(b) Circuit diagram



4.Butterworth Band Stop Filter (BSF) (BRF)

For L.P.F

$$f_c = 2\times10^{3} = \frac{1}{2\pi RC}, \text{ choose} \rightarrow \boxed{C = 0.01\,\mu F}$$

$$\therefore R = \frac{1}{2\pi \times 2\times10^{3} \times 0.01\times10^{-6}} \rightarrow \boxed{R \cong 7.96\,k\Omega}$$

For HPF

$$f_c = 10\times10^{3} = \frac{1}{2\pi \bar{R}\bar{C}}, \text{ choose } \boxed{\bar{C} = 0.01\,\mu F}$$

$$\therefore \bar{R} = \frac{1}{2\pi \times 10\times10^{3} \times 0.01\times10^{-6}} \rightarrow \boxed{\bar{R} = 1.59\,k\Omega}$$

For $n = 4 \rightarrow (s^2 + 1.848s + 1)\cdot(s^2 + 0.765s + 1)$

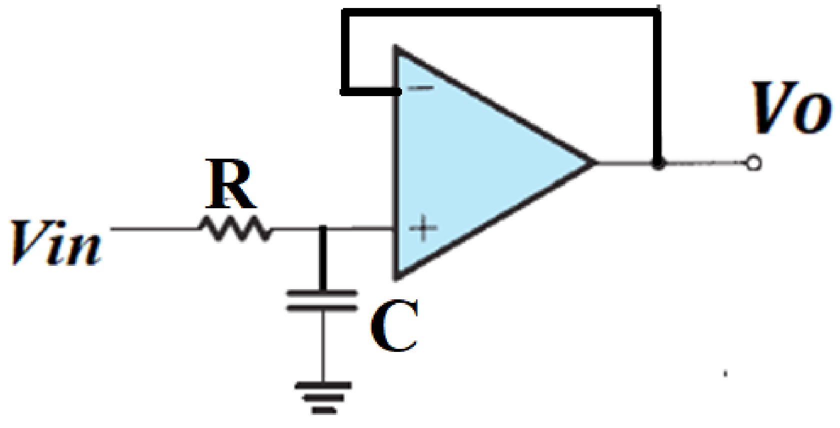
$2K = 1.848$	$2K = 0.765$
$A_o = 3 - 2K = 1.152 = 1 + \frac{R_2}{R_1}$	$A_o = 3 - 2K = 2.235 = 1 + \frac{\grave{R}_2}{\grave{R}_1}$
$\therefore \boxed{\frac{R_2}{R_1} = 0.152}$	$\therefore \boxed{\frac{\grave{R}_2}{\grave{R}_1} = 1.235}$
choose $\boxed{R_1 = 10\,k\Omega}$	choose $\boxed{\grave{R}_1 = 10\,k\Omega}$
$\therefore \boxed{R_2 = 1.52\,k\Omega}$	$\therefore \boxed{\grave{R}_2 = 12.35\,k\Omega}$

Unity Gain Filter (A_m =1)

1. First Order LPF:

$$\text{Transfer Function}$$

$$A_V(S) = \frac{1}{1 + \frac{S}{W_c}}$$



The maximum gain A_m is unity

$W_c = 2\pi f_c$

f_c Is the cut-off frequency

Design Rules:

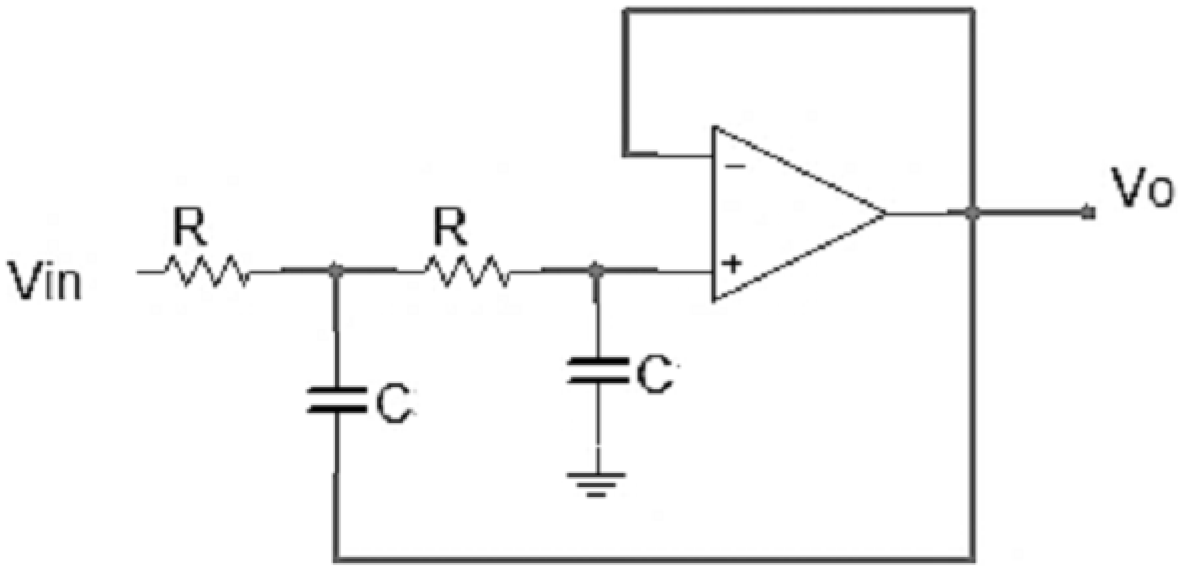
- ❑ Maximum gain $A_m = 1$
- ❑ Cut-Off Frequency $f_c = \frac{1}{2\pi RC}$

Unity Gain Filter (A_m =1)

2. second Order LPF:

$\boldsymbol{Transfer\ Function}$:

$$A_V(S) = \frac{V_o}{Vin} = \frac{1}{(\frac{S}{Wc})^2 + 2K\left(\frac{S}{Wc}\right) + 1}$$



$\boldsymbol{The\ maximum\ gain\ A_m\ is}$ **unity**

W_c = 2π f_c

f_c ***Is the cut-off frequency***

K is the damping ratio

(2K) is the coefficient of (S) in the table

Design Rules:

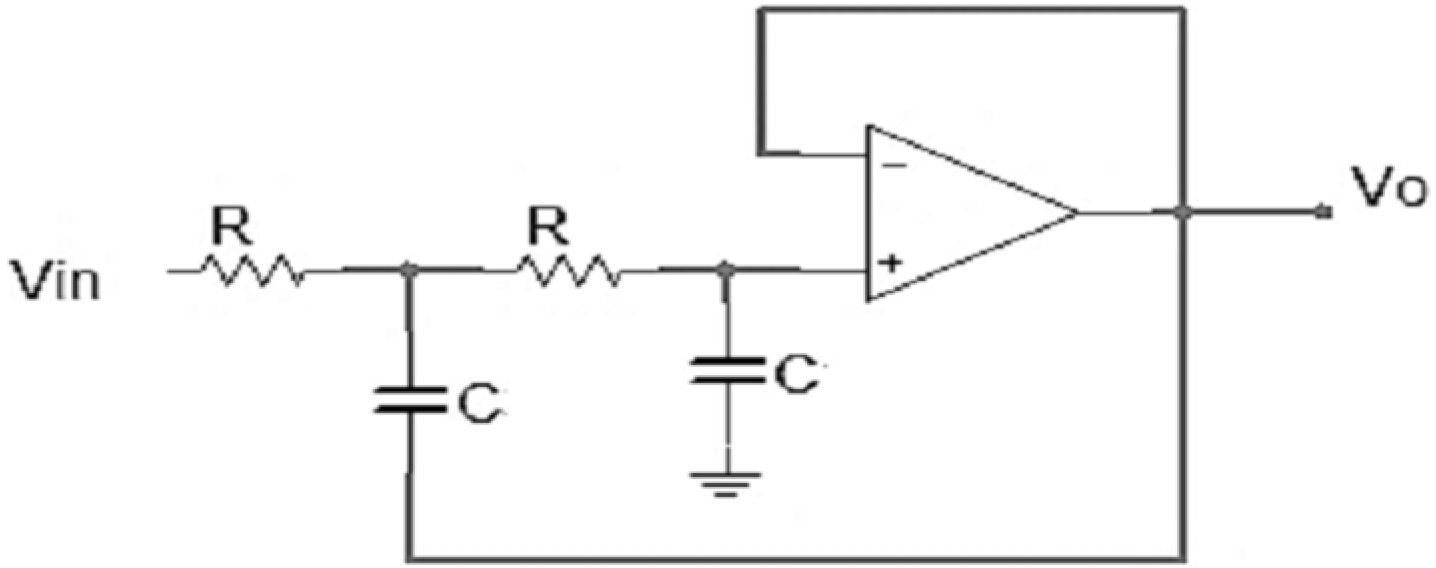
- ❑ $\boldsymbol{Maximum\ gain\ \ A_m = 1}$
- ❑ ***Cut-Off Frequency*** $f_c = \frac{1}{2\pi\ R\ C}$
- ❑ ***Damping Ratio 2K = Coefficient of (S) in the polynomial*** $B_n(S)$.

Unity Gain Filter (A_m =1)

Example:

Analyze the unity gain filter shown :

1. Derive an expression for the filter transfer function.
2. What is the order and type of the filter?
3. Design the filter cut-off frequency of 100Hz



Unity Gain Filter (A_m =1)

Solution:

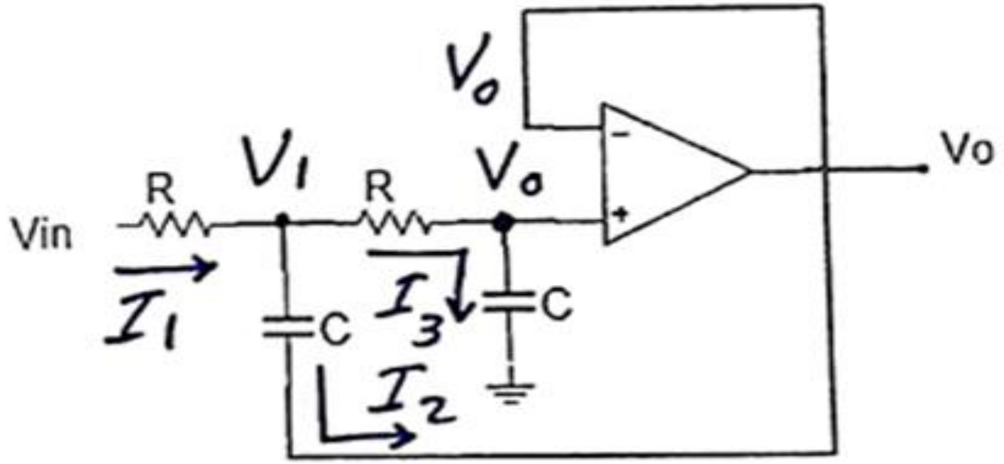
1. Derive an expression for the filter transfer function (Vo/Vin).

$$V_o = I_3 \cdot \frac{1}{SC}$$

$$V_o = \left[\frac{V_1}{R + \frac{1}{SC}}\right] \cdot \frac{1}{SC}$$

$$\therefore V_o = \frac{V_1}{SCR + 1}$$

$$\therefore \boxed{V_1 = (SCR + 1) V_o} \quad \boxed{1}.$$



Unity Gain Filter (A_m =1)

$$\frac{V_{in}-V_1}{R} = \frac{V_1-V_o}{1/sC} + \frac{V_1}{R+\frac{1}{sC}}$$

$$\frac{V_{in}-V_1}{R} = sCV_1 - sCV_o + \frac{sCV_1}{sCR+1} \quad \times R$$

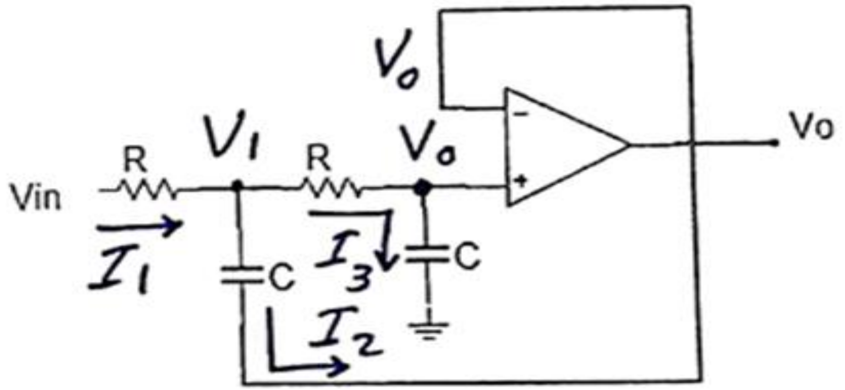
$$V_{in}-V_1 = sCRV_1 - sCRV_o + \frac{sCR}{sCR+1}V_1$$

$$V_{in} = \left[(sCR+1) + \frac{sCR}{sCR+1}\right]V_1 - sCRV_o$$

$$V_{in} = \left[(sCR+1) + \frac{sCR}{sCR+1}\right](sCR+1)V_o - sCRV_o$$

$$V_{in} = \left[(sCR+1)^2 + sCR\right]V_o - sCRV_o$$

$$V_{in} = \left[s^2C^2R^2 + 2sCR + 1 + \cancel{sCR} - \cancel{sCR}\right]V_o$$



$$\therefore \boxed{\frac{V_o}{V_{in}} = \frac{1}{s^2C^2R^2 + 2sCR + 1}}$$

$$\frac{V_o}{V_{in}} = \frac{1}{\left(\frac{s}{1/RC}\right)^2 + 2\left(\frac{s}{1/RC}\right) + 1}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + 2\left(\frac{s}{\omega_o}\right) + 1}}$$

Unity Gain Filter (A_m =1)

(b) What is the order ad type of the filter?

Second-Order
LPF

(c) Design the filter for a cut-off frequency of 100KHz.

$$\omega_o = \frac{1}{RC} = 2\pi f_o$$

$$\therefore f_o = \frac{1}{2\pi RC}$$

choose $\boxed{C = 0.01\,\mu F}$ →

$$\therefore R = \frac{1}{2\pi(100\times10^3)(0.01\times10^{-6})}$$

$\boxed{R = 159.155\,\Omega}$ →