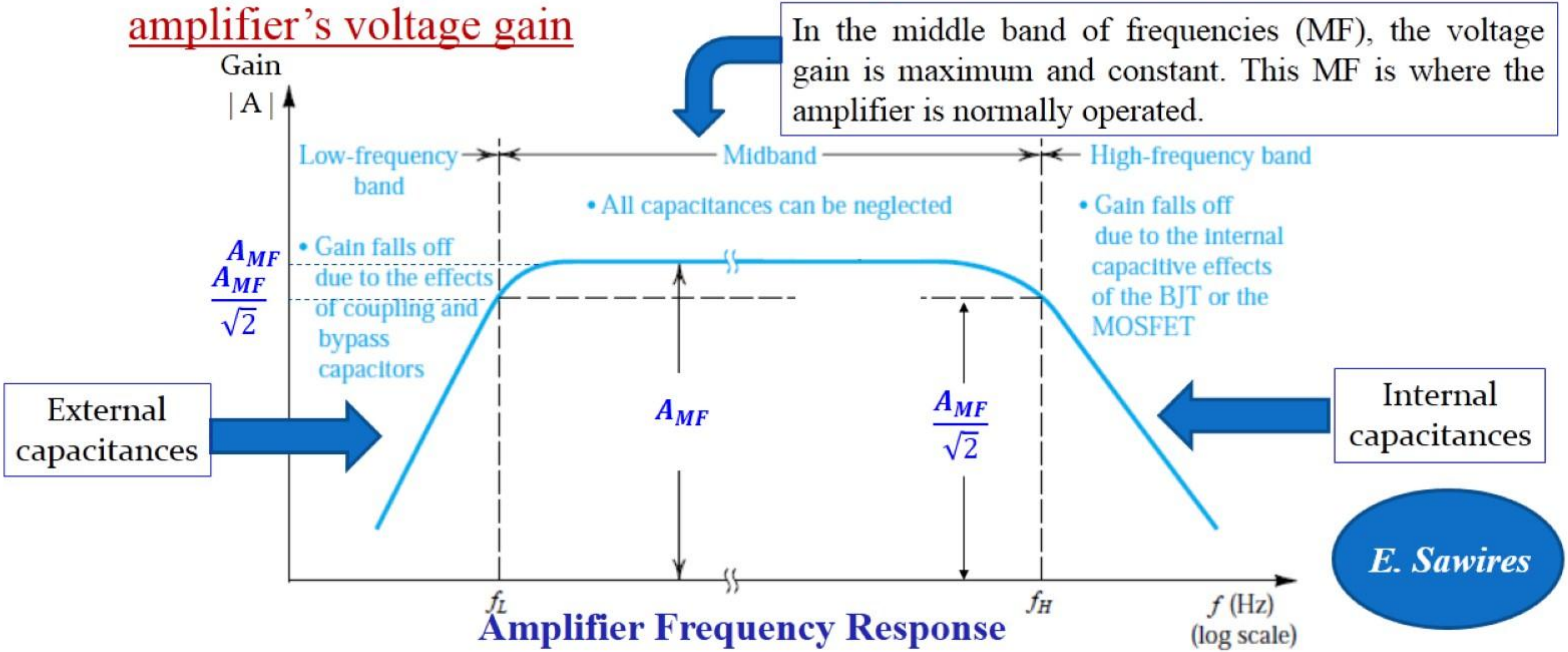
Amplifier Frequency Response

Objective

- We want to study the effect that these capacitances have on the amplifier's voltage gain



Amplifier Frequency Response

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Objective

- **For this study, we will consider that the frequency spectrum is divided into three bands.**

1) **Midband** ⟹ A_{MF}
2) **Low-Frequency Band** ⟹ f_L
3) **High Frequency Band** ⟹ f_H

Band width of Amplifier: the difference between the higher and lower 3-dB cutoff Frequencies .

$$BW = f_H - f_L$$

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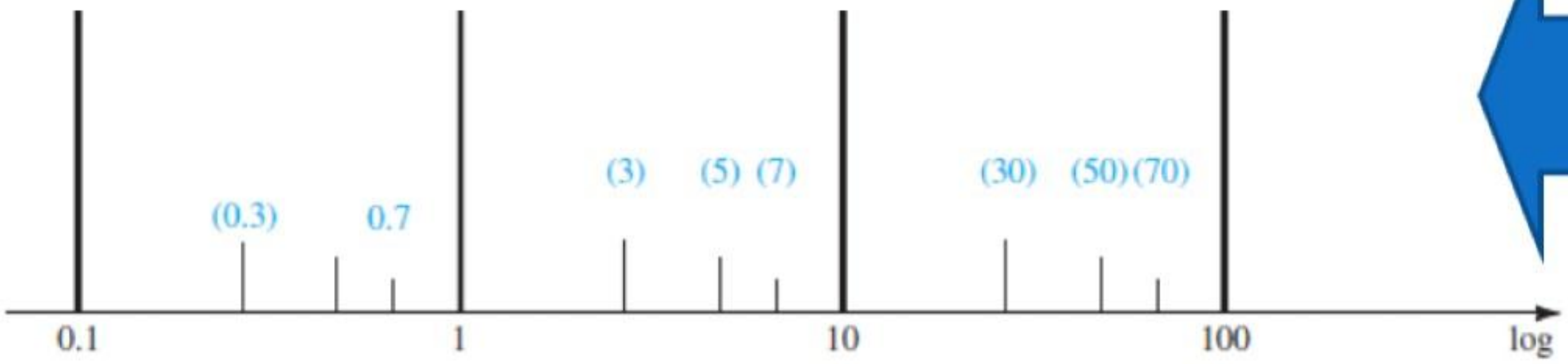
Introduction

Important Tools

Logarithmic Scale



$$
\begin{aligned}
\log_{10} 10^0 &= 0\\
\log_{10} 10 &= 1\\
\log_{10} 100 &= 2\\
\log_{10} 1{,}000 &= 3\\
\log_{10} 10{,}000 &= 4\\
\log_{10} 100{,}000 &= 5\\
\log_{10} 1{,}000{,}000 &= 6\\
\log_{10} 10{,}000{,}000 &= 7\\
\log_{10} 100{,}000{,}000 &= 8
\end{aligned}
$$

etc.

On the log axis the distance from 0.1 to 1 equals the distance from 1 to 10 equals the distance from 10 to 100, and so on.

This will be useful in displaying wide ranges of frequencies, which is always hard in case of using linear scale.

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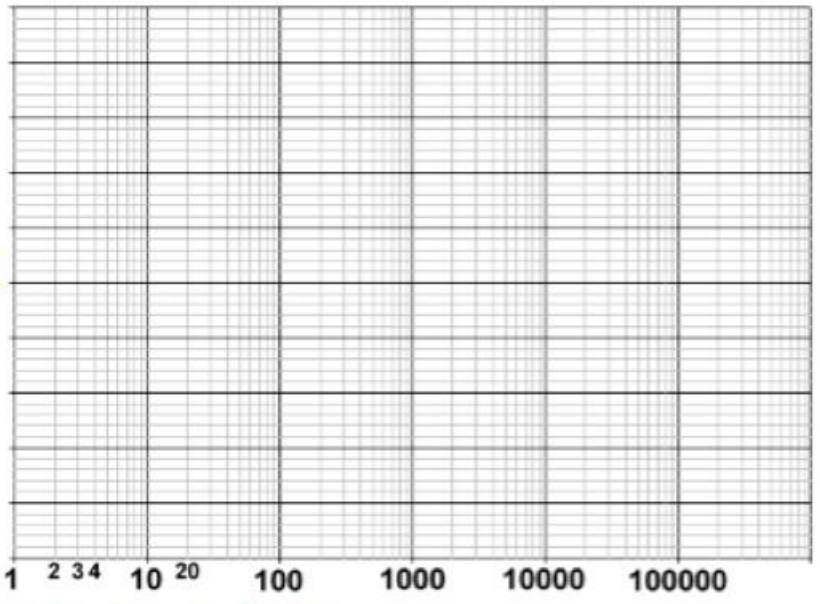
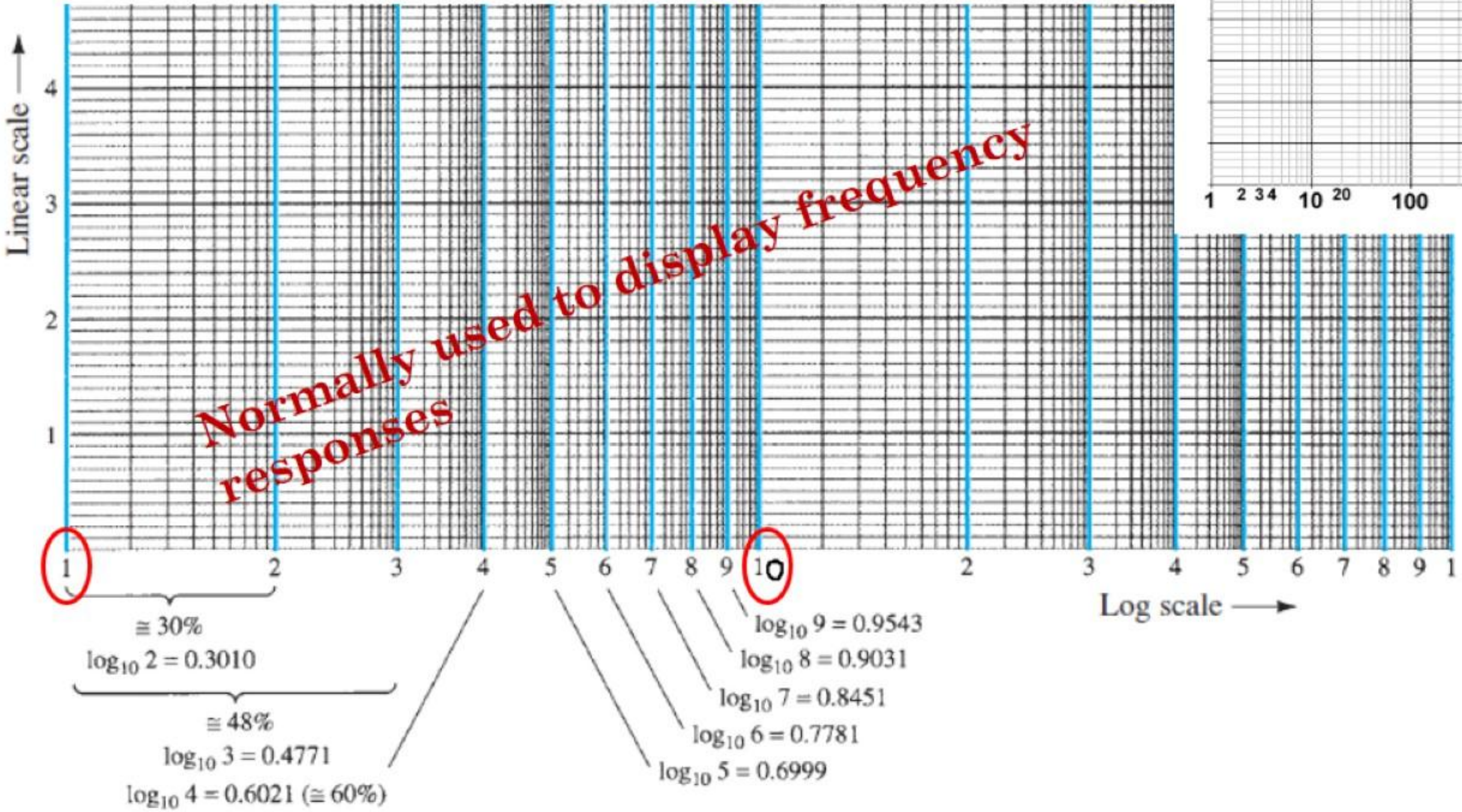
Important Tools

Semi logarithmic Graph





Normally used to display frequency responses

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Important Tools
Decibels

The term bel is derived from the name of Alexander Graham Bell. The relation between two power levels P1, and P2 is given by:

$$G = 10log_{10}\frac{P2}{P1}\ dB$$

Since Power=$(V)^2/R$, then the last equation can be written as:

$$G = 10log_{10}\left(\frac{V_2}{V_1}\right)^2\ dB = 20\ log_{10}\frac{V_2}{V_1}\ dB$$

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Important Tools

Decibels

One of the advantages of using dB is with the cascading stages:

$$|A_{VT}| = |A_{V1}| \text{x} \, |A_{V2}| \text{x} \ldots\ldots \text{x} \, |A_{Vn}|$$

Multiplication of individual gains

$$|A_{VT}| dB = |A_{V1}| dB + |A_{V2}| dB + \ldots\ldots + |A_{Vn}| \text{dB}$$

Summation of individual gains

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Introduction

Single Time Constant, STC, Circuits

Single-time-constant (STC) circuits are those circuits that are composed of or can be reduced to one reactive component (inductance or capacitance) and one resistance. An STC circuit formed of an inductance *L* and a resistance *R* has a time constant τ=L/R . The time constant τ of an STC circuit composed of a capacitance *C* and a resistance *R* is given by $\tau = CR$.

Classification of STC Circuits

STC circuits can be classified into two categories, *low-pass* (LP) and *high-pass* (HP) types, with each category displaying distinctly different signal responses.

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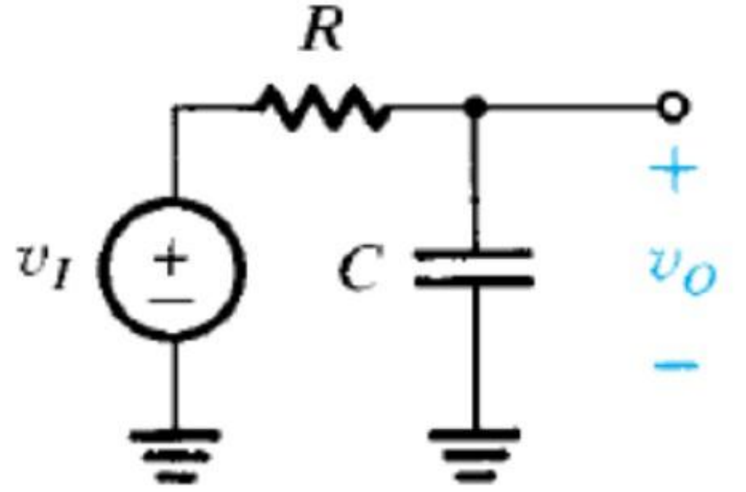
Frequency Response of LPF Circuits

The transfer function $T(s)$ of an STC low-pass circuit can always be written in the form:

$$T(j\omega) = \frac{K}{1 + j\left(\frac{\omega}{\omega_o}\right)}$$

$$T(S) = \frac{K}{1 + \frac{S}{\omega_O}}$$

$$\omega_o = \frac{1}{\tau} = \frac{1}{RC} \quad , \qquad K = 1$$



$$Z_c = \frac{1}{sC} = \frac{1}{j\omega C} = \frac{1}{j2\pi fC}$$

And the magnitude response is:

$$|T(j\omega)| = \frac{K}{\sqrt{1 + (\omega/\omega_o)^2}}$$

$$V_o = Z_c \frac{V_I}{R + Z_c}$$

$$T(j\omega) = V_o / V_I = \frac{Z_c}{R + Z_c}$$

And the phase response is:

$$\phi(\omega) = -tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$T = \frac{1}{1 + j\omega CR} \qquad CR = 1/\omega_0$$

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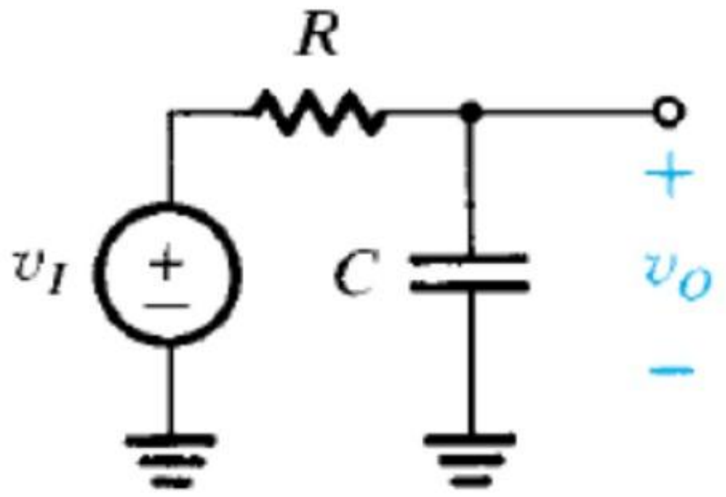
Frequency Response of LPF Circuits

Notice the magnitude is normalized to the gain constant K, and the frequency variable is normalized to ω_o.

$$|T(j\omega)| = \frac{K}{\sqrt{1 + (\omega/\omega_o)^2}}$$



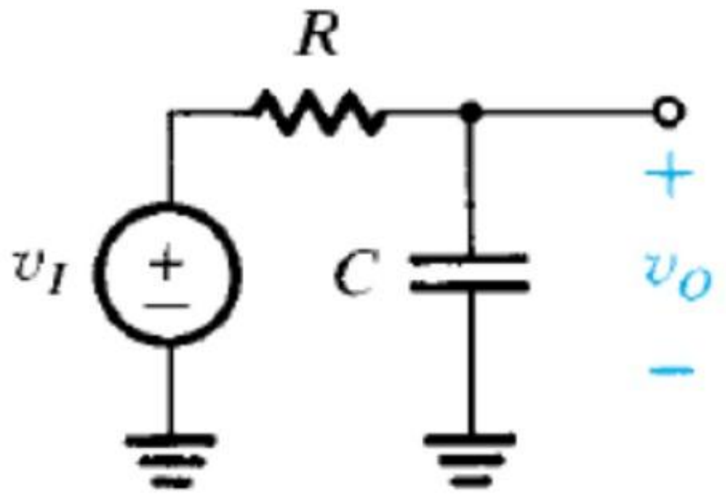
Frequency Response of LPF Circuits

Notice the magnitude is normalized to the gain constant K, and the frequency variable is normalized to ω $_o$.

$$|T(j\omega)| = \frac{K}{\sqrt{1+(\omega/\omega_o)^2}}$$

$$|T(j\omega)|dB = 20\,log\frac{K}{\sqrt{1+(\omega/\omega_o)^2}}$$

$$|T(j\omega)|dB = 20\,logK - 20log\sqrt{1+(\omega/\omega_o)^2}$$



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Frequency Response of LPF Circuits

$$|T(j\omega)|dB = 20\ logK - 20log\sqrt{1 + (\omega/\omega_o)^2}$$

At $\omega = \omega_o$

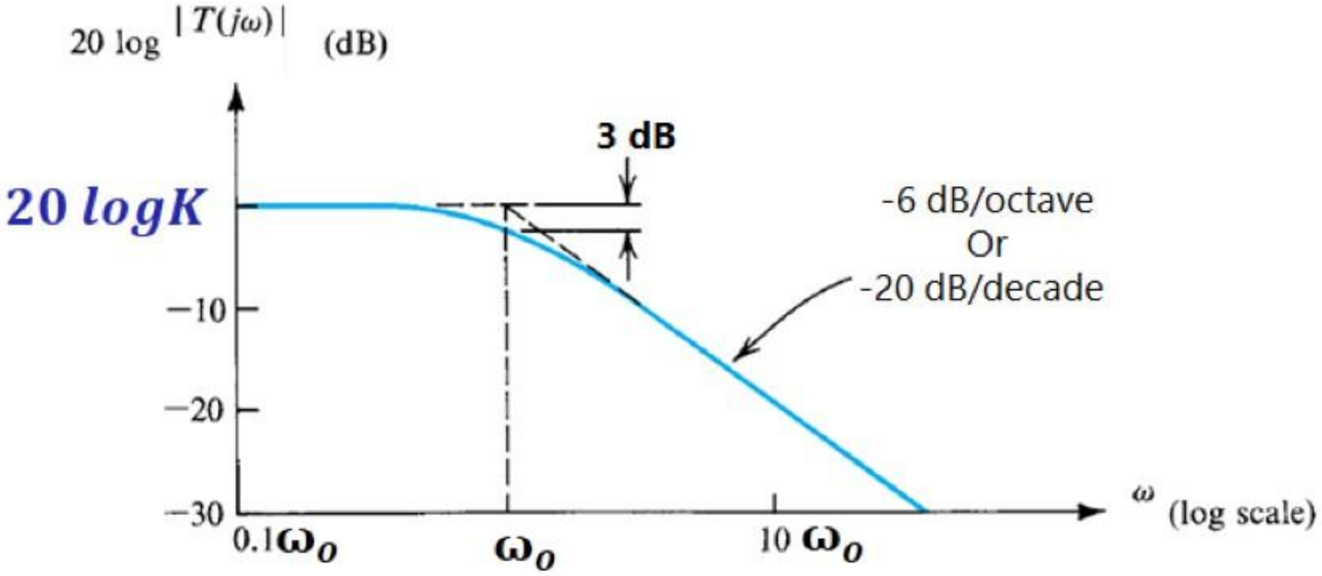
$$|T(j\omega)|dB = 20\ logK - 20log\sqrt{2}$$
$$= 20\ logK - 3\ dB$$

At $\omega = 0.1\ \omega_o$

$$|T(j\omega)|dB = 20\ logK$$

At $\omega = 10\ \omega_o$

$$|T(j\omega)|dB = 20\ logK\ \text{-}20\ dB$$



Magnitude response

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Frequency Response of LPF Circuits

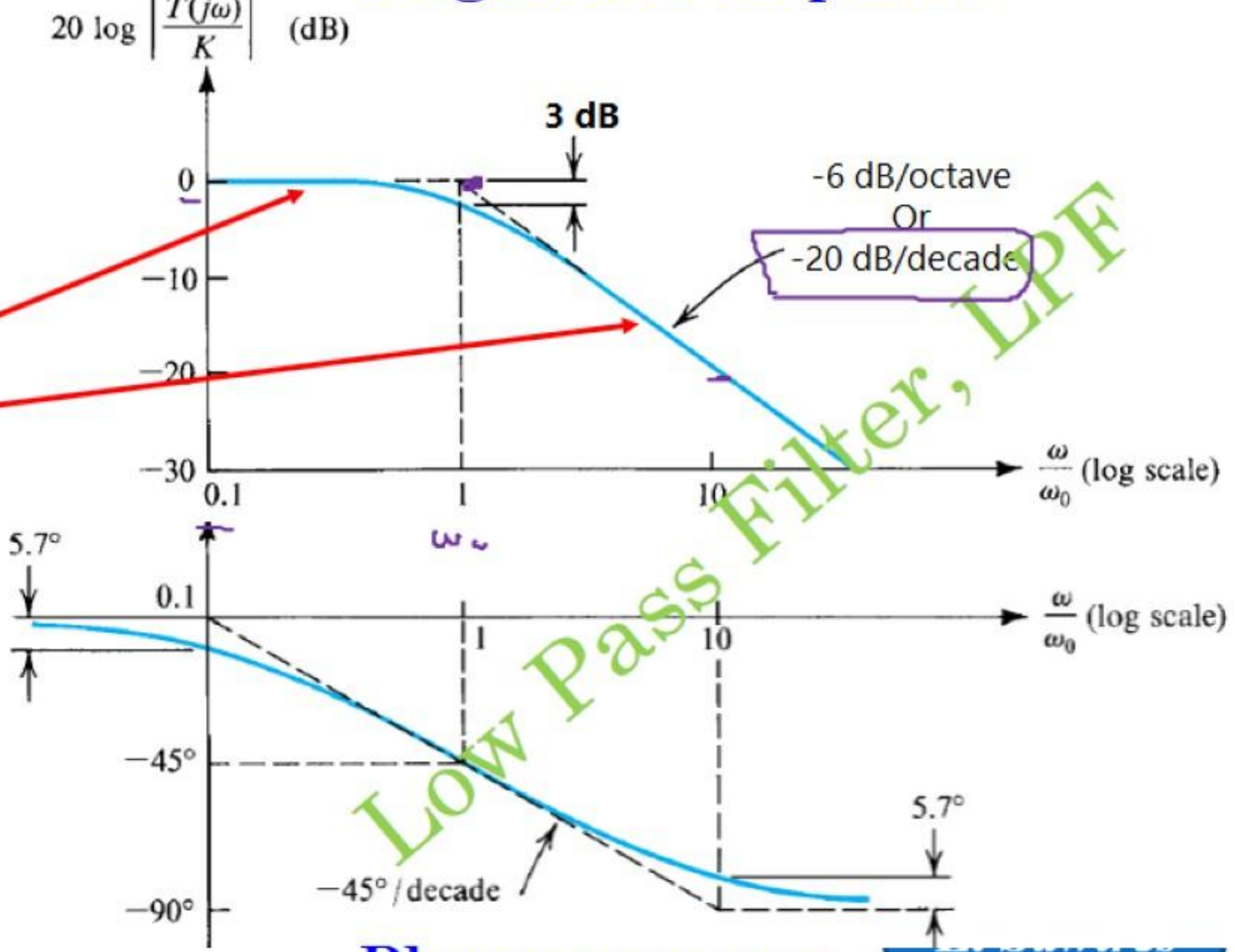
$$\left|\frac{T(j\omega)}{K}\right| = \frac{1}{\sqrt{1+(\omega/\omega_o)^2}}$$

$$\left|\frac{T(j\omega)}{K}\right| dB = 20\,log1 - 20log\sqrt{1+(\omega/\omega_o)^2}$$

Two straight lines asymptotes

0dB for the 1st asymptote, and -20dB/decade for the 2nd asymptote.

Magnitude response



Phase response

STC

Frequency Response of HPF Circuits

$Z_c = \frac{1}{sC} = \frac{1}{j\omega C}$

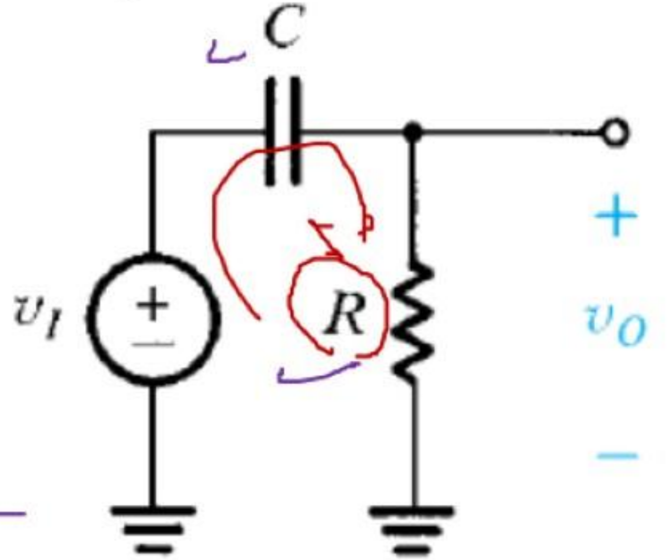
$$\boxed{T(S) = \frac{K}{1+\frac{\omega_o}{S}}} = \frac{KS}{S+\omega_o}$$

$T = V_o / V_I$

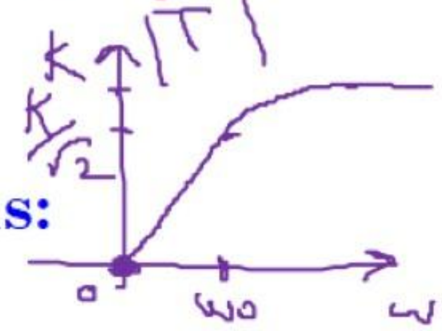
$T(s)$

$T(j\omega)$

$$T(j\omega) = \frac{K}{1 - j\,\frac{\omega_o}{\omega}}$$



$$with\; w_o = \frac{1}{\tau} = \frac{1}{RC} \quad , \qquad K = 1$$



And the magnitude response is:

$$\boxed{|T(j\omega)| = \frac{K}{\sqrt{1 + (\omega_o/\omega)^2}}}$$

$V_o = R \frac{V_I}{Z_c + R}$

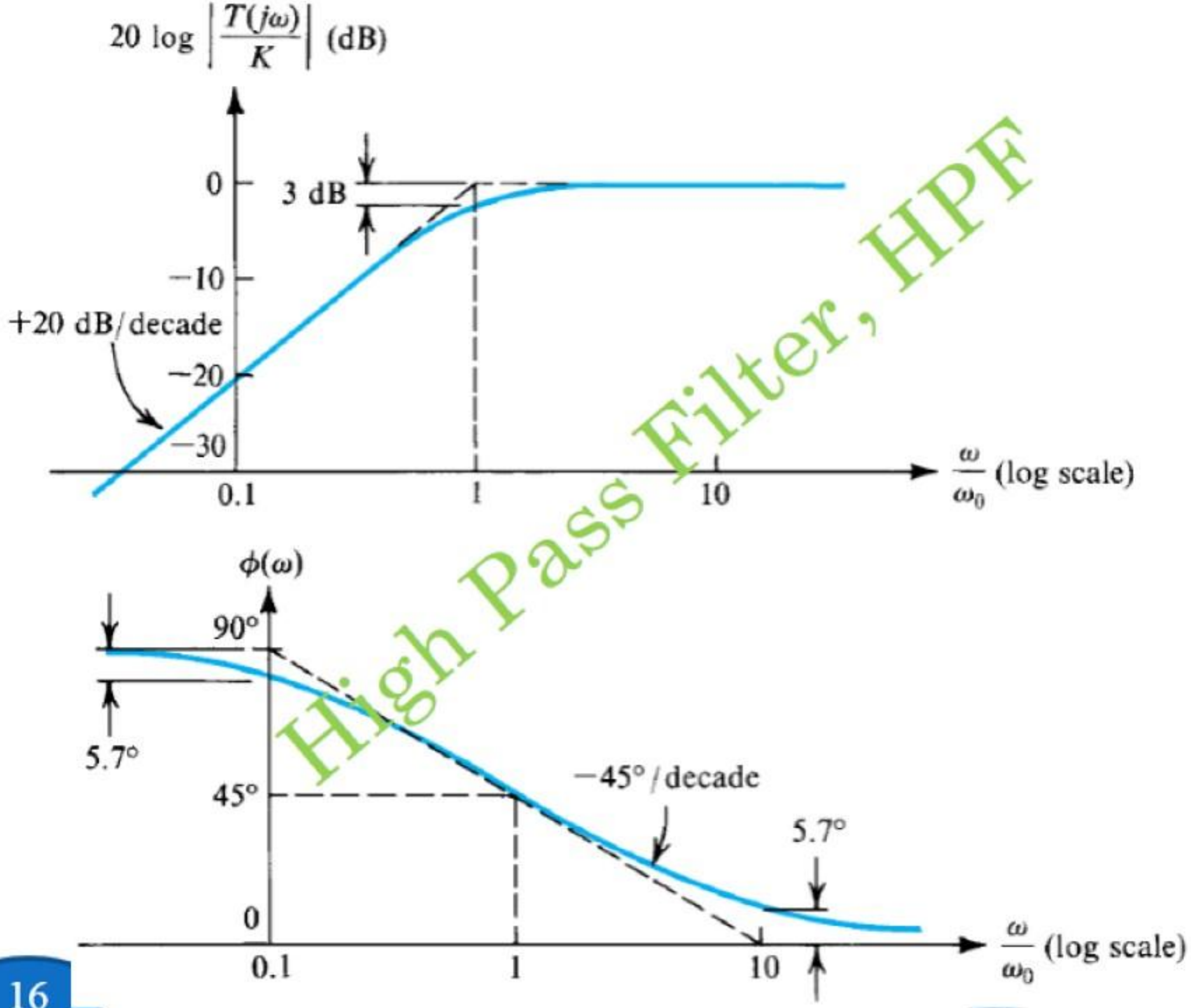
And the phase response is:

$T = V_o / V_I = \frac{R}{Z_c + R} = \frac{R}{\frac{1}{sC} + R}$

$$\emptyset(\omega) = tan^{-1}\left(\frac{\omega_0}{\omega}\right)$$

$$\boxed{T = \frac{1}{1 + \frac{1}{sCR}}}$$

Frequency Response of HPF Circuits

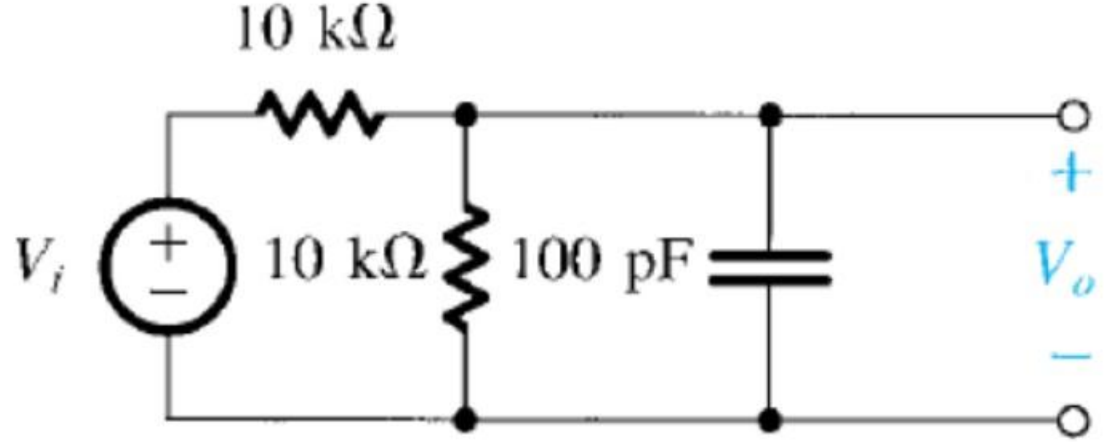


Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\boxed{\frac{K}{1+\frac{S}{\omega_o}}}$	$\frac{K}{1+\frac{\omega_o}{S}} = \frac{KS}{S+\omega_o}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\frac{K}{1+j(\frac{\omega}{\omega_o})}$	$\frac{K}{1-j\,\frac{\omega_o}{\omega}}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_o)^2}}$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega_o/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-tan^{-1}\left(\frac{\omega}{\omega_0}\right)$	$tan^{-1}\left(\frac{\omega_0}{\omega}\right)$
Transmission at $\omega = 0$ (dc)	K	0
Transmission at $\omega = \infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant	$\tau = CR$ or L/R
Bode Plots	in slide #15	in slide #17

Ex:-

What kind of filter is the shown circuit. Find the dc transmission, the gain constant, the corner frequency f_0, and the transmission at f = 2 MHz.



Ans: –6 dB; –6 dB; 318 kHz; –22 dB

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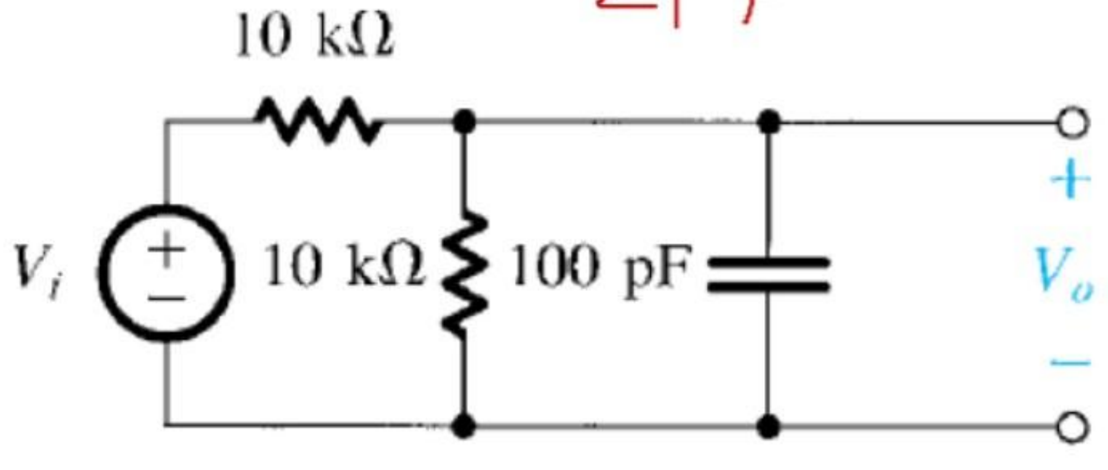
Ans:-

LPF

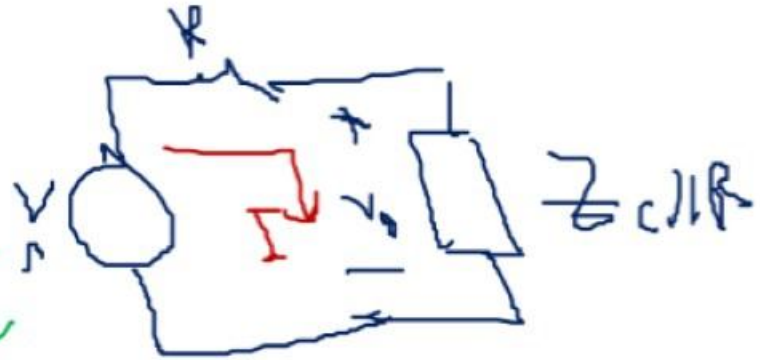


$$T = \frac{V_o}{V_i}$$

$$V_o = (Z_C // R)\,\frac{V_i}{R + (Z_C // R)}$$

$$Z_C = \frac{1}{sC}$$

$$T(s) = \frac{V_o}{V_i} = \boxed{\frac{1/2}{1 + s\,\frac{RC}{2}}} = \frac{K}{1 + \frac{s}{\omega_o}}$$



$$K = 1/2 \qquad \omega_o = \frac{1}{\tau} = \frac{2}{RC} = \frac{2}{10\times10^3 \times 100\times10^{-12}} = 2\pi f_o$$

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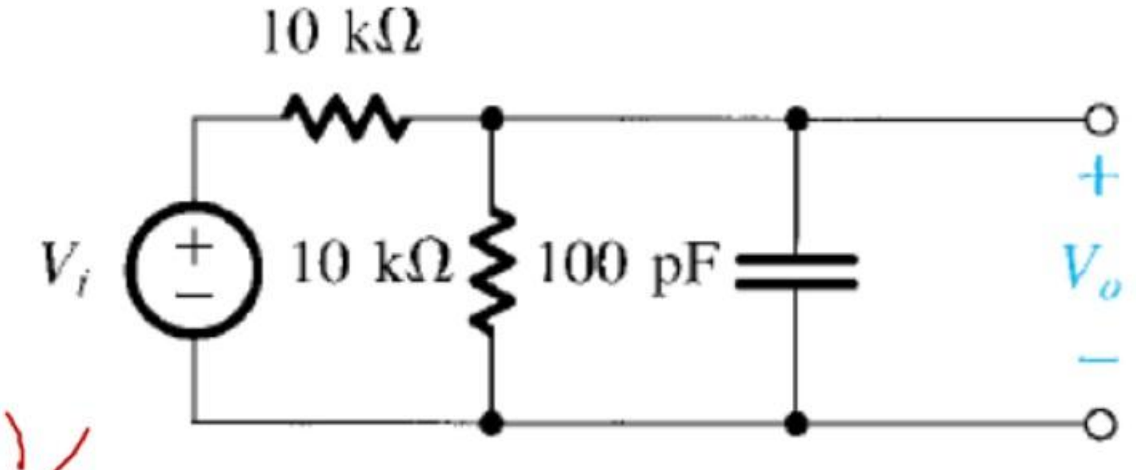
$F_0 = 318.309\ KHz$



at $\omega = 0$

$$K\Big|_{dB} = 20 \log K = 20 \log \tfrac{1}{2}$$

$$= -6\ dB$$

$$|T(j\omega)| = \frac{1/2}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}} = \frac{1/2}{\sqrt{1+\left(\frac{F}{F_0}\right)^2}} = \frac{1/2}{\sqrt{1+\left(\frac{2\times 10^6}{318.3}\right)^2}}$$

at $F = 2\ MHz$ $= 0.78$

$20 \log 0.78 = -22.09\ dB$

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T at $w = 0$

o.c cct

$Z_c = \frac{1}{jwC} = \infty$

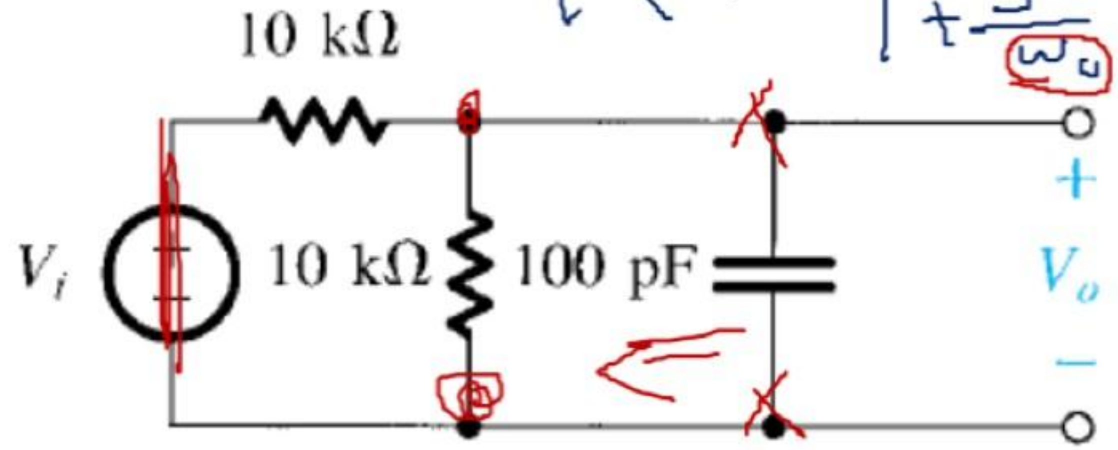
$T\Big|_{at\, w=0} = V_o / V_i = \boxed{1/2 = k}$

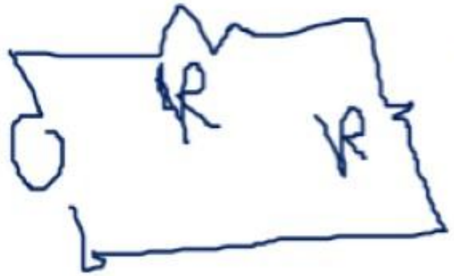
$T\Big|_{at\, w=\infty} = \frac{0}{V_i} = 0$

$Z_c = 0$

LPF

$T(s) = \frac{1/2}{1 + \frac{s}{w_o}} =$



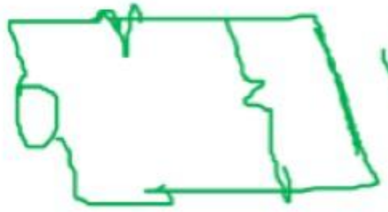


$V_o = \frac{1}{2} V_i$

$w_o = 1/t = \frac{1}{RC}$

$t = R_{eq} C$

$R/2\, C$



$V_o = 0$

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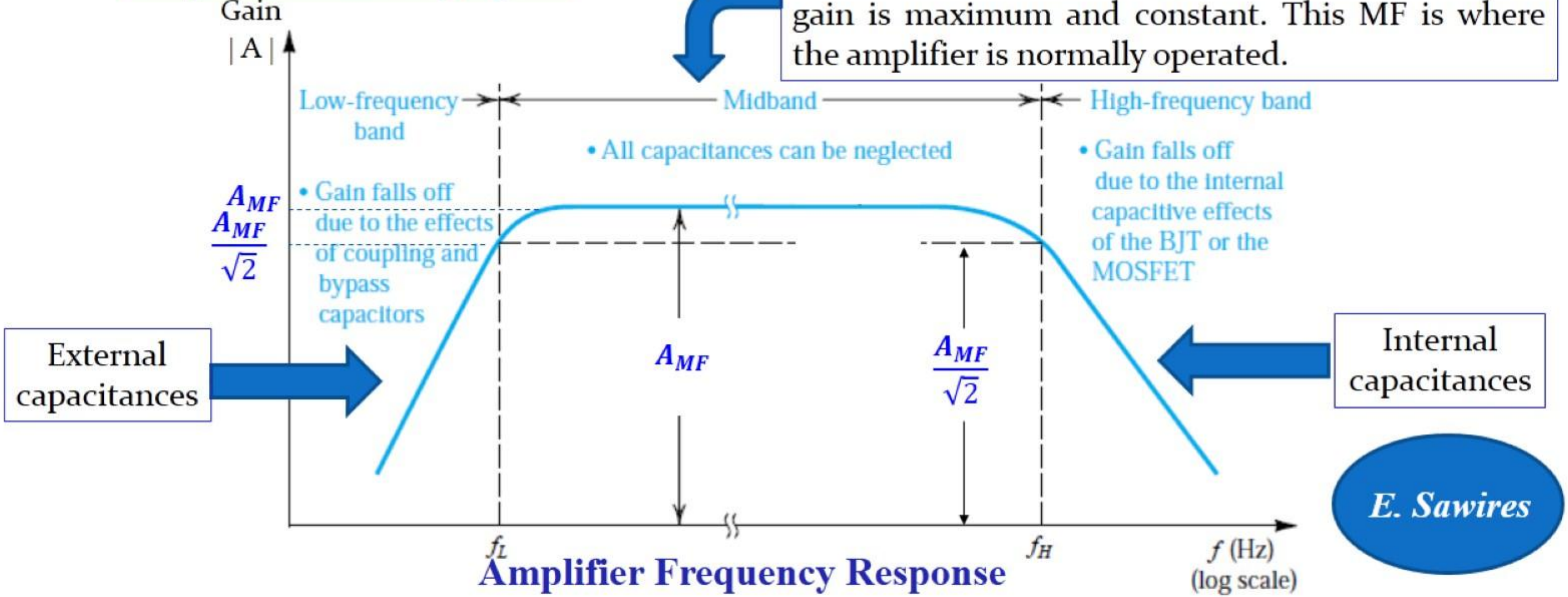
Amplifier Frequency Response

Objective

- We want to study the effect that these capacitances have on the amplifier's voltage gain

In the middle band of frequencies (MF), the voltage gain is maximum and constant. This MF is where the amplifier is normally operated.



Amplifier Frequency Response

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Objective

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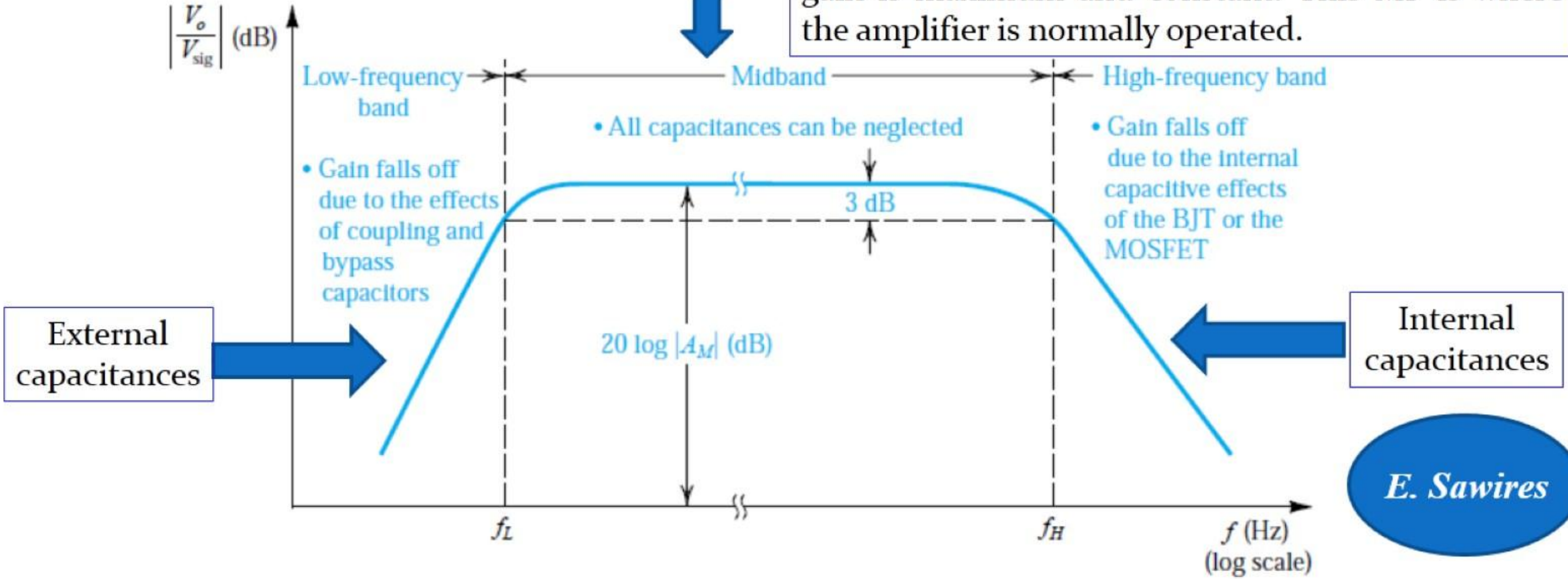
In the middle band of frequencies (MF), the voltage gain is maximum and constant. This MF is where the amplifier is normally operated.



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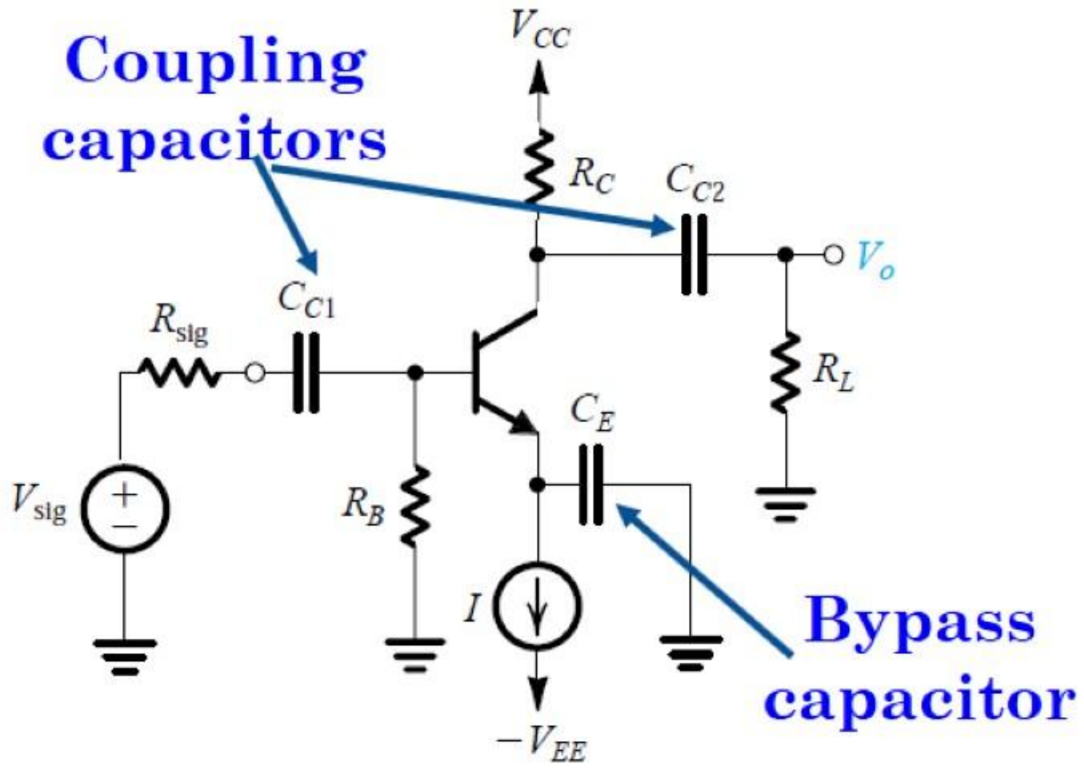
Introduction

External Capacitances (μF)



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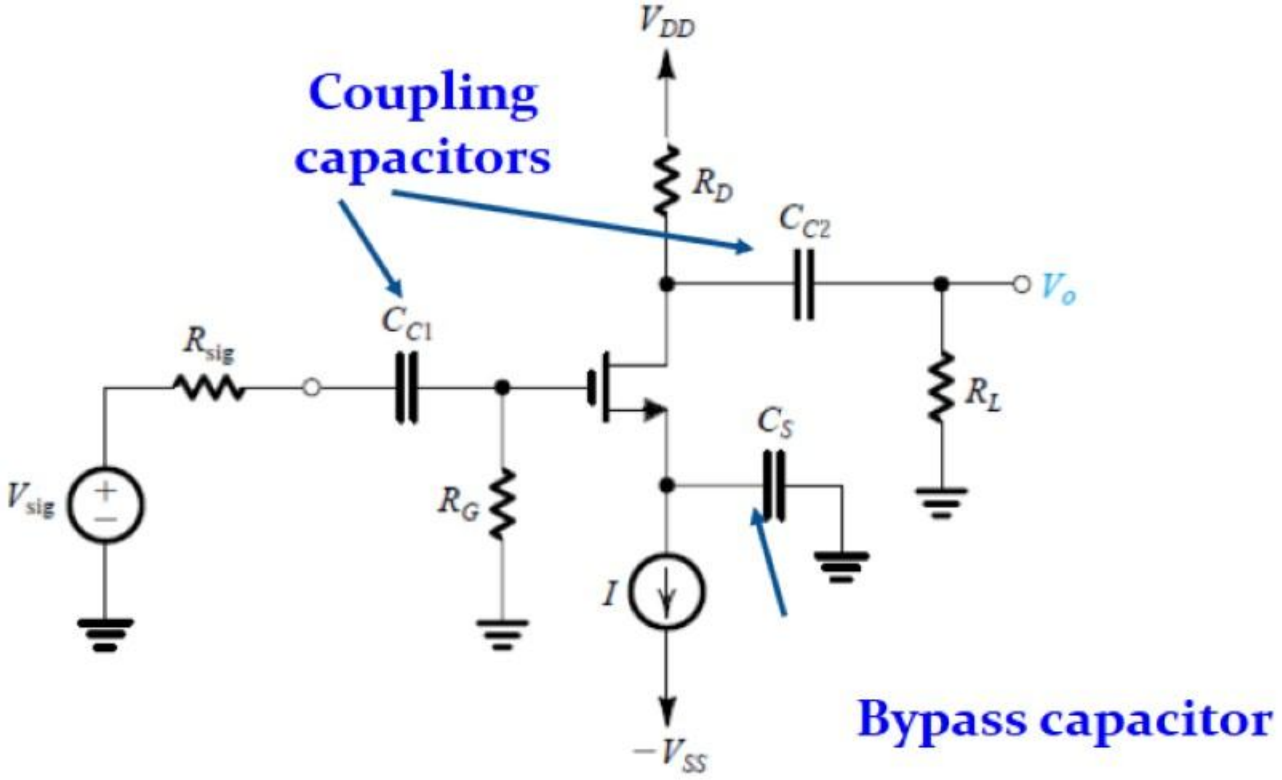
Introduction

External Capacitances (μF)



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Introduction

- **In the previous lectures we studied the voltage amplifiers, without taking in to account any capacitances in the circuit.**
  - The voltage gain was calculated while all **external capacitors** were **Short circuit**
  - The Short circuit is based on the assumption that: the product of the input signal frequency with the capacitor value is infinity

$$Z_{Cext} = \frac{1}{SC_{external}} = \frac{1}{j2\pi f C_{external}} \cong 0$$

  - However, **this is not valid for low frequencies**!

Introduction

- **The physical structure and biasing of the transistors (MOS or BJT) result in Internal capacitances between its terminals.**
  - However, we have considered their impedances as **Open Circuit**

$$Z_{Cint} = \frac{1}{SC_{int}} = \frac{1}{j2\pi f C_{int}} \cong \infty$$

  - However, this is not valid for High frequencies!

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External Capacitances (μF)

Typically, the external capacitors are large (in order of Micro Farad)

$$\boxed{Z_C = \frac{1}{j2\pi f C_{external}}}$$

In LF
capacitors **are not** act
like a (short circuit)

In MF
capacitors act
like a (short circuit)

In HF
capacitors act
like a (short circuit)

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Internal Capacitances (pF)

Typically, the internal capacitors are small (in order of Pico Farad)

$$\boxed{Z_C = \frac{1}{j2\pi f C_{internal}}}$$

In LF	In MF	In HF
capacitors act like a (open circuit)	capacitors act like a (open circuit)	capacitors **are not** act like a (open circuit)

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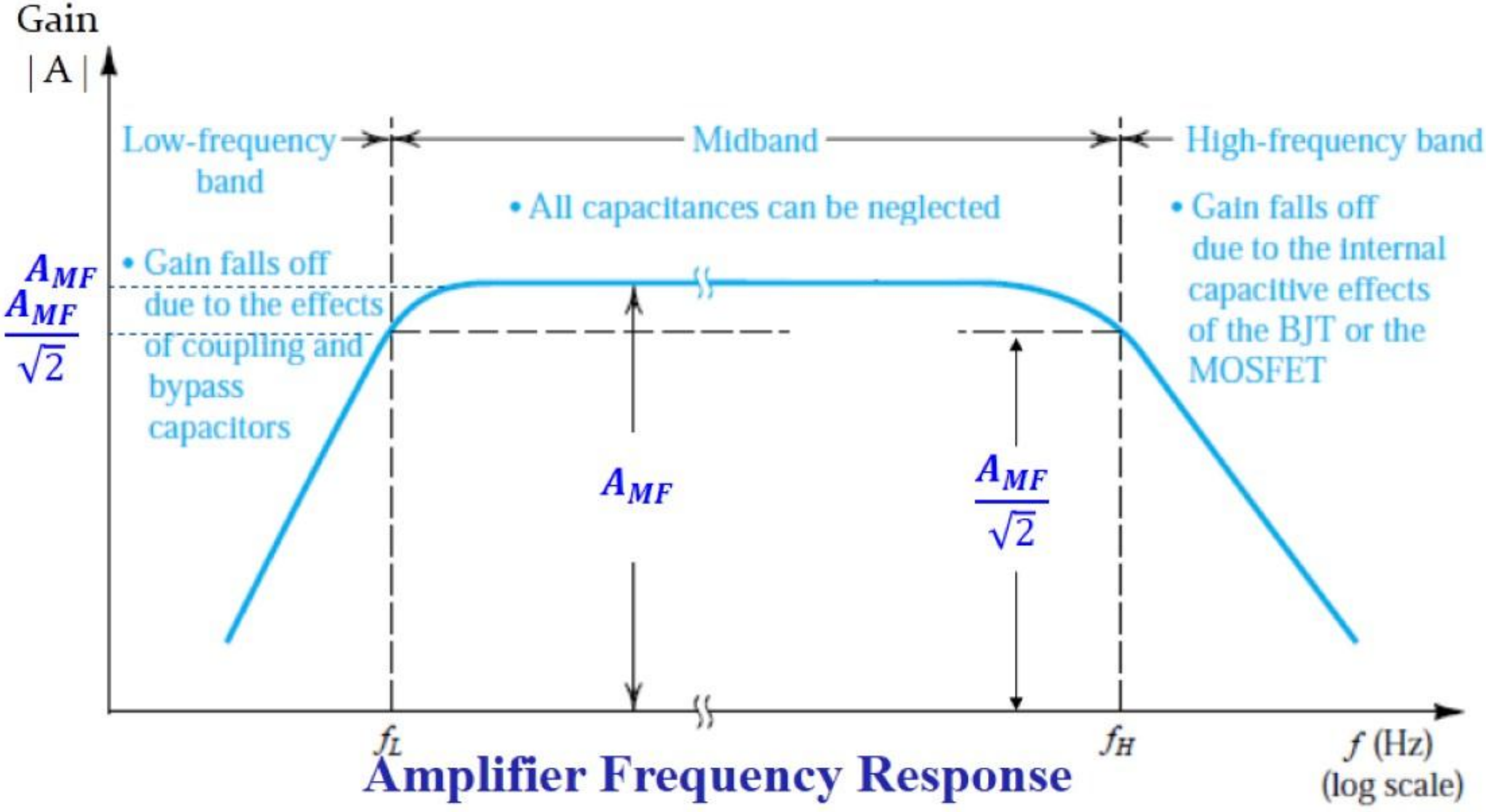
Low Frequency Response



Amplifier Frequency Response

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Low Frequency Response

The gain in the low frequency : $A_{LF}(s)$; S = j ω

(a) Assuming the amplifier has one external capacitor:

$$\boxed{A_{LF}(\mathrm{S}) = A_{MF}\left(\frac{S}{S+\omega_{P_1}}\right) = \frac{A_{MF}}{1+\frac{\omega_{P_1}}{S}} = \frac{A_{MF}}{1+\frac{\omega_L}{j\omega}}}$$

Where A_{MF} is the mid-band gain, and $\omega_L = \omega_{P_1} = 2\pi f_L$ is the lower cut-off radian frequency or the lower 3-dB radian frequency.

The magnitude of the gain in dB is given by: $|A_{LF}|_{\mathrm{dB}} = 20\log|A_{LF}(j\omega)|$

$$= 20\log\frac{A_{MF}}{\sqrt{1+\left(\frac{\omega_L}{\omega}\right)^2}}$$

Note : at $\omega = \omega_L$

$$|A_{LF}| = \frac{A_{MF}}{\sqrt{2}}$$

$$\boxed{|A_{LF}|_{\mathrm{dB}} = (20\log A_{MF}) - 3}$$

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Low Frequency Response

(b) Assuming the amplifier has two independent external capacitors:

$$\boxed{A_{LF}(\mathrm{S}) = A_{MF}\left(\frac{S}{S+\omega_{P_1}}\right)\left(\frac{S}{S+\omega_{P_2}}\right)}$$

$$A_{LF}(\mathrm{S}) = \frac{A_{MF}}{(1+\frac{\omega_{P1}}{S})(1+\frac{\omega_{P2}}{S})}$$

The lower 3-dB frequency is given by :

$$\boxed{f_L = \sqrt{f_{p1}^2 + f_{p2}^2}}$$

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Low Frequency Response

(c) Assuming the amplifier has n independent external capacitors:

Each capacitor will generate a pole and a transmission Zero.

$$A_{LF}(\mathrm{S}) = A_{MF}\left(\frac{S}{S+\omega_{P_1}}\right)\left(\frac{S}{S+\omega_{P_2}}\right)\cdots\left(\frac{S}{S+\omega_{P_n}}\right)$$

$$\boxed{A_{LF}(\mathrm{S}) = \frac{A_{MF}}{\left(1+\frac{\omega_{P1}}{S}\right)\left(1+\frac{\omega_{P2}}{S}\right)\ldots\left(1+\frac{\omega_{Pn}}{S}\right)}}$$

$$\boxed{f_L = \sqrt{f_{p1}^2 + f_{p2}^2 + \cdots f_{pn}^2}}$$

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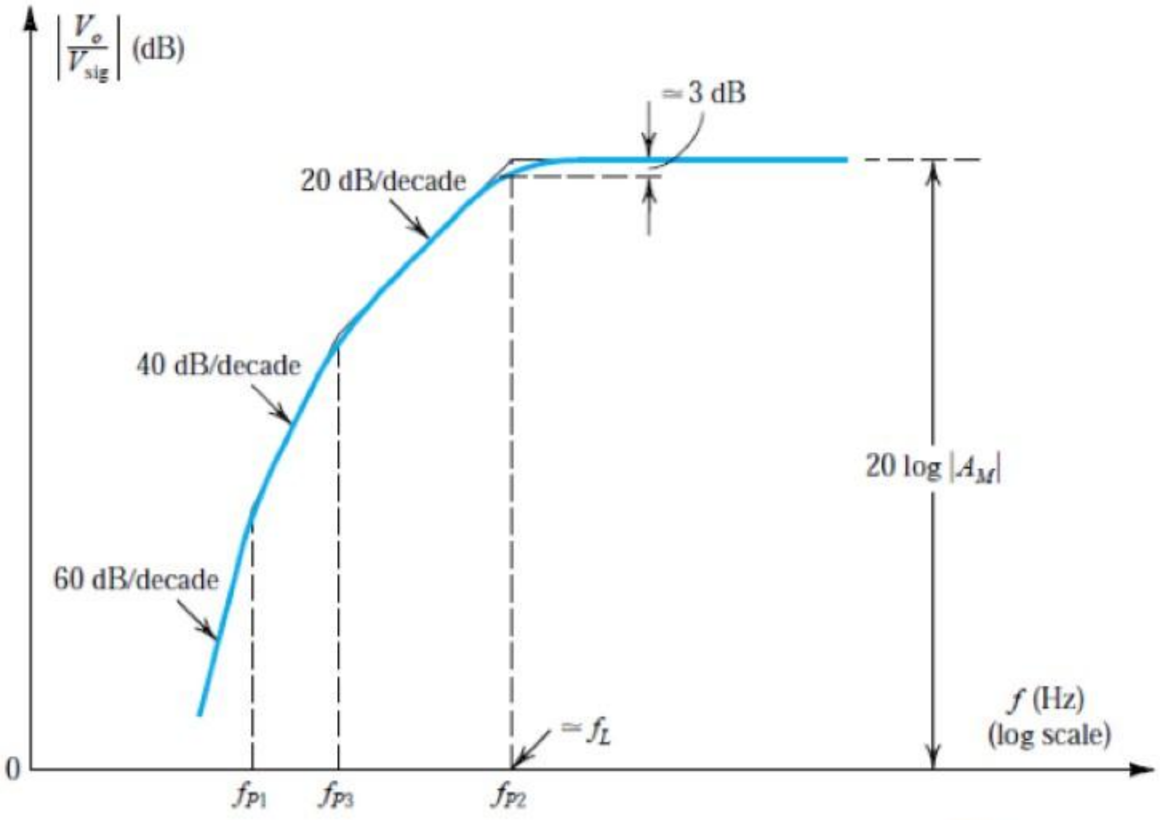
Low Frequency Response

Assuming that the three capacitors are not interacting, we can write the following overall equation:

$$A_{LF}(S) = \frac{V_o}{V_{sig}} = A_M\left(\frac{S}{S+w_{p1}}\right)\left(\frac{S}{S+w_{p2}}\right)\left(\frac{S}{S+w_{p2}}\right)$$

$$A_{LF}(S) = \frac{A_{MF}}{\left(1+\frac{\omega_{P1}}{S}\right)\left(1+\frac{\omega_{P2}}{S}\right)\left(1+\frac{\omega_{P3}}{S}\right)}$$

from which we see that it acquires three poles with frequencies f_{P1}, f_{P2}, and f_{P3}, all in the low frequency band. If the three frequencies are widely separated, their effects will be distinct, as indicated in the figure.



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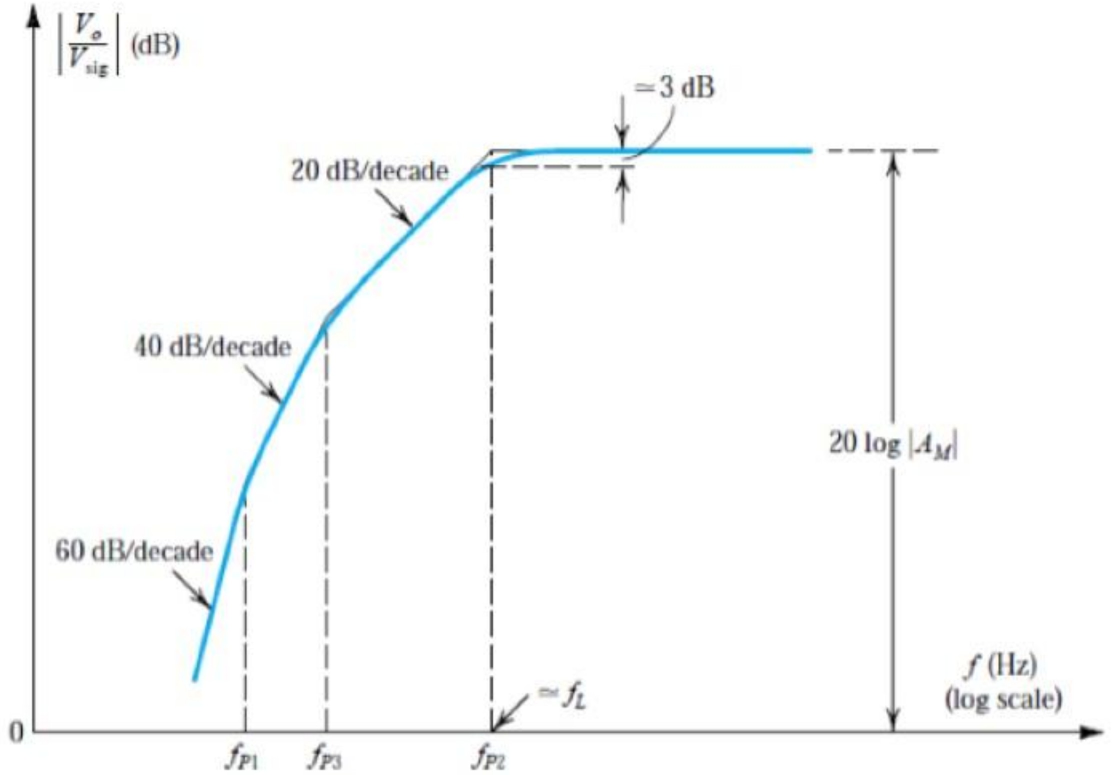
Low Frequency Response

$$f_L = \sqrt{f_{p1}^2 + f_{p2}^2 + f_{p3}^2}$$

OR

The important point to note here is that the lower 3-dB frequency, f_L is determined by the highest of the three pole frequencies (dominant pole) if $f_{p2}/f_{p3} \geq 10$.



Example: Bode Plot for Amplifier with Three External Capacitors

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Calculating the Lower 3-dB frequency

Calculating f_L

- The lower 3-dB frequency is calculated by NOT considering the impedances of the **external capacitances Short Circuit.**
- The external capacitors poles should be calculated by deriving the voltage gain
- However, deriving the small signal model with ‘n’ external capacitors is complicated
  - We will consider each external capacitor at a time while the other external capacitors will be considered Short Circuit
  - Repeat for every capacitor and all the poles will be evaluated
- The internal capacitors while calculating the lower 3-dB frequency are neglected (considered open circuit)

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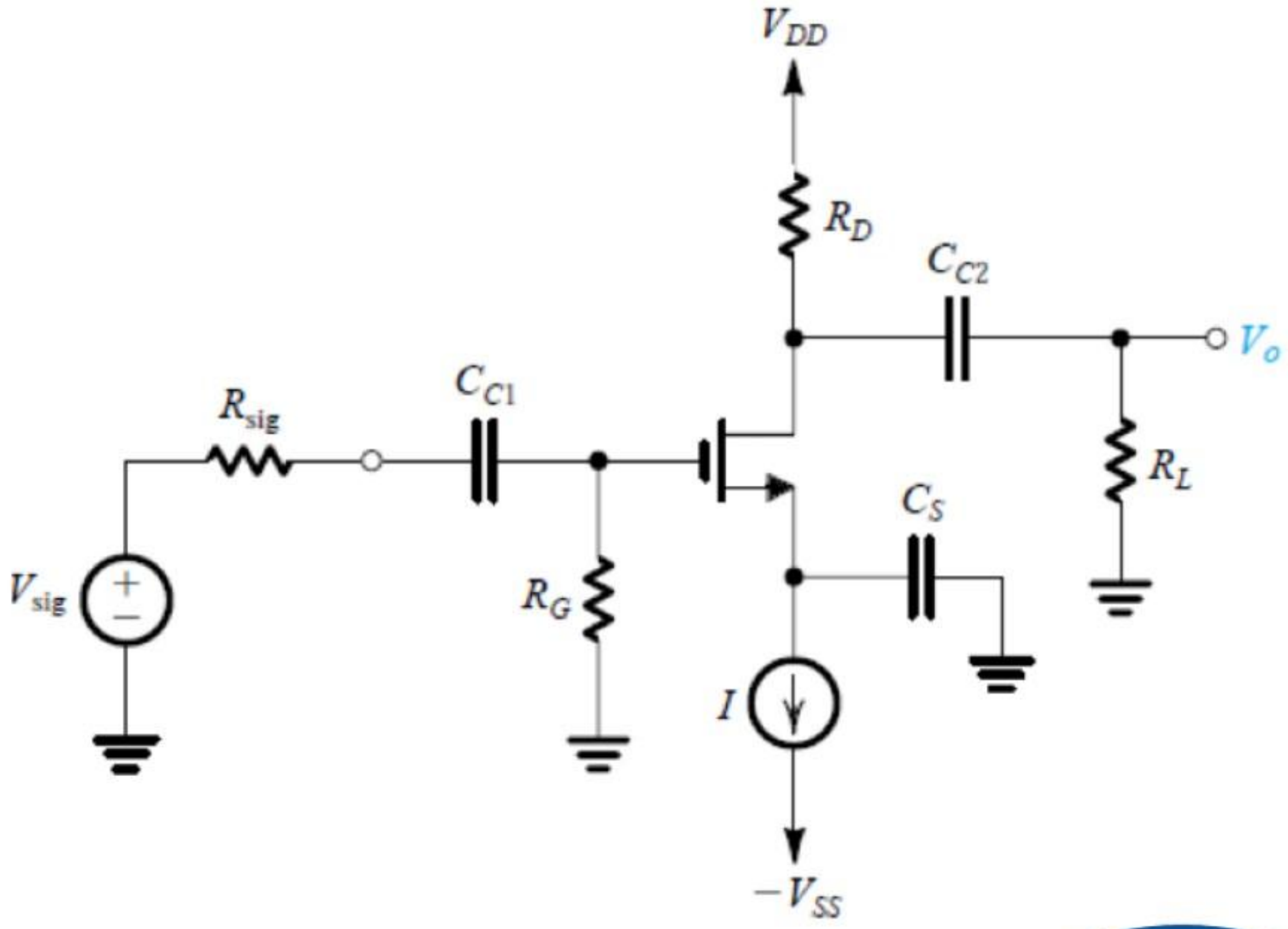
Example

For the shown amplifier :
(a) Find the mid-band voltage gain, and the three break frequencies due to: C_{C1}, C_{C2}, and C_S .
(b) Find the lower 3-dB frequency.



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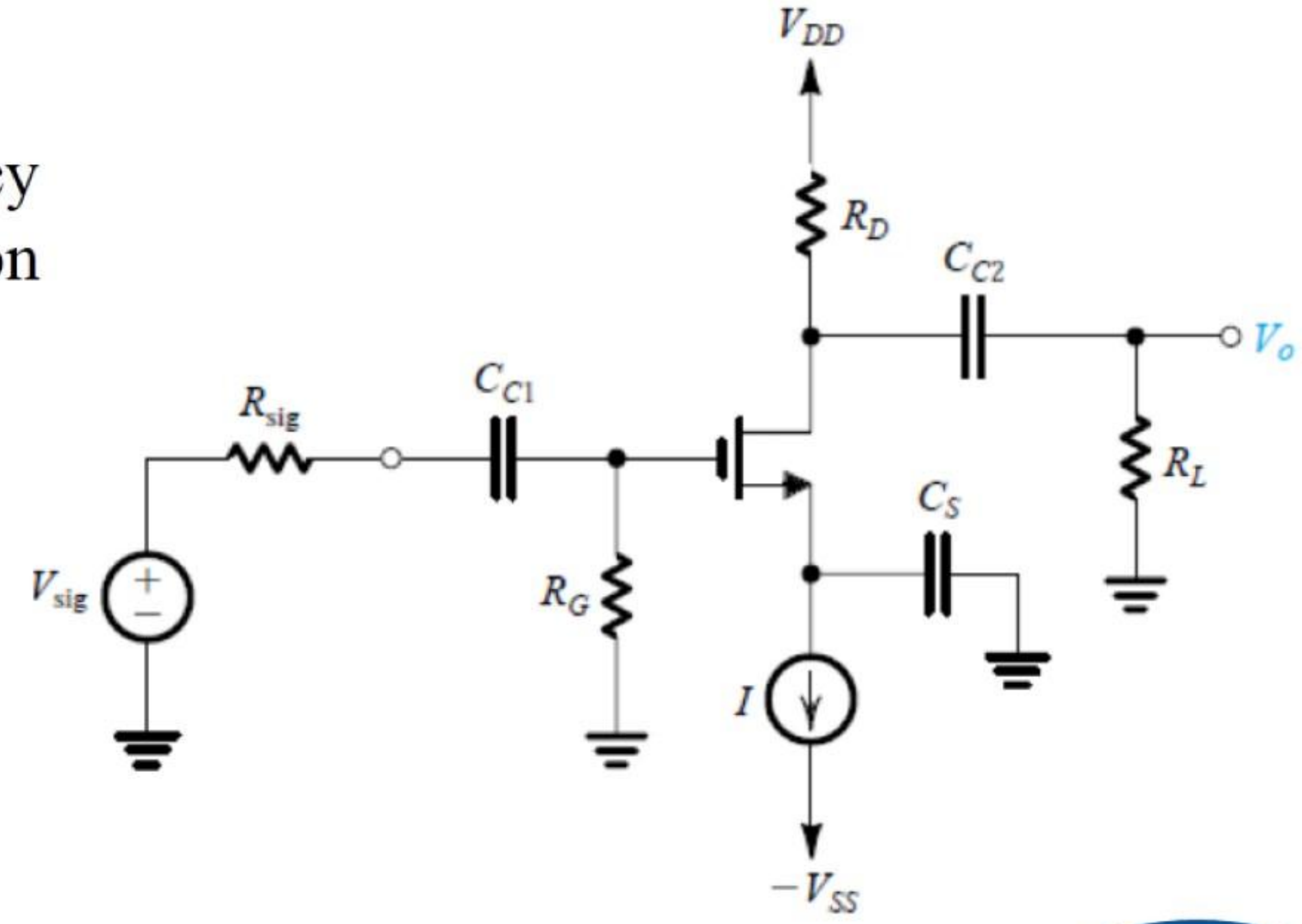
Example

Calculate the Low-frequency band gain for the Common Source Amplifier.



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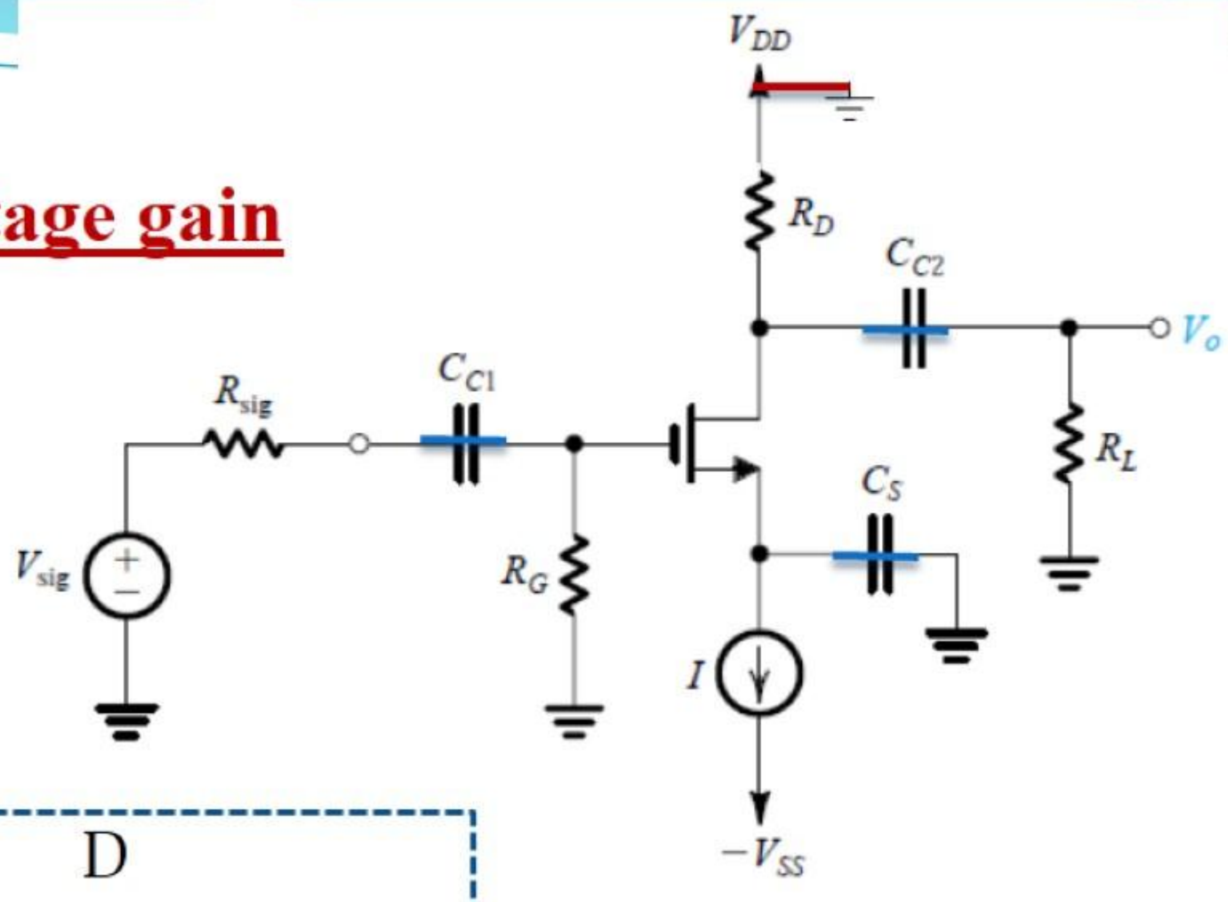
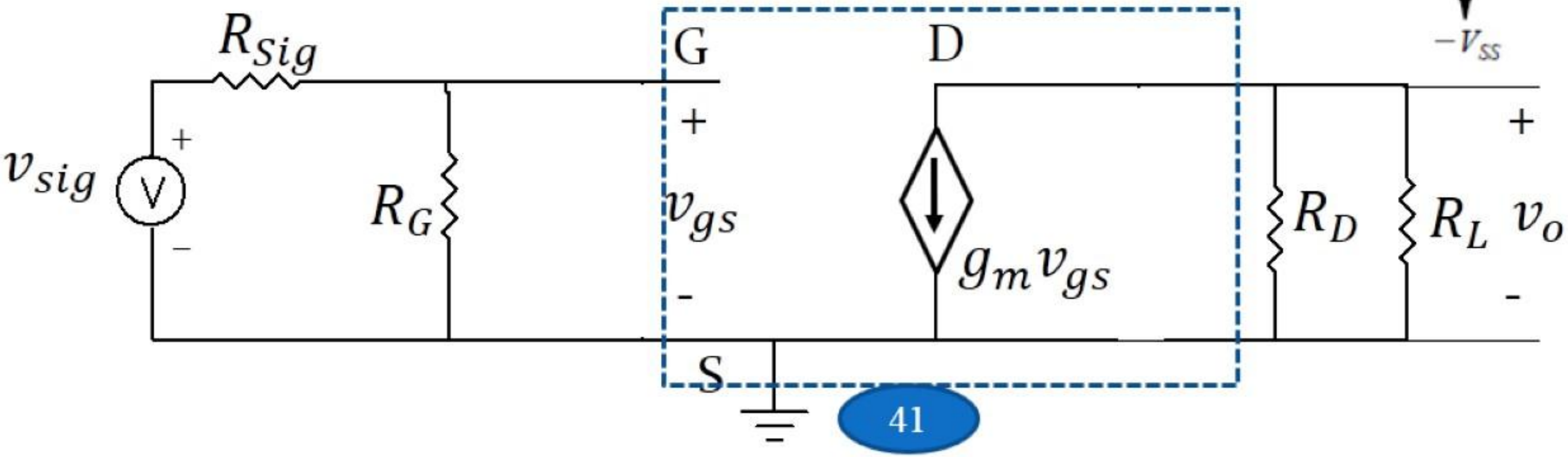
(a) Calculate the mid-band voltage gain





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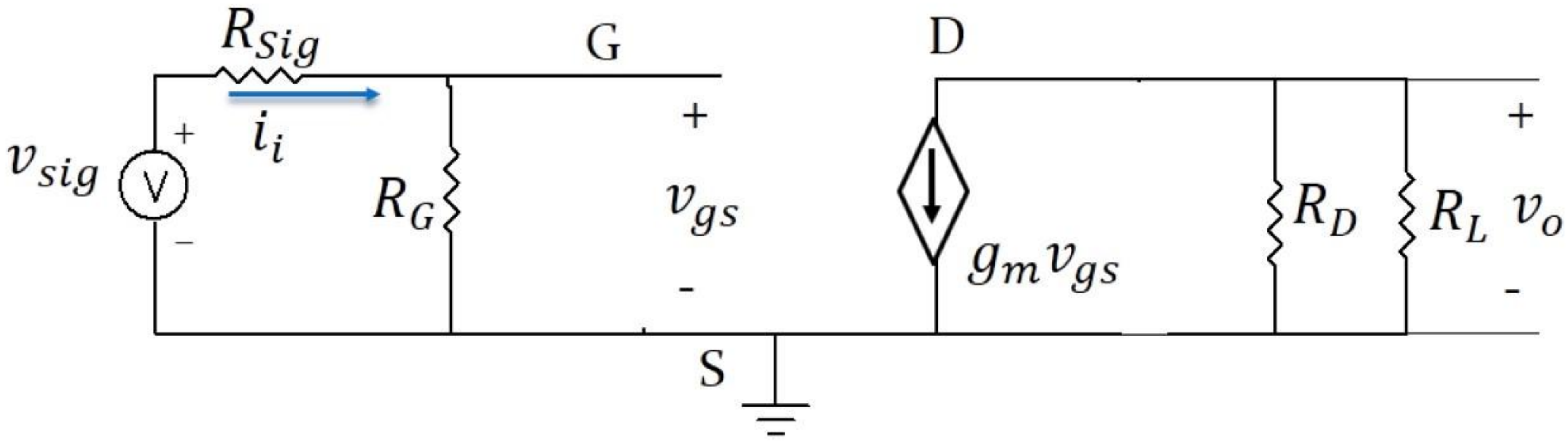


$$v_o = -g_m v_{gs}(R_L//R_D)$$

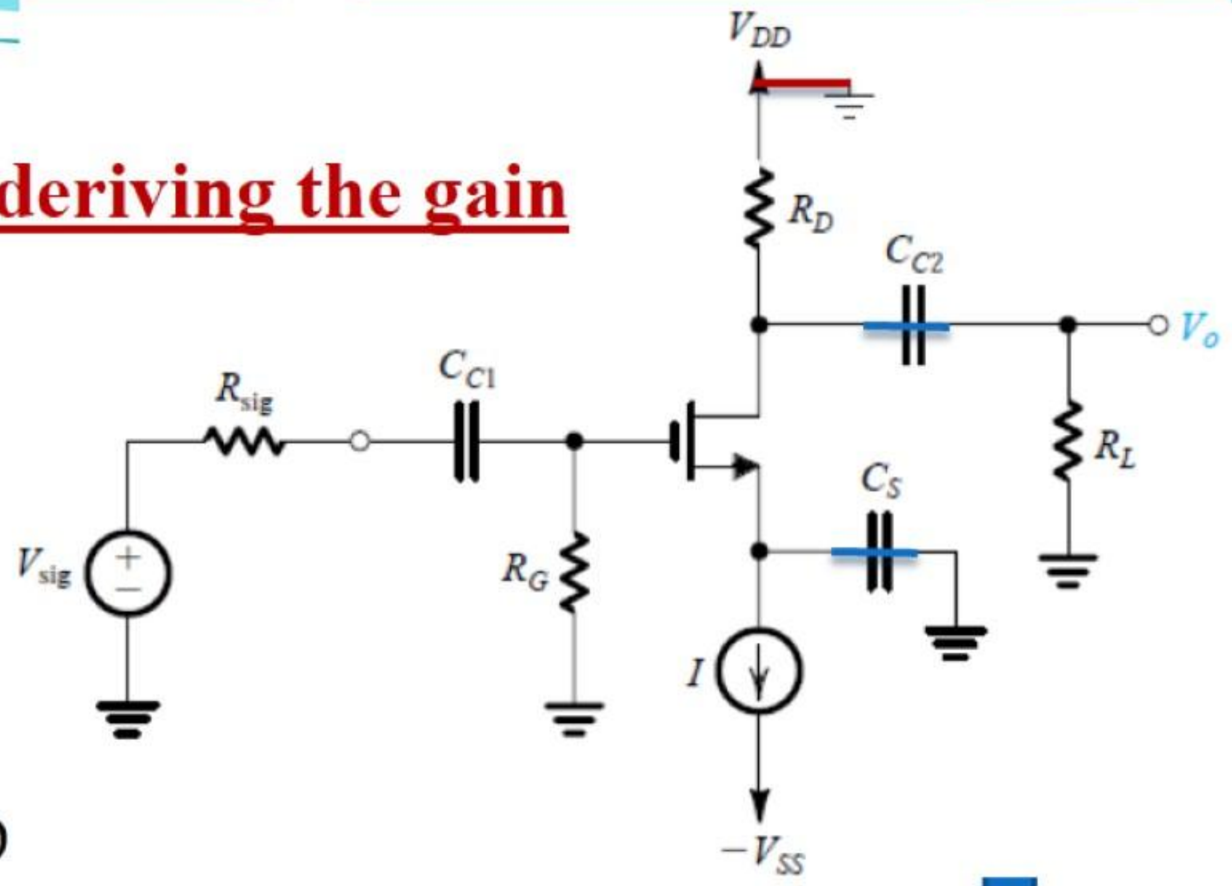
$$v_{gs} = i_i R_G$$

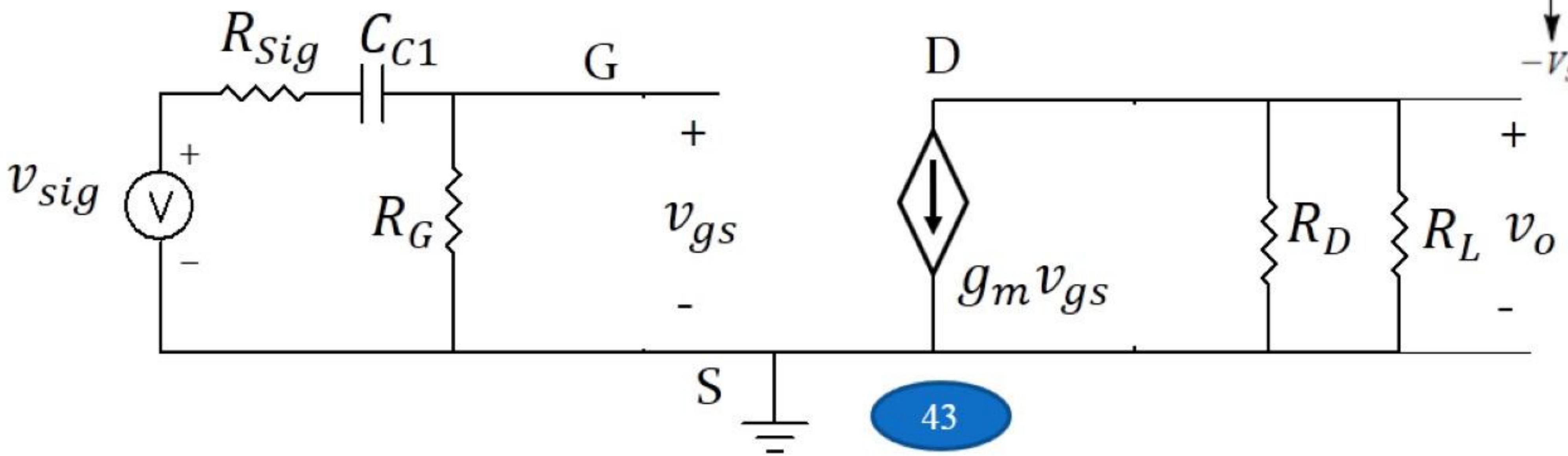
$$i_i = \frac{v_{sig}}{R_{sig} + R_G}$$

$$\frac{v_o}{v_{sig}} = A_{MF} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+R_G]}$$

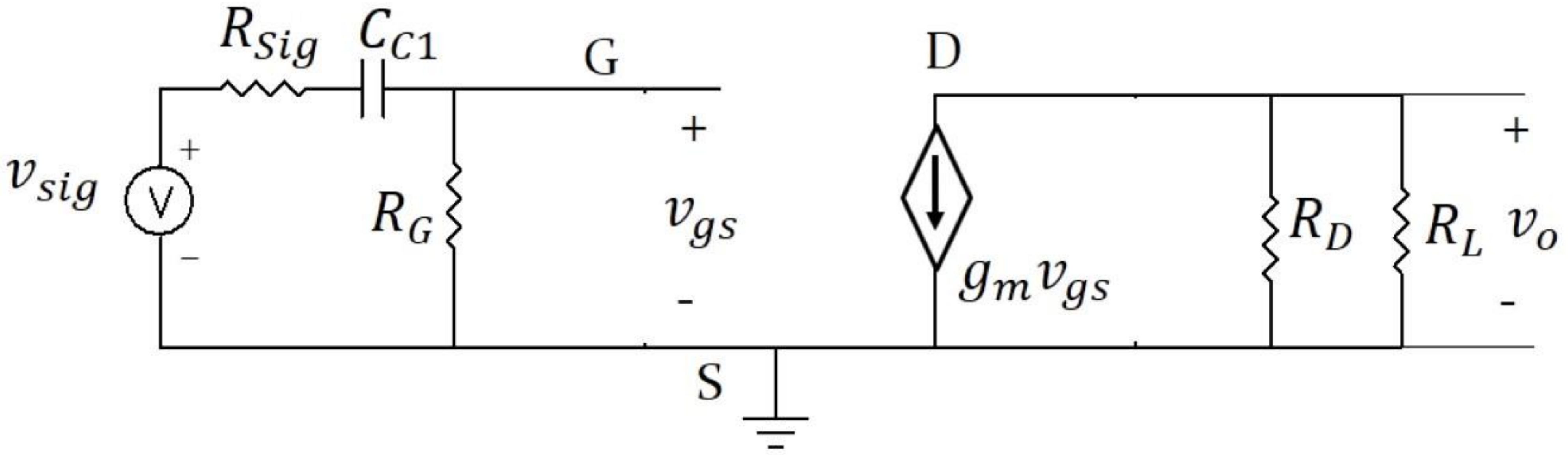
E. Sawires

Calculate the pole due to C_{C1} by deriving the gain (C_{C2}, C_S are neglected)





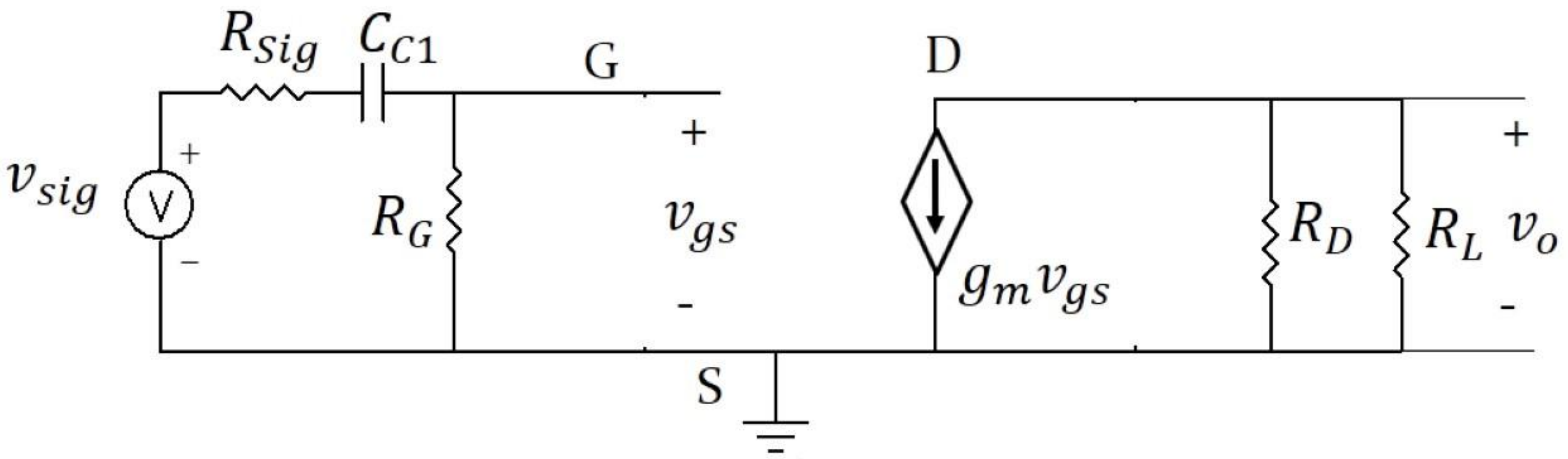
E. Sawires



$$v_o = -g_m v_{gs}(R_L//R_D)$$

$$v_{gs} = i_i\, R_G \qquad i_i = \frac{v_{sig}}{R_{sig} + \frac{1}{SC_{C1}} + R_G}$$

E. Sawires



$$\frac{v_o}{v_{sig}} = \frac{-g_m\,(R_L//R_D\big)R_G}{[R_{sig}+\frac{1}{s\,c_{C1}}+R_G]}$$

E. Sawires

$$\frac{v_o}{v_{sig}} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+\frac{1}{s\,C_{C1}}+R_G]}$$

Note that: The gain of the amplifier in the low frequency can be written as:

$$A_{LF,C1}(\mathrm{S}) = \frac{A_{MF}}{1+\frac{\omega_{P1}}{S}}$$

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+R_G]\left(1+\frac{1}{SC_{C1}[R_{sig}+R_G]}\right)}$$

E. Sawires

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+R_G]}\frac{1}{\left(1+\frac{1}{SC_{C1}[R_{sig}+R_G]}\right)}$$

$$A_{LF,C1}(\mathrm{S}) = \frac{A_{MF}}{1+\frac{\omega_{P1}}{S}}$$

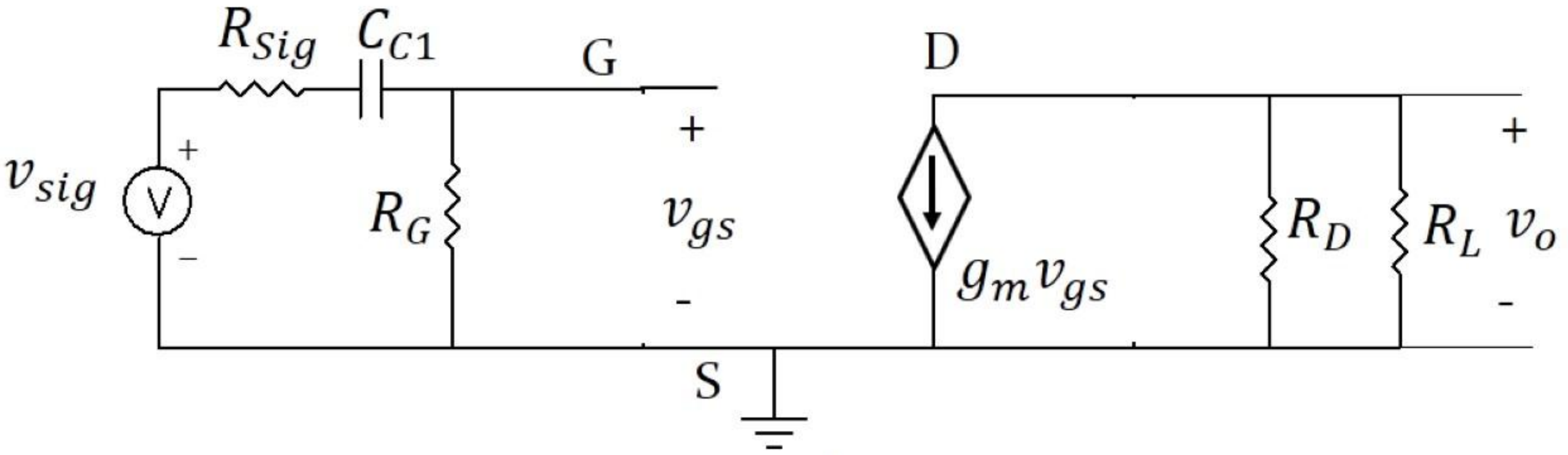
Therefore, the break frequency due to C_{C1} is given by:

$$\omega_{P1} = 2\pi f_{P1} = \frac{1}{C_{C1}[R_{sig}+R_G]} = \frac{1}{\tau}$$

$$\boxed{\tau = C_{C1}R_{eq}}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}\left[R_{sig}+R_G\right]} \longrightarrow \boxed{1}$$

E. Sawires



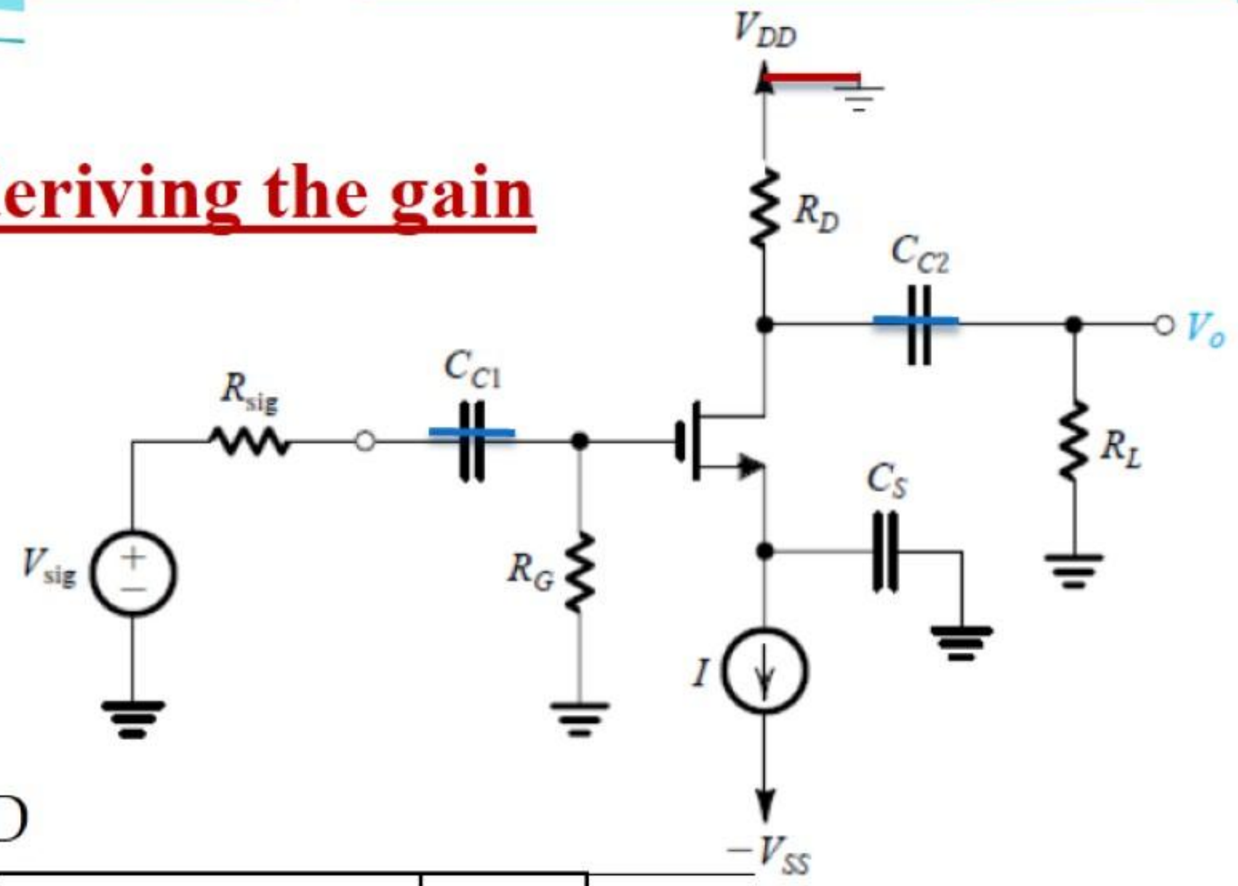
$$\omega_{P1} = 2\pi f_{P1} = \frac{1}{C_{C1}[R_{sig}+R_G]} = \frac{1}{\tau}$$

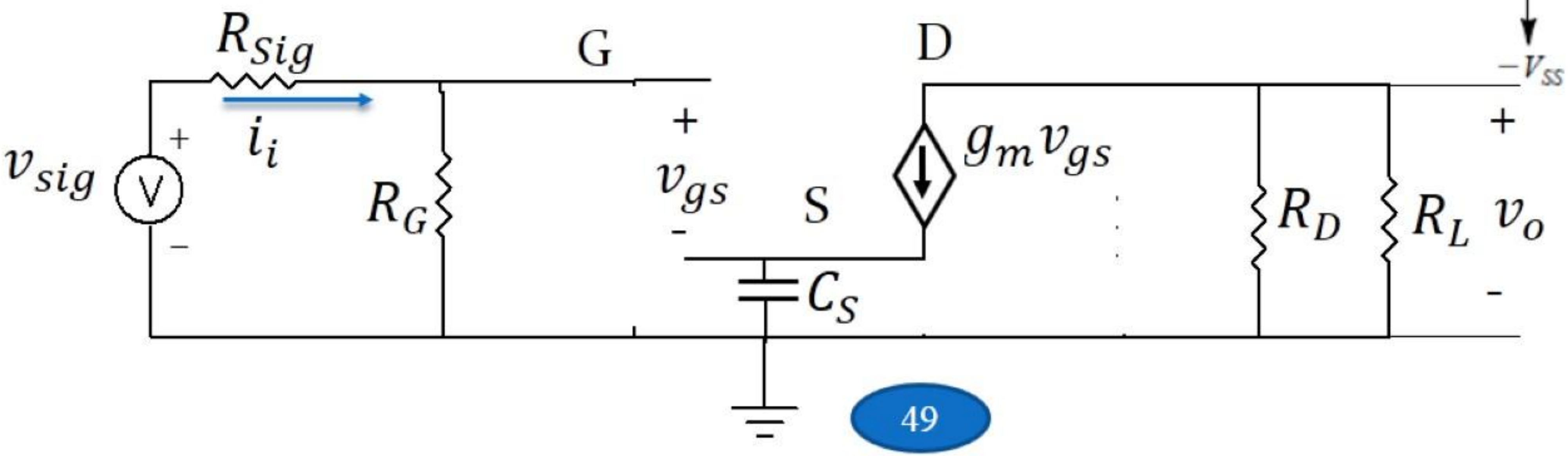
$$\boxed{\tau = C_{C1}R_{eq}}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}\left[R_{sig} + R_G\right]} \longrightarrow \boxed{1}$$

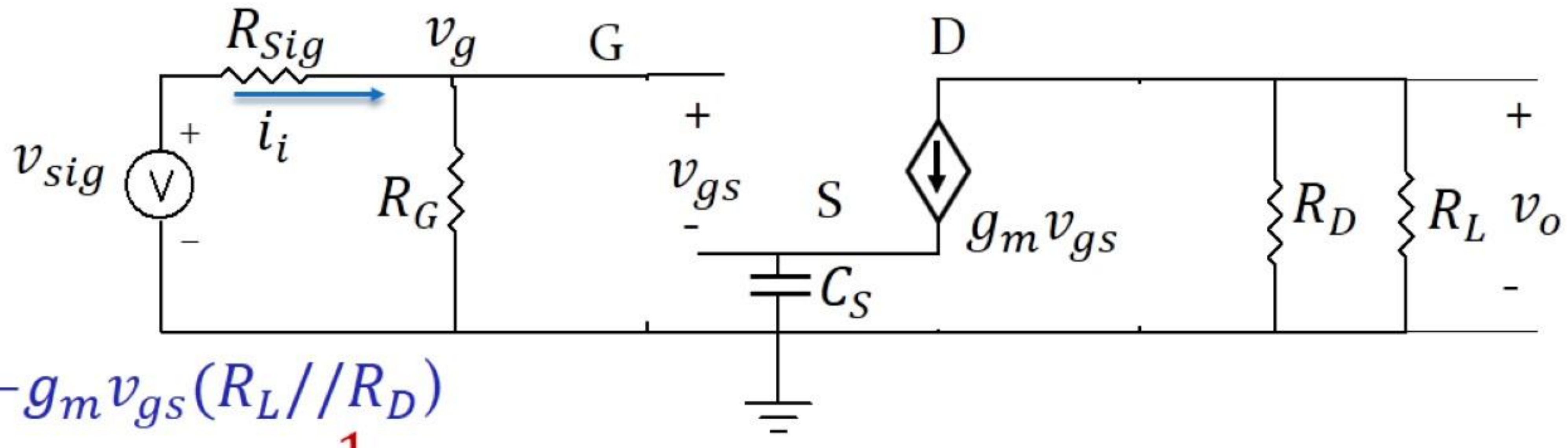
E. Sawires

Calculate the pole due to C_S by deriving the gain (C_{C1}, C_{C2} are neglected)





E. Sawires



$$v_o = -g_m v_{gs}(R_L//R_D)$$

$$v_g = v_{gs} + g_m v_{gs}\frac{1}{SC_S}$$

$$R_G i_i = v_{gs}(1 + g_m\frac{1}{SC_S})$$

$$i_i = \frac{v_{sig}}{R_{sig} + R_G}$$

$$v_o = -g_m(R_L//R_D)\, R_G \frac{v_{sig}}{R_{sig}+R_G}\frac{1}{(1+g_m\frac{1}{SC_S})}$$

E. Sawires

$$v_o = -g_m(R_L//R_D)\, R_G \frac{v_{sig}}{R_{sig}+R_G} \frac{1}{(1+g_m\frac{1}{SC_S})}$$

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+R_G]\left(1+\frac{g_m}{SC_S}\right)}$$

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+R_G]} \frac{1}{\left(1+\frac{g_m}{SC_S}\right)}$$

$$A_{LF,CS}(\mathrm{S}) = \frac{A_{MF}}{1+\frac{\omega_{P2}}{S}}$$

E. Sawires

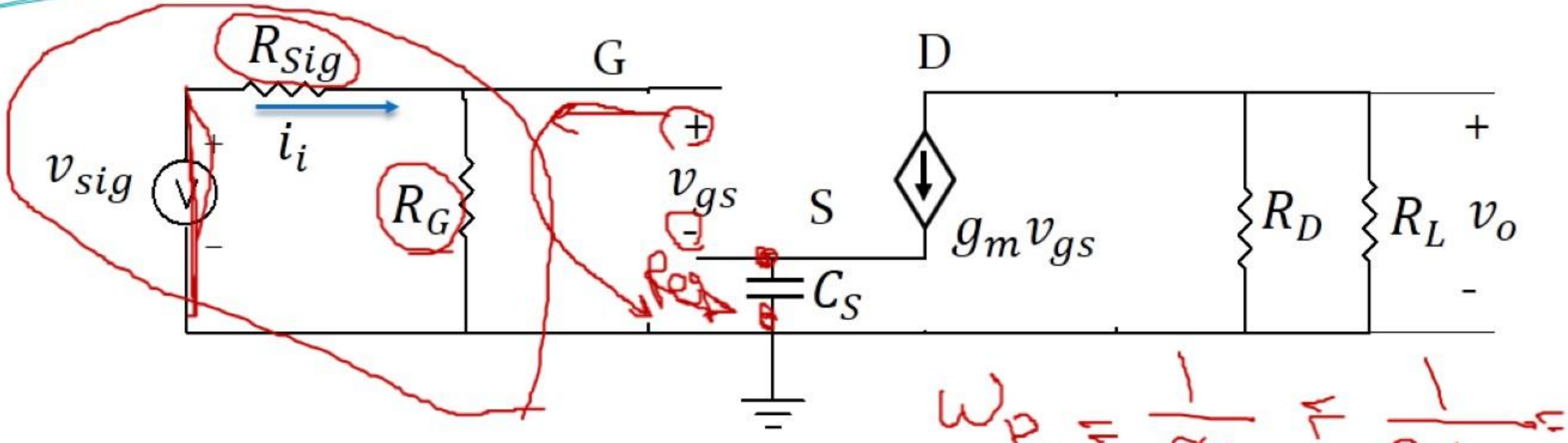
Therefore, the break frequency due to C_S is given by:

$$\omega_{P2} = 2\pi f_{P2} = \frac{g_m}{C_S} = \frac{\mathbf{1}}{\boldsymbol{\tau}}$$

$$\boxed{\boldsymbol{\tau} = \boldsymbol{C_S}\frac{\mathbf{1}}{\boldsymbol{g_m}}}$$

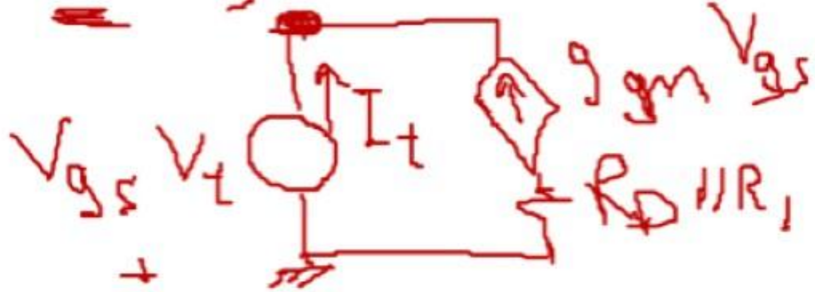
$$f_{P2} = \frac{1}{2\pi C_S(\frac{1}{g_m})} \longrightarrow \boxed{\mathbf{2}}$$

E. Sawires



$$\omega_{P_S} = \frac{1}{\tau} = \frac{1}{C_S \, 1/g_m} = \frac{g_m}{C_S}$$

$$R_{eq} = V_t / I_t = 1/g_m$$



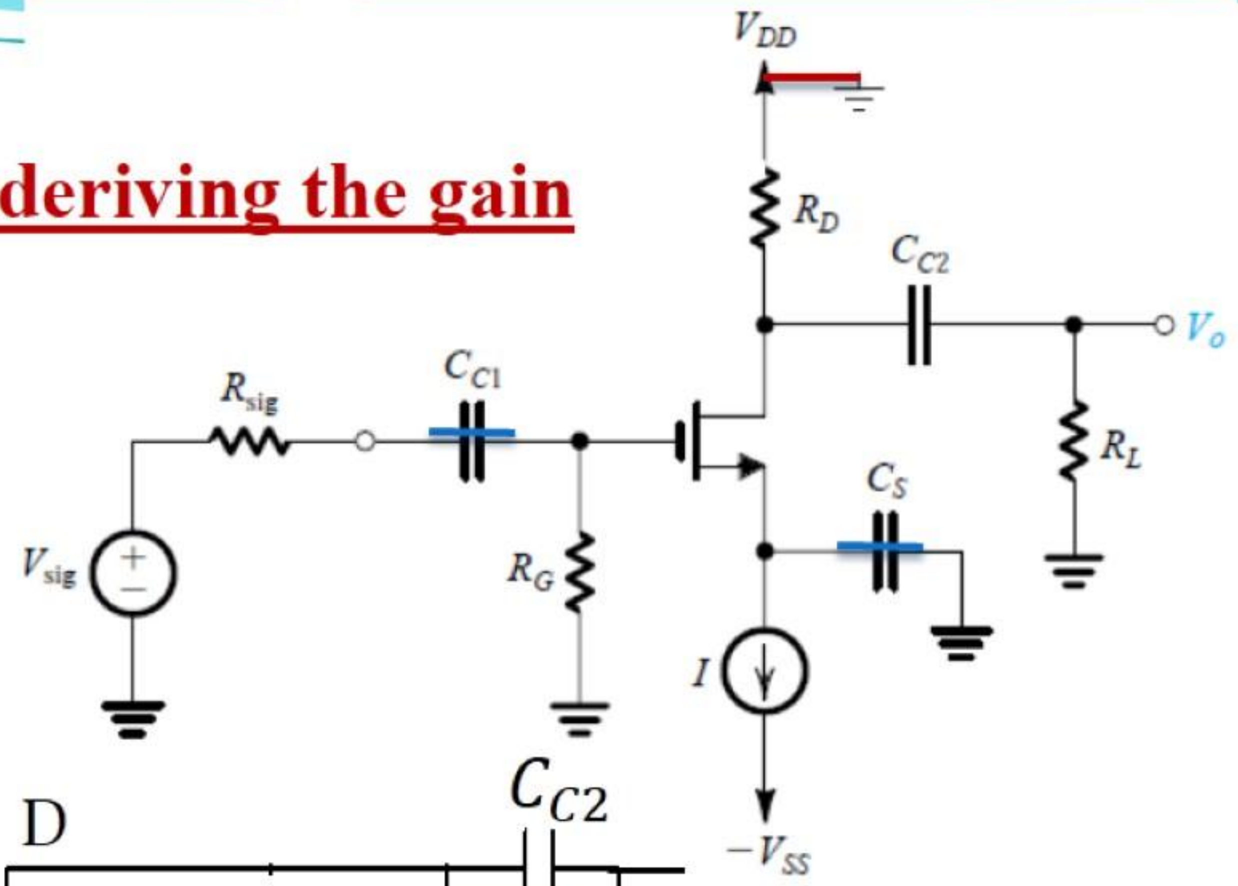
$$I_t + g_m V_{gs} = 0$$

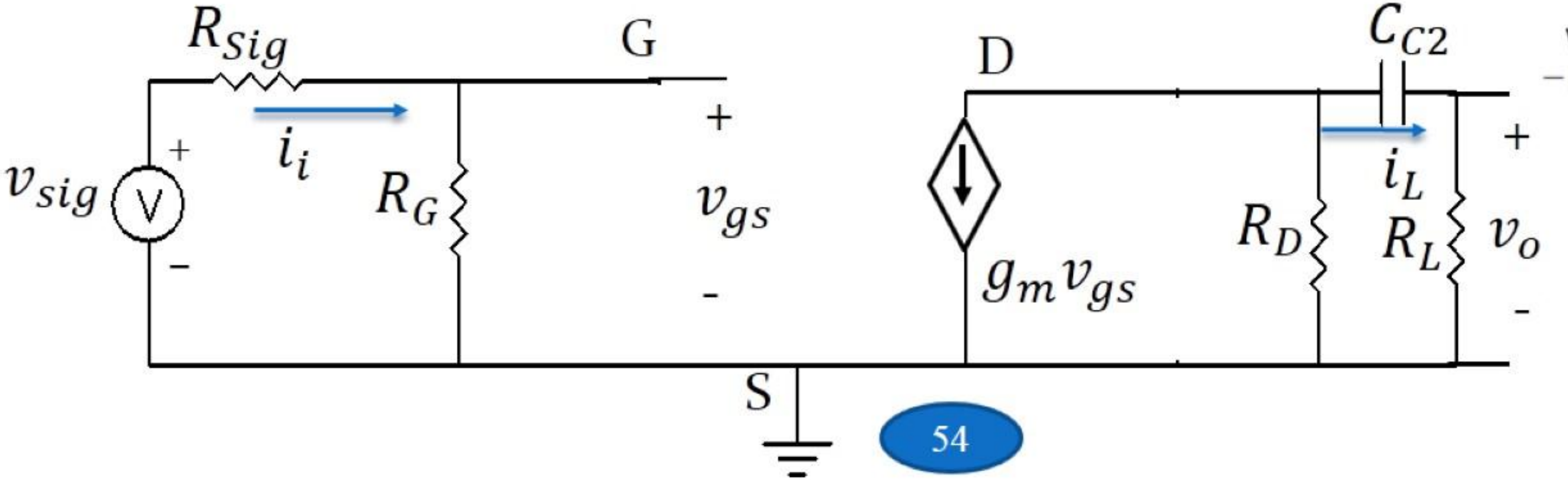
$$I_t - g_m V_t = 0$$

$$R_{eq} = V_t / I_t = 1/g_m$$

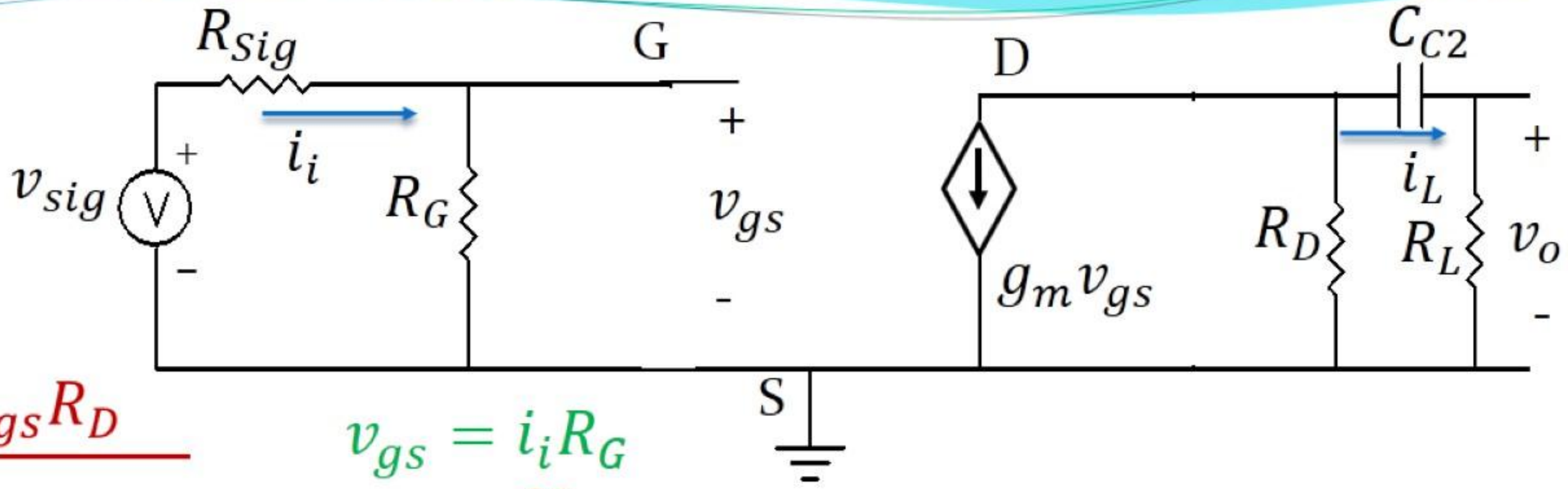
E. Sawires

Calculate the pole due to C_{C2} by deriving the gain (C_{C1}, C_{CS} are neglected)





E. Sawires



$$v_o = i_L R_L$$

$$i_L = \frac{-g_m v_{gs} R_D}{R_D + \frac{1}{SC_{C2}} + R_L}$$

$$v_{gs} = i_i R_G$$

$$i_i = \frac{V_{sig}}{R_{sig} + R_G}$$

$$\frac{v_o}{v_{sig}} = \frac{-g_m R_D R_L R_G}{\left[R_D + \frac{1}{SC_{C2}} + R_L\right]\left[R_{sig} + R_G\right]} = \frac{-g_m R_D R_L R_G}{(R_D + R_L)\left[R_{sig} + R_G\right]\left[1 + \frac{1}{SC_{C2}(R_D + R_L)}\right]}$$

E. Sawires

$$\frac{v_o}{v_{sig}} = \frac{-g_m R_D R_L R_G}{(R_D + R_L)\left[R_{sig} + R_G\right]\left[1 + \frac{1}{SC_{C2}(R_D + R_L)}\right]}$$

$$\frac{v_o}{v_{sig}} = \frac{-g_m (R_D // R_L) R_G}{\left[R_{sig} + R_G\right]\left[1 + \frac{1}{SC_{C2}(R_D + R_L)}\right]}$$

$$A_{LF,C2}(\mathrm{S}) = \frac{A_{MF}}{1 + \frac{\omega_{P3}}{S}}$$

Therefore, the break frequency due to C_{C2} is given by:

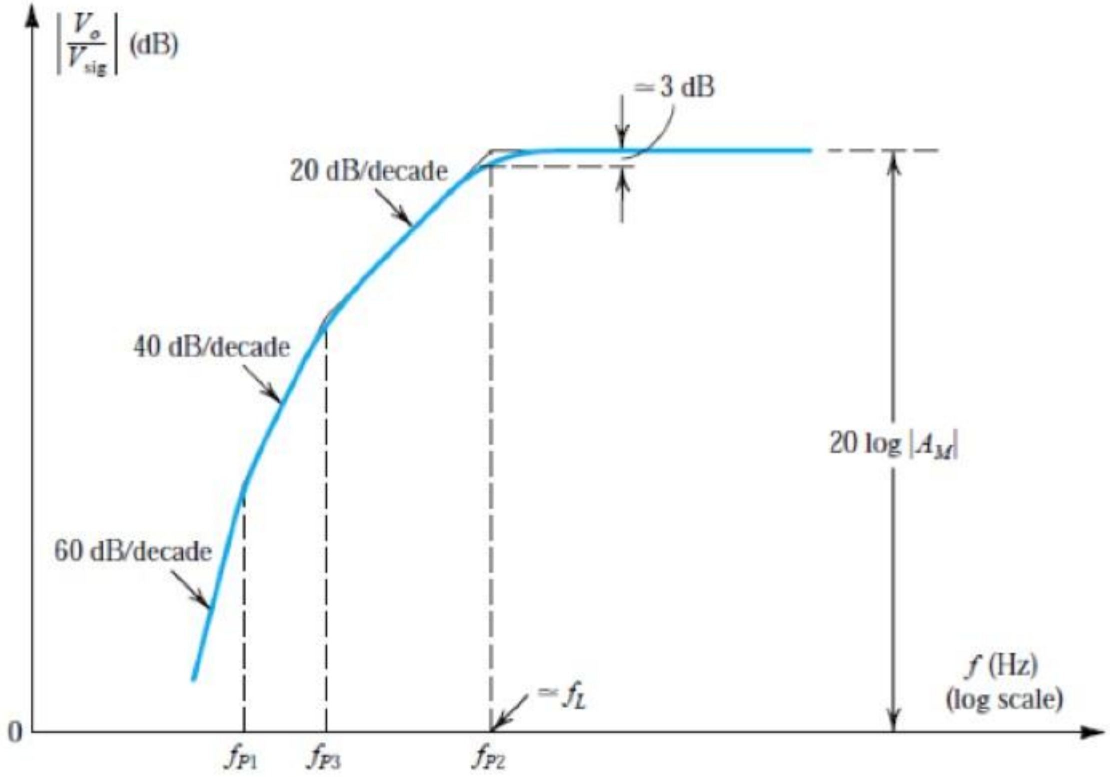
$$\tau = C_{C2}(R_D + R_L)$$

$$f_{P3} = \frac{1}{2\pi C_{C2}\,(R_D + R_L)} \longrightarrow \boxed{3}$$

E. Sawires

(b) Calculate the lower 3-dB frequency

$$f_L = \sqrt{f_{p1}^2 + f_{p2}^2 + f_{p3}^2}$$



$$\frac{V_{out}}{V_{sig}} = A_{LF}(\mathrm{S}) = \frac{A_{MF}}{(1+\frac{\omega_{P1}}{S})(1+\frac{\omega_{P2}}{S})(1+\frac{\omega_{P3}}{S})}$$

E. Sawires

Example:

A CS amplifier has $C_{C1} = C_S = C_{C2} = 1\ \mu F$, $R_G = 10\ M\Omega$, $R_{sig} = 100\ k\Omega$, $g_m = 2\ mA/V$, $R_D = R_L = 10\ k\Omega$. Find A_M, f_{P1}, f_{P2}, f_{P3}, and f_L.

Sol

$$\frac{v_o}{v_{sig}} = A_{MF} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+R_G]} = -9.9 V/V$$

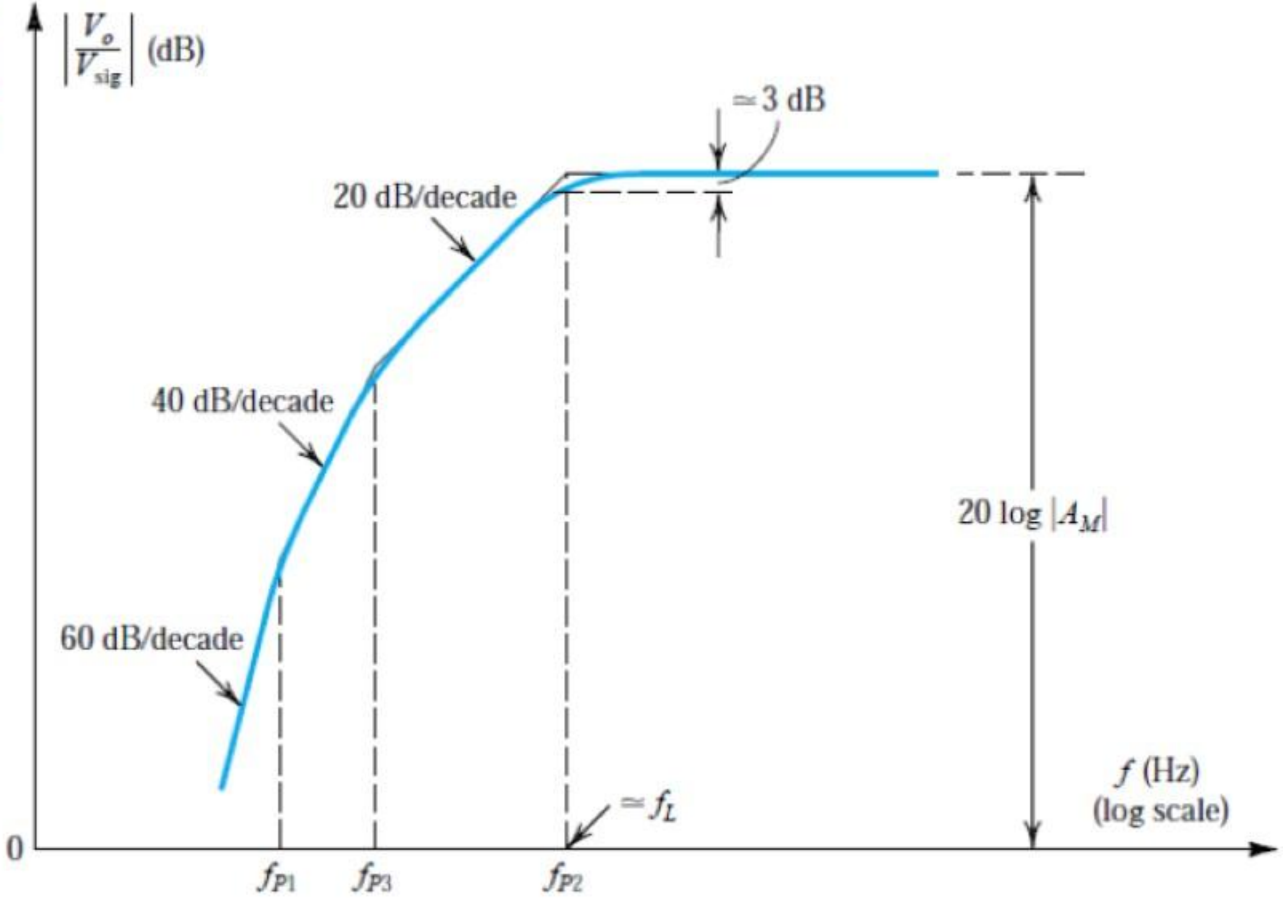
$$f_{P1} = \frac{1}{2\pi C_{C1}[R_{sig}+R_G]} = 0.016\ HZ$$

E. Sawires

Sol

$$f_{P2} = \frac{1}{2\pi C_S \frac{1}{g_m}} = 318 \text{ HZ}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}\,(R_D+R_L)} = 8 \text{ HZ}$$



$$f_L = \sqrt{f_{p1}^2 + f_{p2}^2 + f_{p3}^2} = 318.1\ HZ$$

$$f_L = f_{P2} \cong 318\ HZ$$

E. Sawires

Example:

We wish to select appropriate values for the coupling capacitors C_{C1} and C_{C2} and the bypass capacitor C_S for a CS amplifier for which $R_G = 4.7$ MΩ, $R_D = R_L = 15$ kΩ, $R_{sig} = 100$ kΩ, and $g_m = 1$ mA/V. It is required to have f_L at 100 Hz and that the nearest break frequency be at least a decade lower.

We select C_S so that: $f_{P2} = \frac{1}{2\pi(C_S/g_m)} = f_L$=100Hz ⇒ $C_S = 1.6\mu F$

For $f_{P1} = f_{P3} = 10$ Hz, (decade lower), we obtain:

$$10 = \frac{1}{2\pi C_{C1}(0.1 + 4.7)x10^6} \Rightarrow C_{C1} = 3.3nF$$

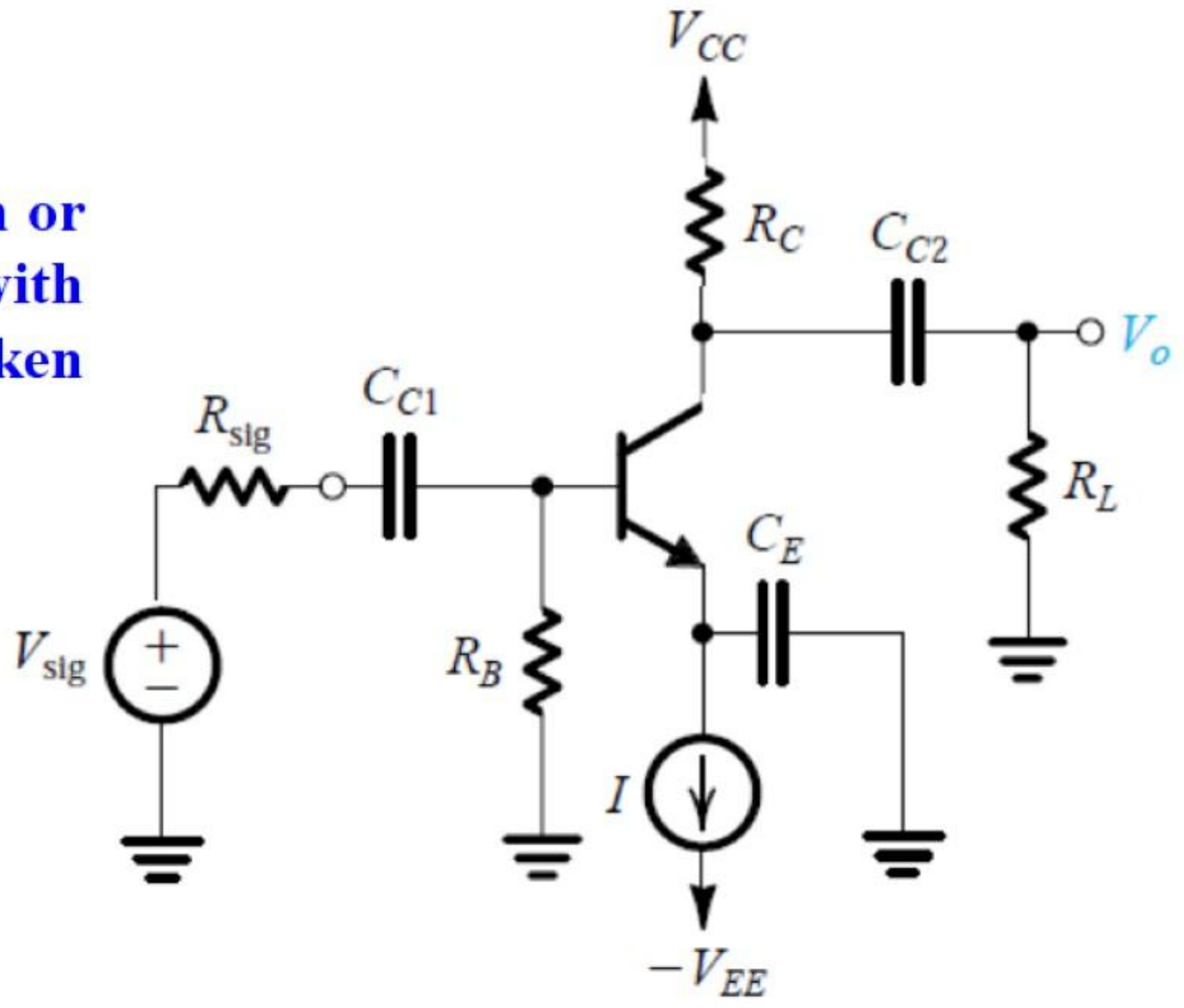
$$10 = \frac{1}{2\pi C_{C2}(15 + 15)x10^3} \Rightarrow C_{C2} = 0.53\mu F$$

E. Sawires

Assignment

Determine the amplifier gain or transfer function Vo/Vsig with these three capacitances taken into account.

Find A_M, f_{P1}, f_{P2}, f_{P3}, and f_L.



E. Sawires

Sol

$$w_{p1} = \frac{1}{C_{C1}\left[(R_B||r_\pi) + R_{sig}\right]}$$

$$w_{p2} = C_E\left(r_e + \frac{R_B||R_{sig}}{\beta + 1}\right)$$

$$w_{p3} = \frac{1}{C_{C2}(R_C + R_L)}$$

$$\frac{V_o}{V_{sig}} = A_M\left(\frac{S}{S + w_{p1}}\right)\left(\frac{S}{S + w_{p2}}\right)\left(\frac{S}{S + w_{p2}}\right)$$

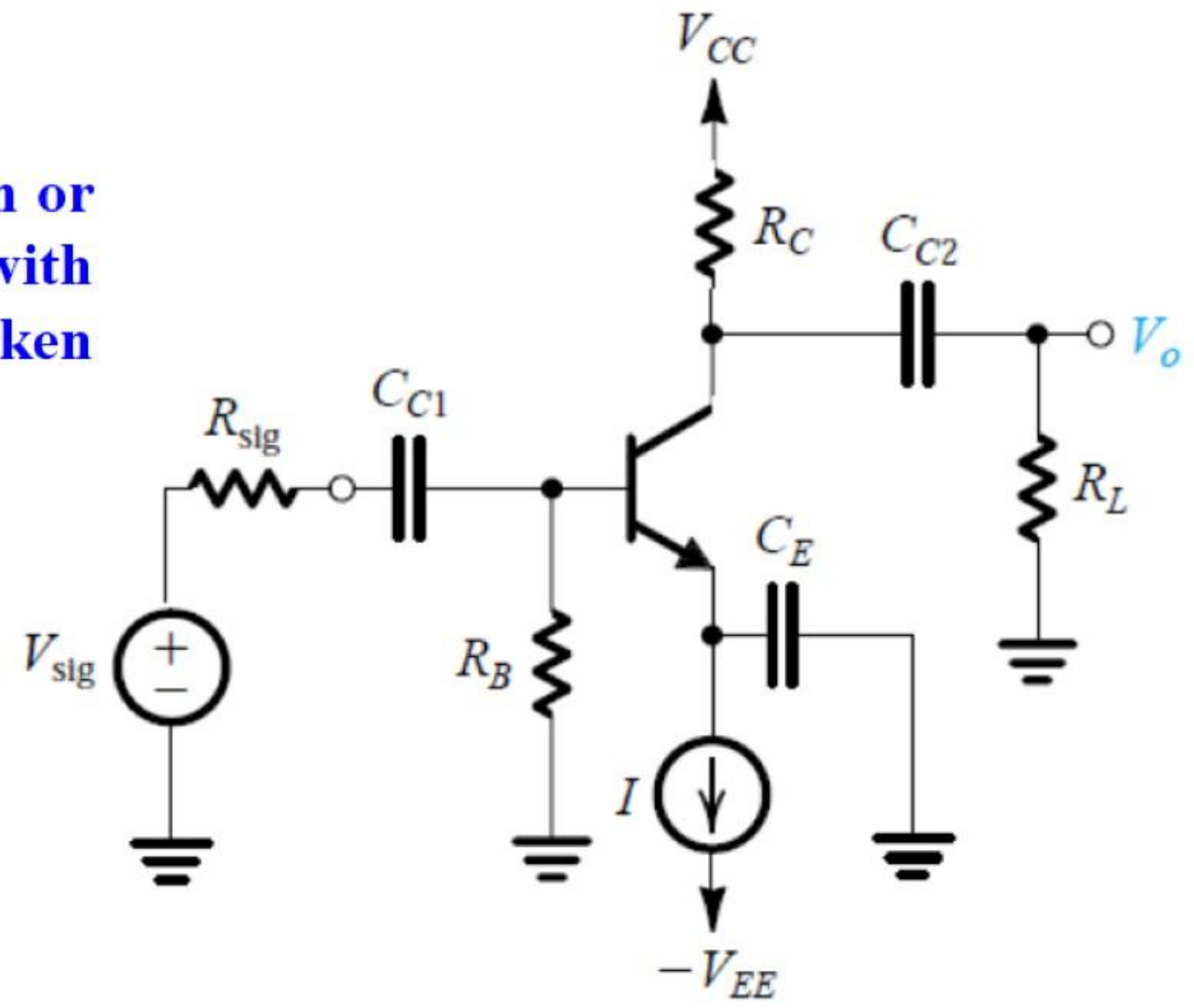
E. Sawires

Low Frequency Response of Common Emitter Amplifier

Low Frequency Response of Common Emitter Amplifier

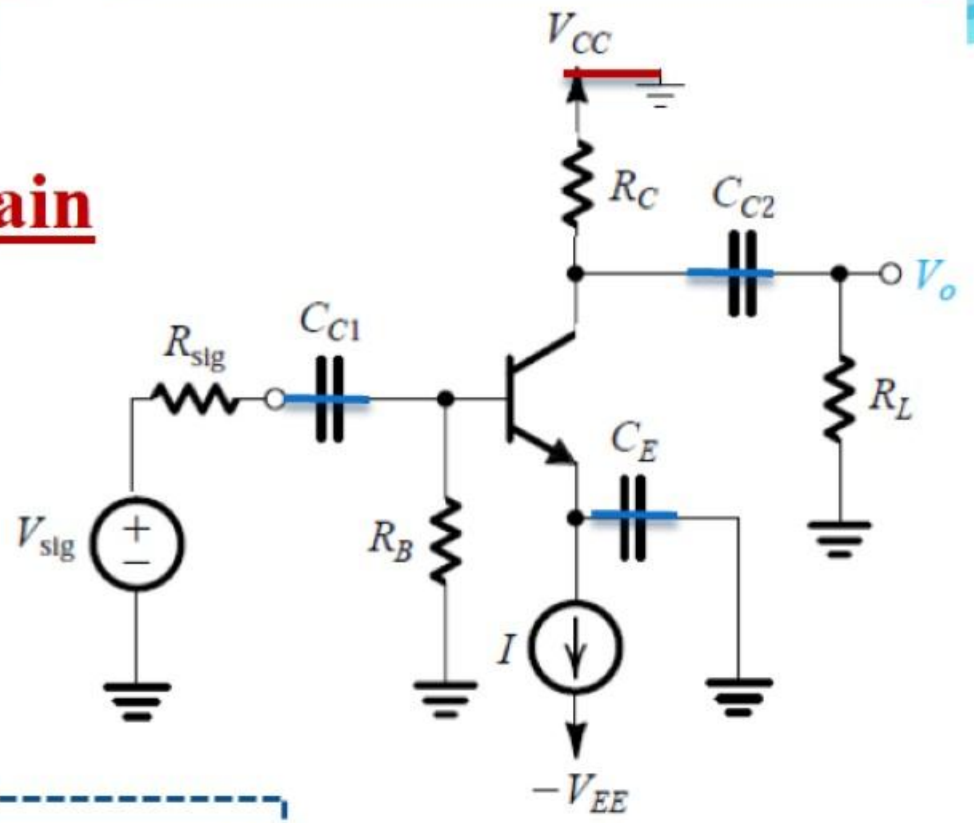
Determine the amplifier gain or transfer function Vo/Vsig with these three capacitances taken into account.

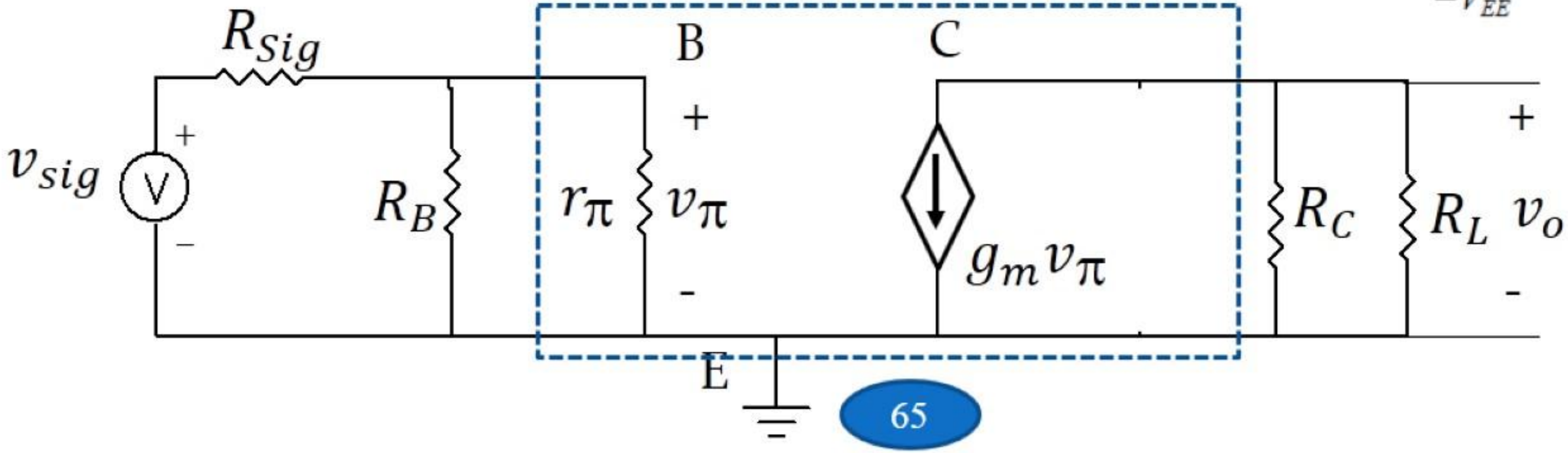
Or: Find A_M, f_{P1}, f_{P2}, f_{P3}, and f_L.



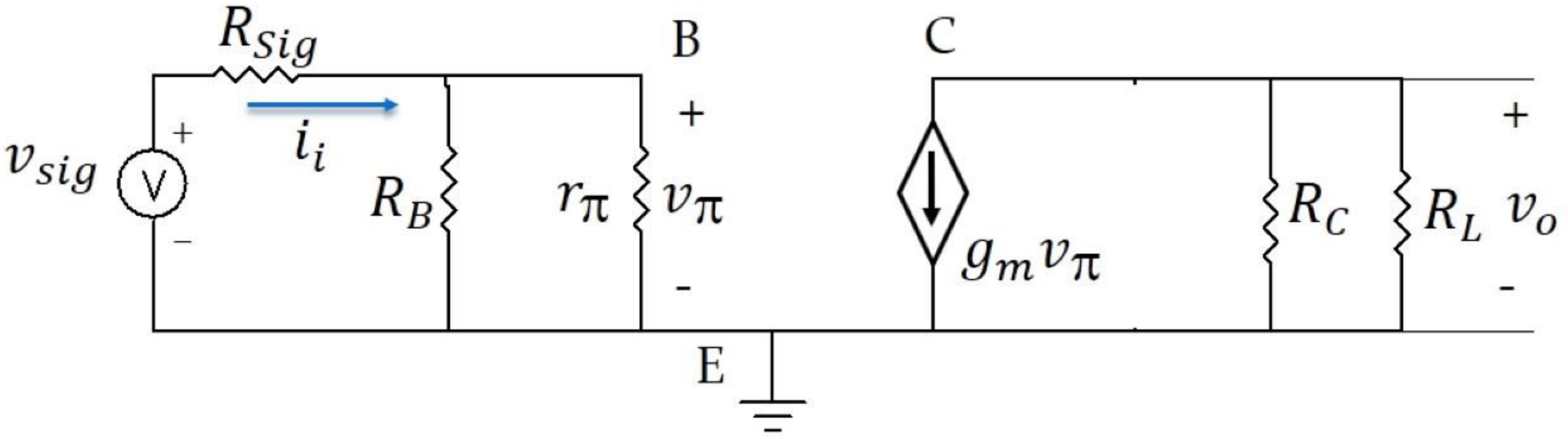
E. Sawires

(a) Calculate the mid-band voltage gain





E. Sawires

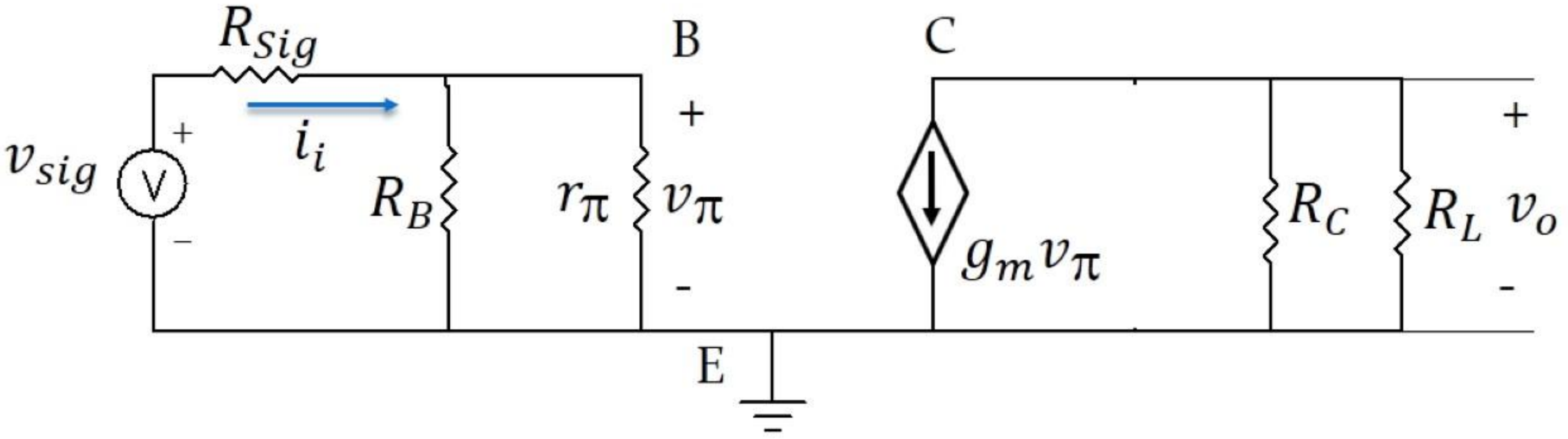


$$v_o = -g_m v_\pi (R_L // R_C)$$

$$v_\pi = i_i (R_B // r_\pi)$$

$$i_i = \frac{v_{sig}}{R_{sig} + (R_B // r_\pi)}$$

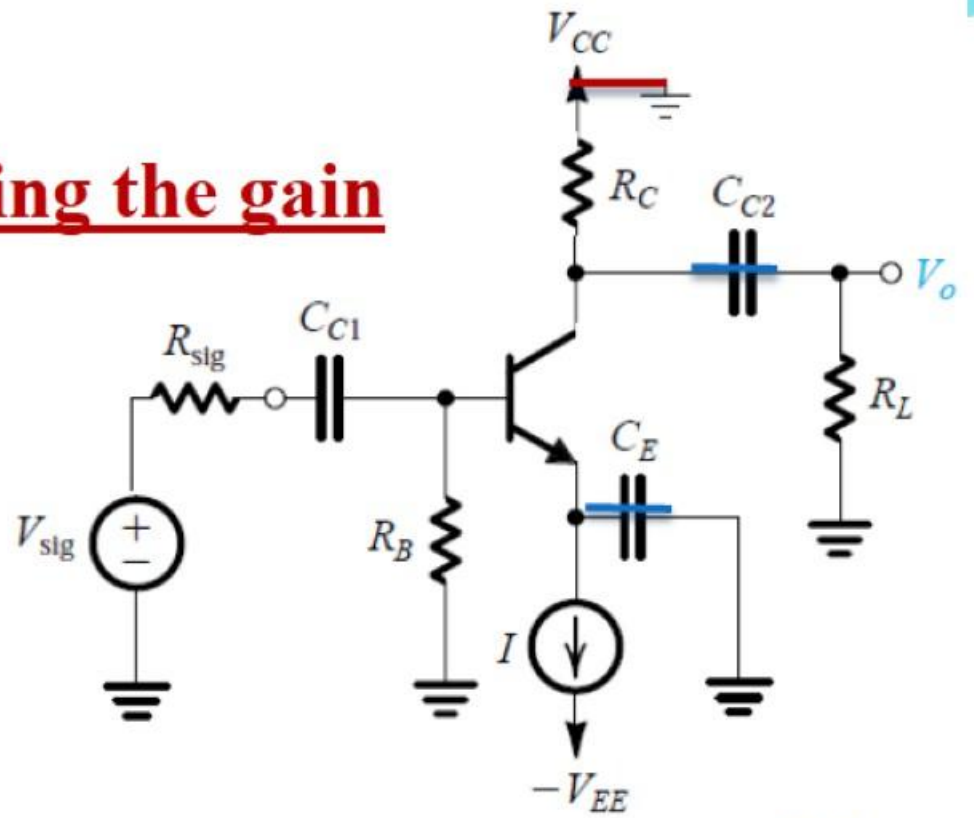
E. Sawires

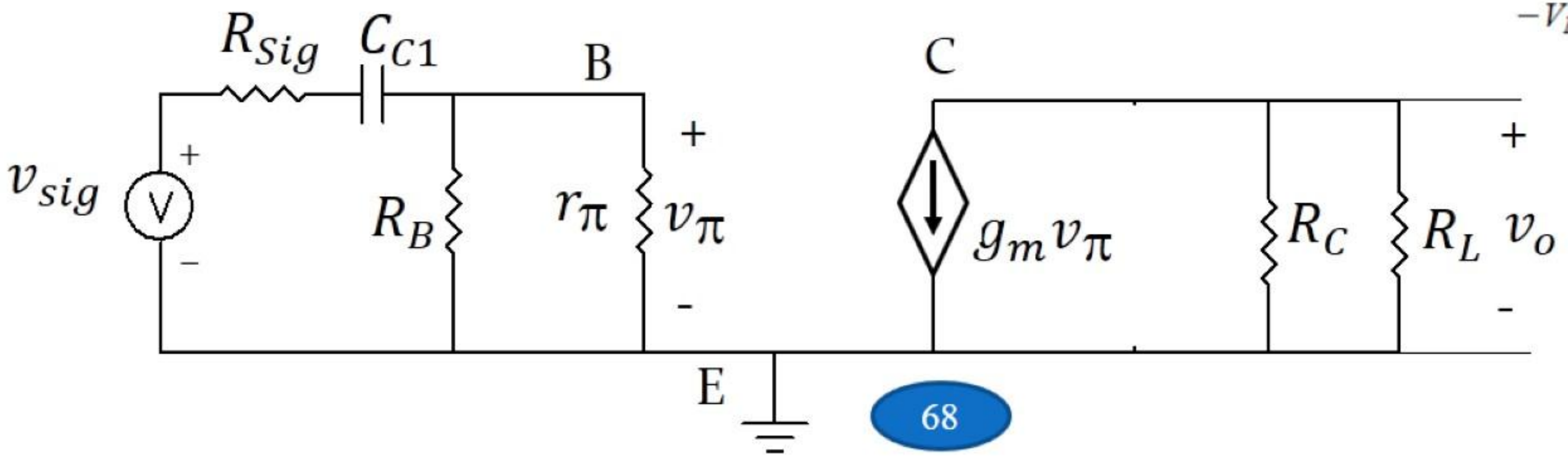


$$\frac{v_o}{v_{sig}} = A_{MF} = \frac{-g_m\,(R_L//R_C)(R_B//r_\pi)}{[R_{sig}+(R_B//r_\pi)]}$$

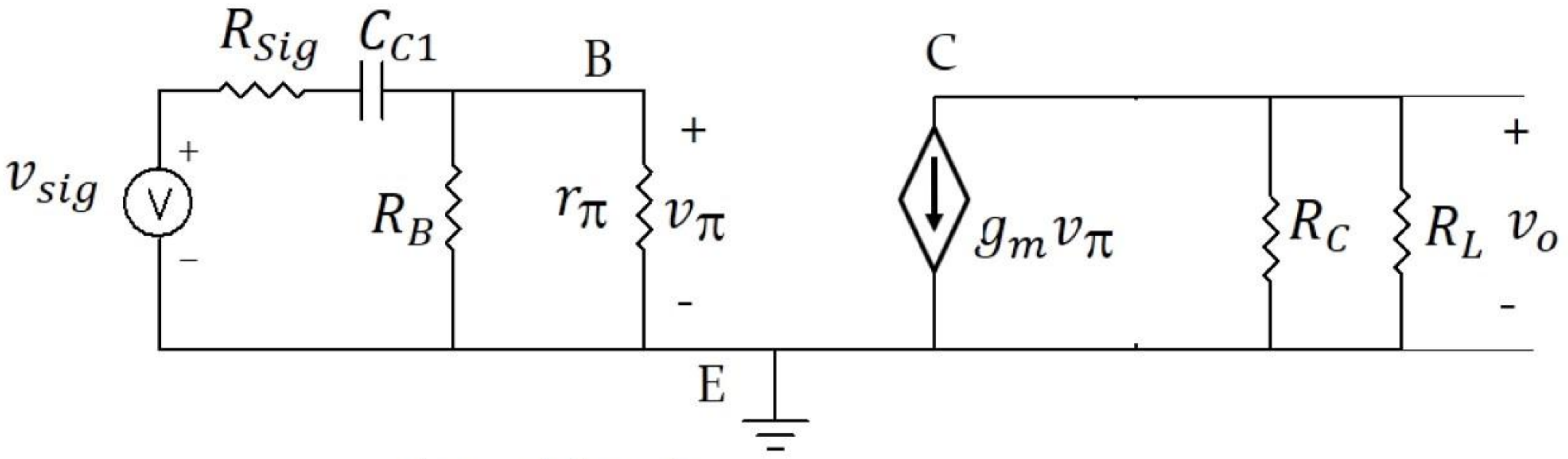
E. Sawires

Calculate the pole due to C_{C1} by deriving the gain (C_{C2}, C_E are neglected)





E. Sawires

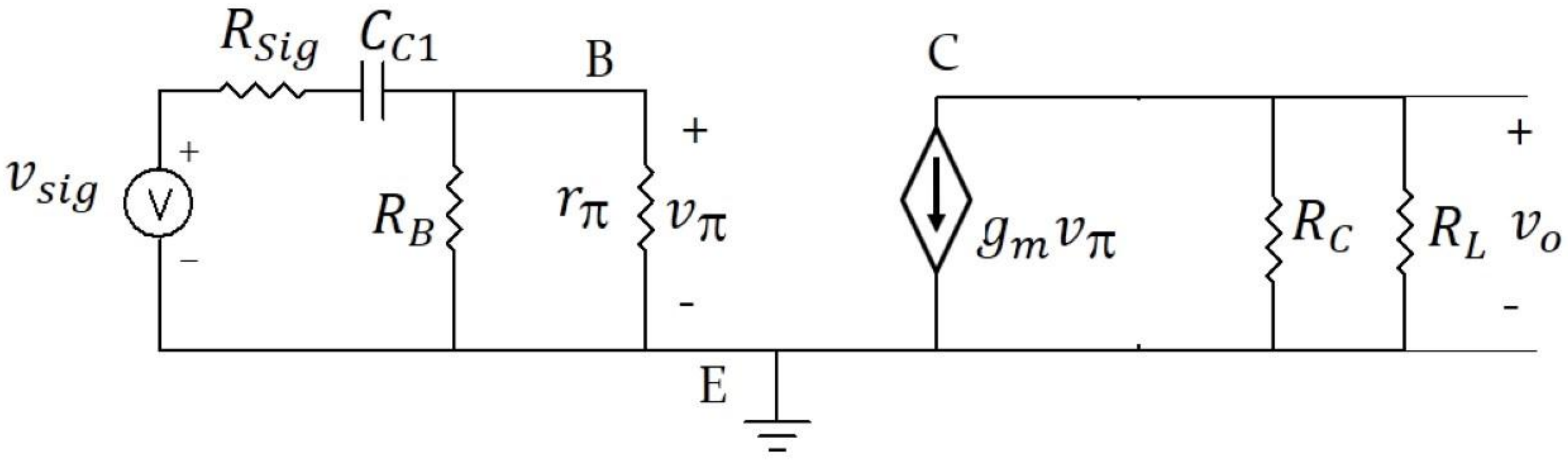


$$v_o = -g_m v_\pi (R_C // R_L)$$

$$v_\pi = i_i (R_B // r_\pi)$$

$$i_i = \frac{v_{sig}}{R_{sig} + \frac{1}{SC_{C1}} + (R_B // r_\pi)}$$

E. Sawires



$$\frac{v_o}{v_{sig}} = \frac{-g_m\,(R_L//R_C)(R_B//r_\pi)}{\left[R_{sig}+\frac{1}{S\,C_{C1}}+(R_B//r_\pi)\right]}$$

E. Sawires

$$\frac{v_o}{v_{sig}} = \frac{-g_m\,(R_L//R_C)(R_B//r_\pi)}{[R_{sig}+\frac{1}{S\,C_{C1}}+(R_B//r_\pi)]}$$

Note that: The gain of the amplifier in the low frequency can be written as:

$$A_{LF,C1}(\mathrm{S}) = \frac{A_{MF}}{1+\frac{\omega_{P1}}{S}}$$

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_C//R_L)R_B//r_\pi}{[R_{sig}+R_B//r_\pi]\left(1+\frac{1}{sC_{C1}[R_{sig}+R_B//r_\pi]}\right)}$$

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_C//R_L)R_B//r_\pi}{[R_{sig}+R_B//r_\pi]}\,\frac{1}{\left(1+\frac{1}{sC_{C1}[R_{sig}+R_B//r_\pi]}\right)}$$

$$A_{LF,C1}(S) = \frac{A_{MF}}{1+\frac{\omega_{P1}}{S}}$$

Therefore, the break frequency due to C_{C1} is given by:

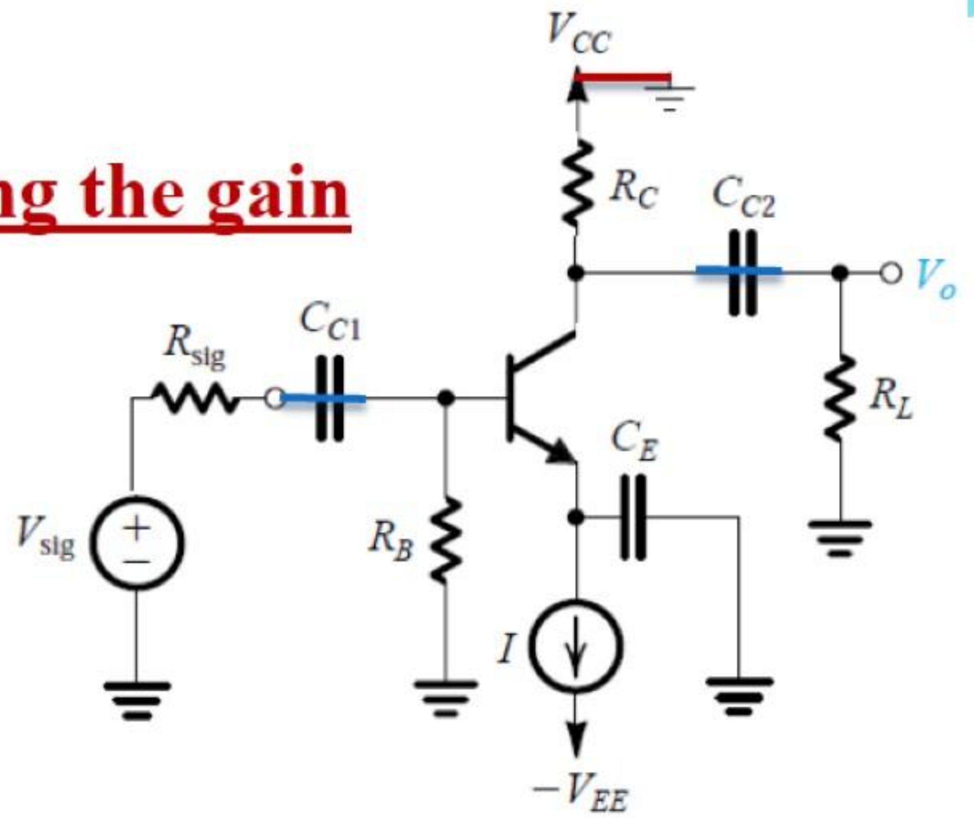
$$\omega_{P1} = 2\pi f_{P1} = \frac{1}{C_{C1}[R_{sig}+(R_B||r_\pi)]} = \frac{1}{\tau}$$

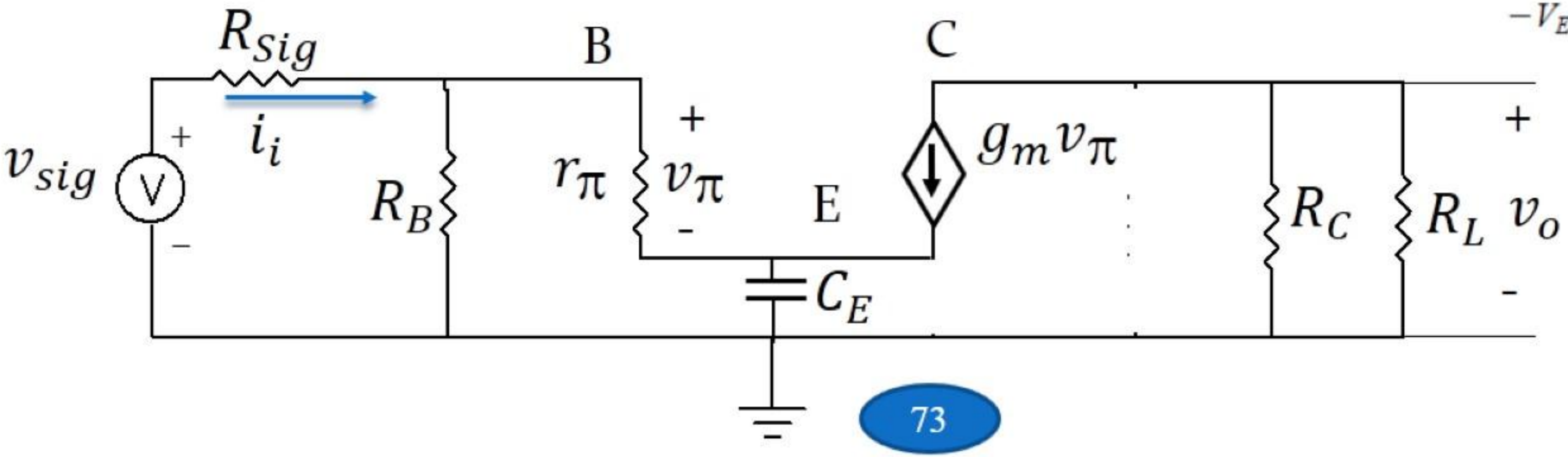
$$\boxed{\tau = C_{C1}R_{eq}}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}[R_{sig}+(R_B||r_\pi)]} \longrightarrow \boxed{1}$$

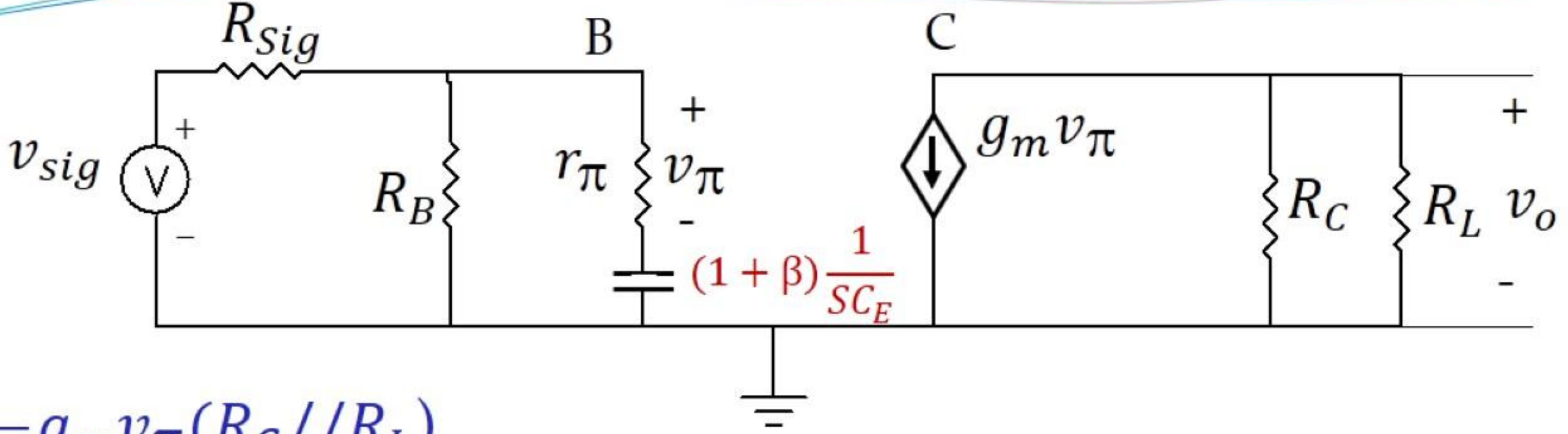
E. Sawires

Calculate the pole due to C_E by deriving the gain (C_{C1}, C_{C2} are neglected)





E. Sawires



$$v_o = -g_m v_\pi (R_C // R_L)$$

$$v_\pi = \frac{v_{sig} R_B}{R_{sig} + R_B} \quad \frac{r_\pi}{(R_{Sig}//R_B) + r_\pi + (1+\beta)\frac{1}{SC_E}}$$

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_C//R_L) R_B r_\pi}{(R_{sig} + R_B)[(R_{Sig} \| R_B) + r_\pi]} \quad \frac{1}{[1 + \frac{(1+\beta)}{SC_E[(R_{Sig} \| R_B) + r_\pi]}]}$$

E. Sawires

$$\frac{v_{out}}{v_{sig}} = \frac{-g_m\,(R_C//R_L)R_B r_\pi}{(R_{sig}+R_B)[(R_{Sig}\|R_B)+r_\pi]}\ \frac{1}{[1+\dfrac{(1+\beta)}{SC_E[(R_{Sig}\|R_B)+r_\pi]}]}$$

$$A_{LF,CE}(\mathrm{S}) = \frac{A_{MF}}{1+\dfrac{\omega_{P2}}{S}}$$

Note

$$\frac{g_m\,(R_C//R_L)R_B//r_\pi}{[R_{sig}+R_B//r_\pi]} = \frac{g_m\,(R_C//R_L)R_B r_\pi}{(R_{sig}+R_B)[(R_{Sig}\|R_B)+r_\pi]}$$

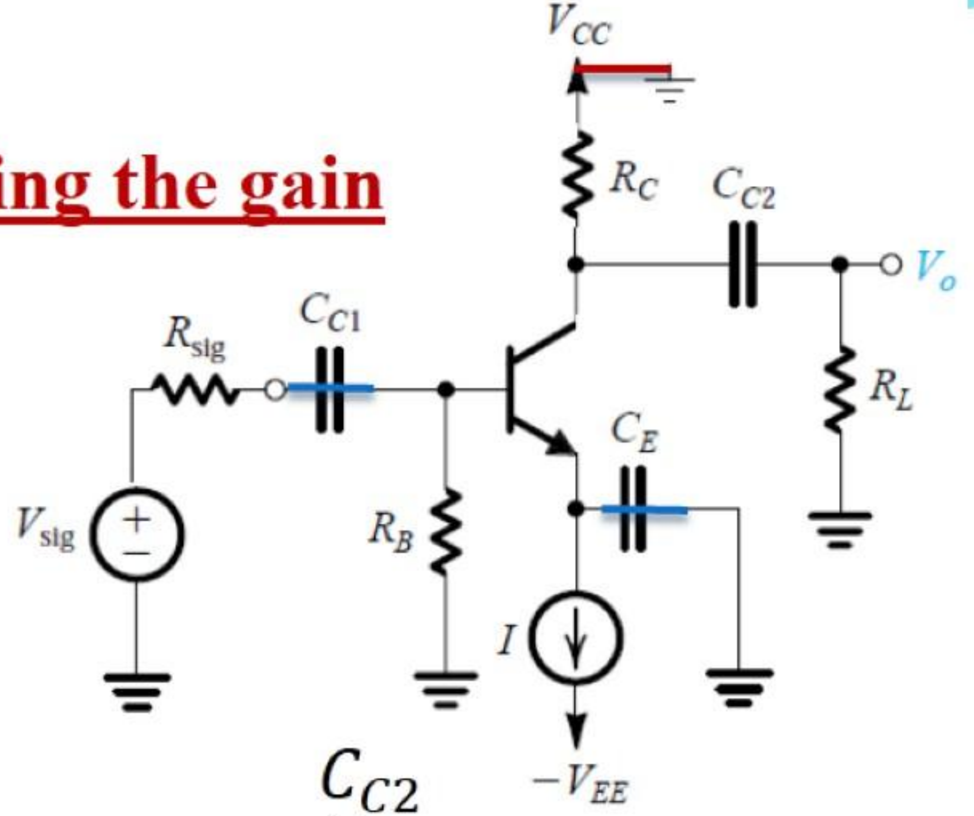
E. Sawires

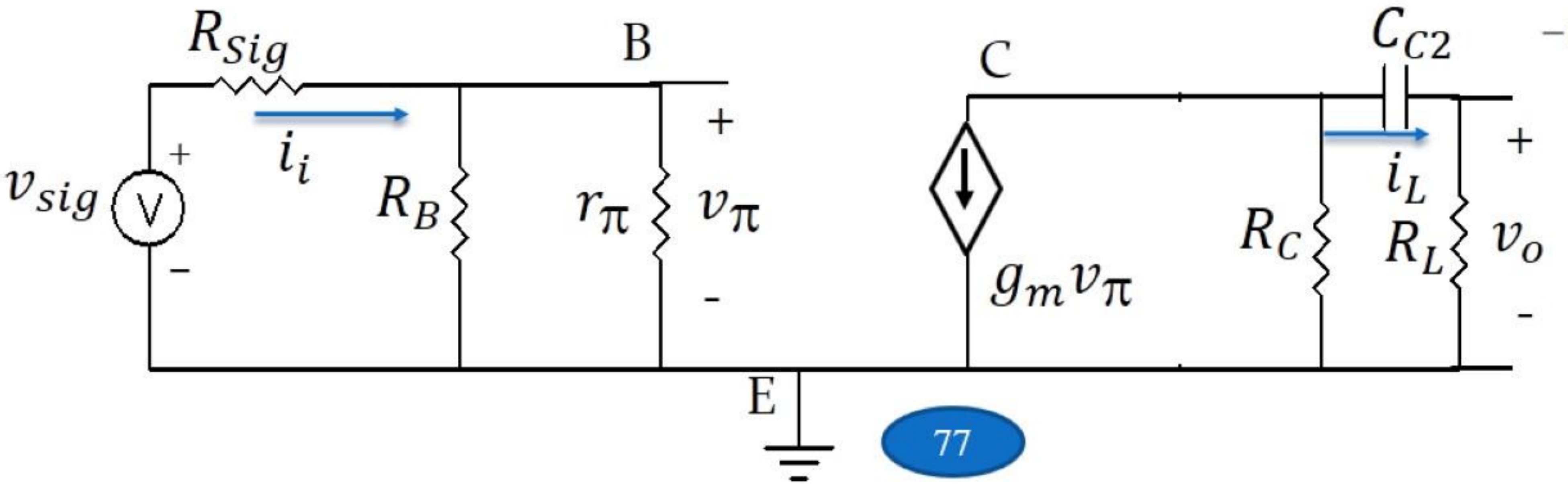
Therefore, the break frequency due to C_E is given by:

$$w_{p2} = 2\pi f_{P2} = \frac{(1+\beta)}{C_E[(R_{Sig}||R_B+r\pi]} = \frac{1}{C_E\left(r_e+\frac{R_B||R_{sig}}{\beta+1}\right)} = \frac{1}{\tau}$$

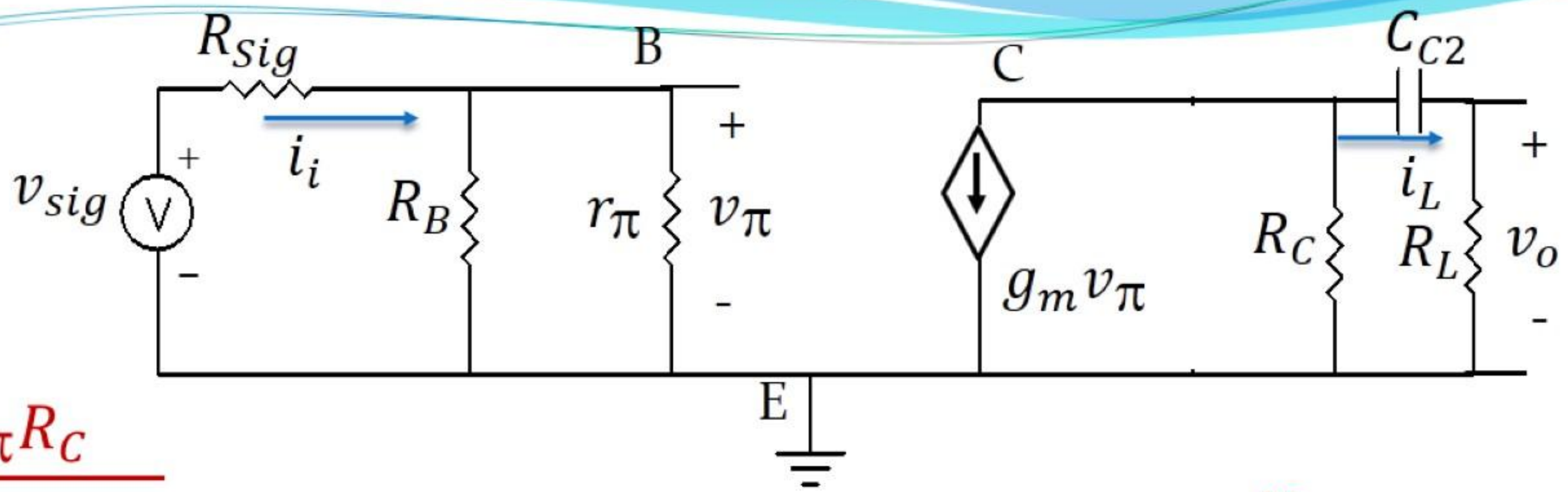
E. Sawires

Calculate the pole due to C_{C2} by deriving the gain (C_{C1}, C_{CE} are neglected)





E. Sawires



$$v_o = i_L R_L$$

$$i_L = \frac{-g_m v_\pi R_C}{R_C + \frac{1}{SC_{C2}} + R_L}$$

$$v_\pi = i_i (R_B \| r_\pi)$$

$$i_i = \frac{V_{sig}}{R_{sig} + (R_B \| r_\pi)}$$

$$\frac{v_o}{v_{sig}} = \frac{-g_m R_C R_L (R_B \| r_\pi)}{\left[R_C + \frac{1}{SC_{C2}} + R_L\right]\left[R_{sig} + (R_B \| r_\pi)\right]}$$

E. Sawires

$$\frac{v_o}{v_{sig}} = \frac{-g_m R_C R_L (R_B || r_\pi)}{(R_C + R_L)\left[R_{sig} + (R_B || r_\pi)\right]\left[1 + \frac{1}{SC_{C2}(R_C + R_L)}\right]}$$

$$\frac{v_o}{v_{sig}} = \frac{-g_m (R_C // R_L)(R_B || r_\pi)}{\left[R_{sig} + (R_B || r_\pi)\right]\left[1 + \frac{1}{SC_{C2}(R_C + R_L)}\right]}$$

$$A_{LF,C2}(\mathrm{S}) = \frac{A_{MF}}{1 + \frac{\omega_{P3}}{S}}$$

Therefore, the break frequency due to C_{C2} is given by:

$$\boxed{\tau = C_{C2}(R_C + R_L)}$$

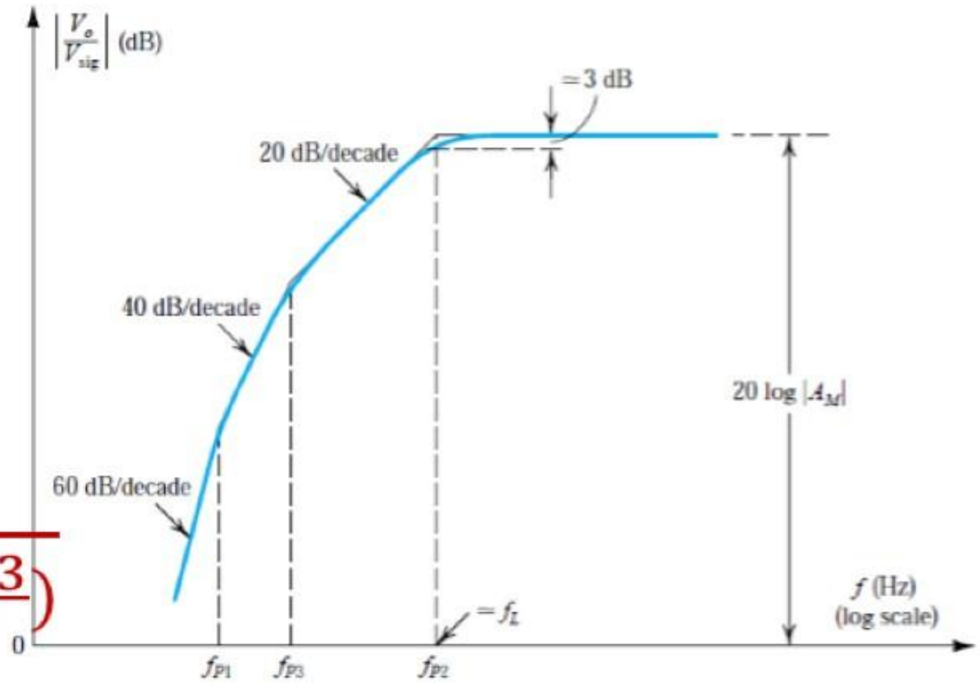
$$f_{P3} = \frac{1}{2\pi C_{C2}\,(R_C + R_L)} \longrightarrow \boxed{3}$$

E. Sawires

(b) Calculate the lower 3-dB frequency

$$f_L = \sqrt{f_{p1}^2 + f_{p2}^2 + f_{p3}^2}$$

$$\frac{V_{out}}{V_{sig}} = A_{LF}(\mathrm{S}) = \frac{A_{MF}}{(1+\frac{\omega_{P1}}{S})(1+\frac{\omega_{P2}}{S})(1+\frac{\omega_{P3}}{S})}$$



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$$w_{p1} = \frac{1}{C_{C1}\left[(R_B||r_\pi) + R_{sig}\right]} \qquad w_{p2} = \frac{1}{C_E\left(r_e + \frac{R_B||R_{sig}}{\beta + 1}\right)}$$

$$w_{p3} = \frac{1}{C_{C2}(R_C + R_L)}$$

$$\frac{V_o}{V_{sig}} = A_M\left(\frac{S}{S + w_{p1}}\right)\left(\frac{S}{S + w_{p2}}\right)\left(\frac{S}{S + w_{p2}}\right)$$

The important point to note here is that the 3-dB frequency f_L *is determined by the highest of the three pole frequencies*. This is usually the pole caused by the bypass capacitor C_E, simply because the resistance that it sees is usually quite small. Thus, even if one uses a large value for C_E, f_{P2} is usually the highest of the three pole frequencies.

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High Frequency Response

Introduction

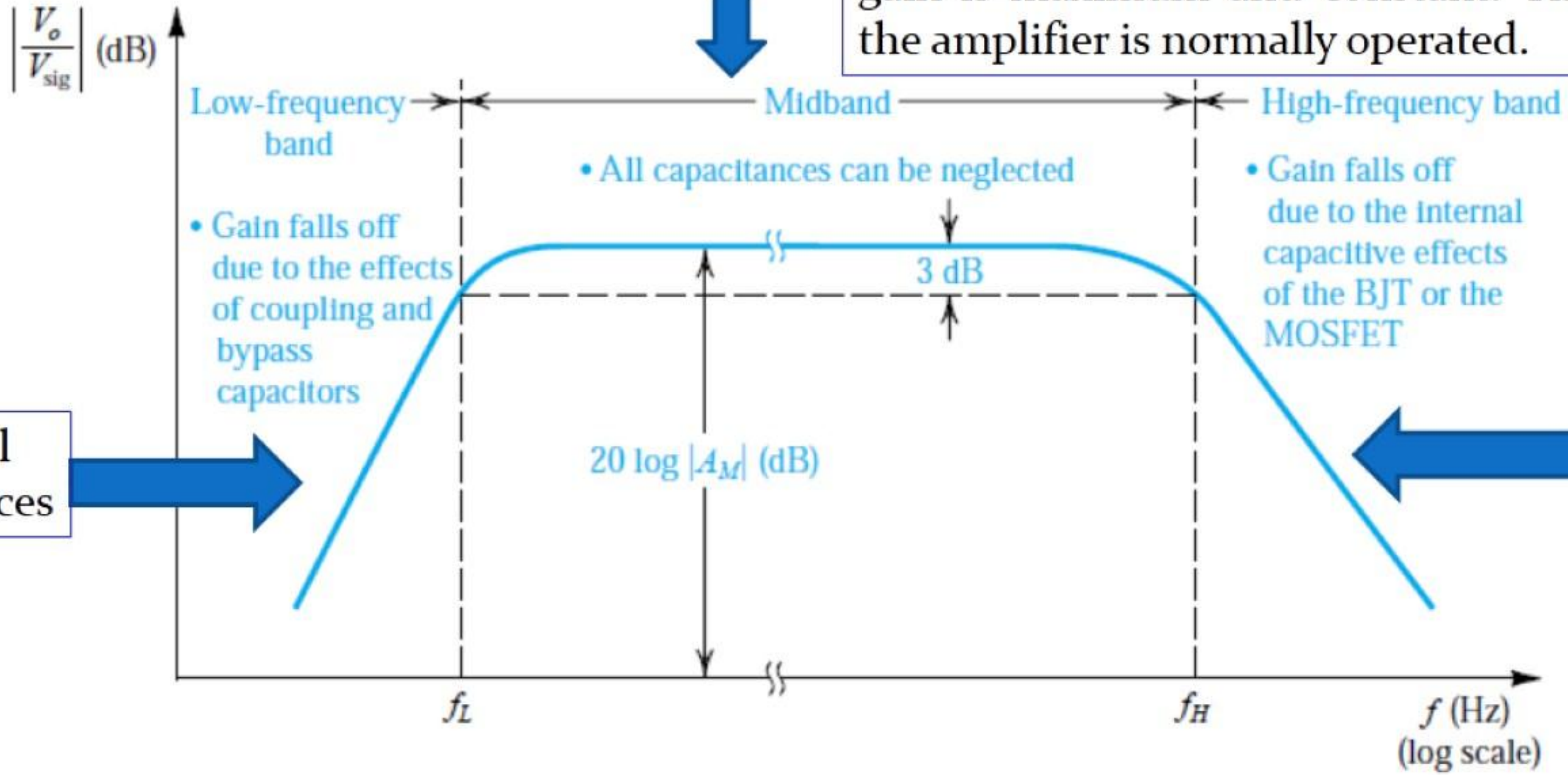
Objective

➢ We want to study the effect that these capacitances have on the amplifier's voltage gain

In the middle band of frequencies (MF), the voltage gain is maximum and constant. This MF is where the amplifier is normally operated.

External capacitances

Internal capacitances



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External Capacitances (µF)

Typically, the external capacitors are large (in order of Micro Farad)

$$\boxed{Z_C = \frac{1}{j2\pi f C_{external}}}$$

In LF	In MF	In HF
capacitors **are not** act like a (short circuit)	capacitors act like a (short circuit)	capacitors act like a (short circuit)

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Internal Capacitances (pF)

Typically, the internal capacitors are small (in order of Pico Farad)

$$\boxed{Z_C = \frac{1}{j2\pi f C_{internal}}}$$

In LF	In MF	In HF
capacitors act like a (open circuit)	capacitors act like a (open circuit)	capacitors **are not** act like a (open circuit)

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- **The physical structure and biasing of the transistors (MOS or BJT) result in Internal capacitances between its terminals.**
  - However, we have considered their impedances as **Open Circuit**

$$Z_{Cint} = \frac{1}{SC_{int}} = \frac{1}{j2\pi f C_{int}} \cong \infty$$

  - However, **this is not valid for High frequencies!**

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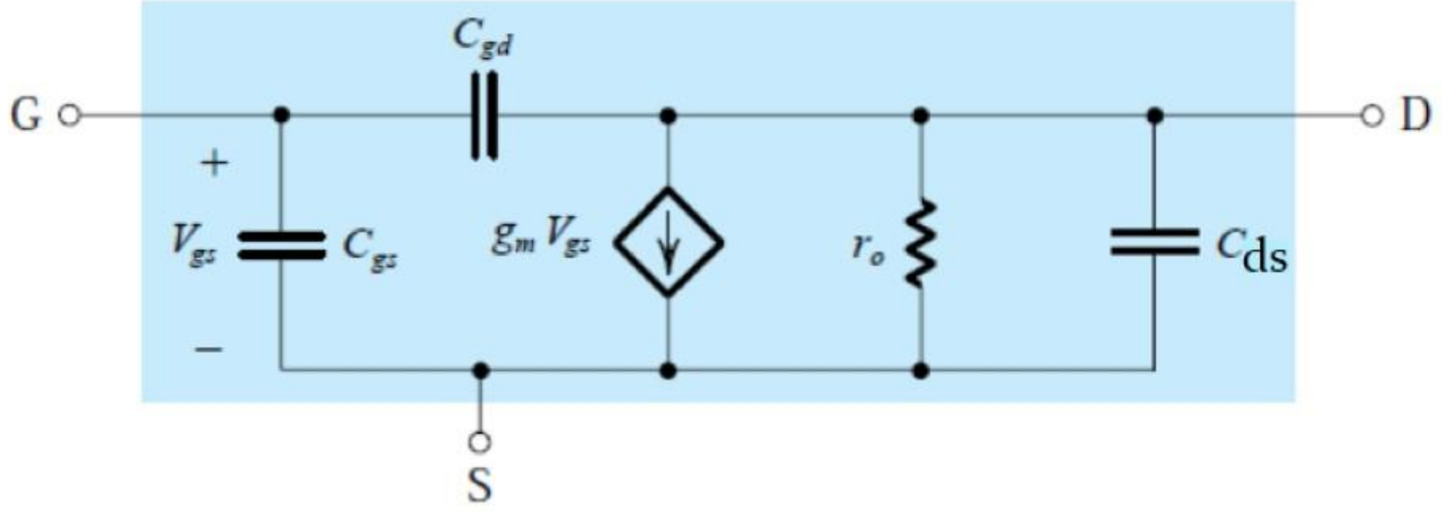
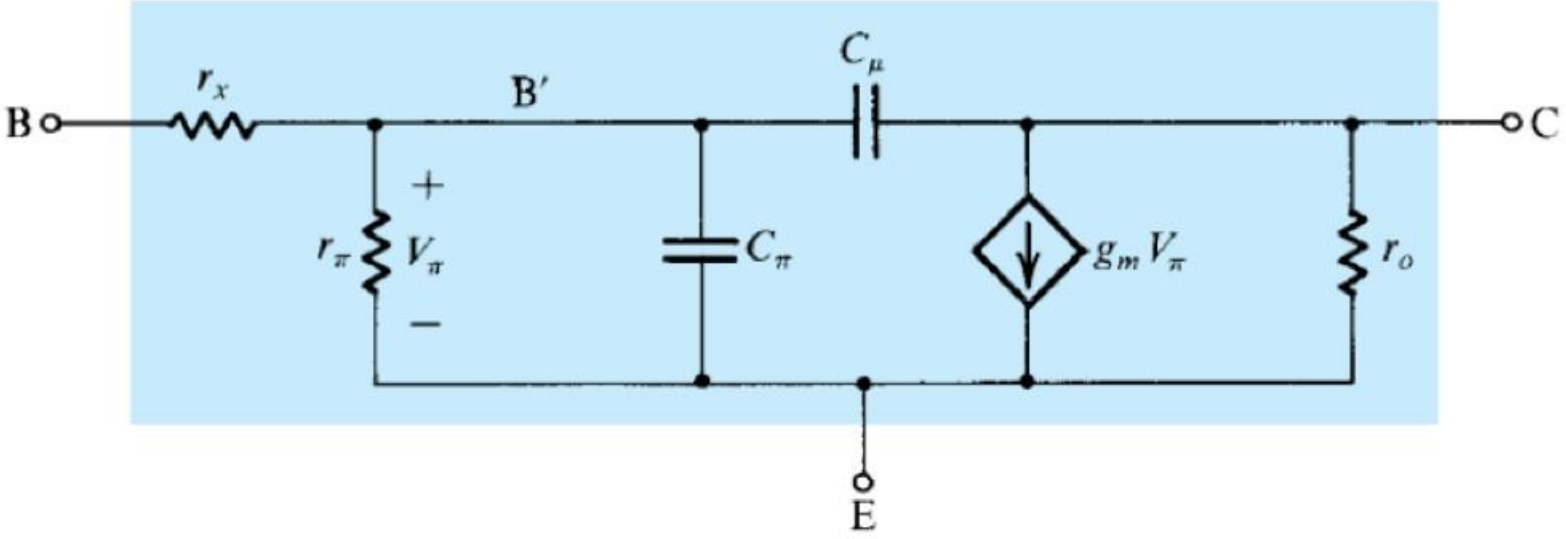
High-Frequency Model of the MOSFET and the BJT



The High-Frequency Model of MOSFET



The high Frequency Hybrid π Model Of The BJT

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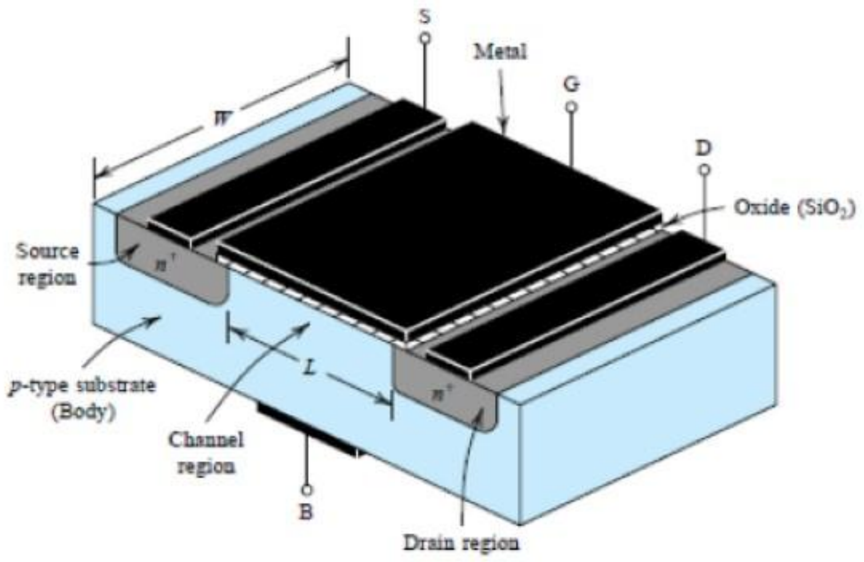
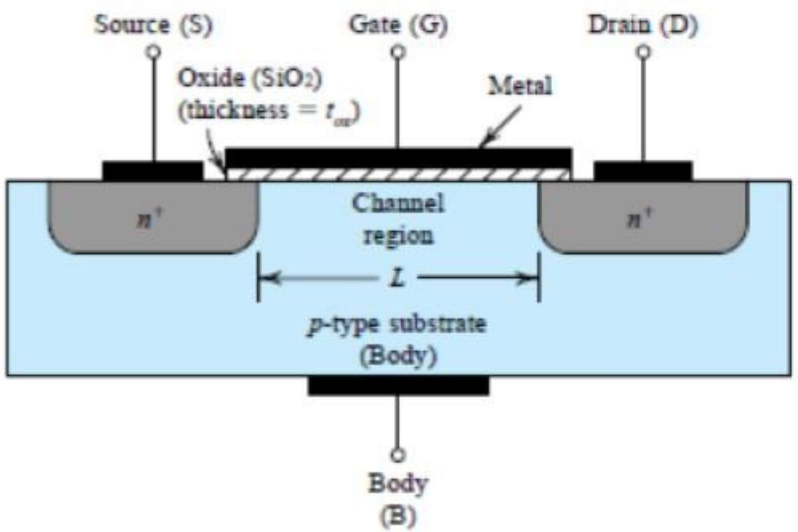
The High-Frequency Model of MOSFET

- There are basically two types of internal capacitance in the MOSFET.

1-The gate capacitive effect: The gate electrode (polysilicon) forms a parallel-plate capacitor with the channel, with the oxide layer serving as the capacitor dielectric.

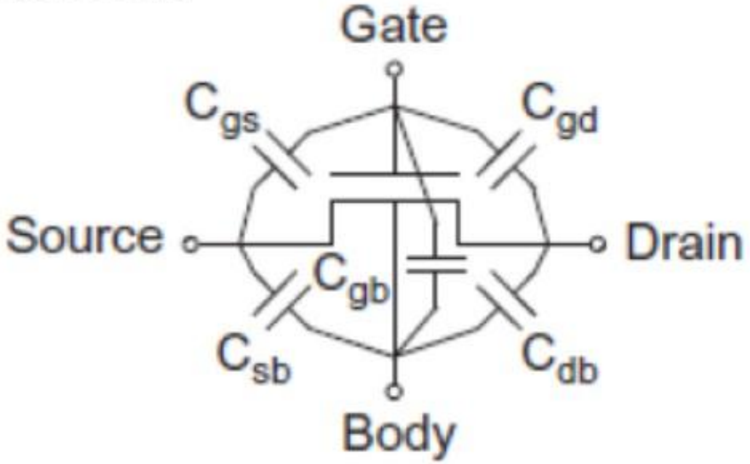
2-The source-substrate and drain-substrate depletion-layer capacitances: These are the capacitances of the reverse-biased *pn* junctions formed by the $n+$ source region (also called the source diffusion) and the *p*-type substrate and by the $n+$ drain region (the drain diffusion) and the substrate.





The High-Frequency Model of MOSFET

- The gate capacitive effect can be modeled by the three capacitances C_{gs}, C_{gd}, and C_{gb}.
- These two capacitive effects can be modeled by including capacitances in the MOSFET model between its four terminals, G, D, S, and B. There will be five capacitances in total: C_{gs}, C_{gd}, C_{gb}, C_{sb}, and C_{db}, where the subscripts indicate the location of the capacitances in the model.



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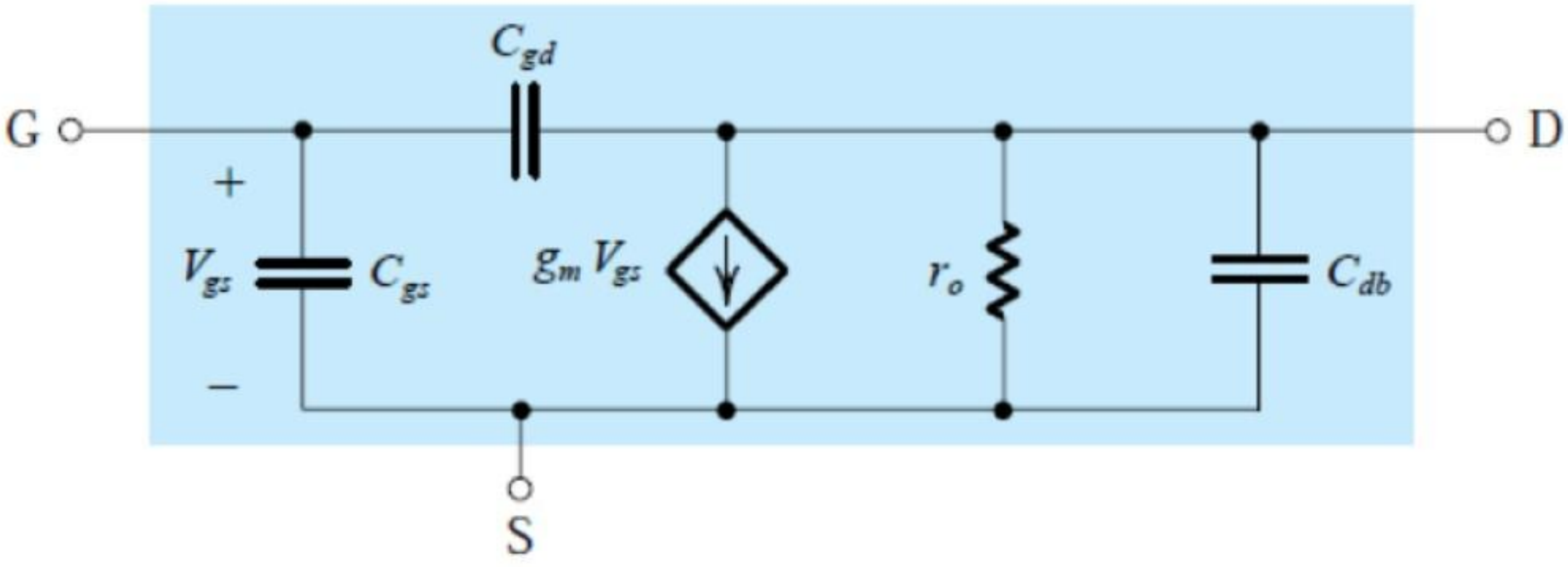
The High-Frequency Model of MOSFET

- When the source is connected to the body, the model simplifies considerably, as shown in Fig.



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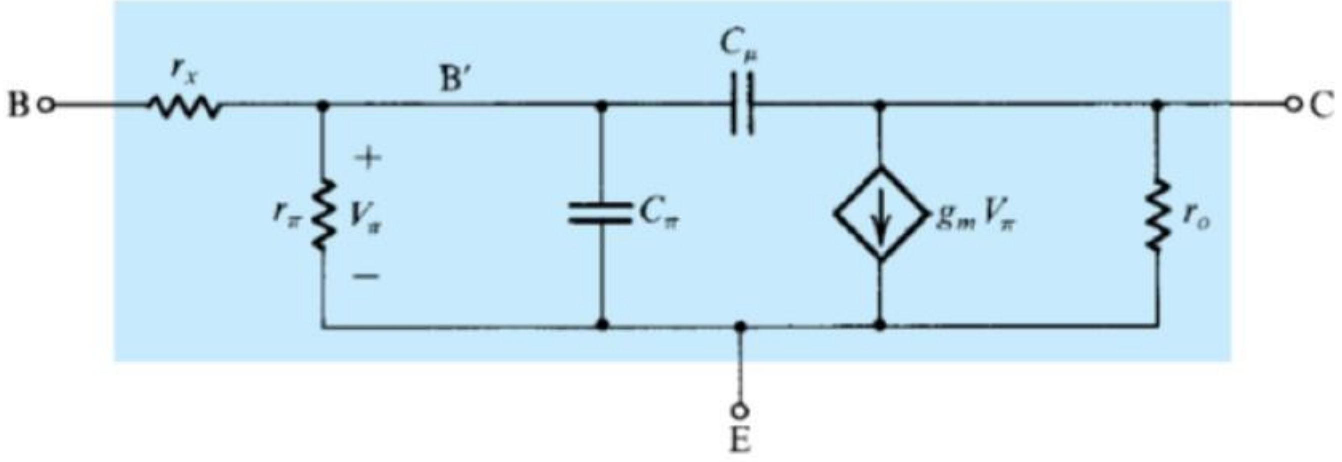
High Frequency Hybrid πModel Of The BJT

- **Specifically, there are two main capacitances added:**

1-**The collector base revers biased depletion capacitance** C_μ, Typically, C_μ is in the range of a fraction of a picofarad to a few picofarads. It depends on the junction voltage V_{CB} and always given in the transistor datasheet.



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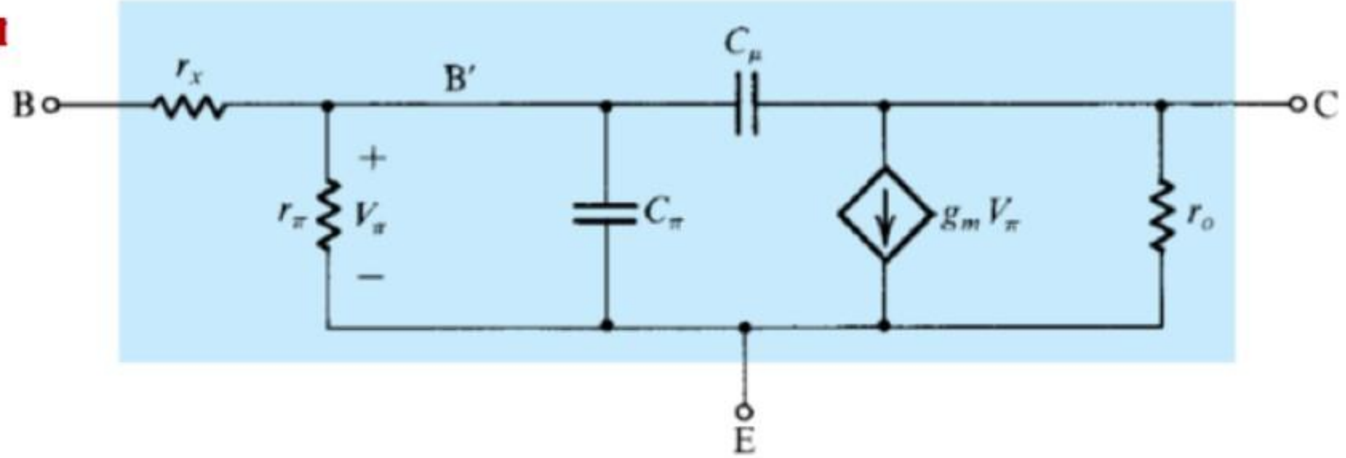
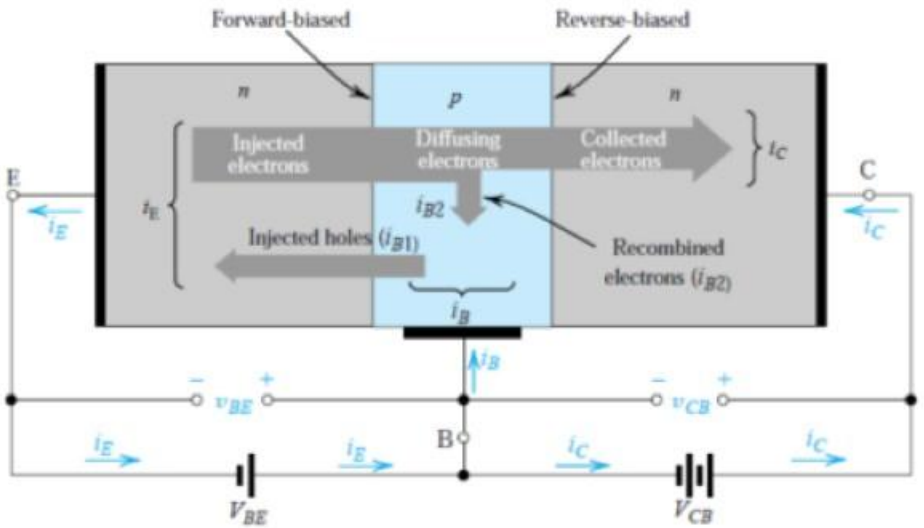
High Frequency Hybrid πModel Of The BJT

- **Specifically, there are two main capacitances added:**

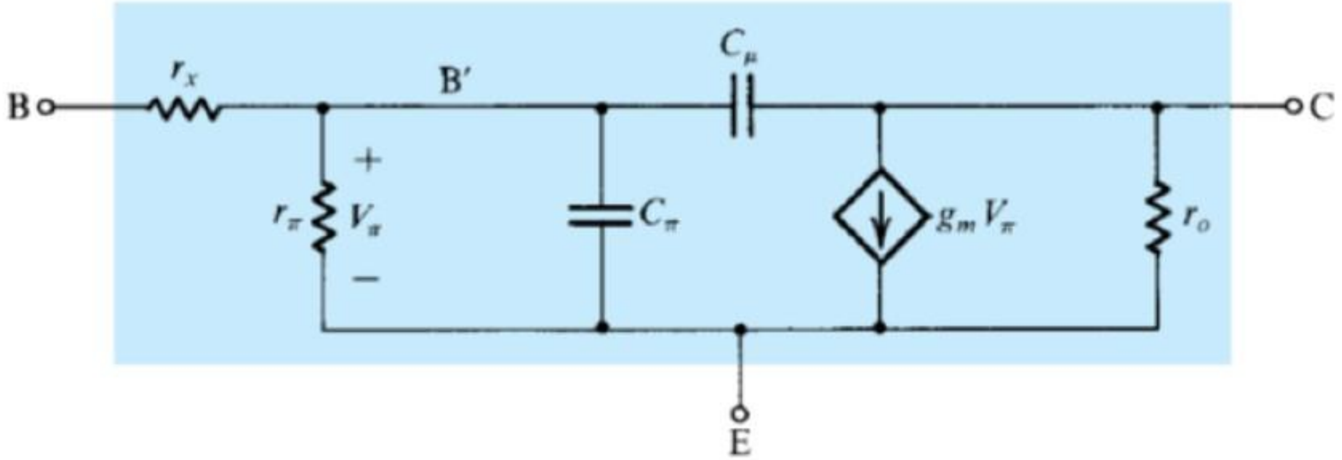
2- **The emitter–base capacitance** $C\pi =$ Cde+ Cje, it is in the range of a few picofarads to a few tens of picofarads.



- **The Base-Charging or Diffusion Capacitance** Cde, is the small signal diffusion capacitance, it is due to accumulation of the diffused electrons in the base region.
- **The Base–Emitter Junction Capacitance** Cje is the forward base depletion capacitance. It is given in the transistor datasheet.



High Frequency Hybrid πModel Of The BJT



3- **rx is the resistance of the silicon material of the base region**. rx is a few tens of ohms, and its value depends on the current level in a rather complicated manner. rx <<rπ , its effect is negligible at low frequencies. Its presence is felt, however, at high frequencies.

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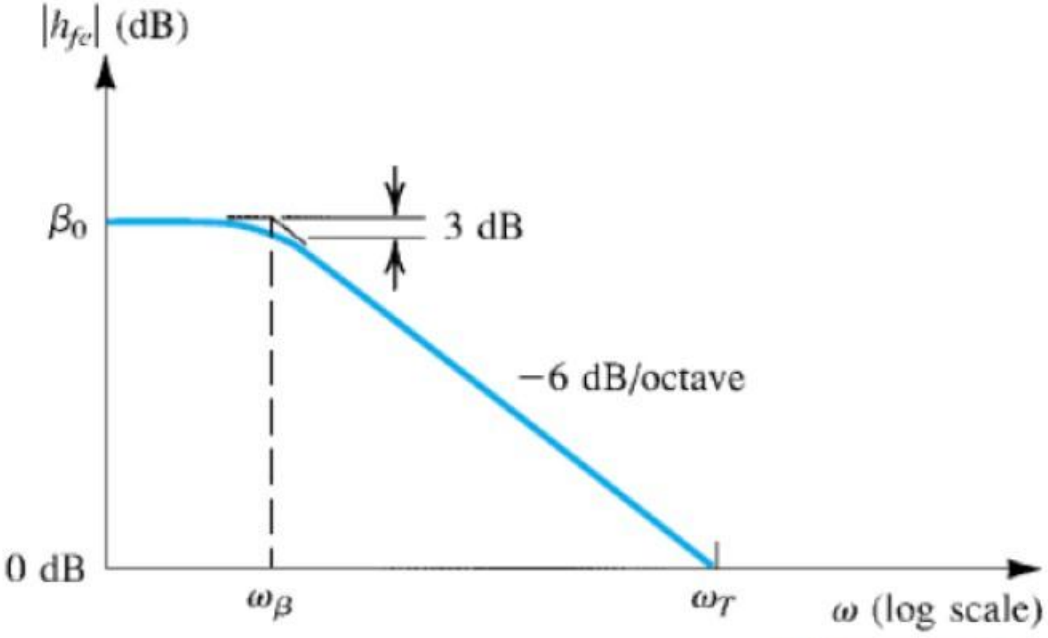
High Frequency Hybrid πModel Of The BJT

- The transistor data sheets do not usually specify the value of $C\pi$, rather, the behavior of β (or h_{fe}) versus frequency is normally given by:

$$h_{fe} = \frac{\beta_o}{1 + S(C_\pi + C_\mu)r_\pi}$$



- Where $\beta_o = g_m r_\pi$ is the low-frequency value of β.Thus h_{fe} has a single-pole (or STC) response with a 3-dB frequency at $\omega = \omega_\beta$,given by:

$$w_\beta = \frac{1}{(C_\pi + C_\mu)r_\pi}$$

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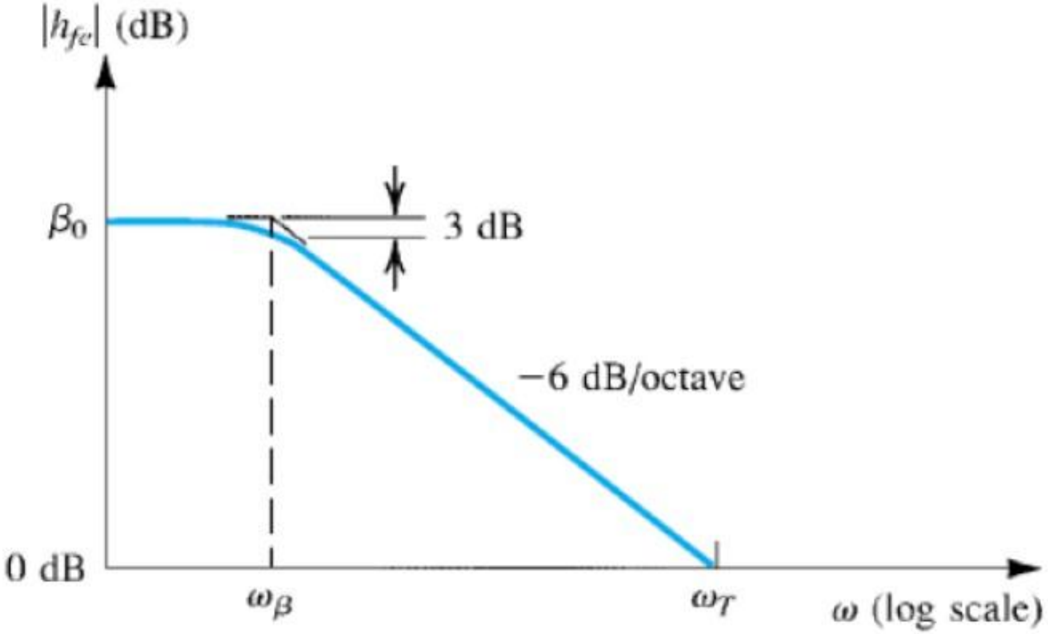
High Frequency Hybrid πModel Of The BJT

The **unity-gain bandwidth** ω_T, is given by

$$w_T = \beta_o w_\beta = \frac{g_m}{(C_\pi + C_\mu)}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$



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High Frequency Response

High Frequency Response

The gain in the high frequency : $A_{HF}(s)$; S = j ω

(a) Assuming the amplifier has one internal capacitor:

$$\boxed{A_{HF}(\mathrm{S}) = \frac{A_{MF}}{1+\frac{S}{\omega_{p1}}} = \frac{A_{MF}}{1+\frac{S}{\omega_{H}}}}$$

Where A_{MF} is the mid-band gain, and $\omega_H = 2\pi f_H = \omega_{P1}$ is the higher cut-off radian frequency or the higher 3-dB radian frequency.

The magnitude of the gain in dB is given by:

$$|A_{HF}(j\omega)|\mathrm{dB} = 20\log|A_{HF}(j\omega)| = 20\log\frac{A_{MF}}{\sqrt{1+\left(\frac{\omega}{\omega_H}\right)^2}}$$

at $\omega = \omega_H$

$|A_{HF}|\mathrm{dB} = 20\log A_{MF}$ -3 dB

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(b) Assuming the amplifier has two independent internal capacitors:

$$\boxed{A_{HF}(S) = \frac{A_{MF}}{(1+\frac{S}{\omega_{P1}})(1+\frac{S}{\omega_{P2}})}}$$

$$\omega_H = \frac{1}{\sqrt{\left(\frac{1}{\omega_{P1}}\right)^2 + \left(\frac{1}{\omega_{P2}}\right)^2}}$$

The higher 3-dB frequency is given by :

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2}}$$

(c) Assuming the amplifier has n independent internal capacitors:

$$A_{HF}(S) = \frac{A_{MF}}{\left(1+\frac{S}{\omega_{P1}}\right)\left(1+\frac{S}{\omega_{P2}}\right)\ldots\ldots\left(1+\frac{S}{\omega_{Pn}}\right)}$$

The higher 3-dB frequency is given by :

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P1}}\right)^2+\left(\frac{1}{f_{P2}}\right)^2+\ldots+\left(\frac{1}{f_{Pn}}\right)^2}}$$

If one of the poles is much smaller than the others, then it will be the dominant pole and other poles can be neglected.

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Calculating the Higher 3-dB frequency

Calculating f_H

- The higher cutoff 3-dB frequency is calculated by NOT considering the impedances of the **internal capacitances Open Circuit.**
- The internal capacitors poles can be calculated by deriving the voltage gain
- The external capacitors while calculating the higher 3-dB cutoff frequency are neglected (considered short circuit)

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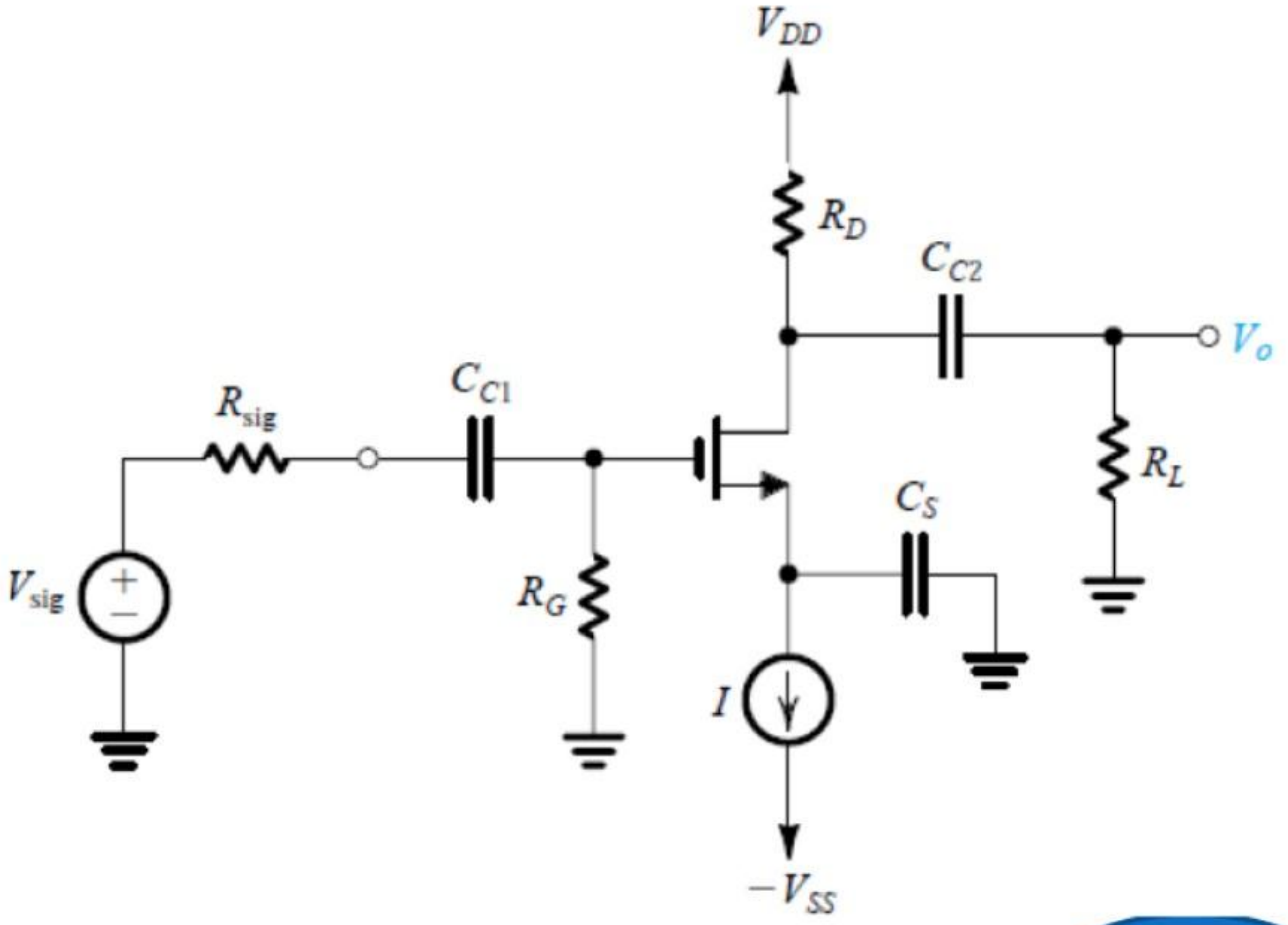
High Frequency Response of CS Amplifier

Example

For the shown amplifier : find the midband voltage gain and the higher 3-dB frequency, f_H.



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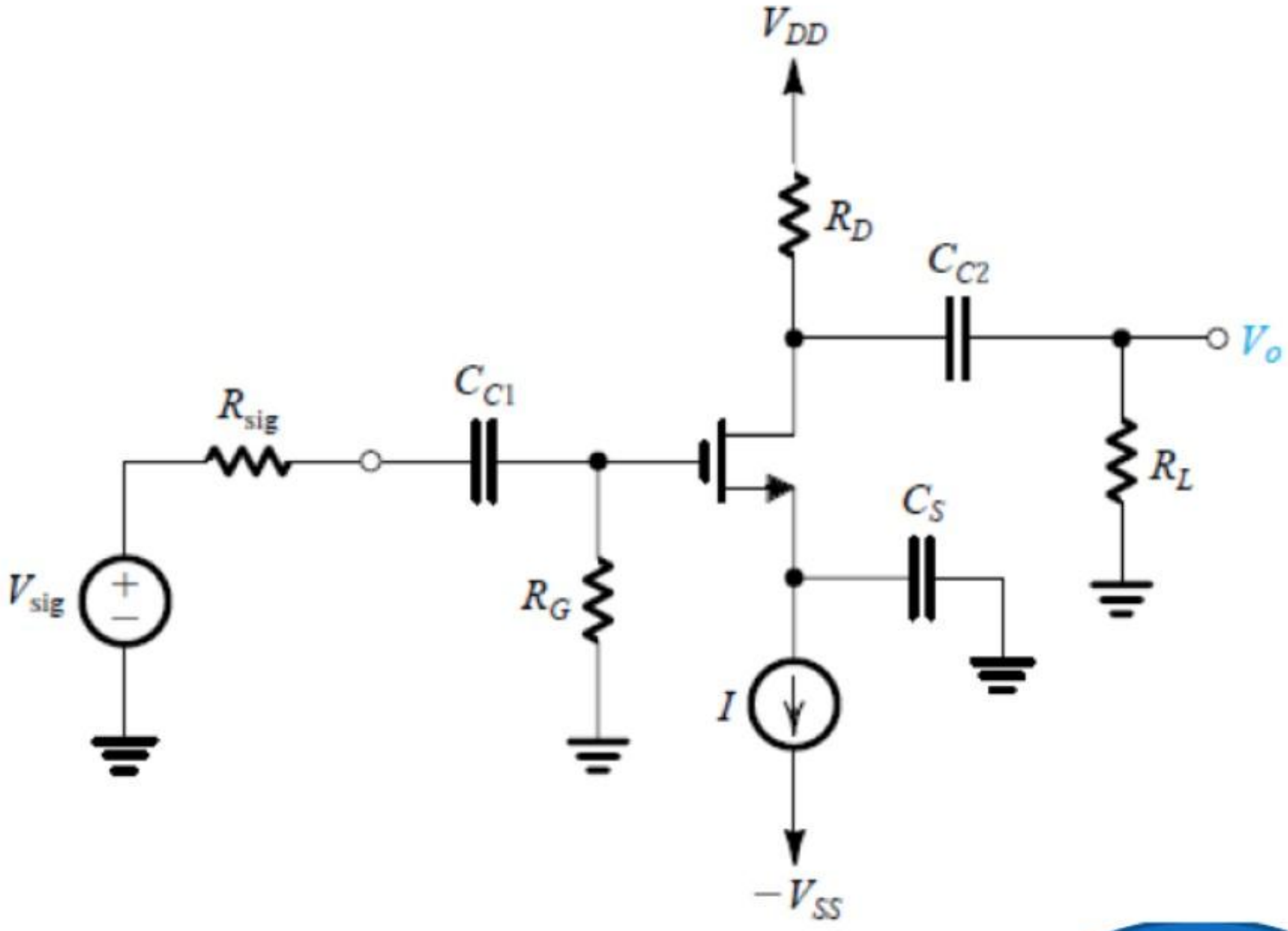
Example

Calculate the high-frequency band gain for the Common Source Amplifier.



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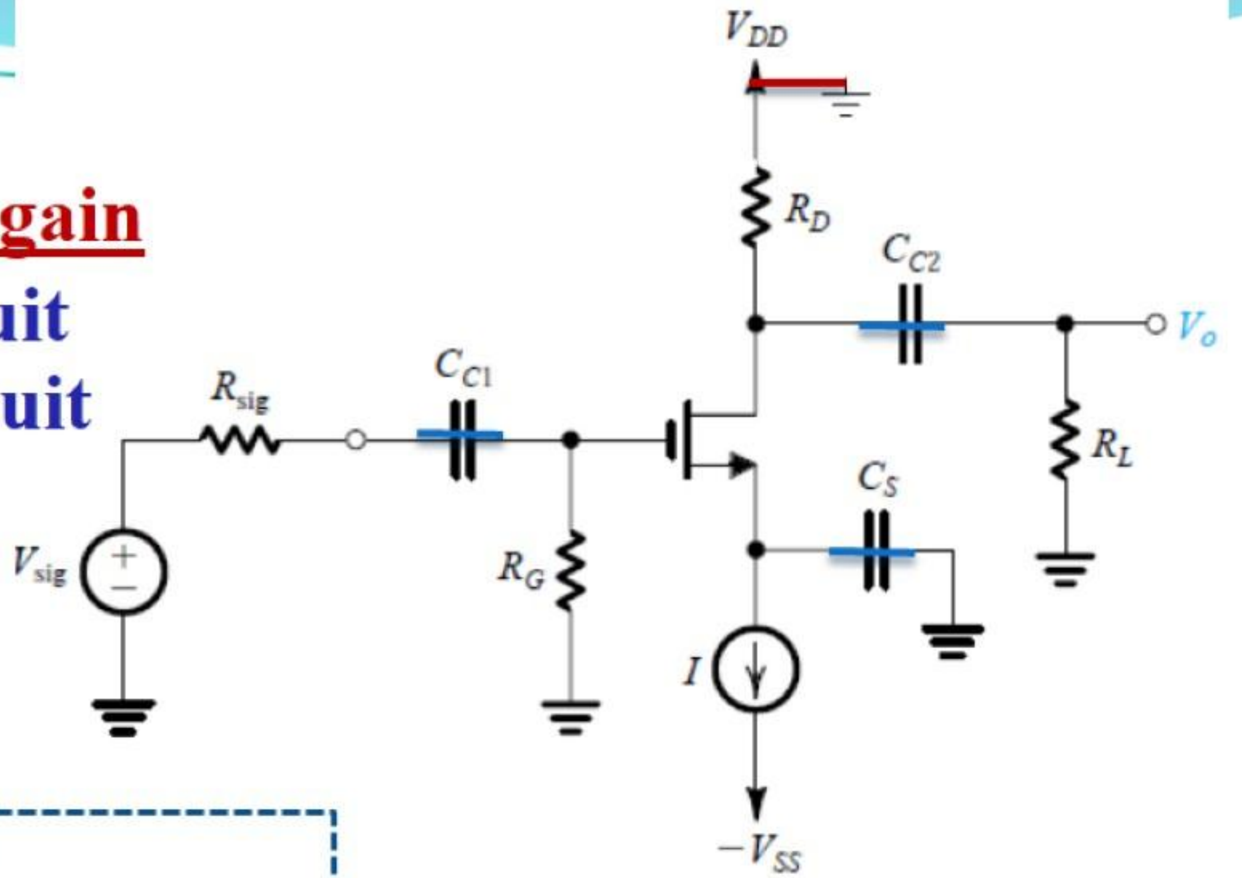
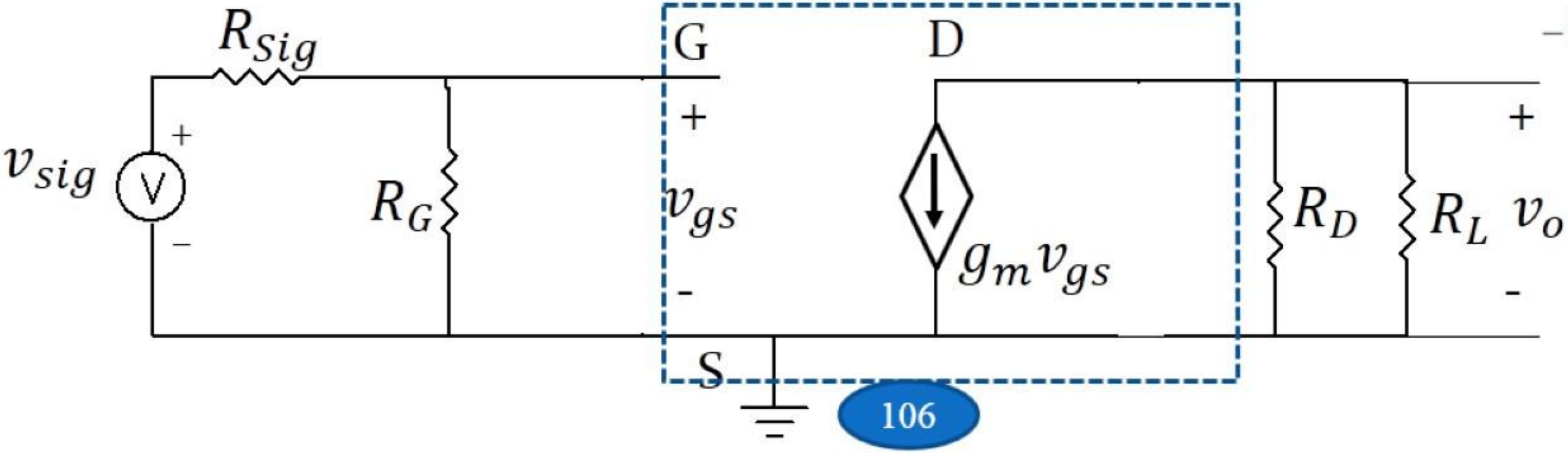
(a) Calculate the mid-band voltage gain

internal capacitors $\Longrightarrow$ open circuit
external capacitors $\Longrightarrow$ short circuit





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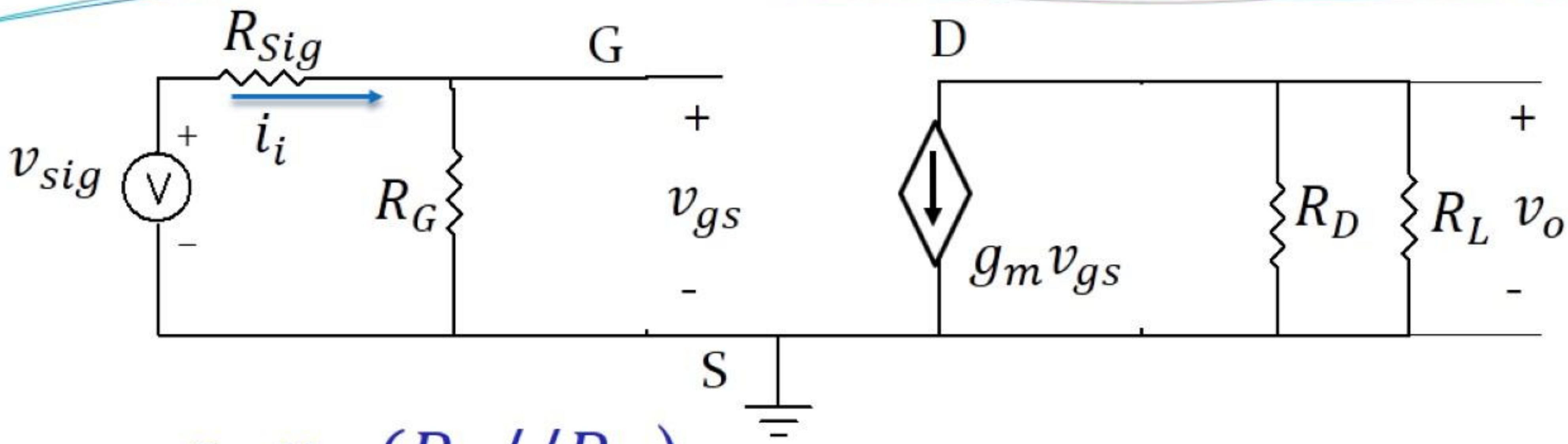

$$v_o = -g_m v_{gs}(R_L//R_D)$$

$$v_{gs} = i_i R_G$$

$$i_i = \frac{v_{sig}}{R_{sig} + R_G}$$

$$\frac{v_o}{v_{sig}} = A_{MF} = \frac{-g_m\,(R_L//R_D)R_G}{[R_{sig}+R_G]}$$

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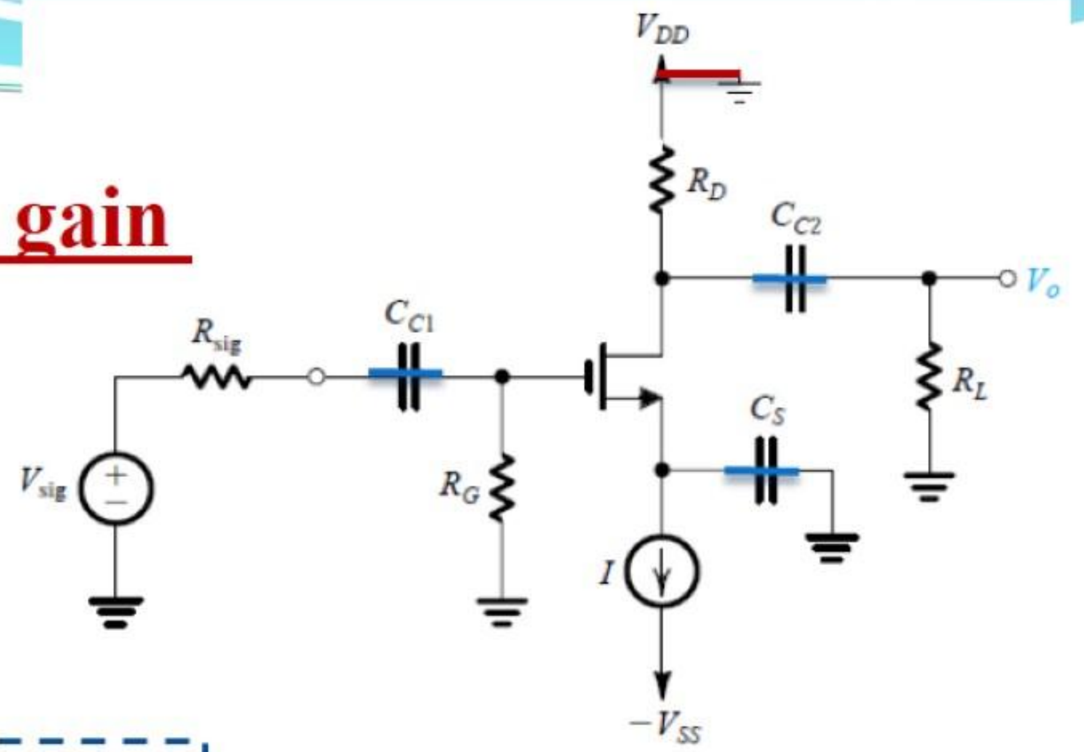
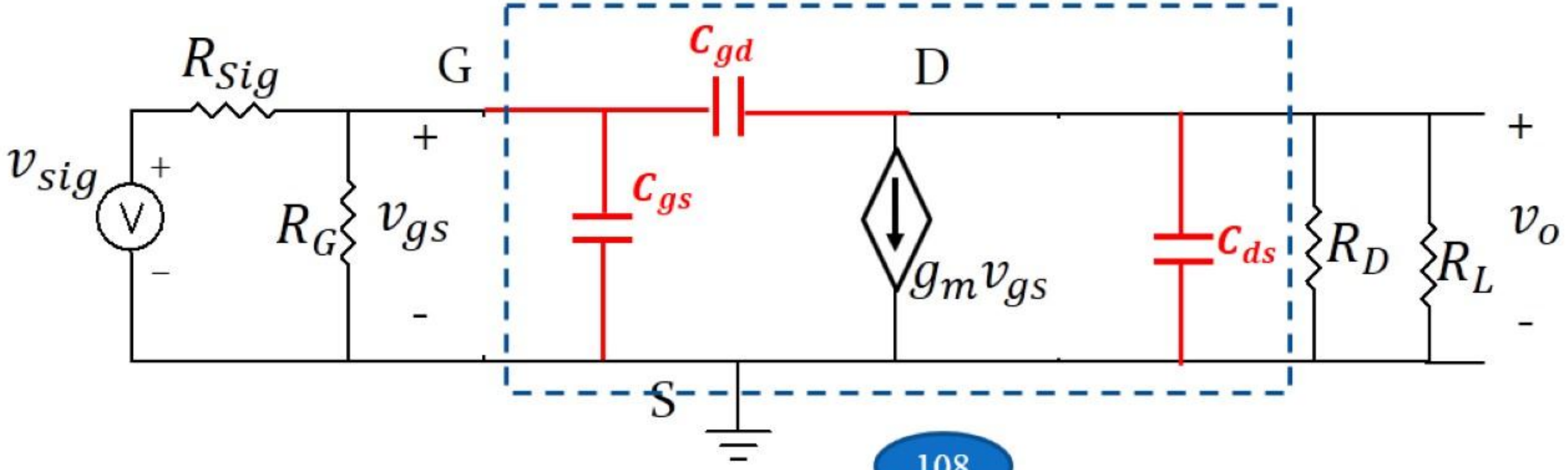
(b) Calculate the high-frequency band gain

external capacitors ⟹ **short circuit**





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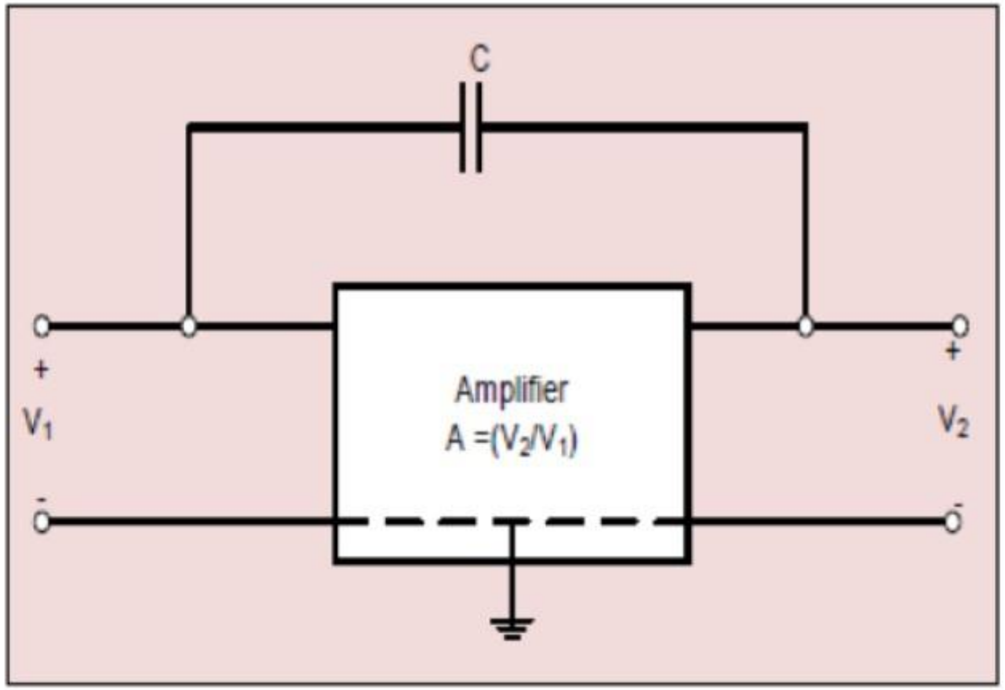
Miller's Theory:



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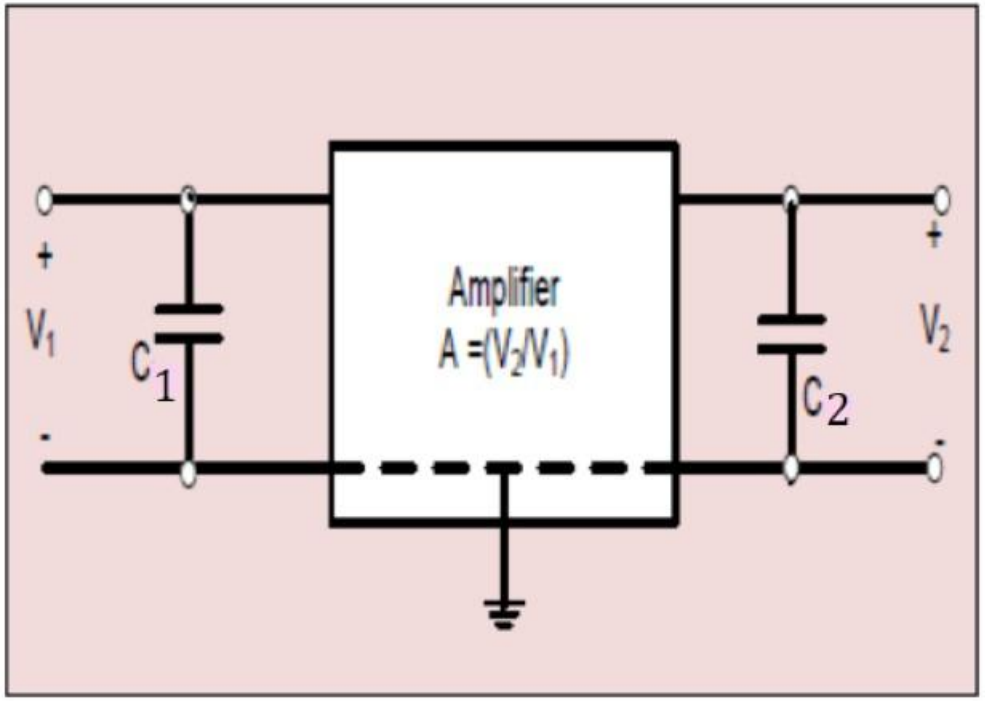

$C_1 = C(1\text{-}A)$

$C_2 = C(1\text{-}\frac{1}{A})$

Where:
A is calculated at f=0.

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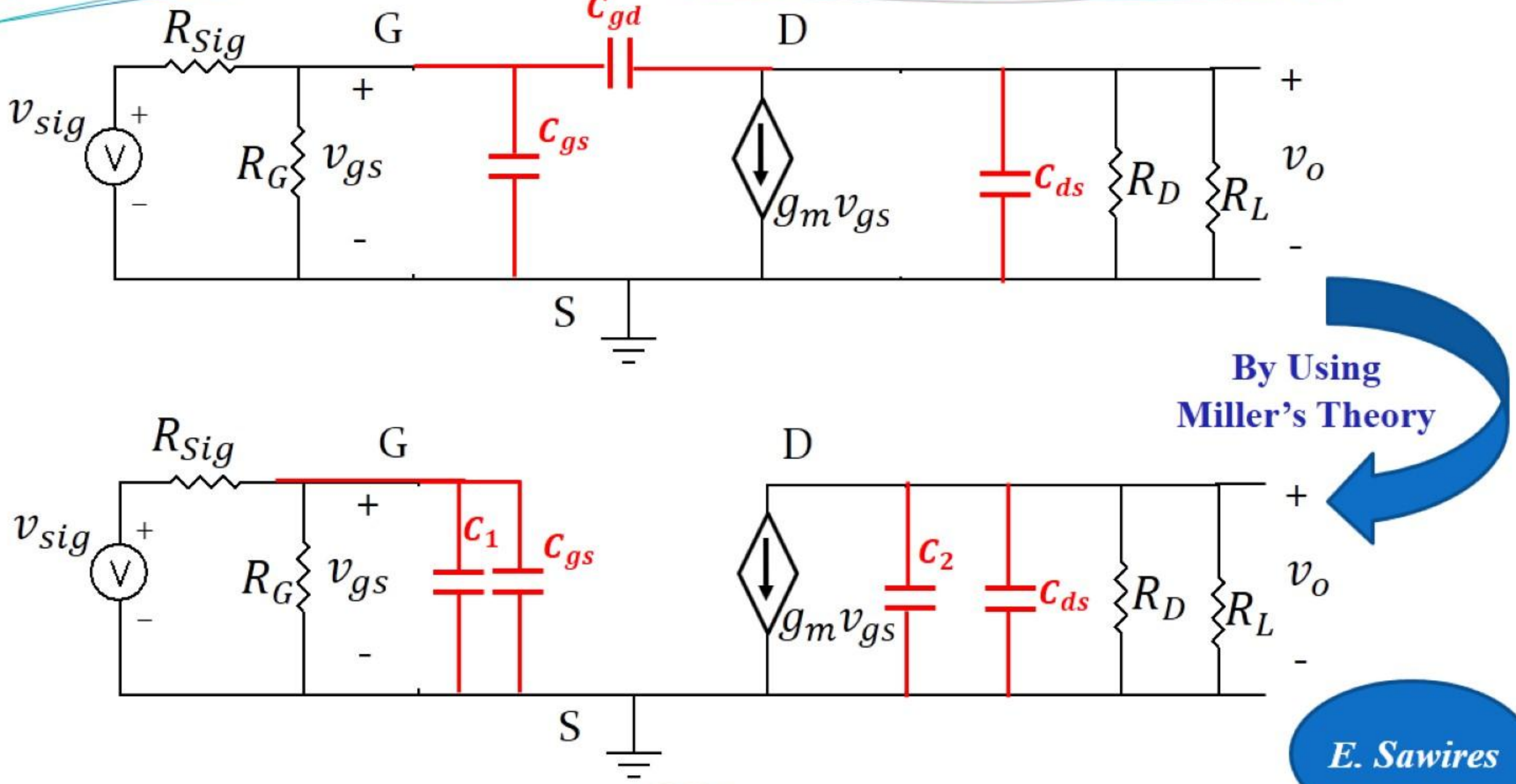

R_{Sig}
G
C_{gd}
D
v_{sig}
R_G
v_{gs}
C_{gs}
$g_m v_{gs}$
C_{ds}
R_D
R_L
v_o
S
By Using
Miller's Theory
R_{Sig}
G
D
v_{sig}
R_G
v_{gs}
C_1
C_{gs}
$g_m v_{gs}$
C_2
C_{ds}
R_D
R_L
v_o
S

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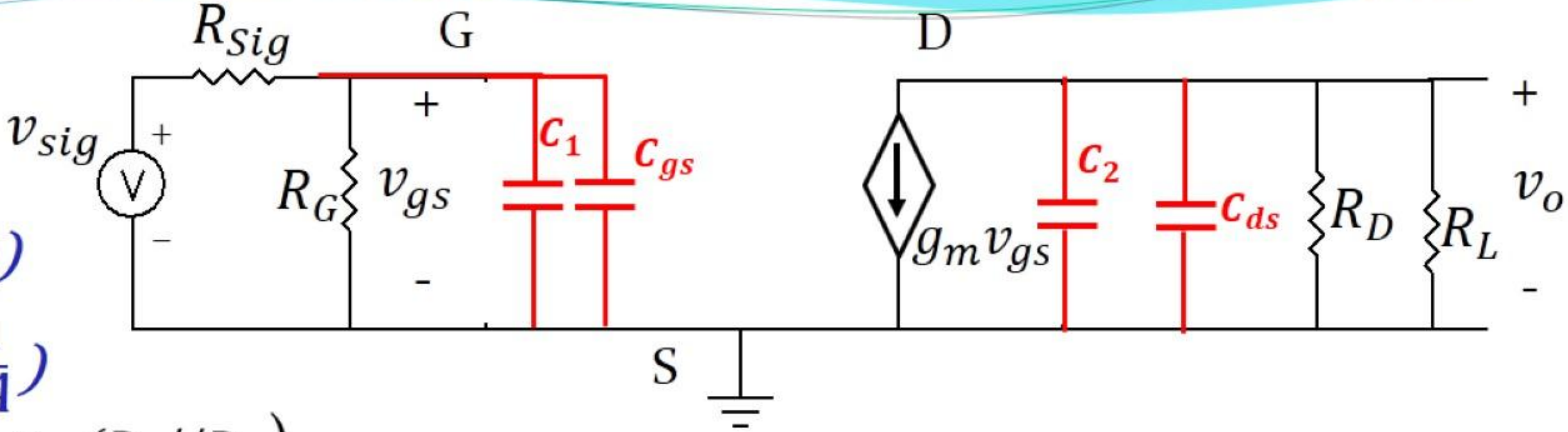
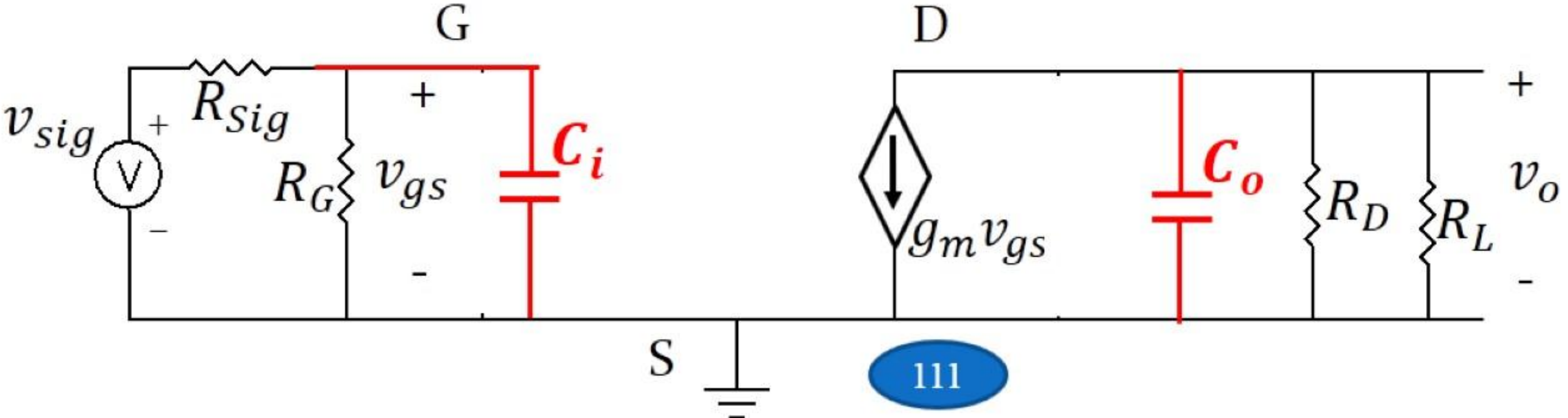


$C_1 = C_{gd}(1\text{-}A)$

$C_2 = C_{gd}\left(1\text{-}\frac{1}{A}\right)$

$$A = \left.\frac{v_D}{v_G}\right|_{\text{at f=0}} = \frac{-g_m v_{gs}(R_L//R_D)}{v_{gs}} = -g_m(R_L//R_D)$$



$C_{in} = C_{gs} + C_1$

$C_o = C_{ds} + C_2$

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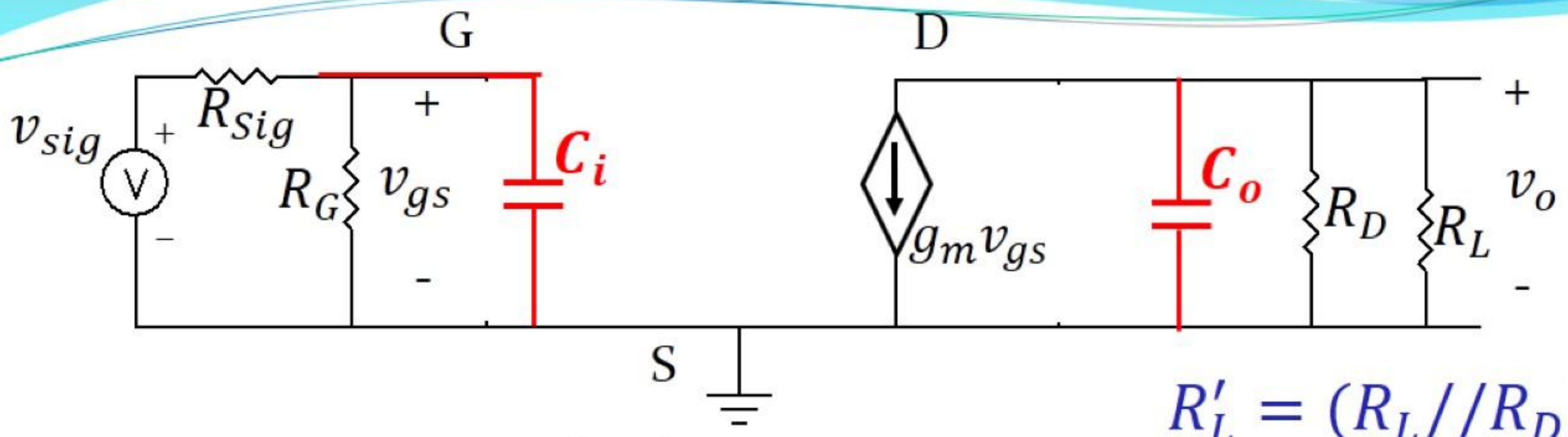

$$R'_L = (R_L // R_D)$$

$$v_o = -g_m v_{gs} (R'_L) // \left(\frac{1}{SC_o}\right)$$

$$v_o = -g_m v_{gs} R'_L \frac{1}{(1 + SC_o R'_L)}$$

$$v_o = -g_m v_{gs} R'_L \frac{1}{1+\frac{S}{\omega_{Po}}} \longrightarrow \boxed{1}$$

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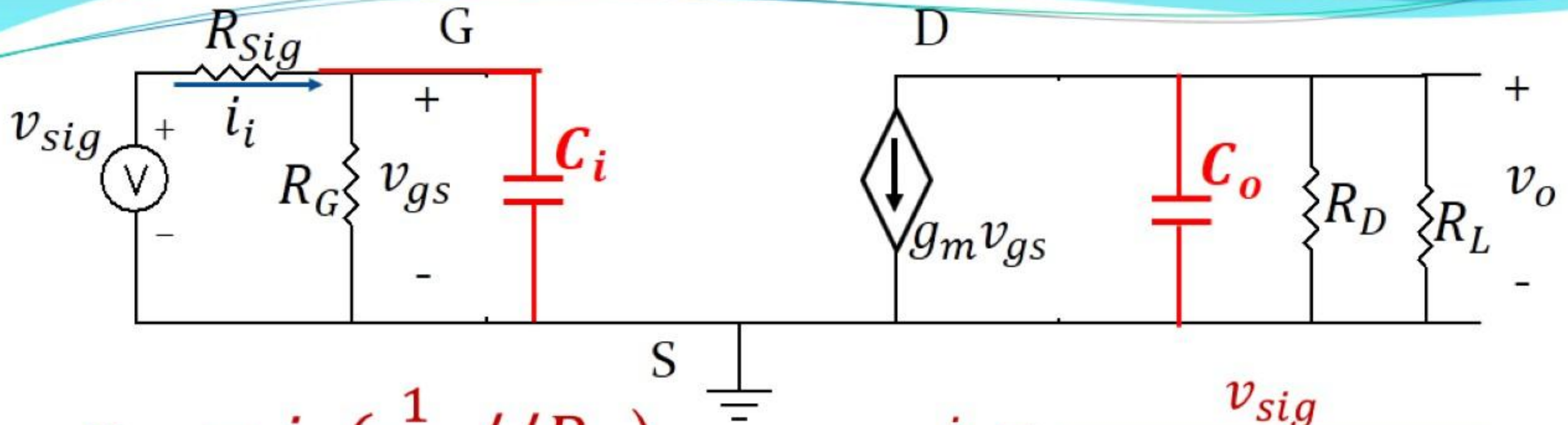

$$v_{gs} = i_i \left(\frac{1}{SC_i} // R_G\right) \qquad i_i = \frac{v_{sig}}{R_{sig} + \left(\frac{1}{SC_i} // R_G\right)}$$

$$v_{gs} = \frac{v_{sig}\left(\frac{1}{SC_i} // R_G\right)}{R_{sig} + \left(\frac{1}{SC_i} // R_G\right)} = \frac{R_G v_{sig}}{\left(R_G + R_{sig}\right)\left[1 + \frac{SC_i R_G R_{sig}}{R_G + R_{sig}}\right]}$$

$$= \left(\frac{R_G v_{sig}}{R_G + R_{sig}}\right)\frac{1}{1 + SC_i\left(R_G // R_{sig}\right)} = \left(\frac{R_G v_{sig}}{R_G + R_{sig}}\right)\frac{1}{1 + \frac{S}{\omega_{Pi}}} \longrightarrow \boxed{2}$$

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From (1) and (2):

$$\frac{v_o}{v_{sig}} = A_{HF} = \frac{-g_m R_L' R_G \frac{1}{(1+SC_o R_L')}}{(R_G+R_{sig})[1+SC_i(R_G//R_{sig})]}$$

$$\frac{v_o}{v_{sig}} = A_{HF} = \frac{-g_m R_L' R_G}{(R_G+R_{sig})} \frac{1}{[1+SC_i(R_G//R_{sig})][1+SC_o R_L']}$$

$$A_{HF}(\mathrm{S}) = \frac{A_{MF}}{(1+\frac{S}{\omega_{Pi}})(1+\frac{S}{\omega_{Po}})}$$

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$$\omega_{Pi} = 2\pi f_{Pi} = \frac{1}{C_i[R_G//R_{sig}]} = \frac{1}{\tau} = \frac{1}{C_i R_{eq}}$$

$$f_{Pi} = \frac{1}{2\pi C_i[R_G//R_{sig}]}$$

$$\omega_{Po} = 2\pi f_{Po} = \frac{1}{C_o[R_L']} = \frac{1}{\tau} = \frac{1}{C_o R_{eq}}$$

$$f_{Po} = \frac{1}{2\pi C_o[R_L']}$$

The higher 3-dB frequency is given by :

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{pi}}\right)^2 + \left(\frac{1}{f_{Po}}\right)^2}}$$

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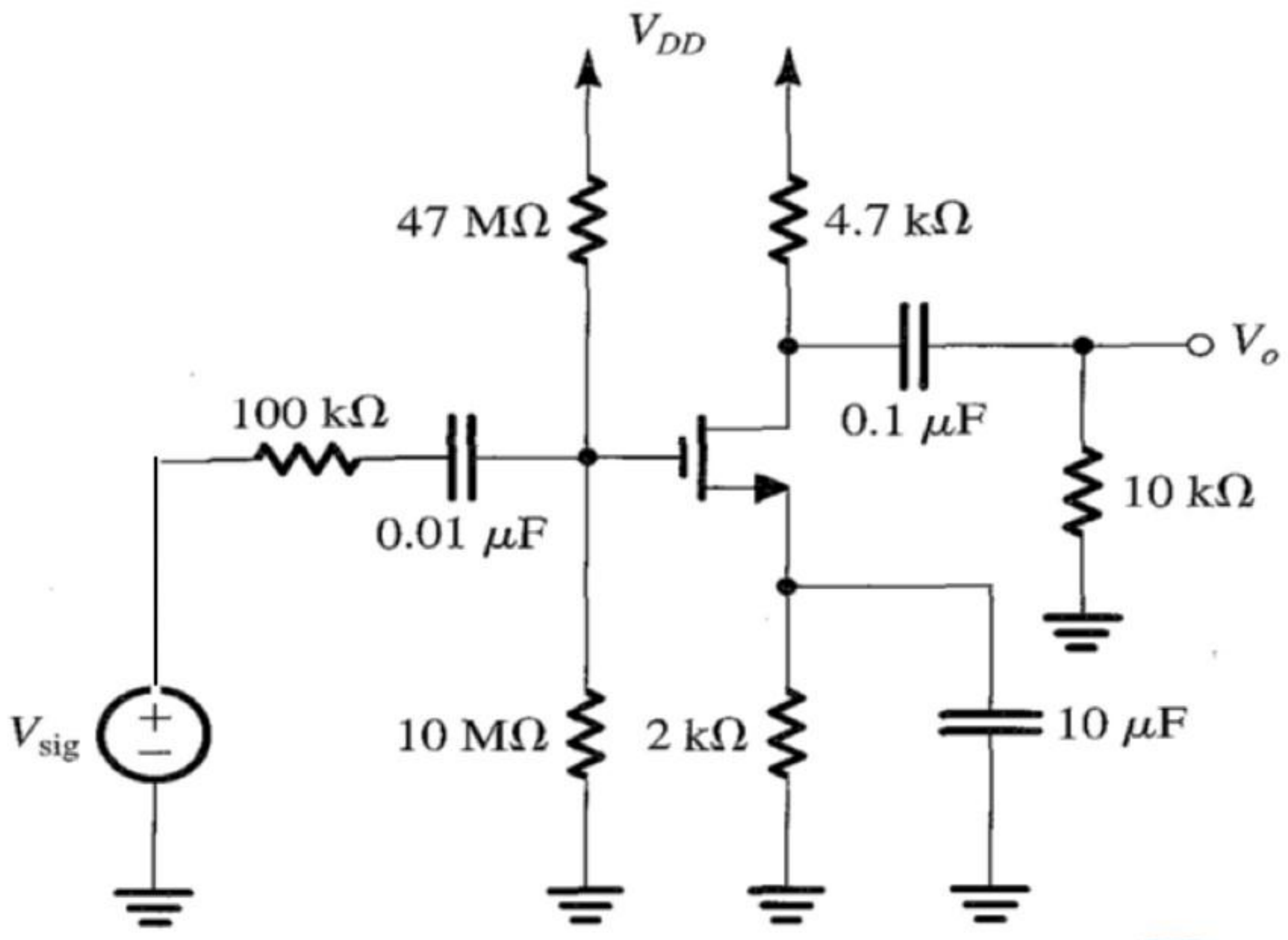
Example

Find the mid band gain A_M and the higher 3-dB frequency f_H of a CS amplifier fed with a signal source having an internal resistance Rsig= 100 kΩ. The amplifier has g_m = 1 mA/V and r_0 = 100 kΩ. Find A_M. If C_{gs} = 1 pF and C_{gd} = 0.2 pF, find f_H.



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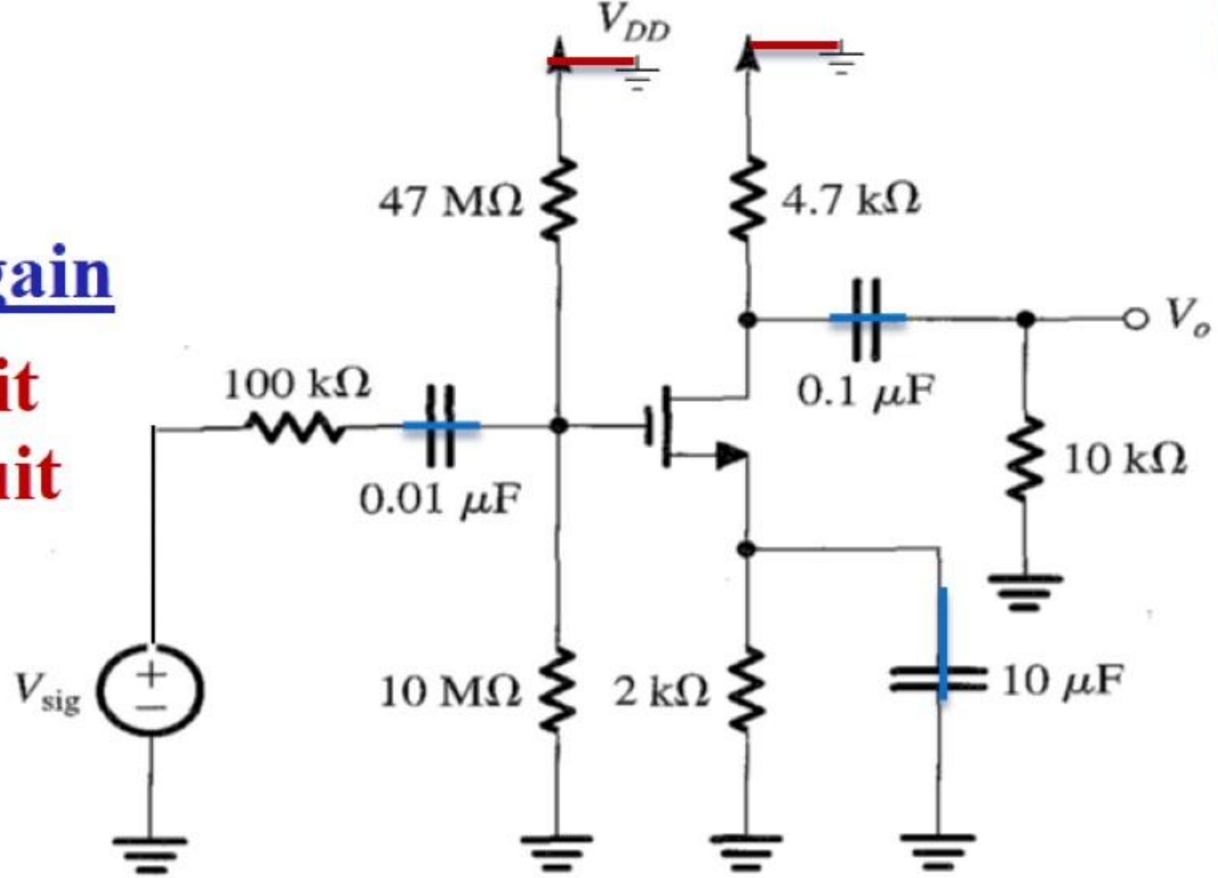
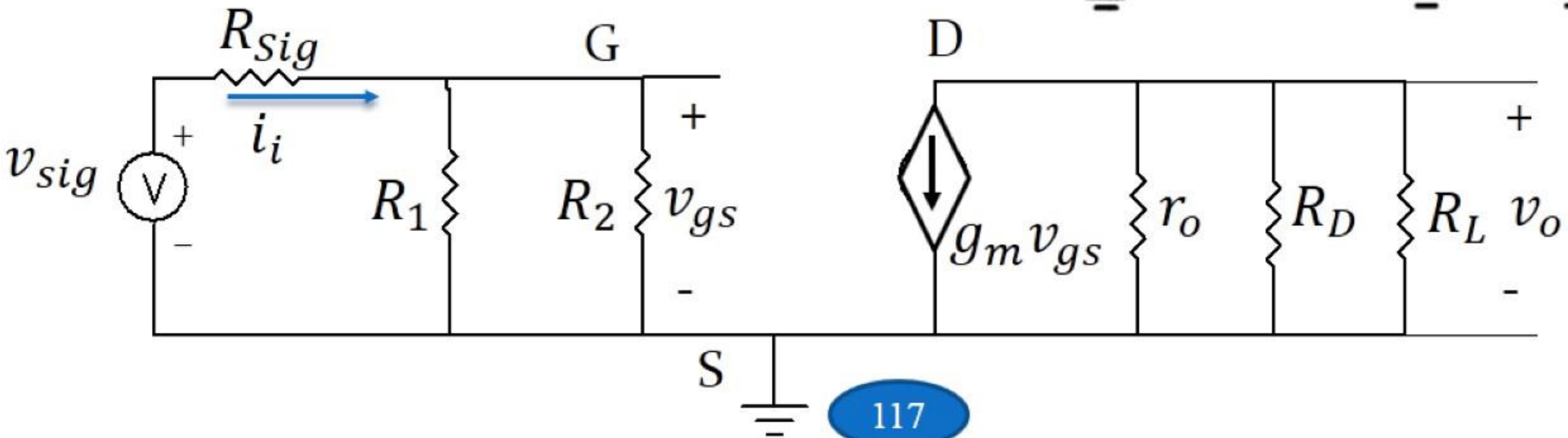
(a) Calculate the mid-band voltage gain

internal capacitors ⟹ open circuit
external capacitors ⟹ short circuit





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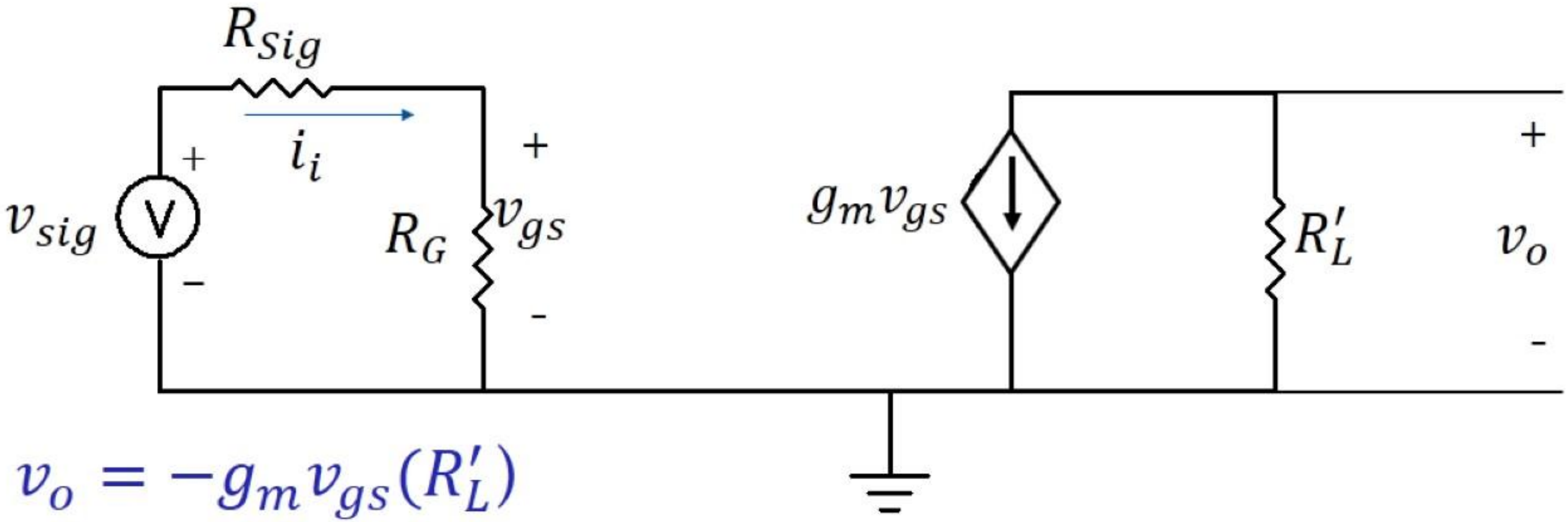
$R_G = (R_1//R_2)$ $\qquad$ $R_L' = (R_L//R_D//r_o)$



$$v_o = -g_m v_{gs}(R_L')$$

$$v_{gs} = i_i\,(R_G)$$

$$i_i = \frac{v_{sig}}{R_{sig} + (R_G)}$$

$$\frac{v_o}{V_{sig}} = A_{MF} = \frac{-g_m\, R_G R_L'}{[R_{sig} + R_G]} = \text{-3.06 V/V}$$

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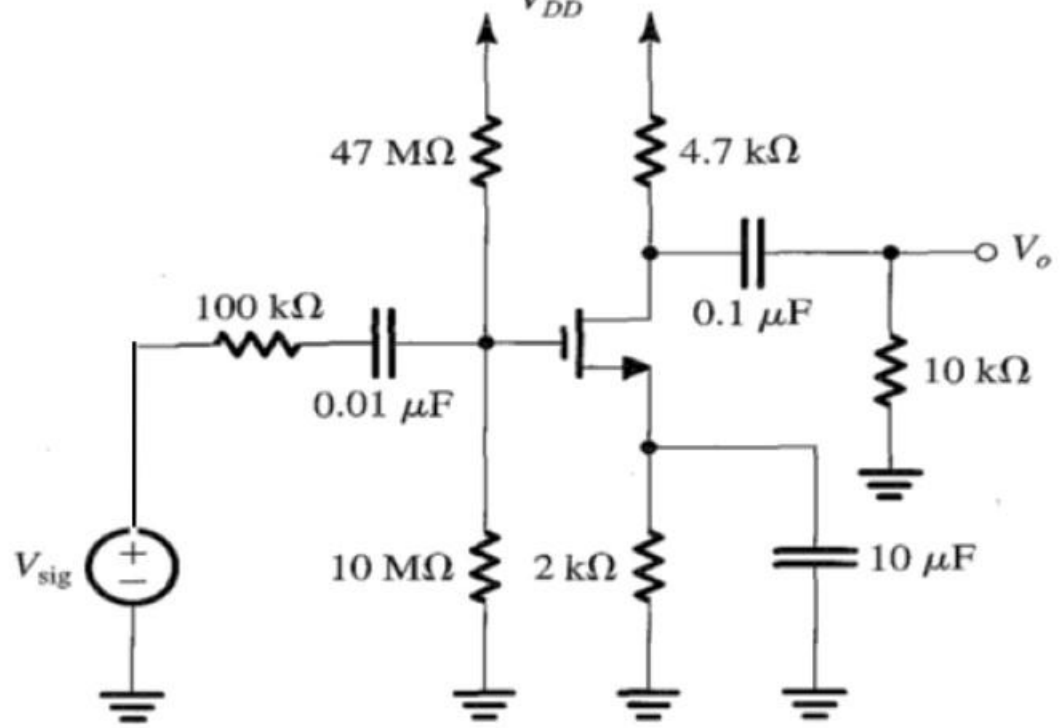
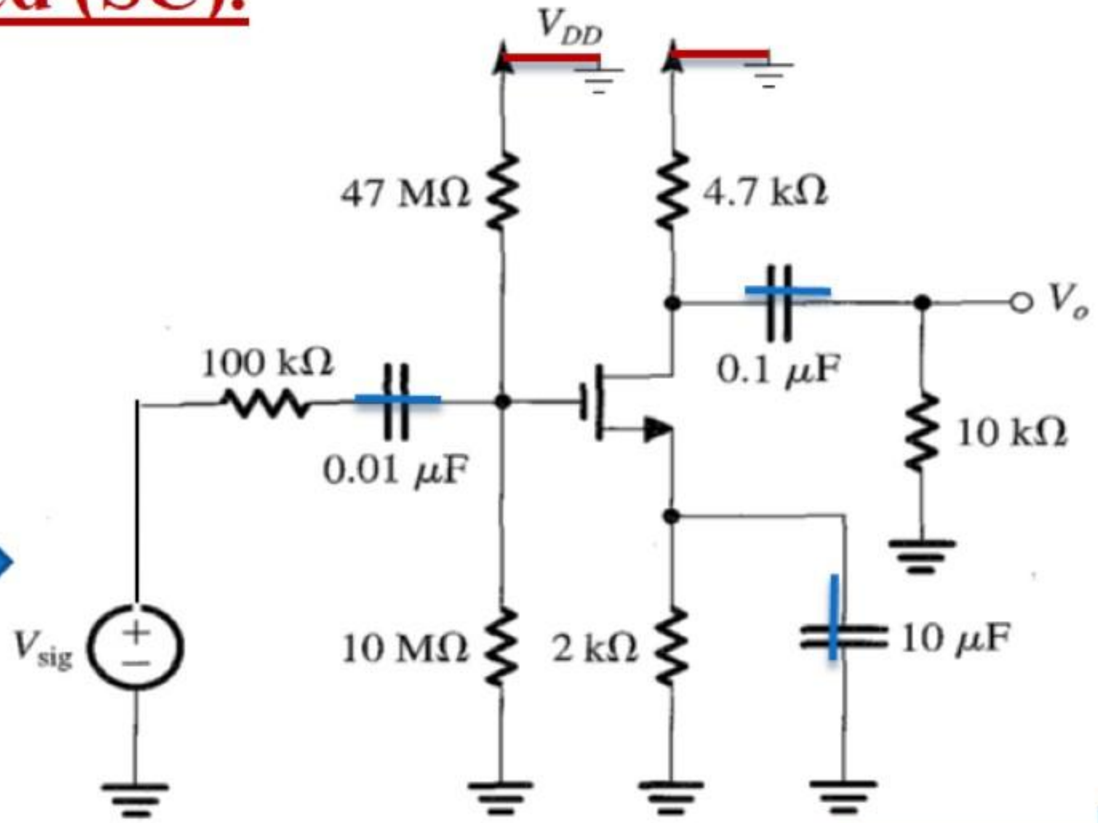
(b) In the high frequency, internal capacitors are taking into account and external capacitors are neglected (SC).





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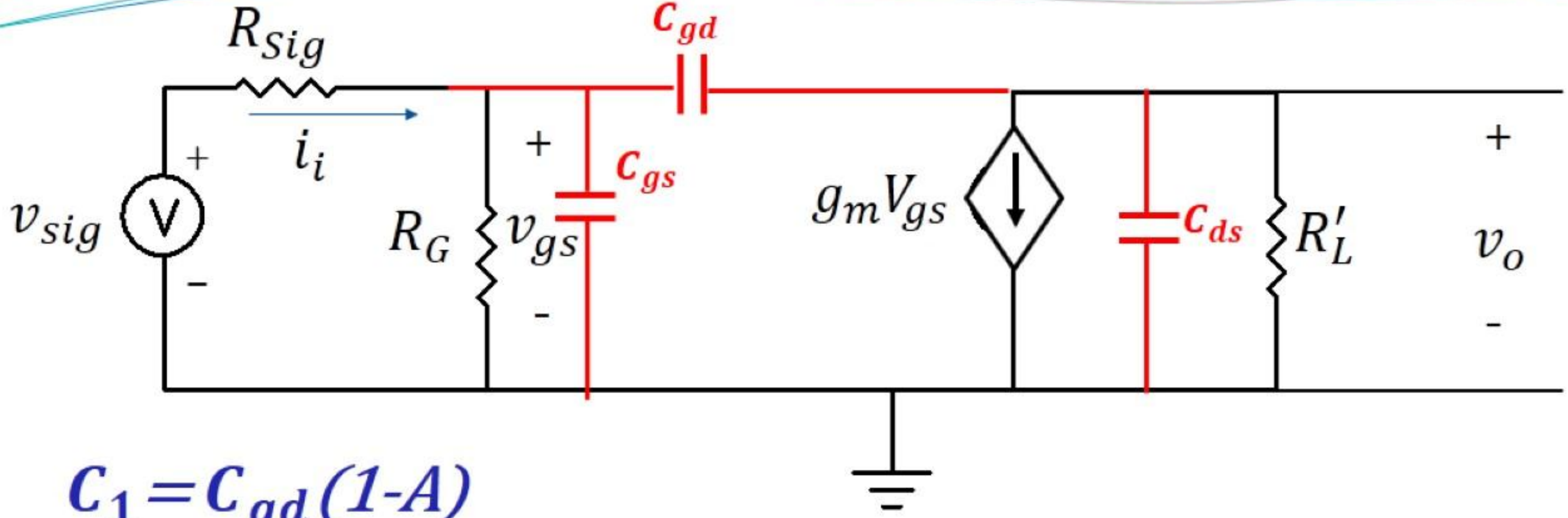

$$C_1 = C_{gd}(1\text{-}A)$$

$$C_2 = C_{gd}\left(1\text{-}\frac{1}{A}\right)$$

$$A = \left.\frac{v_D}{v_G}\right|_{\text{at } f=0} = \frac{-g_m v_{gs} R_L'}{v_{gs}} = -g_m R_L'$$

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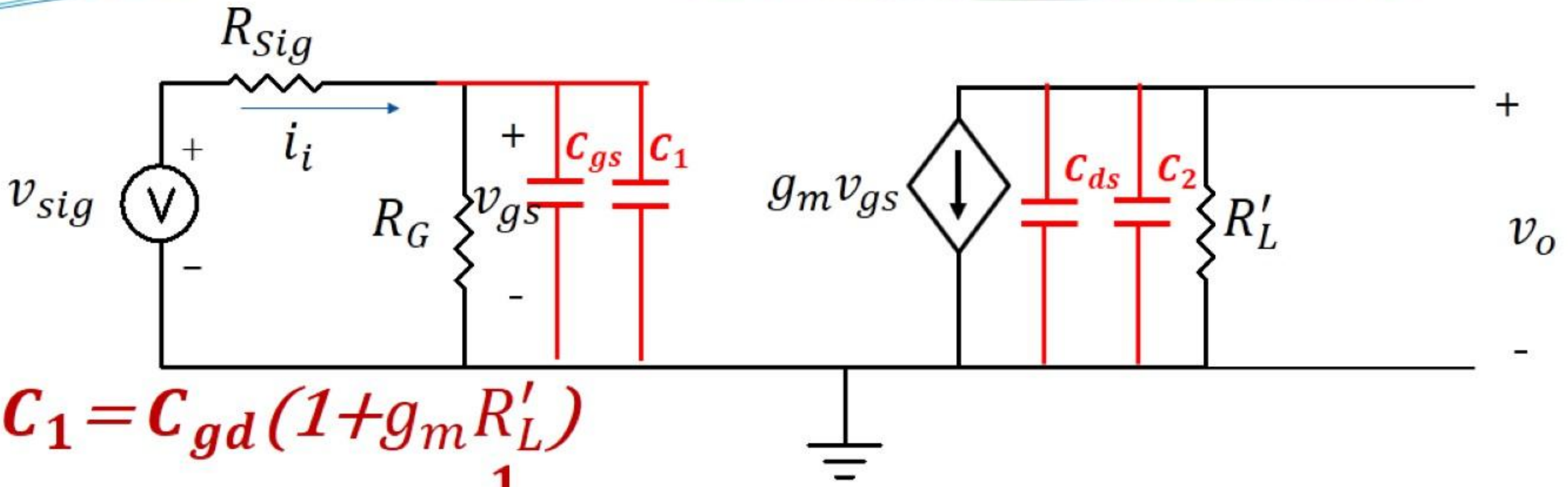

$$C_1 = C_{gd}(1+g_m R'_L)$$

$$C_2 = C_{gd}\left(1+\frac{1}{g_m R'_L}\right)$$

$$C_i = C_{gs} + C_1 = 1+0.2(1+1\,X\,3.098) = 1.82\ pf$$

$$C_o = C_{ds} + C_2 = C_2 = 0.2\left(1+\frac{1}{1X3.098}\right) = 0.265\ pf$$

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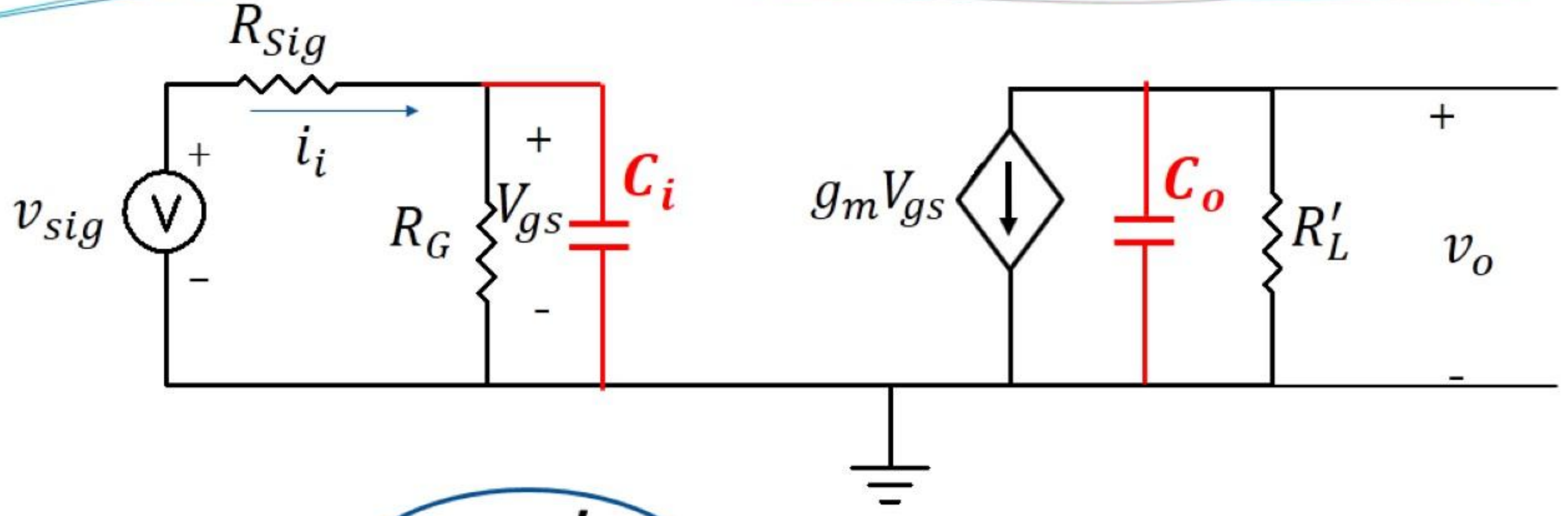


$$\frac{V_{out}}{V_{sig}} = A_{HF} = \frac{-g_m R'_L R_G}{(R_G + R_{sig})} \frac{1}{[1 + SC_i(R_G // R_{sig})][1 + SC_o R'_L]}$$

$$A_{HF}(\mathrm{S}) = \frac{A_{MF}}{(1 + \frac{S}{\omega_{Pi}})(1 + \frac{S}{\omega_{Po}})}$$

E. Sawires

$$\omega_{Pi} = 2\pi f_{Pi} = \frac{1}{C_i\left[R_G//R_{sig}\right]}$$

$$f_{Pi} = \frac{1}{2\pi C_i\left[R_G//R_{sig}\right]} = 875 \text{ KHZ}$$

$$\omega_{Po} = 2\pi f_{Po} = \frac{1}{C_o\left[R_L'\right]}$$

$$f_{Po} = \frac{1}{2\pi C_o\left[R_L'\right]} = 194 \text{ MHZ}$$

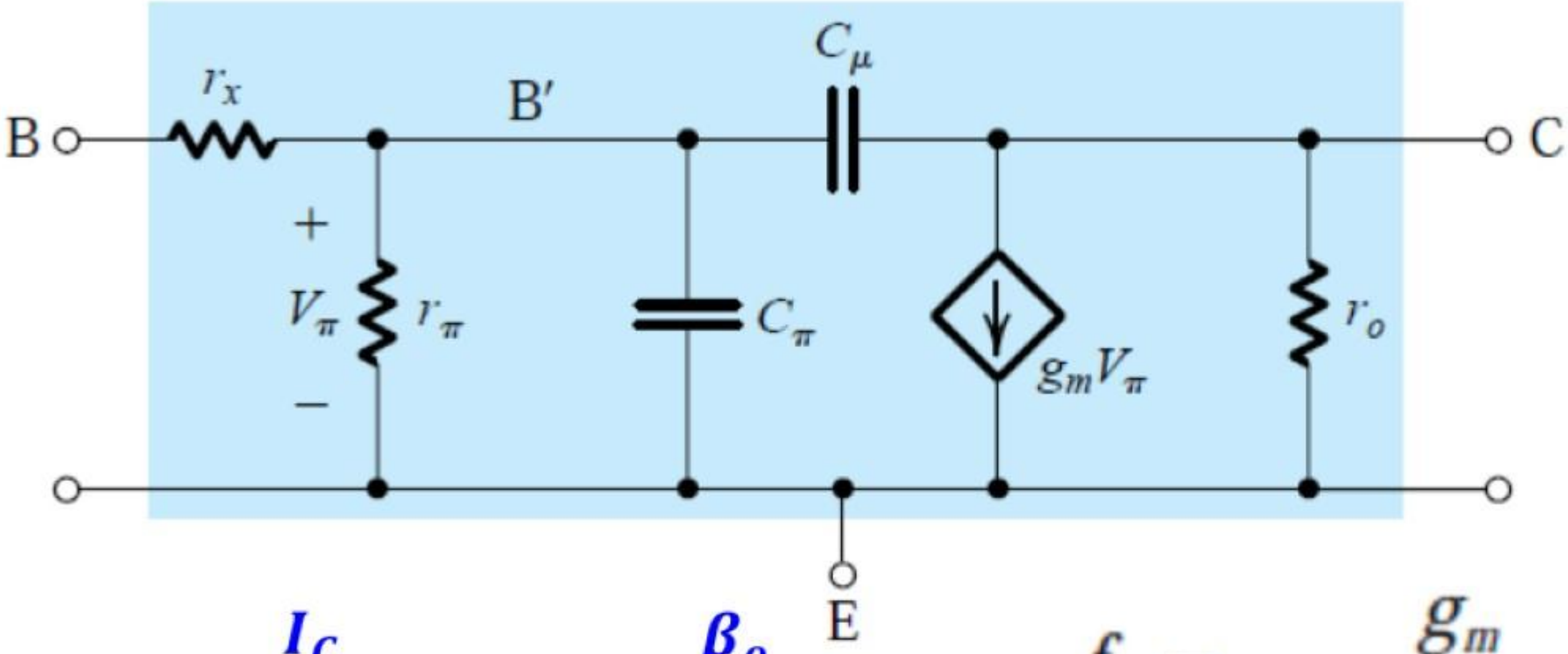
The higher 3-dB frequency is given by :

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{Pi}}\right)^2 + \left(\frac{1}{f_{Po}}\right)^2}} = 874.99 \text{ KHZ} \cong f_{pi}$$

E. Sawires

High Frequency Response of CE Amplifier

The BJT High-Frequency Model



$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{\beta_o}{g_m} \qquad f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$r_o = \frac{|V_A|}{I_C} \qquad C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

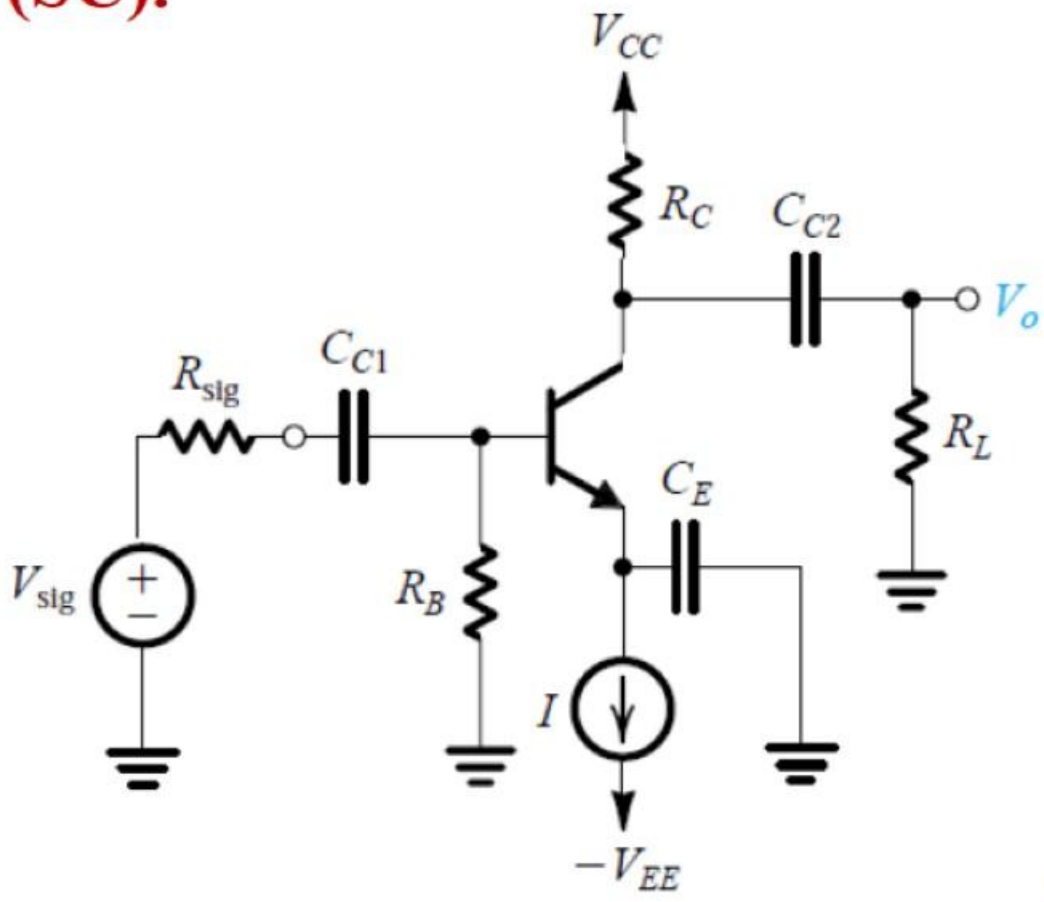
E. Sawires

High Frequency Response of Common Emitter Amplifier

(b) In the high frequency, internal capacitors are taking into account and external capacitors are neglected (SC).

AC Analysis

- **Eliminate the dc sources by replacing each dc voltage source with a short circuit and each dc current source with an open circuit.**



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High Frequency Response of Common Emitter Amplifier

(b) In the high frequency, internal capacitors are taking into account and external capacitors are neglected (SC).

AC Analysis

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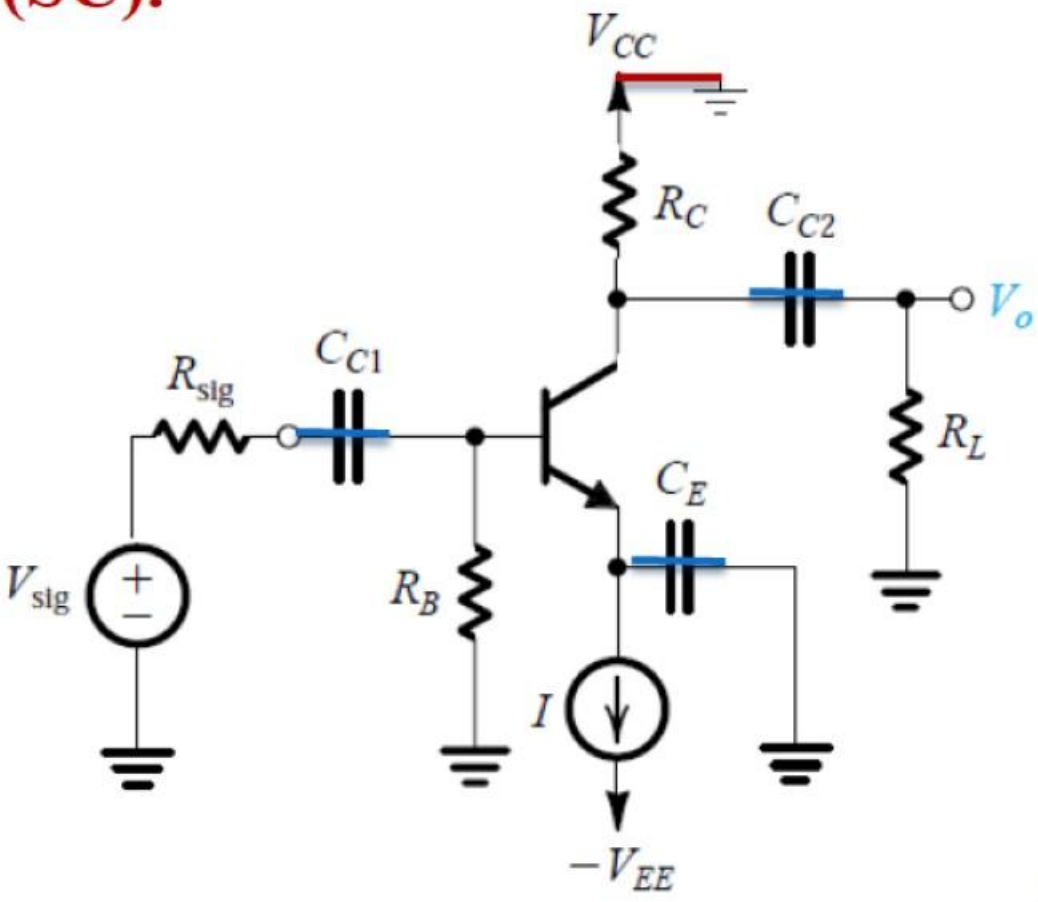

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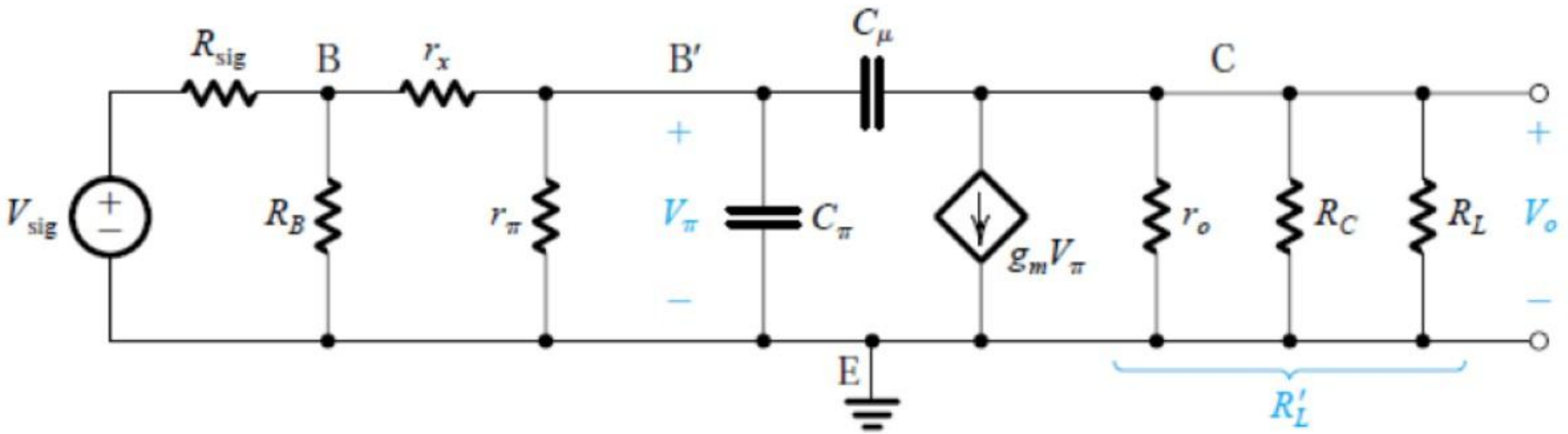
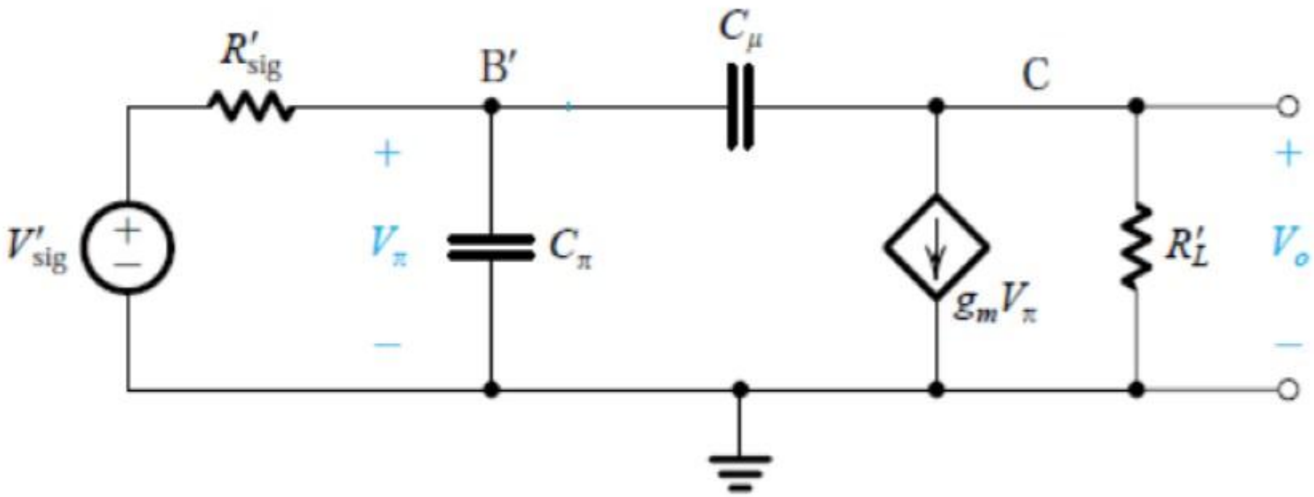
➢ Figure shows the high-frequency equivalent circuit of a CE amplifier.





Thévenin theorem at the input side

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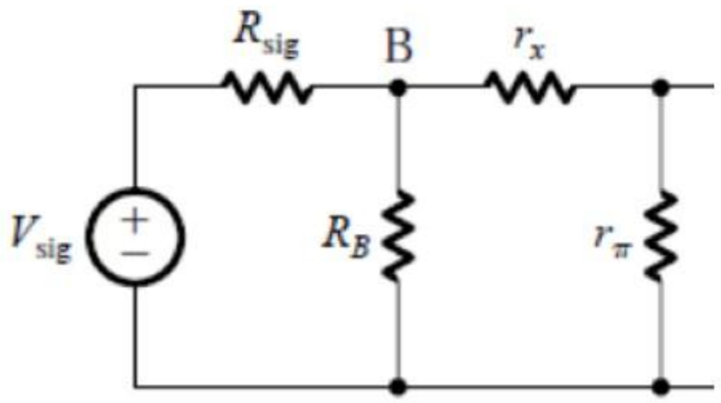
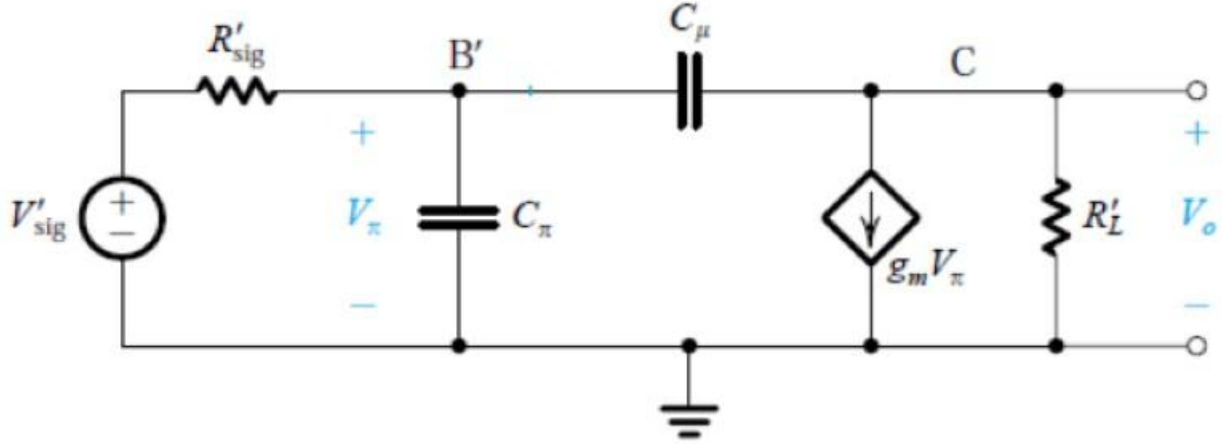
- Figure shows the high-frequency equivalent circuit of a CE amplifier.

$$V'_{sig} = V_{sig}\frac{R_B}{R_B + R_{sig}}\frac{r_\pi}{r_\pi + r_x + (R_{sig}\|R_B)}$$

$$R'_{sig} = r_\pi \| [r_x + (R_B \| R_{sig})]$$

$$R'_L = r_o \| R_C \| R_L$$





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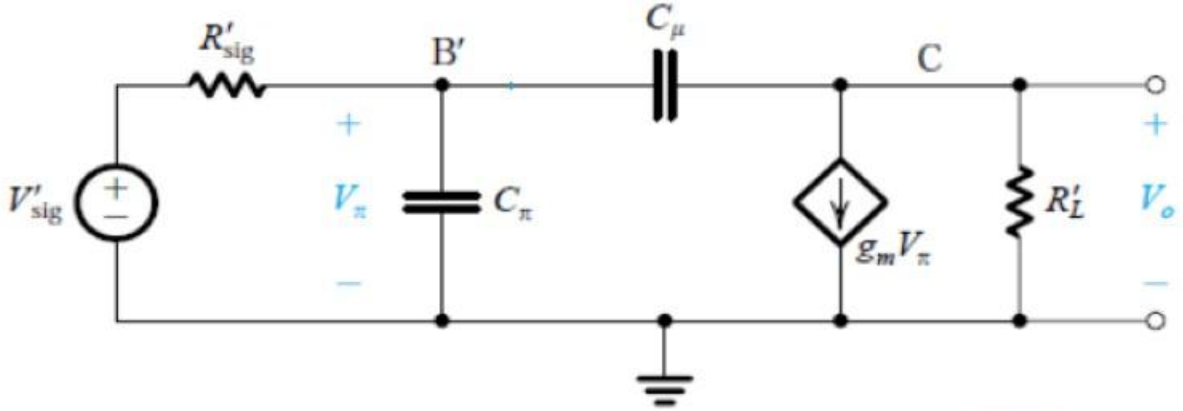
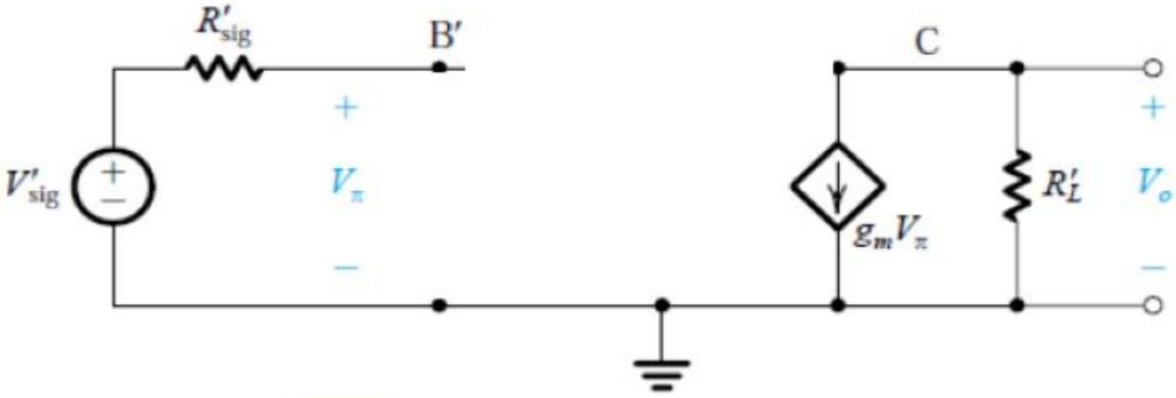
(a) Calculate the mid-band voltage gain

The equivalent circuit in Fig. can be used to obtain the midband gain by setting C_π and C_μ to zero.



$$v_o = -g_m V_\pi (R'_L)$$

$$V_\pi = V'_{Sig}$$



E. Sawires

(a) Calculate the mid-band voltage gain

The equivalent circuit in Fig. can be used to obtain the midband gain by setting C_π and C_μ to zero.

$$\frac{v_o}{v_{sig}} = A_{MF} = \frac{-g_m\,(R_C//R_L)R_B r_\pi}{(R_{sig}+R_B)[(R_{Sig}\|R_B)+r_\pi]}$$

$$v_o = -g_m V_\pi (R_L')$$

$$V_\pi = V_{Sig}'$$

$$V_{\text{sig}}' = V_{\text{sig}} \frac{R_B}{R_B + R_{\text{sig}}} \frac{r_\pi}{r_\pi + r_x + (R_{\text{sig}} \| R_B)}$$

$$A_M = \frac{V_o}{V_{\text{sig}}} = -\frac{R_B}{R_B + R_{\text{sig}}} \frac{r_\pi}{r_\pi + r_x + (R_{\text{sig}} \| R_B)} (g_m R_L')$$

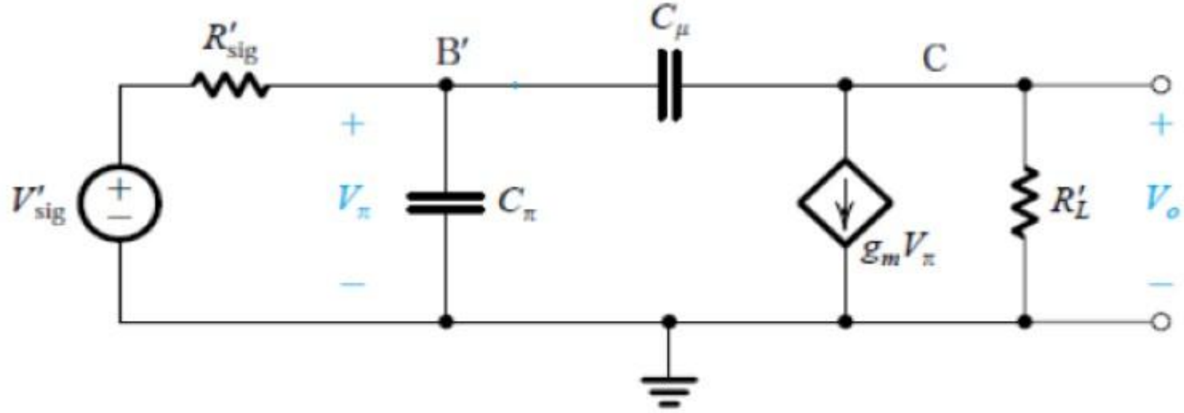
E. Sawires

(b) In the high frequency, internal capacitors are taking into account and external capacitors are neglected (SC).

$$C_1 = C_\mu (1\text{-}A)$$

$$C_2 = C_\mu \left(1\text{-}\frac{1}{A}\right)$$

$$A = \left.\frac{V_C}{V_{B'}}\right|_{\text{at } f=0} = \frac{-g_m V_\pi R_L'}{V_\pi} = -g_m R_L'$$

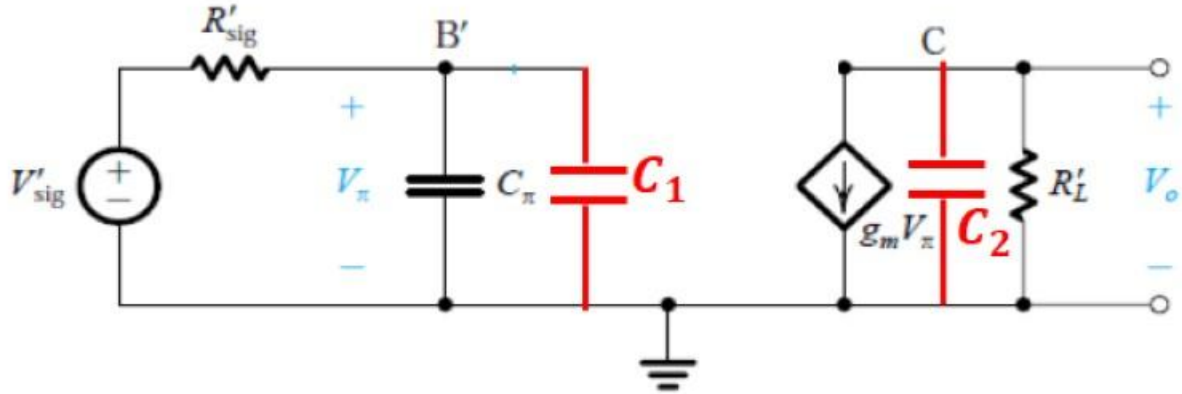


$$C_1 = C_\mu (1 + g_m R_L')$$

$$C_2 = C_\mu \left(1 + \frac{1}{g_m R_L'}\right)$$

$$C_i = C_\pi + C_1$$

$$C_i = C_\pi + C_\mu (1 + g_m R_L')$$



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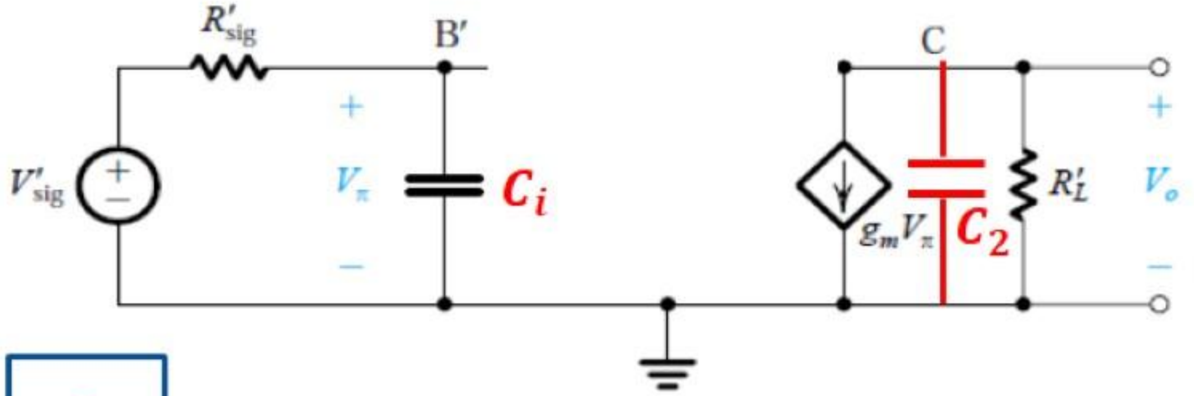
$$v_o = -g_m V_\pi (R'_L)\ //\left(\frac{1}{SC_2}\right)$$

$$v_o = -g_m V_\pi R'_L \frac{1}{(1+SC_2R'_L)}$$

$$v_o = -g_m V_\pi R'_L \frac{1}{1+\frac{S}{\omega_{Po}}} \longrightarrow \boxed{1}$$



$$V_\pi = i_i\left(\frac{1}{SC_i}\right)$$

$$i_i = \frac{v'_{sig}}{R'_{sig}+\frac{1}{SC_i}}$$

$$V_\pi = v'_{sig}\frac{1}{(1+SC_iR'_{Sig})} \longrightarrow \boxed{2}$$

E. Sawires

$$v_o = -v'_{sig} g_m R'_L \frac{1}{(1+SC_2R'_L)(1+SC_iR'_{Sig})}$$

$$V'_{\text{sig}} = V_{\text{sig}} \frac{R_B}{R_B + R_{\text{sig}}} \frac{r_\pi}{r_\pi + r_x + (R_{\text{sig}} \| R_B)}$$

$$A_M = \frac{V_o}{V_{\text{sig}}} = -\frac{R_B}{R_B + R_{\text{sig}}} \frac{r_\pi}{r_\pi + r_x + (R_{\text{sig}} \| R_B)}(g_m R'_L)$$

$$A_{HF}(\mathrm{S}) = \frac{A_{MF}}{(1+\frac{S}{\omega_{Pi}})(1+\frac{S}{\omega_{Po}})}$$

$$\omega_{Pi} = 2\pi f_{Pi} = \frac{1}{C_i R'_{Sig}} \qquad f_{Pi} = \frac{1}{2\pi C_i R'_{Sig}}$$

$$\omega_{Po} = 2\pi f_{Po} = \frac{1}{C_2 R'_L} \qquad f_{Po} = \frac{1}{2\pi C_2 R'_L}$$

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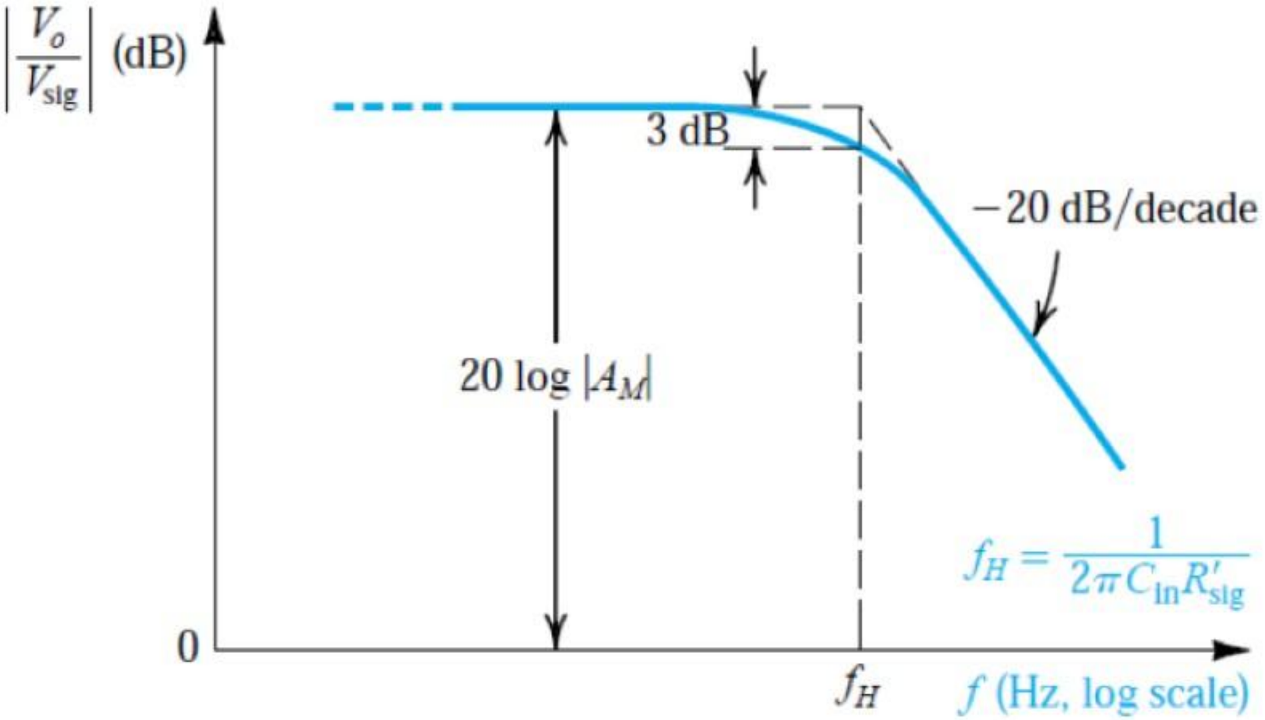
$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{Pi}}\right)^2 + \left(\frac{1}{f_{Po}}\right)^2}} \cong f_{pi}$$



$$R'_{sig} = r_\pi \,\|\, [r_x + (R_B \,\|\, R_{sig})]$$

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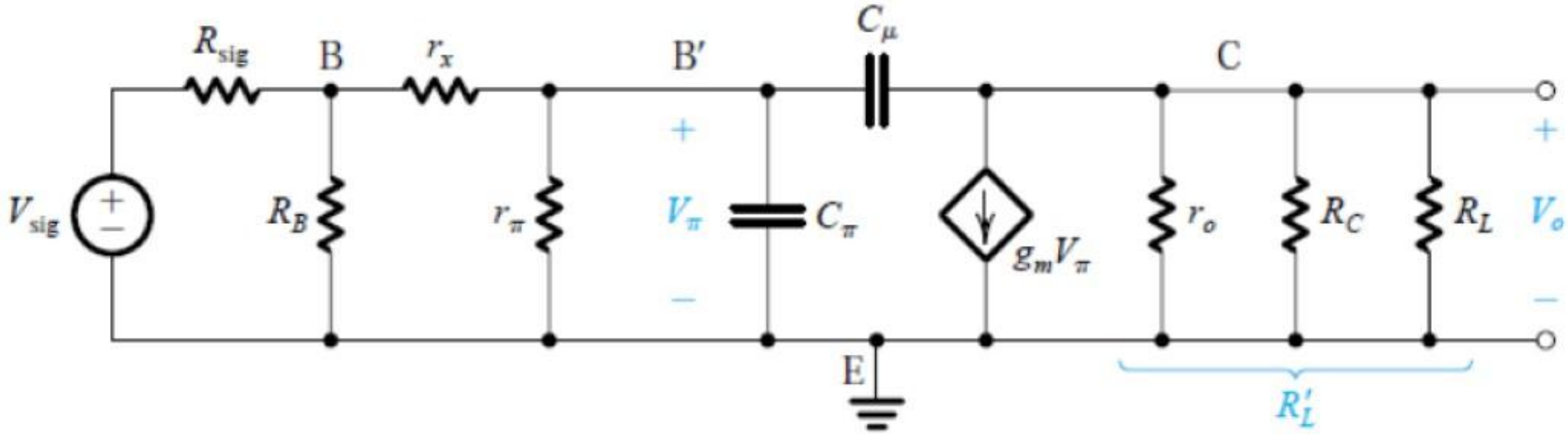
- Observe that $\boldsymbol{C_{in}}$ is simply the sum of $\boldsymbol{C_{\pi}}$ and the Miller capacitance $\boldsymbol{C_{\mu}}(1+g_m R_L')$. The resistance R_{sig}' seen by $\boldsymbol{C_{in}}$ can be easily found from the equivalent circuit in Fig. (slide 131) as follows: Reduce $\boldsymbol{V_{sig}}$ to zero, "grab hold" of the terminals B′ and E and look back (to the left). You will see $\boldsymbol{r_{\pi}}$ in parallel with $\boldsymbol{r_x}$, which is in series with $(\boldsymbol{R_B} \| \boldsymbol{R_{sig}})$. This way of finding the resistance "seen by a capacitance" is very useful.



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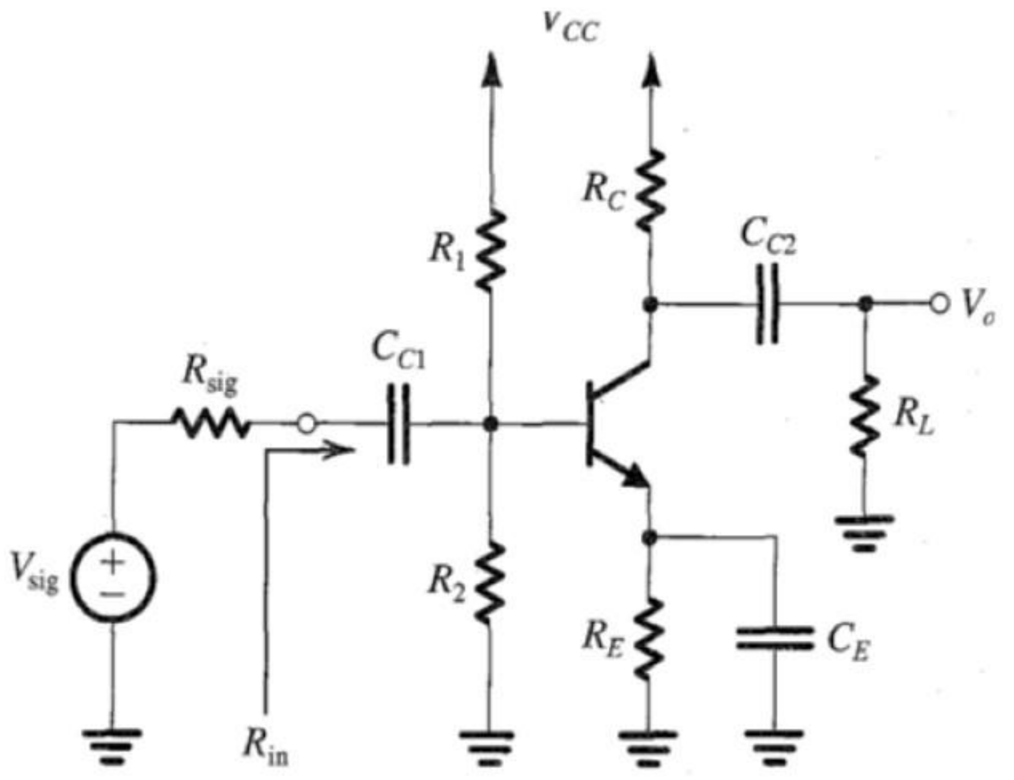
Example

- Consider the common-emitter amplifier of Fig. under the following conditions: $R_{sig} = 5$ kΩ, $R_1 = 33$ kΩ, $R_2 = 22$ kΩ, $R_E = 3.9$ kΩ, $R_C = 4.7$ kΩ, $R_L = 5.6$ kΩ, $V_{CC} = 5$ V. The dc emitter current can be shown to be $I_E \approx 0.3$ mA, at which $\beta_o = 120$, $r_o = 300$ kΩ, and $r_x = 50$ Ω. Find the input resistance R_{in} and the midband gain A_M. If the transistor is specified to have $f_T = 700$ MHz and $C\mu = 1$ pF, find the upper 3-dB frequency f_H. (V_T= 25mV)



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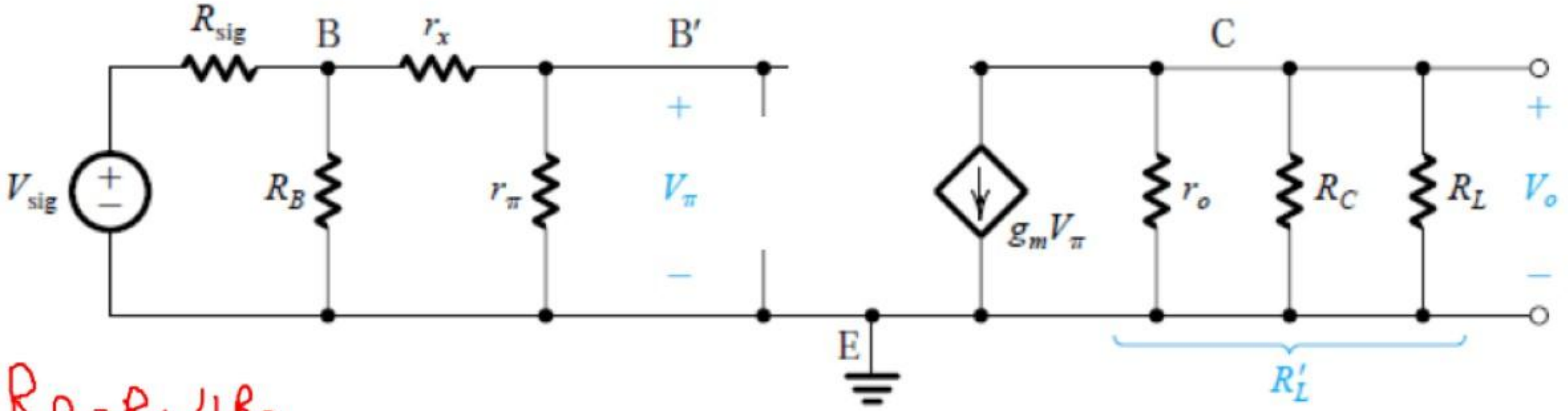
- Figure shows the equivalent circuit of a CE amplifier.



$R_B = R_1 // R_2$

$g_m = \frac{I_c}{V_T} = 12 \text{ mA/V}$

$r_\pi = \beta_o / g_m = \frac{120}{12} = 10 k\Omega$

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$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)}(g_m R_L') = -16.11 \text{ V/V}$$

$$R'_{sig} = r_\pi \| [r_x + (R_B \| R_{sig})] = 2.69\ k\Omega$$

$$R'_L = r_o \| R_C \| R_L = 2.53\ k\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12 \times 10^{-3}}{2\pi \times 700 \times 10^6}$$

$$= 2.73\ pf$$

$$C_\pi = 1.73\ pf$$

$$C_i = C_\pi + C_\mu(1 + g_m R'_L) = 33\ pf$$

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$$f_{Pi} = \frac{1}{2\pi C_i R'_{Sig}} = 1.79\ \text{MHz}$$

$$C_2 = C_\mu\left(1+\frac{1}{g_m R'_L}\right) = 1.03\ pf$$

$$f_{Po} = \frac{1}{2\pi C_2 R'_L} = 60.9\ \text{MHz}$$

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{Pi}}\right)^2 + \left(\frac{1}{f_{Po}}\right)^2}} \cong f_{pi} = 1.79\ \text{MHz}$$

E. Sawires

Thank You

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Have a Wonderful Semester