Electronic Systems

Active Filters

Lecture 7

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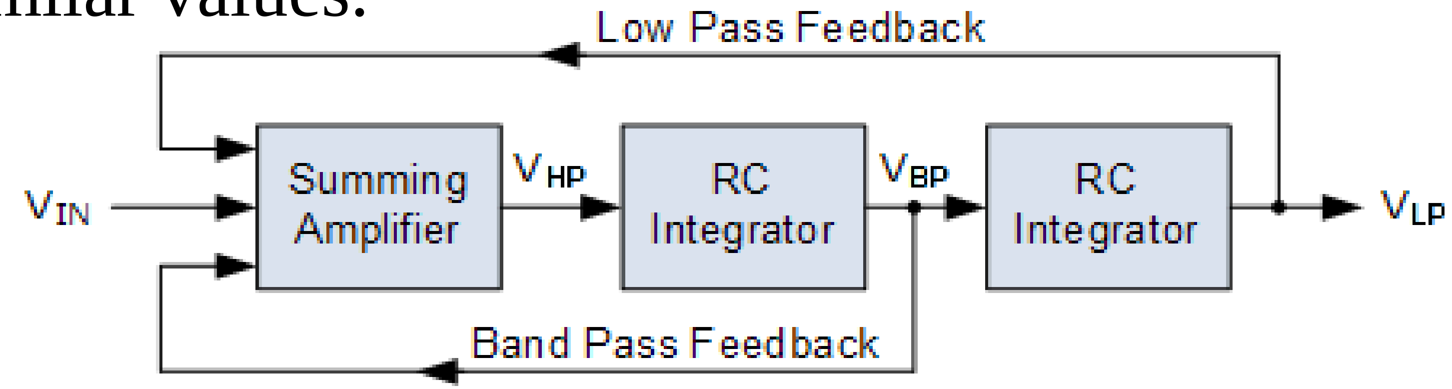
The Active Filters Contents:

State Variable Filter

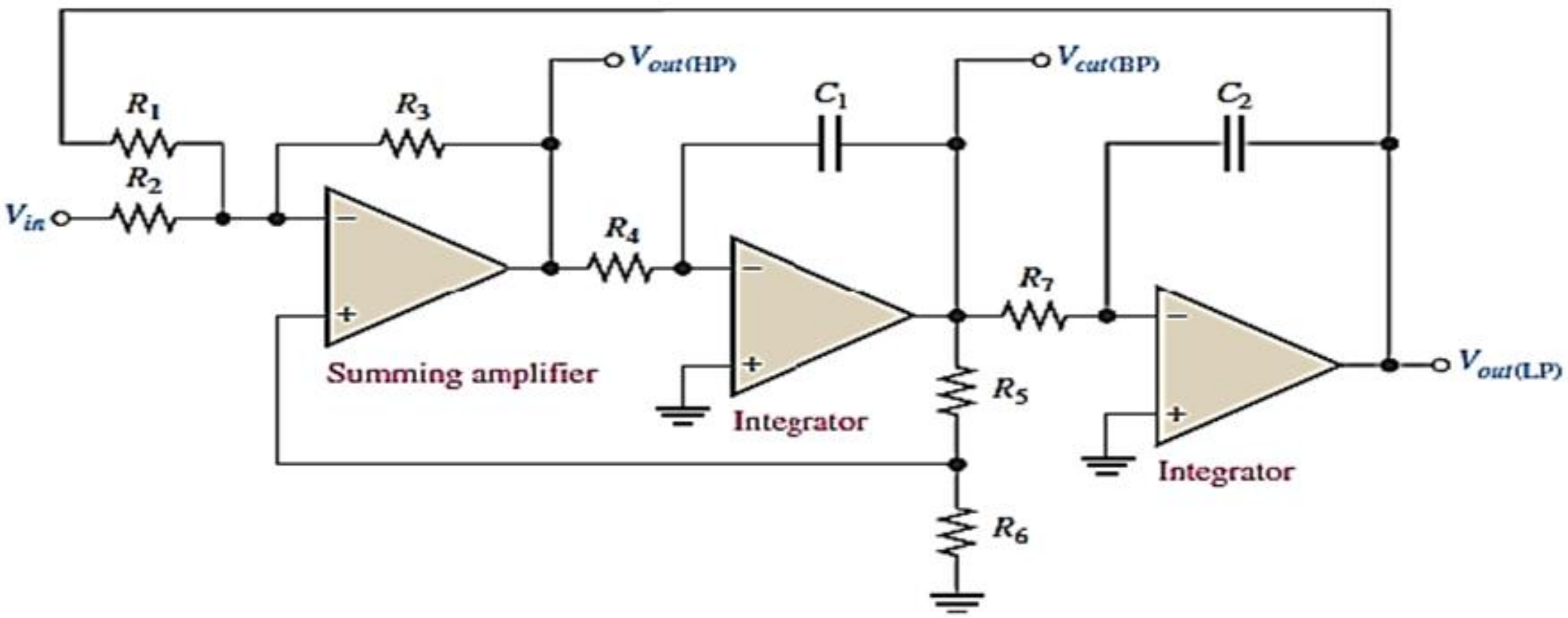
State Variable Filter (Update for KHN Filter)

- The state variable is also called universal active filter is generally used for bandpass projects applications.
- It can be seen in the figure that it comprises a summing amplifier and 2 operational amplifier integrators which are attached in cascaded configuration to create 2nd order filter configuration.
- Though it is used as a bandpass filter the state variable arrangement also gives the low pass and high pass outputs. The mid-frequency is adjusted by the RC circuitry in these to integrators circuits.
- When it operates as a bandpass filter the critical frequency of the integrators are generally has similar values.



State Variable Filter

- *State variable filters* are second-order RC active filters consisting of two identical op-amp integrators with each one acting as a first-order, single-pole low pass filter, a summing amplifier around which we can set the filters gain and its damping feedback network. The output signals from all three op-amp stages are fed back to the input allowing us to define the state of the circuit.



State Variable Filter

- One of the main advantages of a state variable filter design is that all three of the filters main parameters, Gain (A), corner frequency f_{C}, and the quality factor Q can be adjusted or set independently without affecting the filters performance.

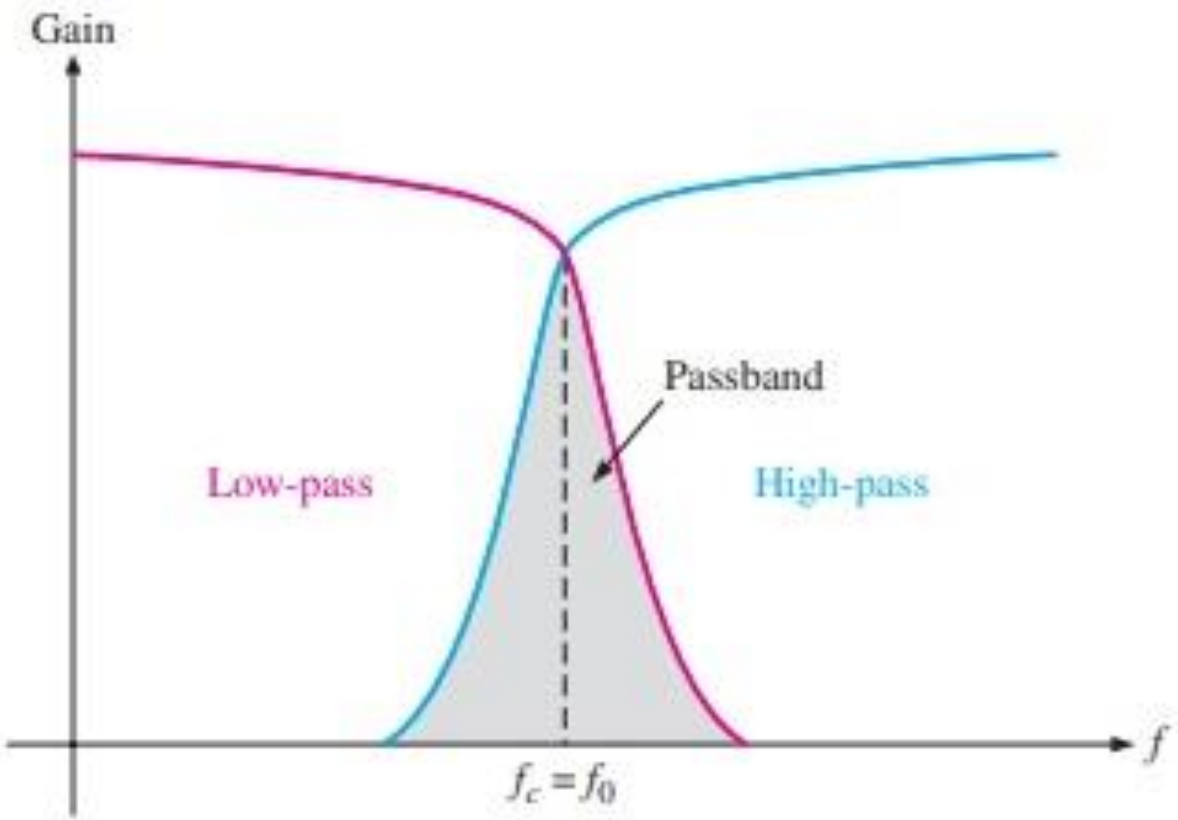
- In fact if designed correctly, the -3dB corner frequency, (fc) point for both the low pass amplitude response and the high pass amplitude response should be identical to the center frequency point of the band pass stage.

- That is $f_{LP(-3dB)}$ equals $f_{HP(-3dB)}$ which equals $f_{BP(center)}$.

State Variable Filter

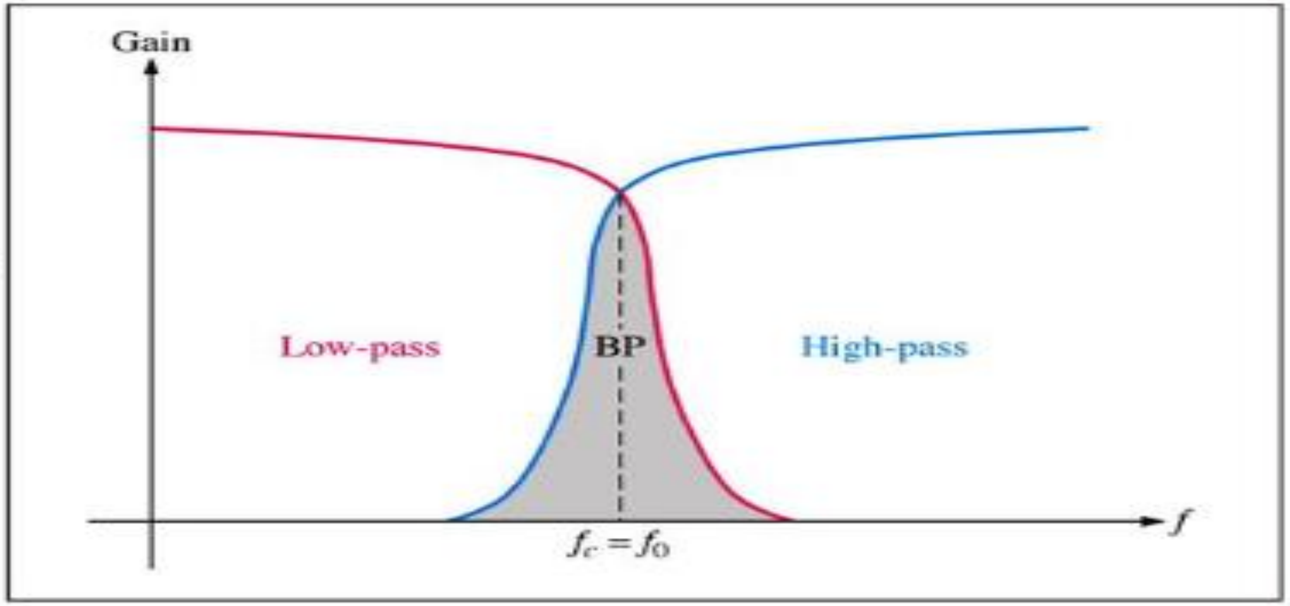
- Over the fc the low pass response diminishes so stopping the input signal from moving through the integrators. As a quencequence the bandpass filter results has a sharp peak at fc.

$$f_c = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2}$$



State Variable Filter

- It consists of a summing amplifier and two integrators.
- It has outputs for low-pass, high-pass, and band-pass.
- The center frequency is set by the integrator RC circuits.
- The critical frequency of the integrators usually made equal
- R_5 and R_6 set the Q (bandwidth).



- The band-pass output peaks sharply the center frequency giving it a high Q.
- The Q is set by the feedback resistors R_5 and R_6 according to the following equations :

$$\boxed{Q = \frac{1}{3}\left(1 + \frac{R_5}{R_6}\right)}$$

$$f_c = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2}$$

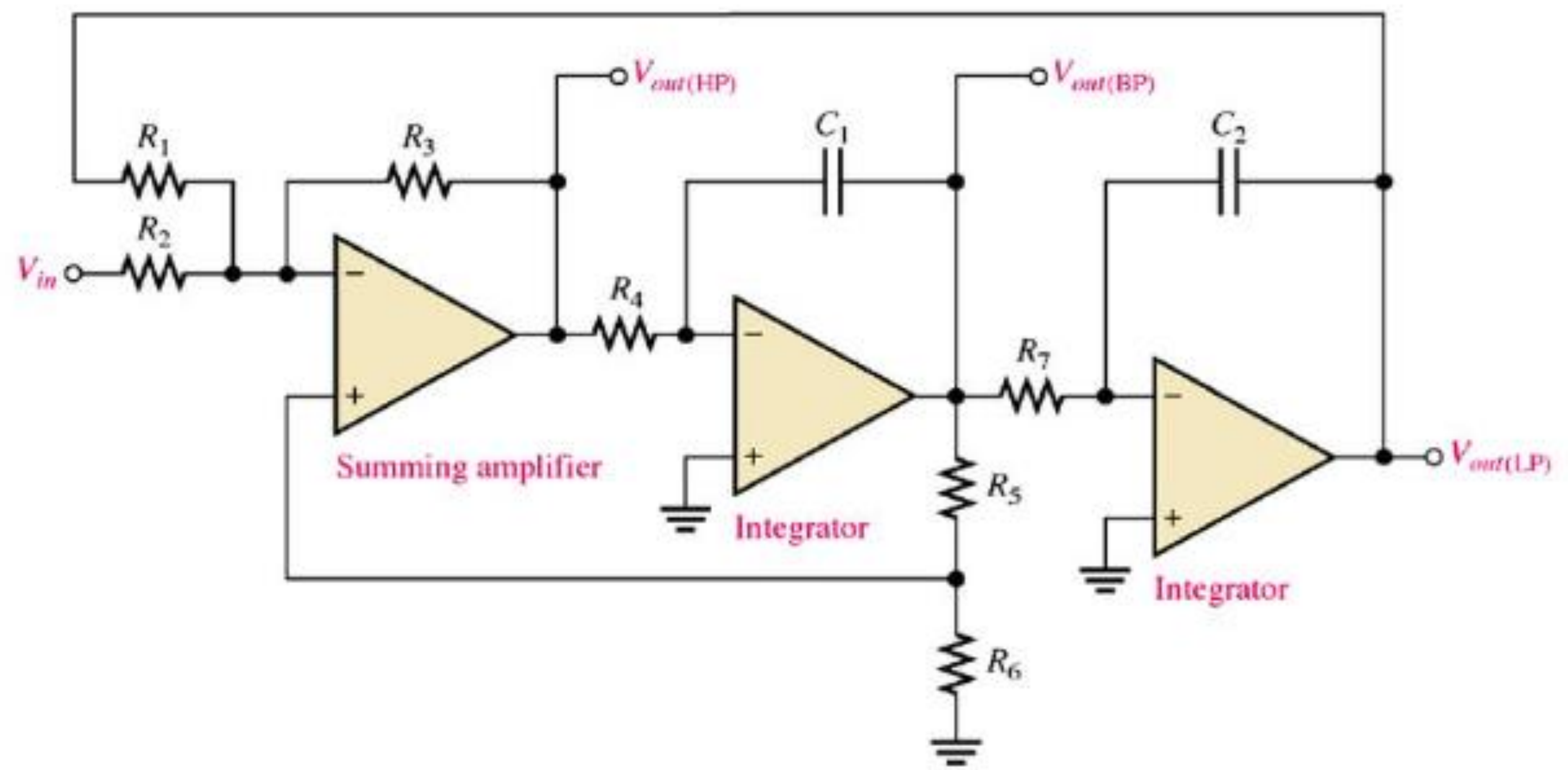
State Variable Filter (Update for KHN Filter)

- The KHN biquad filter is like to the state variable filter with the difference that it comprises of the integrator, with the inverting amplifier and another category of the integrator.

- Such differences in the arrangements among the KHN biquad and stable variable filter arrangement cause in some operation difference though both permit a large value of Q.

- In a KHN biquad filter, the Q is dependent on the critical frequency though in-state variable filter it is only the reverse the bandwidth is dependent and the Q is non-dependent on the critical frequency.

State Variable Filter



State Variable Filter

$$* \quad V_x = \left(\frac{V_{BP}}{R_5+R_6}\right)\cdot R_6$$

$$\boxed{V_x = \frac{R_6}{R_5+R_6} V_{BP} = K\, V_{BP}} \quad (1) \qquad K = \frac{R_6}{R_5+R_6}$$

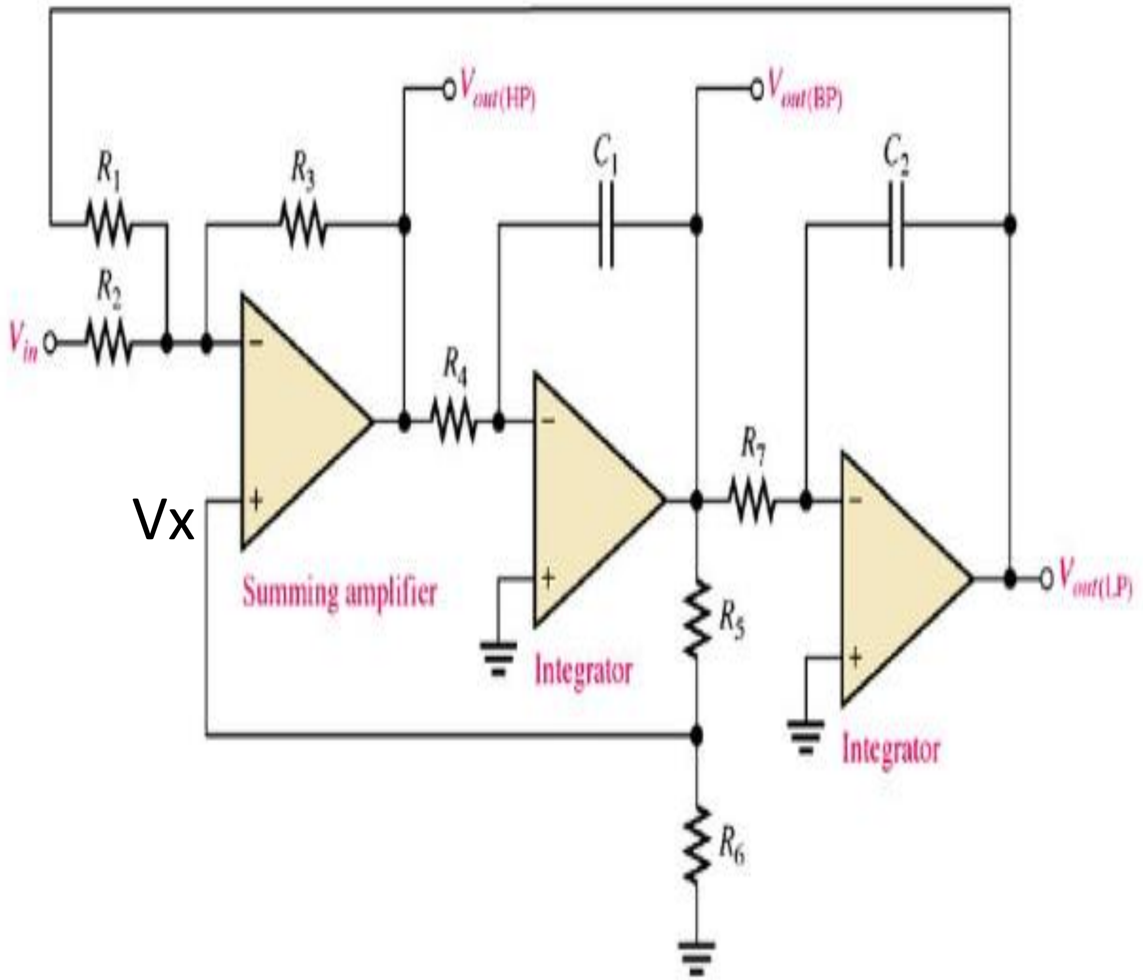
$$* \quad V_{hp} = -\frac{R_3}{R_1} V_{LP} - \frac{R_3}{R_2} V_{in} + \left(1+\frac{R_3}{R_1 \| R_2}\right) V_x$$

$$\boxed{V_{hp} = -\frac{R_3}{R_1} V_{LP} - \frac{R_3}{R_2} V_{in} + \left(1+\frac{R_3}{R_1 \| R_2}\right) K\, V_{BP}} \quad (2)$$

$$* \quad \boxed{V_{BP} = -\frac{1}{sC_1R_4} V_{hp}} \quad (3)$$

$$* \quad V_{LP} = \left(-\frac{1}{sC_1R_4}\right)\left(-\frac{1}{sC_2R_7}\right) V_{hp}$$

$$\boxed{V_{LP} = \frac{1}{s^2 C_1 C_2 R_4 R_7} V_{hp}} \quad (4)$$



State Variable Filter

Sub. into ②

$$\therefore V_{hp} = -\frac{R_3}{R_1}\frac{1}{s^2 C_1 C_2 R_4 R_7} V_{hp} - \frac{R_3}{R_2} V_{in}$$

$$+ \left(1 + \frac{R_3}{R_1 \| R_2}\right) K \frac{-1}{s C_1 R_4} V_{hp}$$

$$V_{hp}\left[1 + \frac{R_3/R_1}{s^2 C_1 C_2 R_4 R_7} + \frac{K\left(1+\frac{R_3}{R_1\|R_2}\right)}{s C_1 R_4}\right] = -\frac{R_3}{R_2} V_{in}$$

$$V_{hp}\left[\frac{s^2 C_1 C_2 R_4 R_7 + \frac{R_3}{R_1} + K\left[1+\frac{R_3}{R_1\|R_2}\right] s C_2 R_7}{s^2 C_1 C_2 R_4 R_7}\right] = -\frac{R_3}{R_2} V_{in}$$

$$\therefore T_{HP} = \frac{V_{hp}}{V_{in}} = \frac{-\frac{R_3}{R_2} s^2 C_1 C_2 R_4 R_7}{s^2 C_1 C_2 R_4 R_7 + K\left[1+\frac{R_3}{R_1\|R_2}\right] s C_2 R_7 + \frac{R_3}{R_1}}$$

$$T_{HP} = \frac{-\frac{R_3}{R_2} s^2 \cancel{C_1 C_2 R_4 R_7}}{\cancel{C_1 C_2 R_4 R_7}\left[s^2 + \frac{K\left(1+\frac{R_3}{R_1\|R_2}\right)}{C_1 \cancel{C_2} R_4 \cancel{R_7}} s \cancel{C_2 R_7} + \frac{R_3}{R_1}\frac{1}{C_1 C_2 R_4 R_7}\right]}$$

State Variable Filter

$$T_{HP} = \frac{-\frac{R_3}{R_2} s^2}{s^2 + \frac{1}{C_1 R_4} K \left(1 + \frac{R_3}{R_1 \| R_2}\right) s + \frac{R_3}{R_1} \frac{1}{C_1 C_2 R_4 R_7}}$$

$$\frac{R_3}{R_1} = 1, \boxed{R_3 = R_1}$$

$$T_{HP} = \frac{-\frac{R_3}{R_2} s^2}{s^2 + \frac{1}{C_1 R_4} K \left(1 + \frac{R_3}{R_1 \| R_2}\right) s + \frac{1}{C_1 C_2 R_4 R_7}}$$

$$T_{HP} = \frac{-a s^2}{s^2 + \left(\frac{\omega_0}{a}\right) s + \omega_0^2}$$

$$\therefore \omega_0 = \frac{1}{\sqrt{C_1 C_2 R_4 R_7}}$$

$$\boxed{f_0 = \frac{1}{2\pi \sqrt{C_1 C_2 R_4 R_7}}}$$

$$C_1 = C_2 = C, \quad R_4 = R_7 = R, \quad \omega_0 = \frac{1}{C_1 R_4} = \frac{1}{C_2 R_7}$$

$$\boxed{f_0 = \frac{1}{2\pi R C}} = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2}$$

State Variable Filter

$$\frac{\omega_0}{Q} = \frac{1}{C_1 R_4} \frac{R_6}{R_5+R_6} \left[1 + \frac{R_3}{\frac{R_1 R_2}{R_1+R_2}}\right]$$

$$\frac{\omega_0}{Q} = \frac{1}{C_1 R_4} \frac{1}{\frac{R_5}{R_6}+1} \left[1 + \frac{R_1+R_2}{R_2}\right]$$

Choose $\boxed{R_1 = R_2}$

$$\frac{\omega_0}{Q} = \frac{1}{C_1 R_4} \frac{1}{\frac{R_5}{R_6}+1} [1+2]$$

$$\frac{\omega_0}{Q} = \frac{1}{C_1 R_4} \frac{3}{\frac{R_5}{R_6}+1}$$

$$\therefore \quad Q = C_1 R_4 \, \omega_0 \, \frac{1}{3}\left[1 + \frac{R_5}{R_6}\right]$$

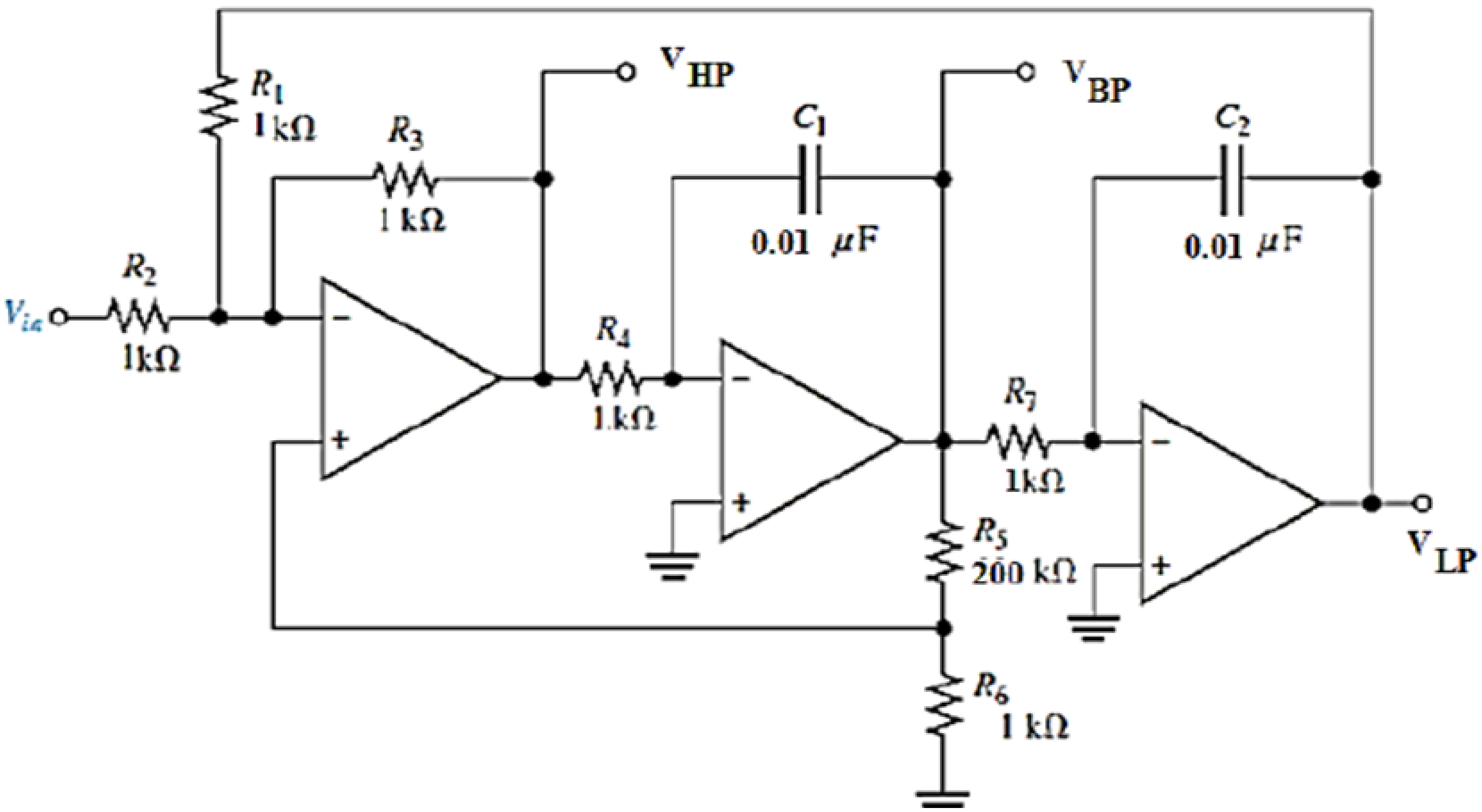
$$Q = C_1 R_4 \frac{1}{C_1 R_4} \frac{1}{3}\left[1 + \frac{R_5}{R_6}\right]$$

$$Q = \frac{1}{3}\left[1 + \frac{R_5}{R_6}\right]$$

State Variable Filter

- **Example 1:**

Determine the center frequency, Q, and BW for the state-variable filter shown.



State Variable Filter

Solution For each integrator,

$$f_c = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2} = \mathbf{15.92KHz}$$

The center frequency is approximately equal to the critical frequencies of integrators.

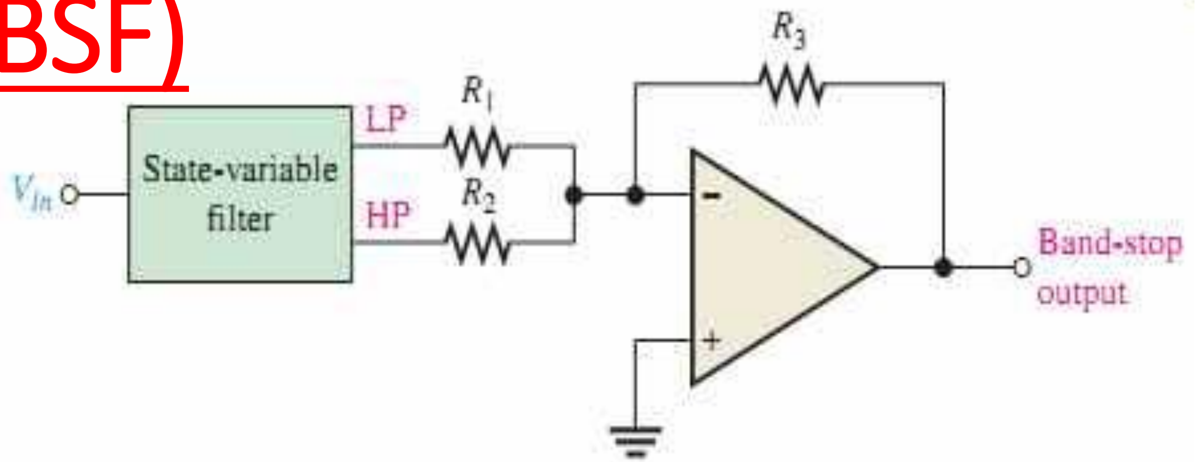
$$f_0 = f_c = \mathbf{15.92KHz}$$

$$Q = \frac{1}{3}\left(\frac{R_5}{R_6} + 1\right) = \frac{1}{3}\left(\frac{200\,\text{k}\Omega}{1.0\,\text{k}\Omega} + 1\right) = 67$$

$$BW = \frac{f_0}{Q} = \frac{15.92\text{KHz}}{67} = 237.5\text{Hz}$$

State Variable Notch Filter (BSF)

- Summing the low-pass and the high-pass responses of the state-variable filter with a summing amplifier creates a band-stop filter.
- One important application of this filter is minimizing the 60 Hz "hum" in audio systems by setting the center frequency to 60 Hz.

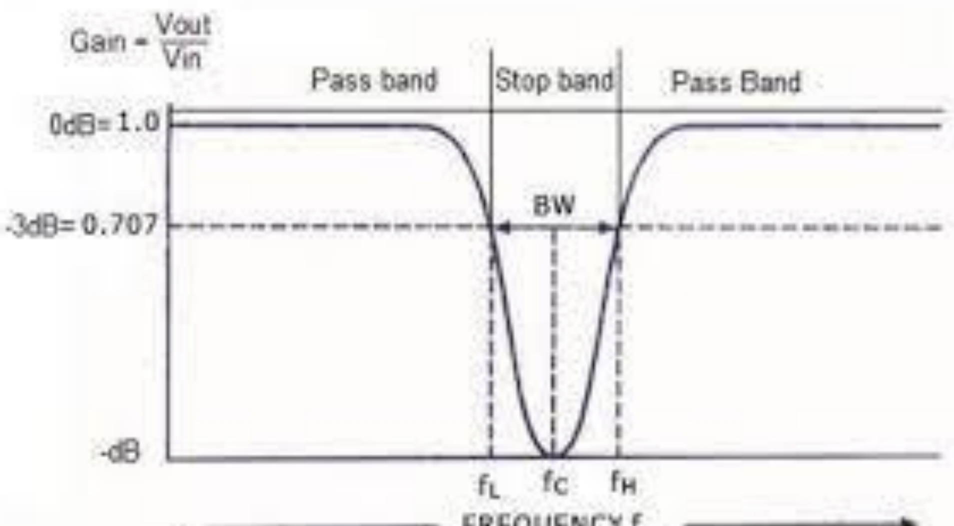


$$H(s) = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

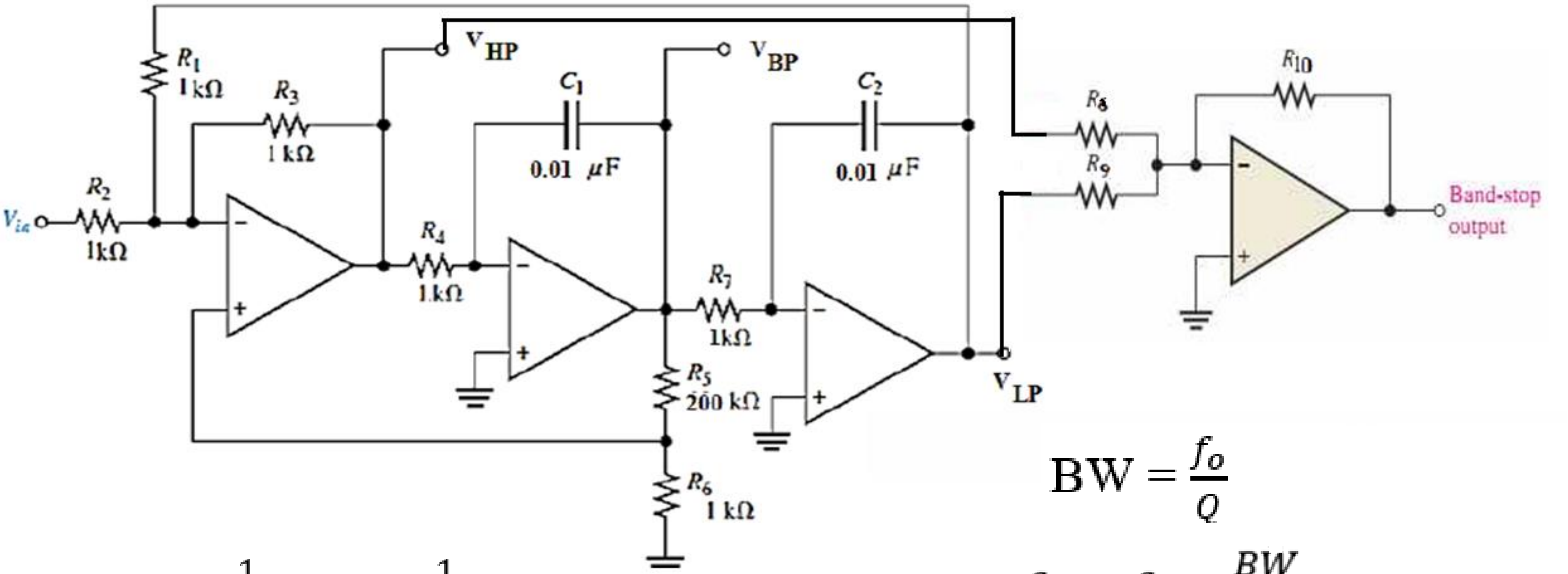
$$H_{LP} = \frac{a_o}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

$$H_{HP} = \frac{a_2 s^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

$$H_{BP} = \frac{a_1 s}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$



State Variable Notch Filter (BSF)



$$\mathrm{BW} = \frac{f_o}{Q}$$

$$f_o = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2}$$

$$f_l = f_o - \frac{BW}{2}$$

$$f_H = f_o + \frac{BW}{2}$$

State Variable Notch Filter (BSF)

- **Example2:**

For the state variable Notch filter shown in last slide:

1- Calculate the notch frequency f_o.

2- Design R5 and R6 for a quality factor Q of 20.

3- Calculate the lower and upper cut-off frequencies.

- Solution:

$$f_o = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2} = \frac{1}{2\pi(10^4)(0.1x10^{-6})} = 159.15\ Hz$$

$$Q = 20 = \frac{1}{3}[\frac{R_5}{R_6} + 1]$$

$\frac{R_5}{R_6} = 59$ $\quad$ $choose$ R_6 =1KΩ, R_5= 59

$$\text{BW} = \frac{f_o}{Q} = 8Hz$$

$f_l = f_o - \frac{BW}{2}$ =155.15Hz $\quad$ $f_H = f_o + \frac{BW}{2}$ =163.15Hz