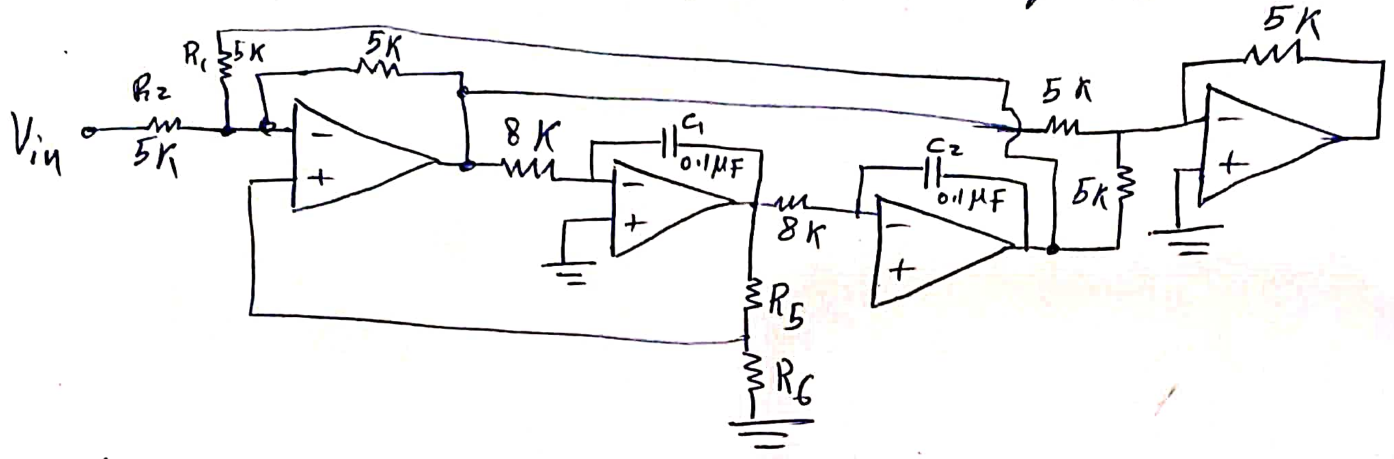
1. For the state variable BSF shown.
- calculate the center frequency (f_o).
- Design the value of R_5 & R_6 for a quality factor (Q) of 20!



sol :

$$f_o = \frac{1}{2\pi R_4 C_1} = \frac{1}{2\pi R_7 C_2} = \frac{1}{2\pi (8*10^3)(0.1*10^{-6})}$$

$$= 198.94 \text{ Hz}$$

$$\because Q = \frac{1}{3}\left[1 + \frac{R_5}{R_6}\right]$$

$$\Rightarrow 20 = \frac{1}{3} + \frac{1}{3}\frac{R_5}{R_6}$$

$$\Rightarrow \frac{R_5}{R_6} = 59 \Rightarrow \text{let } R_6 = 1K\Omega$$

$$\Rightarrow R_5 = 59 K\Omega$$

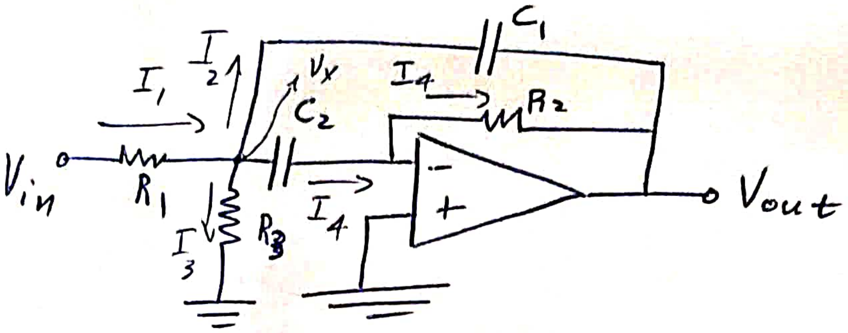
$$\Rightarrow B.W = \frac{f_o}{Q} = \frac{198.94}{20} = 9.947 \text{ Hz}$$

$$\Rightarrow f_l = f_o - \frac{BW}{2} = 198.94 - \frac{9.947}{2} = 193.96 \text{ Hz}$$

$$\Rightarrow f_h = f_o + \frac{BW}{2} = 198.94 + \frac{9.947}{2} = 203.91 \text{ Hz}$$

2. For the multiple feedback BPF shown

- Derive the Transfer function (V_o/V_{in})
- prove that:



$$f_o = \frac{1}{2\pi\sqrt{(R_1 \| R_3) R_2 C_1 C_2}}$$

$$R_1 = \frac{Q}{2\pi f_o C A_o}$$

$$R_2 = \frac{Q}{\pi f_o C}$$

$$R_3 = \frac{Q}{2\pi f_o C(2Q^2 - A_o)} \quad , \quad A_o = \frac{R_2}{2R_1}$$

sol:

$$\because I_4 = V_x s C_2 = -\frac{V_{out}}{R_2}$$

$$\Rightarrow \boxed{V_x = -\frac{V_{out}}{sC_2 R_2}} \quad \dots (1)$$

* KCL at node X:

$$\Rightarrow I_1 = I_2 + I_3 + I_4$$

$$\Rightarrow \frac{V_{in} - V_x}{R_1} = \frac{V_x - V_{out}}{\left(\frac{1}{sC_1}\right)} + V_x s C_2 + \frac{V_x}{R_3}$$

$$\Rightarrow \frac{V_{in}}{R_1} = V_x \left[\frac{1}{R_1} + \frac{1}{R_3} + sC_1 + sC_2\right] - V_{out} s C_1$$

$$\Rightarrow V_{in} = V_x \left[1 + \frac{R_1}{R_3} + sR_1(C_1 + C_2)\right] - V_{out} s C_1$$

sub. from (1):

$$\Rightarrow V_{in} = -V_{out}\left[\frac{1}{sC_2 R_2}\right]\left[\left(\frac{R_1 + R_3}{R_3} + sR_1(C_1 + C_2)\right) + R_2 C_2 (s^2 C_1 R_1)\right]$$

$$\Rightarrow A_v = \frac{V_{out}}{V_{in}}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-sC_2R_2}{\frac{R_1+R_3}{R_3} + sR_1(C_1+C_2) + s^2C_1C_2R_2R_1}$$

$$= \frac{-s\,\frac{\cancel{C_2}\cancel{R_2}}{C_1\cancel{C_2}\cancel{R_2}R_1}}{s^2 + s\,\frac{\cancel{R_1}(C_1+C_2)}{C_1C_2\cancel{R_1}R_2} + \frac{R_1+R_3}{R_1R_2R_3C_1C_2}}$$

$$= \frac{-s\left(\frac{1}{R_1C_1}\right)}{s^2 + s\,\frac{C_1+C_2}{R_2C_1C_2} + \frac{R_1+R_3}{R_1R_2R_3C_1C_2}}$$

$$= \frac{as}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$ (standard form for BPF)

By comparing our TF with the standard form:

$$\Rightarrow \omega_0^2 = \frac{R_1+R_3}{R_1R_2R_3C_1}$$

$$\Rightarrow \omega_0 = \sqrt{\frac{1}{(R_1//R_3)R_2}\cdot\frac{1}{C_1C_2}}$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{(R_1//R_3)R_2C_1C_2}}} = \frac{1}{2\pi C\sqrt{(R_1//R_3)R_2}}$$

$\hookrightarrow$ if $C_1 = C_2 = C$

$$\therefore \frac{\omega_0}{Q} = \frac{C_1+C_2}{R_2C_1C_2} = \frac{2\cancel{C}}{R_2C^{\cancel{2}}}$$

$$\Rightarrow R_2 = \frac{2Q}{\omega_0 C}$$

$$\Rightarrow \boxed{R_2 = \frac{\cancel{2}Q}{\cancel{2}\pi f_0 C} = \frac{Q}{\pi f_0 C}}$$

* Lets find gain (A_o) at w_o :

$$\Rightarrow \left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{a\,jw}{(jw)^2 + jw\left(\frac{w_o}{Q}\right) + w_o^2}\right|$$

$$\Rightarrow \left|\frac{V_{out}}{V_{in}}\right|_{w=w_o} = \frac{|a|\,w_o}{\sqrt{(\cancel{w_o^2} - \cancel{w_o^2})^2 + \left(\frac{w_o^2}{Q}\right)^2}} = \frac{a\,w_o}{\frac{w_o^{\cancel{2}}}{Q}} = \frac{|a|Q}{w_o}$$

* By comparing TF with the standard form :

$$\Rightarrow |a| = \frac{1}{R_1C_1} \quad , \quad \frac{w_o}{Q} = \frac{C_1+C_2}{R_2C_1C_2}$$

- if $c_1 = c_2 = C$:

$$\Rightarrow |a| = \frac{1}{R_1C} \qquad \Rightarrow \frac{w_o}{Q} = \frac{2\cancel{C}}{R_2C^{\cancel{2}}} = \frac{2}{R_2C}$$

$$\Rightarrow A_o = \frac{|a|Q}{w_o} = \frac{|a|}{\left(\frac{w_o}{Q}\right)} = \frac{1/R_1\cancel{C}}{\left(\frac{2}{R_2\cancel{C}}\right)} = \frac{R_2}{2R_1}$$

$$\Rightarrow \boxed{A_o = \frac{R_2}{2R_1}}$$

$$\because A_o = \frac{\frac{1}{R_1C}\,Q}{w_o} \Rightarrow R_1 = \frac{Q}{w_oCA_o}$$

$$\Rightarrow \boxed{R_1 = \frac{Q}{2\pi f_o C A_o}}$$

$$\because w_o^2 = \frac{R_1+R_3}{R_1R_2R_3C_1C_2}$$

* sub. by R_1 & R_2 :

$$\Rightarrow (R_1R_2R_3C^2)w_o^2 = R_1+R_3 \Rightarrow \left(\frac{Q}{\cancel{w_o}CA_o}\right)\left(\frac{2Q}{\cancel{w_oC}}\right)R_3\cancel{C^2}\cancel{w_o^2} = R_1+R_3$$

$$\Rightarrow \frac{2Q^2}{A_o}R_3 = R_1+R_3 = \frac{Q}{w_oCA_o} + R_3$$

$$\Rightarrow \frac{2Q^2}{A_0} R_3 = \frac{Q}{\omega_0 C A_0} + R_3$$

$$\Rightarrow R_3 \left[\frac{2Q^2}{A_0} - 1\right] = \frac{Q}{\omega_0 C A_0}$$

$$\Rightarrow R_3 \left[\frac{2Q^2 - A_0}{\cancel{A_0}}\right] = \frac{Q}{\omega_0 C \cancel{A_0}}$$

$$\boxed{R_3 = \frac{Q}{\omega_0 C\left[2Q^2 - A_0\right]} = \frac{Q}{2\pi f_0 C\,(2Q^2 - A_0)}}$$