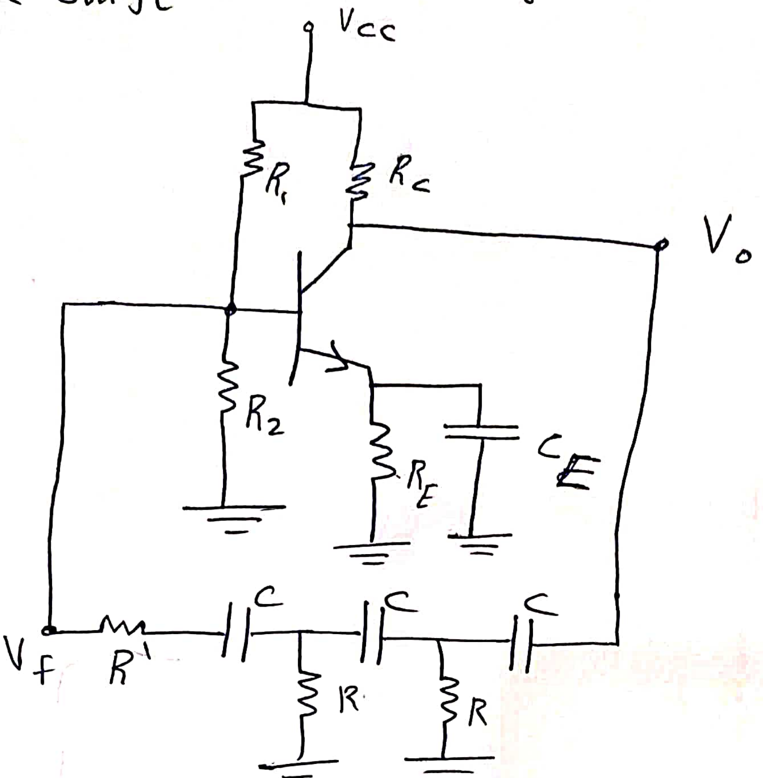
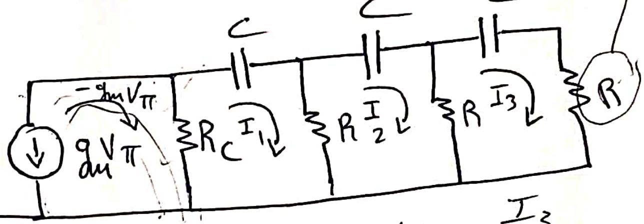
BJT phase shift oscillator :



$\underbrace{R' + (r_\pi // R_1 // R_2)}_{R}$

should be equal to R

* Breaking the f/b loop & taking I_b as i/P & having I_3 as o/p current $\Longrightarrow$



$(R' + (r_\pi // R_1 // R_2))$

$\hookrightarrow \underbrace{A\beta = \dfrac{I_3}{I_B}}$ we will depend on current gain here (more simple in this case)

* using mesh analysis :

$$\Rightarrow R_c(I_1 + g_m V_\pi) + \frac{I_1}{sC} + R(I_1 - I_2) = 0$$

$$\Rightarrow I_1\left[R_c + R + \frac{1}{sC}\right] + I_2[-R] = -g_m V_\pi R_c \qquad (1)$$

$$\Rightarrow (I_2 - I_1)R + \frac{I_2}{sC} + R(I_2 - I_3) = 0$$

$$\Rightarrow I_1[-R] + I_2\left[2R + \frac{1}{sC}\right] + I_3[-R] = 0 \qquad (2)$$

$$\Rightarrow R[I_3 - I_2] + \frac{I_3}{sC} + I_3 R = 0$$

$$\Rightarrow I_2[-R] + I_3\left[2R + \frac{1}{sC}\right] = 0 \qquad (3)$$

$$\Rightarrow \Delta = \begin{array}{ccc} + & - & + \\ \end{array}$$

$$\Rightarrow \Delta = \begin{vmatrix} R_c + R + \frac{1}{sC} & -R & 0 \\ -R & 2R + \frac{1}{sC} & -R \\ 0 & -R & 2R + \frac{1}{sC} \end{vmatrix} = \left(R_c + R + \frac{1}{sC}\right)\left[\overbrace{\left(2R + \frac{1}{sC}\right)^2}^{4R^2 + \frac{4R}{sC} + \frac{1}{(sC)^2}} - R^2\right] + R\left[-R\left(2R + \frac{1}{sC}\right)\right]$$

$$\Rightarrow \Delta = R_c\left(3R^2 + \frac{4R}{sC} + \left(\frac{1}{sC}\right)^2\right) + R\left(3R^2 + \frac{4R}{sC} + \left(\frac{1}{sC}\right)^2\right) + \frac{1}{sC}\left[3R^2 + \frac{4R}{sC} + \left(\frac{1}{sC}\right)^2\right] - R^2\left(2R + \frac{1}{sC}\right)$$

$$\Delta = R_c\left(3R^2 + \frac{4R}{sC} + \left(\frac{1}{sC}\right)^2\right) + R^3 + \frac{6R^2}{sC} + \frac{5R}{(sC)^2} + \frac{1}{(sC)^3}$$

$$\Rightarrow \Delta = R_c\left[3R^2 + \frac{4R}{j\omega C} + \left(-\frac{1}{\omega^2C^2}\right)\right] + R^3 + \frac{6R^2}{j\omega C} - \frac{5R}{\omega^2C^2} - \frac{1}{j\omega^3C^3}$$

$$\Rightarrow \boxed{\Delta = \left(3R^2R_c - \frac{R_c}{\omega^2C^2} + R^3 - \frac{5R}{\omega^2C^2}\right) + j\left(-\frac{4RR_c}{\omega C} - \frac{6R^2}{\omega C} + \frac{1}{\omega^3C^3}\right)}$$

$$\Rightarrow \Delta_3 = \begin{vmatrix} R_c + R + \frac{1}{sC} & -R & -g_m V_\pi R_c \\ -R & 2R + \frac{1}{sC} & 0 \\ 0 & -R & 0 \end{vmatrix}$$

$$= -g_m V_\pi R_c\left[R^2\right] = -g_m V_\pi R_c R^2$$

$$\Rightarrow A\beta = \frac{I_3}{I_b}$$

$$\Rightarrow A\beta = 1$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-g_m V_\pi R_c R^2}{\left(3R^2R_c - \frac{R_c}{\omega^2C^2} + R^3 - \frac{5R}{\omega^2C^2}\right) + j\left(-\frac{4RR_c}{\omega C} - \frac{6R^2}{\omega C} + \frac{1}{\omega^3C^3}\right)}$$

$$\Rightarrow I_b = \frac{V_\pi}{r_\pi}$$

$$\Rightarrow A\beta = \frac{I_3}{I_b} = \frac{-\overbrace{g_m r_\pi}^{\beta} R_c R^2}{\left(3R^2R_c - \frac{R_c}{\omega^2C^2} + R^3 - \frac{5R}{\omega^2C^2}\right) + j\left(-\frac{4RR_c}{\omega C} - \frac{6R^2}{\omega C} + \frac{1}{\omega^3C^3}\right)}$$

$$\Rightarrow -\frac{4RR_c}{\omega_0 C} - \frac{6R^2}{\omega_0 C} + \frac{1}{\omega_0^3C^3} = 0$$

$$\Rightarrow -\omega_0^2C^2\left[4RR_c + 6R^2\right] + 1 = 0$$

$$\Rightarrow \omega_0^2C^2 = \frac{1}{4RR_c + 6R^2} \Rightarrow \omega_0 = \frac{1}{RC\sqrt{6 + 4\frac{R_c}{R}}}$$

$$\Rightarrow A\beta = 1$$

at ω_0:

$$\Rightarrow \frac{-\beta R_C R^2}{3R^2R_C - \frac{R_C}{\omega_0^2C^2} + R^3 - \frac{5R}{\omega_0^2C^2}} = 1$$

$$\Rightarrow \frac{-\beta}{3 - \frac{1}{\omega_0^2C^2R^2} + \frac{R}{R_C} - \frac{5R}{\omega_0^2R^2C^2R_C}} = 1$$

$$\Rightarrow \beta = -3 + \frac{1}{\omega_0^2R^2C^2} - \frac{R}{R_C} + \frac{5R}{\omega_0^2R^2C^2R_C}$$

$\because \omega_0 = [RC\sqrt{6+4R_C/R}]^{-1}$

$$\Rightarrow \beta = -3 + \frac{\cancel{R^2C^2}(6+4\frac{R_C}{R})}{\cancel{R^2C^2}} - \frac{R}{R_C} + \frac{5R(\cancel{R^2C^2})(6+4\frac{R_C}{R})}{\cancel{R^2C^2}\,R_C}$$

$$= -3 + 6 + 4\frac{R_C}{R} - \frac{R}{R_C} + \frac{5R(6+4R_C/R)}{R_C}$$

$$= 3 + 4\frac{R_C}{R} - \frac{R}{R_C} + \frac{30R}{R_C} + 20$$

$$\boxed{\beta = 23 + 4\frac{R_C}{R} + \frac{29R}{R_C}}$$

$\hookrightarrow$ min value for β