Electronic - III

Lecture (4)

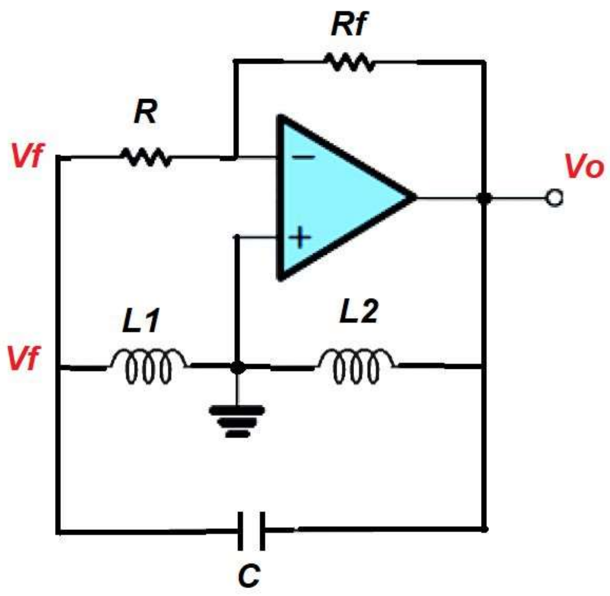
Oscillators

Crystal oscillator

Fall 2020

LC Oscillators

Hartley Oscillator

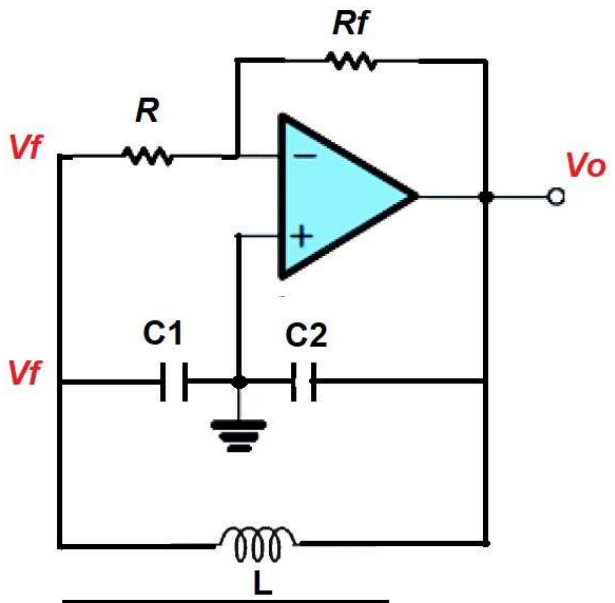


$$f_o = \frac{1}{2\pi\sqrt{L_T C}}$$

$$L_T = L_1 + L_2$$

$$\frac{R_f}{R} = \frac{L_2}{L_1}$$

Colpitts Oscillator



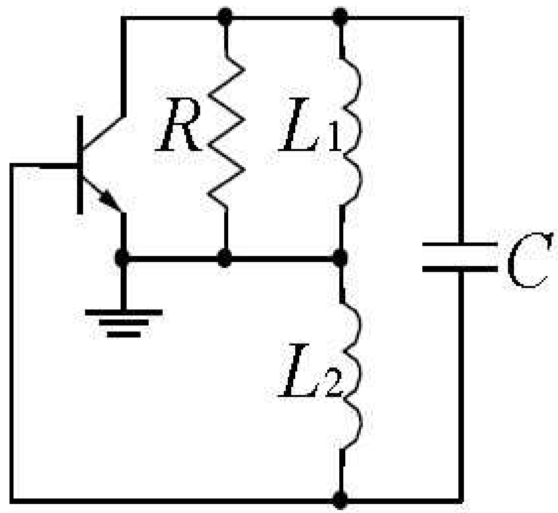
$$f_o = \frac{1}{2\pi\sqrt{L\,C_T}}$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$\frac{Rf}{R} = \frac{C2}{C1}$$

LC Oscillators with BJT Amplifier

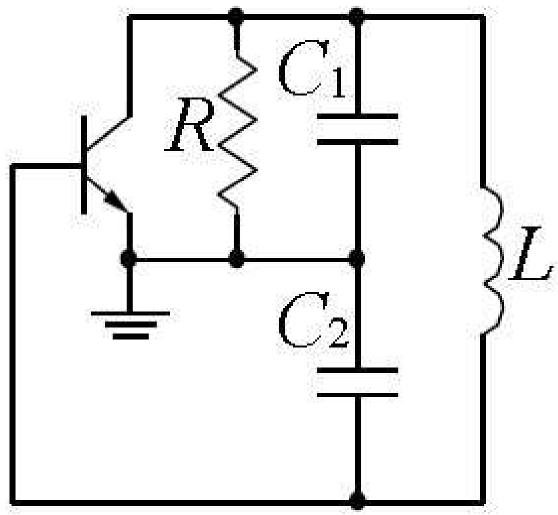
Hartley Oscillator



$$\omega_o = \frac{1}{\sqrt{(L_1 + L_2)C}}$$

$$g_m = \frac{L_1}{RL_2}$$

Colpitts Oscillator



$$\omega_o = \frac{1}{\sqrt{LC_T}} \quad C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$g_m = \frac{C_2}{RC_1}$$

Feedback Circuit Affects the Frequency of Oscillation

Loading of the Feedback Circuit Affects the Frequency of Oscillation

$$f_r = \frac{1}{2\pi\sqrt{LC_T}}\sqrt{\frac{Q^2}{Q^2+1}}$$

Where $Q = \frac{f_o}{B.W}$

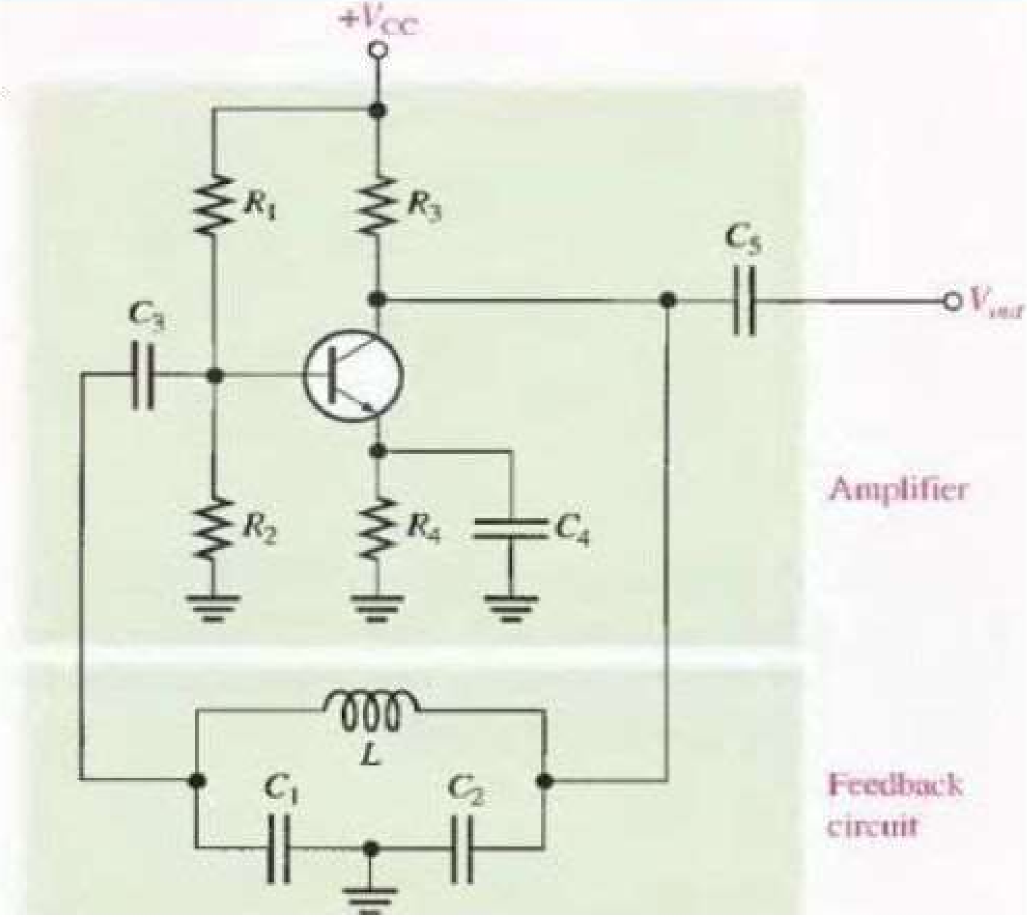
The quality Factor

A basic Colpitts oscillator with a BJT as the gain element

Colpitts Oscillator

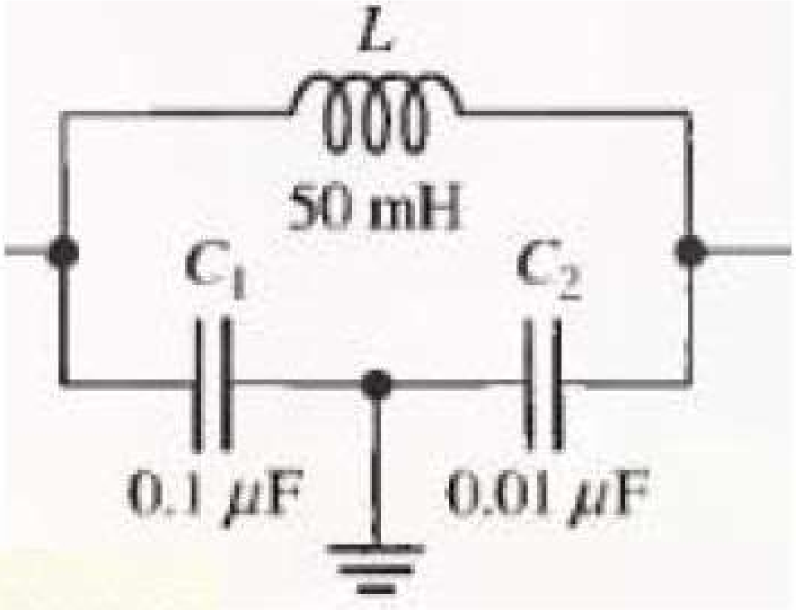
$$\omega_o = \frac{1}{\sqrt{LC_T}}$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$



Example

Determine the frequency for the oscillator. Assume there is negligible loading on the feedback circuit and that its Q is greater than 10.

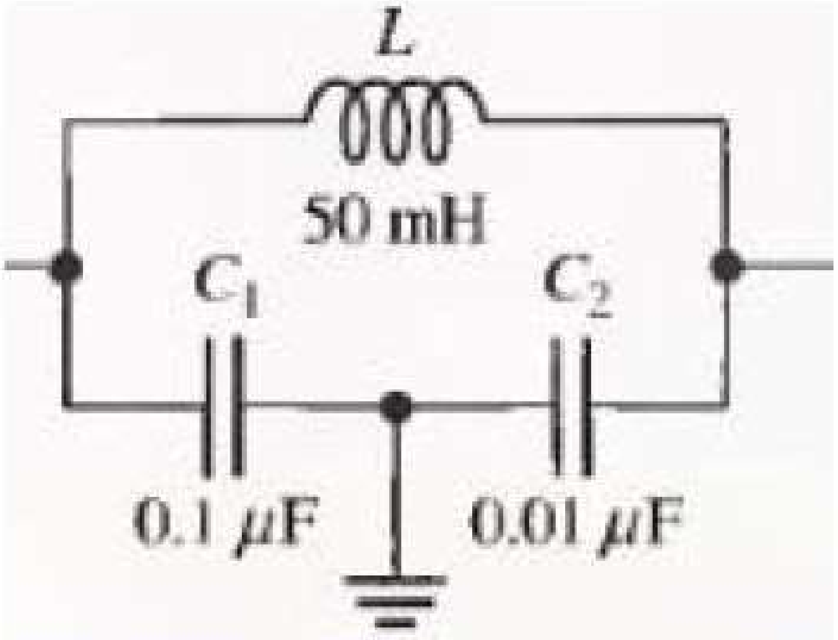


$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{(0.1\ \mu\text{F})(0.01\ \mu\text{F})}{0.11\ \mu\text{F}} = 0.0091\ \mu\text{F}$$

$$f_r \cong \frac{1}{2\pi\sqrt{LC_T}} = \frac{1}{2\pi\sqrt{(50\ \text{mH})(0.0091\ \mu\text{F})}} = 7.46\ \text{kHz}$$

Example

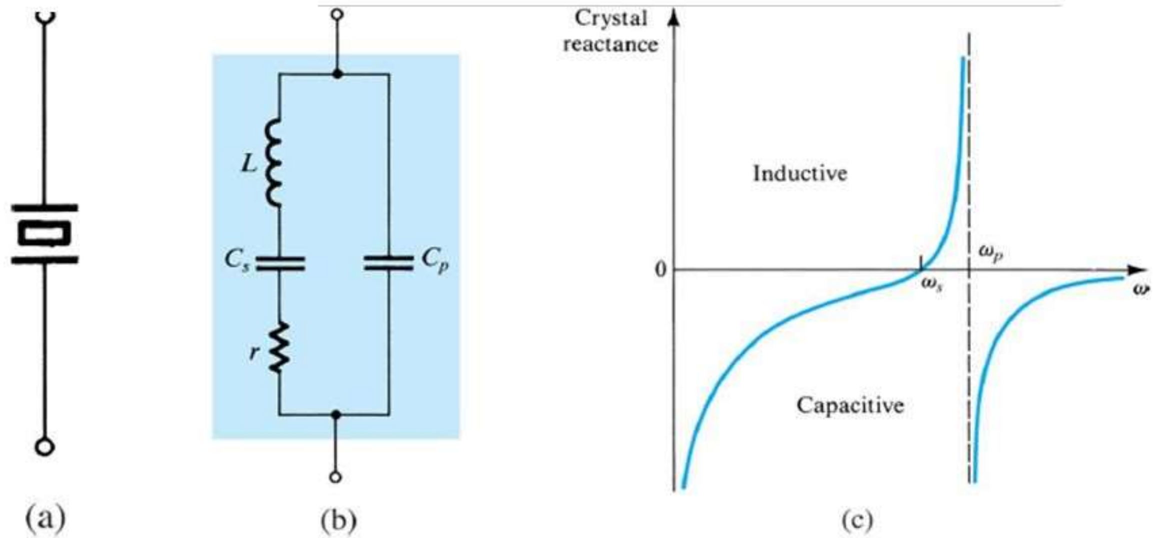
Find the frequency if the oscillator is loaded to a point where the Q drops to 8



$$f_r = \frac{1}{2\pi\sqrt{LC_T}}\sqrt{\frac{Q^2}{Q^2+1}} = (7.46\ \text{kHz})(0.9923) = 7.40\ \text{kHz}$$

Crystal Oscillators

A piezoelectric crystal, such as quartz, exhibits electromechanical-resonance characteristics that are very stable (with time and temperature) and highly selective (having very high Q factors). The resonance properties are characterized by a large inductance L (as high as hundreds of henrys), a very small series capacitance C_s (as small as 0.0005 pF), a series resistance r representing a Q factor $\omega_0 L/r$ that can be as high as a few hundred thousand, and a parallel capacitance C_p (a few picofarads). Capacitor C_p represents the electrostatic capacitance between the two parallel plates of the crystal. Note that $C_p \gg C_s$.



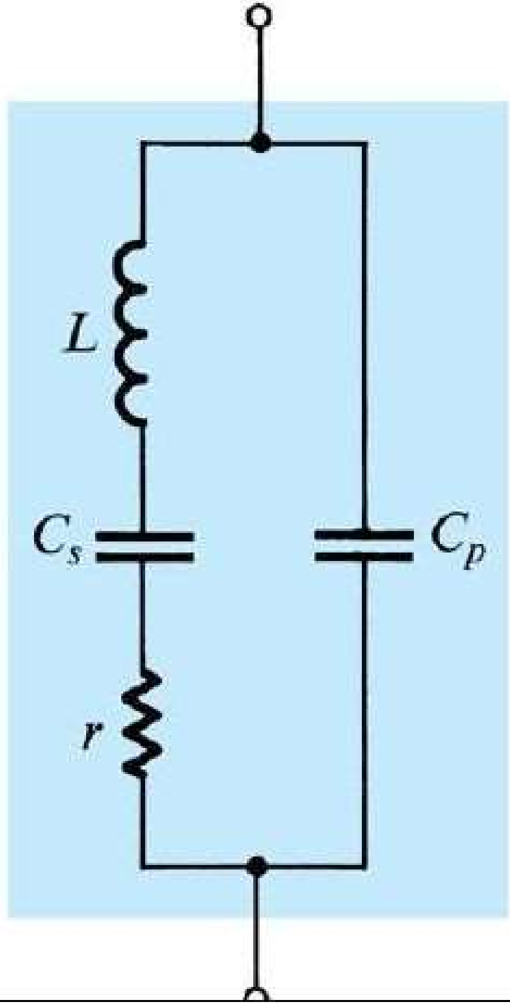
(a) Circuit symbol, **(b)** Equivalent circuit, **(c)** Crystal reactance versus frequency [note that, neglecting the small resistance (r) to get high Quality $Z_{crystal} = jX(w)$

Piezoelectric Crystal Oscillator

Since the Q factor is very high, we may neglect the resistance r and express the crystal impedance as

$$Z(s) = 1 \Big/ \left[sC_p + \frac{1}{sL + 1/sC_s} \right]$$

$$Z(s) = \frac{1}{sC_p} \frac{s^2 + (1/LC_s)}{s^2 + [(C_p + C_s)/LC_sC_p]}$$



Piezoelectric Crystal Oscillator

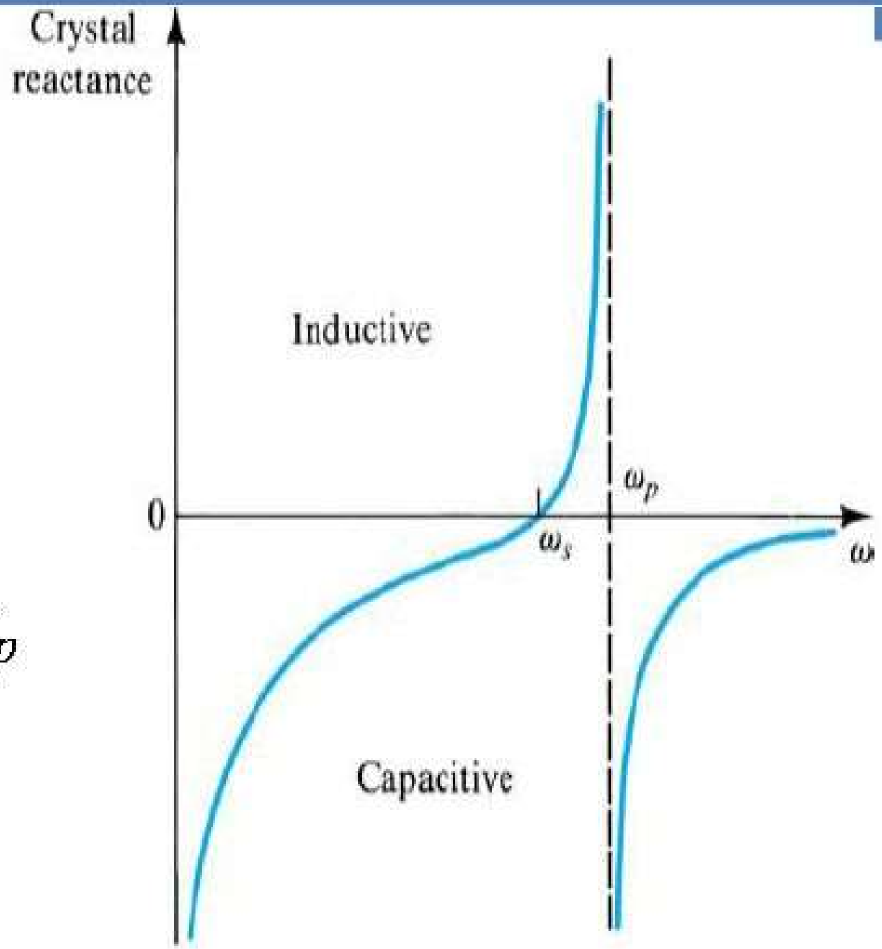
we see that the crystal has two resonance

a series resonance at w_s

$$\omega_s = 1/\sqrt{LC_s}$$

a parallel resonance at w_p

$$\omega_p = 1\Big/\sqrt{L\left(\frac{C_s C_p}{C_s + C_p}\right)}$$



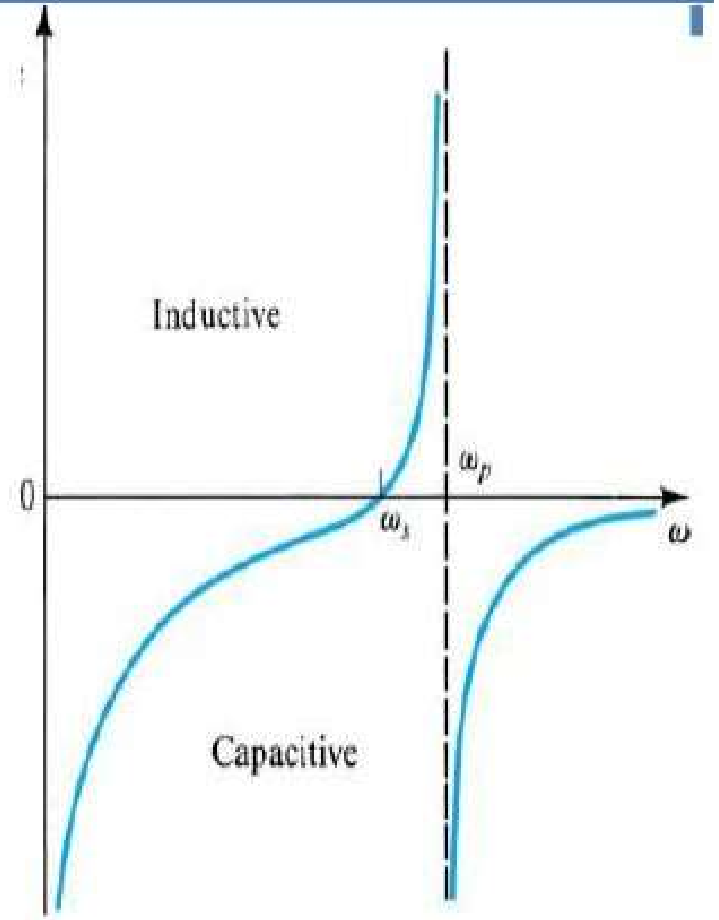
Piezoelectric Crystal Oscillator

For $s = jw$ we can write

$$Z(j\omega) = -j\frac{1}{\omega C_p}\left(\frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_p^2}\right)$$

The two resonance frequencies are very close when

$$C_p \gg C_s$$

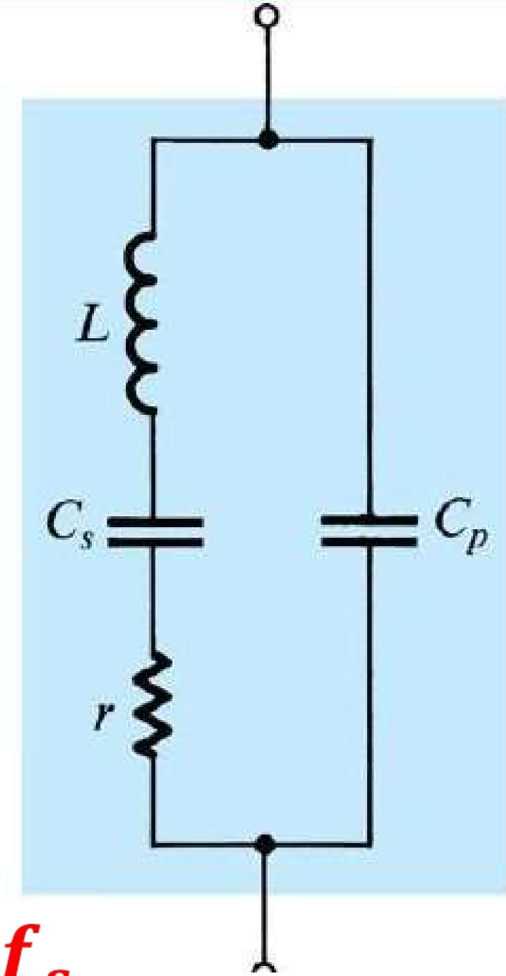


Piezoelectric Crystal Oscillator

The two resonance frequencies are very close when

$$C_p \gg C_s$$

$$\boxed{\omega_0 \simeq 1/\sqrt{LC_s} = \omega_s}$$



Quality Factor (Q)

$$Q = \frac{w_o L}{r}$$

$$w_o = 2\pi f_o = 2\pi f_s$$

$$Z = \frac{-jX_p[jX_L - jX_s]}{-jX_p + jX_L - jX_s} = \frac{X_p X_L - X_p X_s}{j[X_L - X_p - X_s]}$$

We have two resonance frequencies:

1. **Series Resonance Frequency (Z=r=0)**

$$X_p X_L - X_p X_s = 0$$

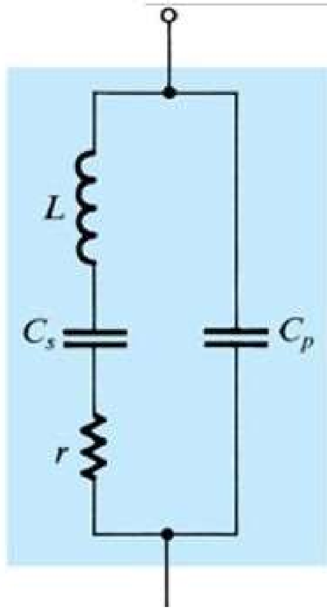
$$X_p X_L = X_p X_s$$

$$X_L = X_s$$

$$w_s L = \frac{1}{w_s C_s}$$

$$w_s = \frac{1}{\sqrt{L\,C_s}}$$

$$f_s = \frac{1}{2\pi\sqrt{L\,C_s}}$$



$$Z = \frac{-jX_p[jX_L - jX_s]}{-jX_p + jX_L - jX_s} = \frac{X_pX_L - X_pX_s}{j[X_L - X_p - X_s]}$$

2. Parallel Resonance Frequency (Z= ∞)

$$X_L - X_p - X_s = 0$$

$$X_L = X_p - X_s$$

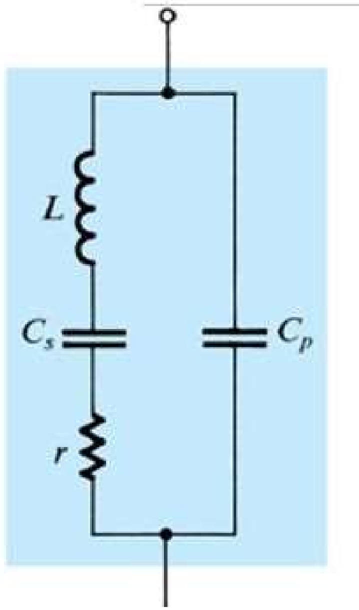
$$w_pL = \frac{1}{w_pC_s} + \frac{1}{w_pC_p}$$

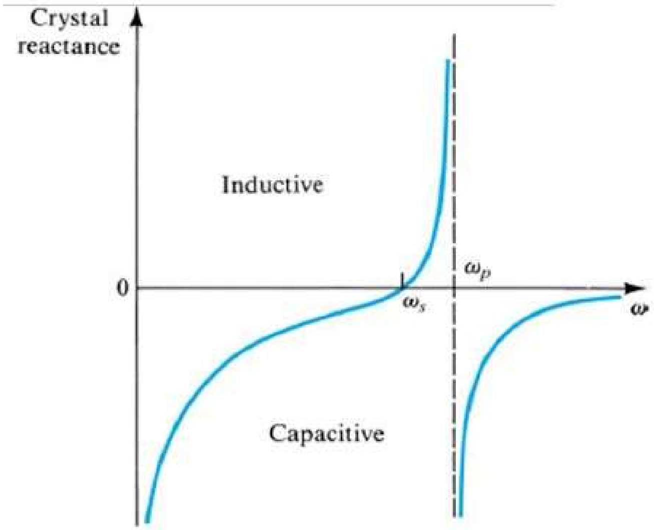
$$w_p{}^2L = \frac{1}{C_s} + \frac{1}{C_p} = \frac{C_p + C_s}{C_pC_s} = \frac{1}{\frac{C_pC_s}{C_p + C_s}}$$

$$w_p{}^2 = \frac{1}{L[\frac{C_pC_s}{C_p + C_s}]}$$

$$f_p = \frac{1}{2\pi\sqrt{L[\frac{C_pC_s}{C_p + C_s}]}}$$

Since $C_s << C_p$, then, $f_p \approx f_s$





Example:

Design a 5MHz crystal oscillator with C_s= 0.0005 PF , $C_p = 10\text{PF}$,
R = 120Ω and L = 2H.

1. Calculate The Series-Resonance Frequency (f_s).
2. Calculate The Parallel-Resonance Frequency (f_p).
3. Claculate the quality factor (Q).

Solution:

1. Calculate The Series-Resonance Frequency (f_s).

$$f_s = \frac{1}{2\pi\sqrt{L\,C_s}} = 5.0329 \text{ MHz}$$

2. Calculate The Parallel-Resonance Frequency (f_p).

$$f_p = \frac{1}{2\pi\sqrt{L[\frac{C_p C_s}{C_p + C_s}]}} = 5.033\text{MHz}$$

3. Claculate the quality factor (Q).

$$Q = \frac{w_o L}{r} = \frac{2\pi f_o\, L}{r} \qquad , \ f_o = 5\text{MHz}$$

Q = 524000.

A Pierce crystal oscillator utilizing a CMOS inverter as an amplifier.