Electronic Systems

Active Filters

Lecture 6

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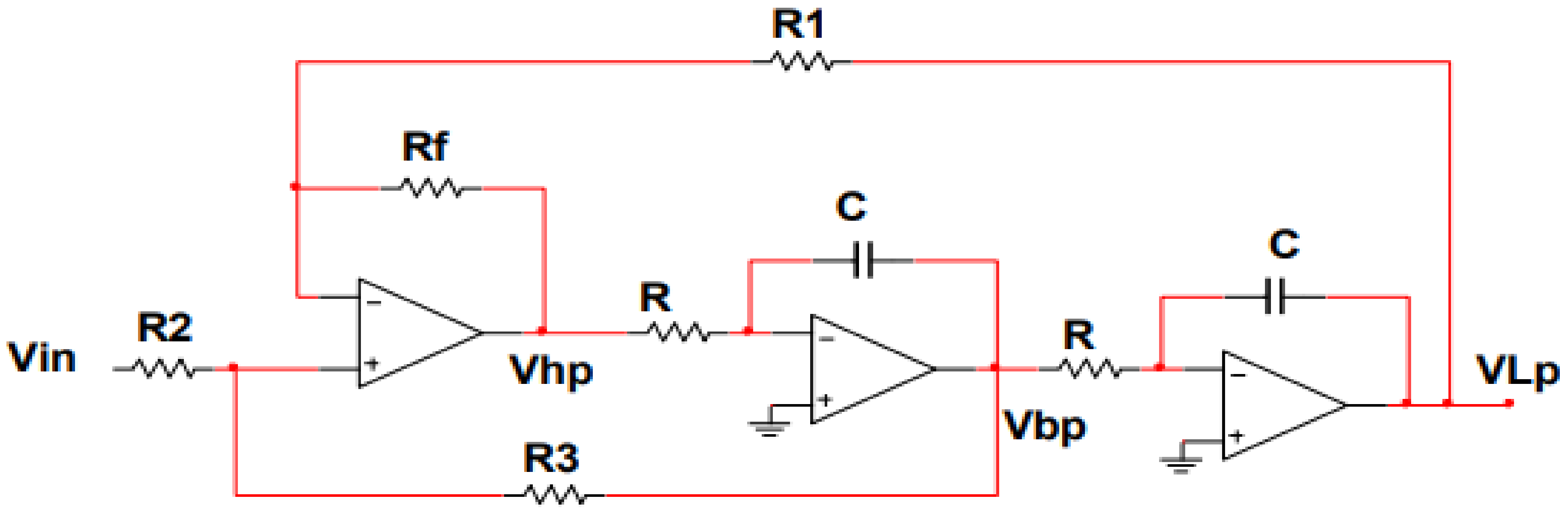
The Active Filters Contents:

1. Introduction to Filters.
2. Low Pass Filter.
3. High Pass Filter.
4. Band Pass Filter.
5. Butterworth Filter.
6. Chebyshev Filter.
7. Bessel Filter.
8. **KHN Biquad Filter.**
9. Multiple Feedback Filters.
10. State Variable Filters.

(KHN) Biquad filter (Summary)



Design Equations

* $\boxed{\frac{R_F}{R_1} = 1}$
* $\boxed{K = 2 - \frac{1}{Q}}$
* $\boxed{\frac{R_3}{R_2} = 2Q - 1}$
* $\boxed{f_o = \frac{1}{2\pi RC}}$
* For BPF

$$\boxed{Q = \frac{f_o}{B.W}}$$

$$f_L = f_o - \frac{B.W}{2}$$

$$f_H = f_o + \frac{B.W}{2}$$

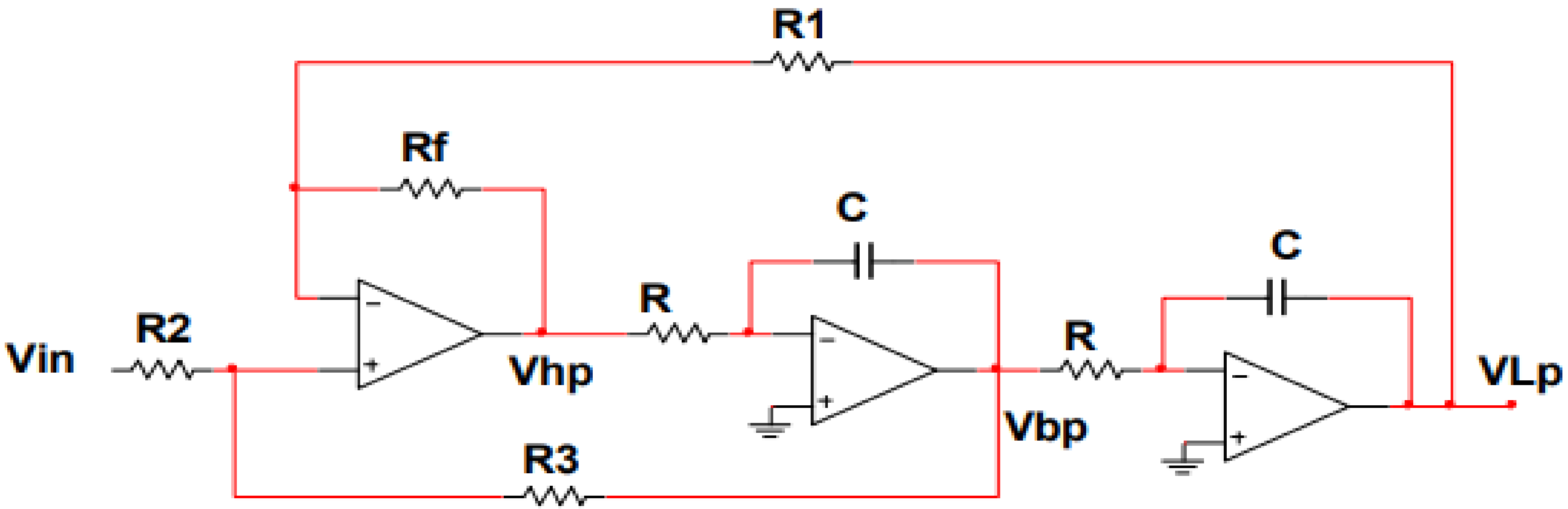
(KHN) Biquad filter

Example 1:

Design a KHN Filter to realize a BPF with center frequency of 10 KHz and bandwidth of 100Hz.

Calculate the center frequency gain (Am). Hint: use 1 nF capacitor.

Solution



Solution:

* $f_o = 10\times10^3 = \dfrac{1}{2\pi RC}$, $\boxed{C = 1nF}$

$\therefore\ R = \dfrac{1}{2\pi[10\times10^3][1\times10^{-9}]} = 15920\ \Omega$

$\boxed{R = 15.92\ k\Omega}$

* $\dfrac{R_F}{R_1} = 1 \rightarrow$ choose $\boxed{R_1 = 10k\Omega \rightarrow \therefore R_F = 10k\Omega}$

* Quality-Factor $Q = \dfrac{f_o}{B.W} = \dfrac{10\times10^3}{100} = 100$

$\boxed{Q = 100}$

* $K = 2 - \dfrac{1}{Q} = 2 - \dfrac{1}{100} \rightarrow \therefore \boxed{k = 1.99}$

* $\dfrac{R_3}{R_2} = 2Q - 1 = 2\times100 - 1$

$\dfrac{R_3}{R_2} = 199 \rightarrow$ choose $\therefore \boxed{\begin{array}{l} R_2 = 1k\Omega \\ R_3 = 199k\Omega \end{array}}$

How to get the center Frequency gain (Am)?

$$\frac{V_{hp}}{V_{in}} = \frac{K s^2}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2}$$

$$\therefore V_{hp} = \frac{K s^2}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2} V_{in}$$

but $V_{bp} = -\frac{\omega_o}{s} V_{hp}$

$$\therefore V_{bp} = \frac{K s^2}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2} \left(-\frac{\omega_o}{s}\right) V_{in}$$

$$\therefore \frac{V_{bp}}{V_{in}} = \frac{-K\omega_o s}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2} = H_{bp} = T_{bp}$$

$$\therefore H_{bp}(j\omega) = \frac{-jK\omega_o\omega}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right) j\omega + \omega_o^2}$$

$$T_{bp}(j\omega) = \frac{-jK\omega_o\omega}{-\omega^2 + j\frac{\omega_o\omega}{Q} + \omega_o^2}$$

$$T_{bp}(j\omega) = \frac{-jK\omega_o\omega}{[\omega_o^2 - \omega^2] + j\frac{\omega_o\omega}{Q}}$$

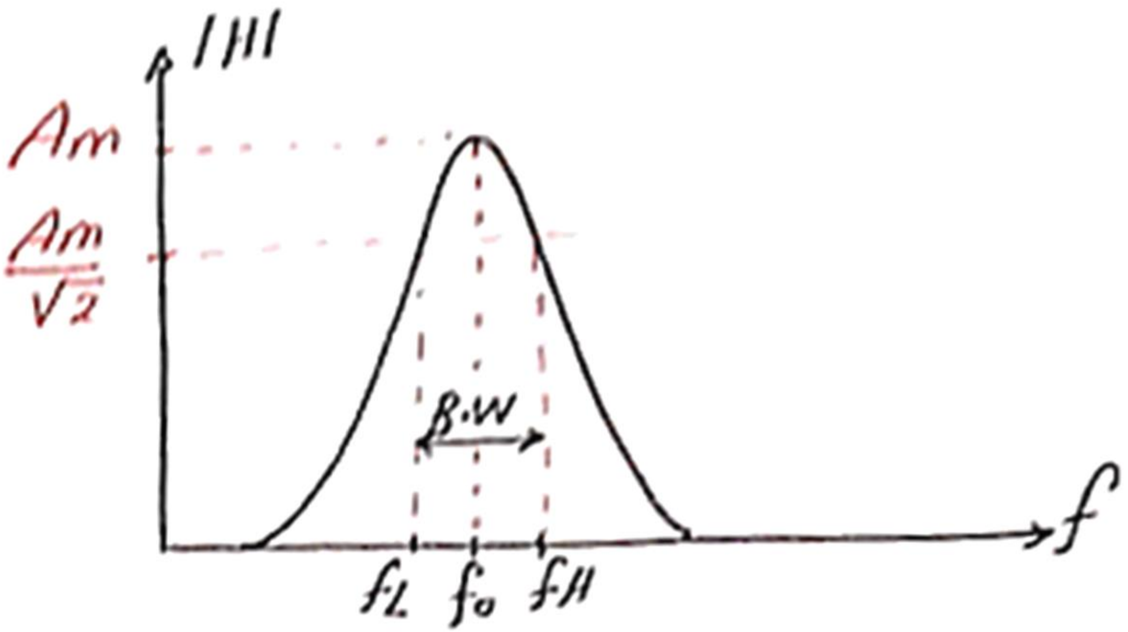
$$|T_{bp}| = \frac{K\omega_o\omega}{\sqrt{[\omega_o^2 - \omega^2]^2 + \left(\frac{\omega_o\omega}{Q}\right)^2}}$$

at $\omega = \omega_o$, $|T_{bp}| = A_m$ max. gain

$$\therefore A_m = \frac{K\omega_o . \omega_o}{\sqrt{\cancel{[\omega_o^2 - \omega_o^2]^2} + \left(\frac{\omega_o\omega_o}{Q}\right)^2}} = \frac{K\omega_o^2}{\frac{\omega_o^2}{Q}}$$

$$\boxed{A_m = K . Q}$$

- Center Frequency gain(Max. gain)



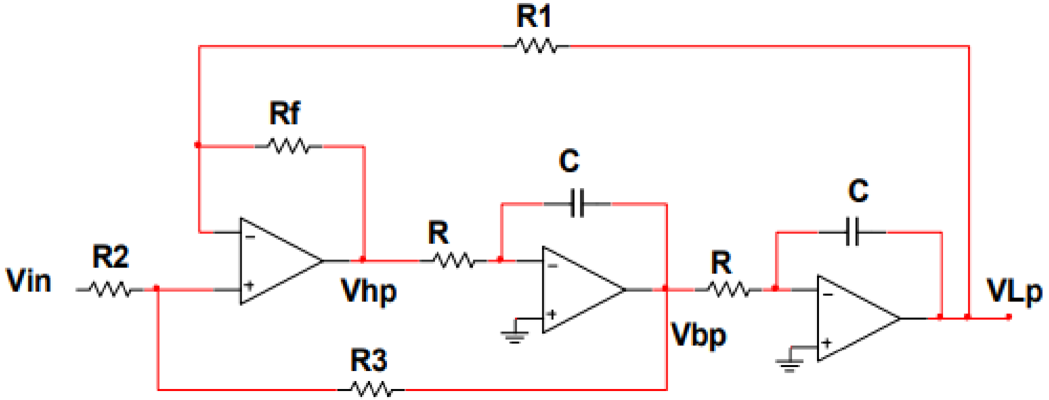
$$\times\ A_m = k.Q = 1.99 \times 10$$

$$\boxed{A_m = 199}$$

Example (2):

Design the KHN Filter to realize a HPF with a cut-off frequency 10KHz(Use C of 1 nF).

Solution:



$$H_{HP} = \frac{K s^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

$$|T_{hp}| = \frac{K\omega^2}{\sqrt{(\omega_o^2-\omega^2)^2+\left(\frac{\omega_o\omega}{Q}\right)^2}}$$

at $\omega = \omega_o$ $\quad |T_{hp}| = \frac{K\omega_o^2}{\frac{\omega_o^2}{Q}} = K.Q$

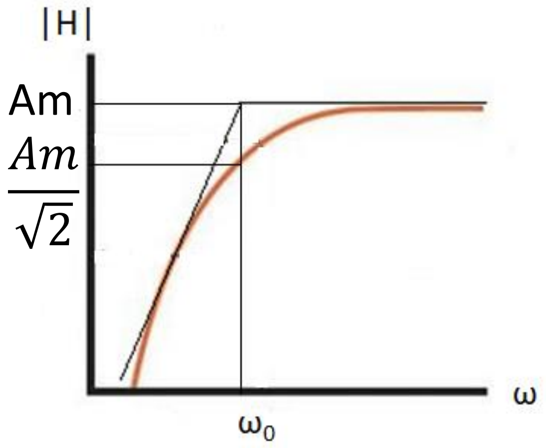
$$\frac{Am}{\sqrt{2}} = K.Q$$

but $T_{hp} = \left.\frac{K}{1 + \frac{\omega_o}{Q}\frac{1}{s} + \frac{\omega_o^2}{s^2}}\right|_{s=\infty} = Am = K$

$\therefore \frac{K}{\sqrt{2}} = K.Q \rightarrow Q = \frac{1}{\sqrt{2}} = 0.707$

* $K = 2 - \frac{1}{Q} = 0.586$, $\frac{R_3}{R_1} = 2Q - 1 = 0.414$

let $R_1 = 10k\Omega$ $\therefore R_3 = 4.14 k\Omega$



* $f_o = 10^4 = \frac{1}{2\pi RC}$

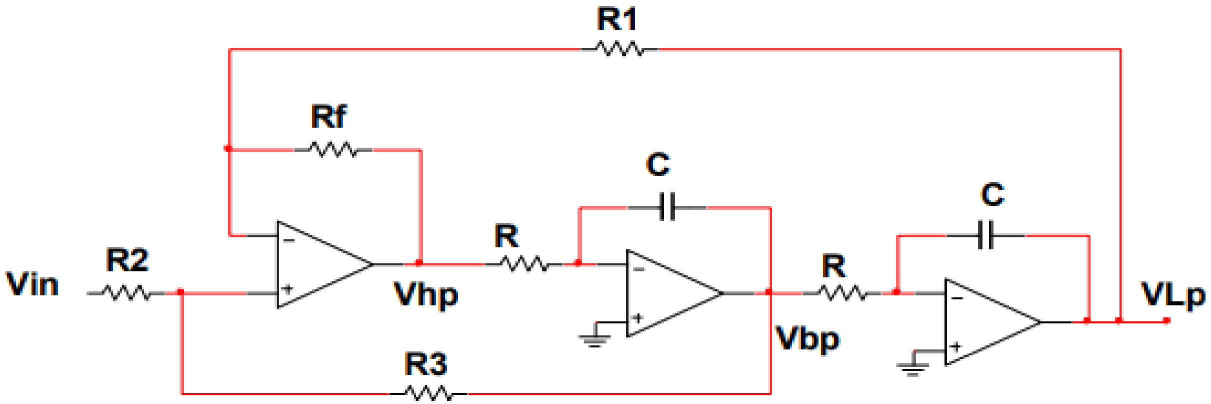
$$10^4 = \frac{1}{2\pi R(1\times10^{-9})}$$

$$\boxed{R = 15.92 k\Omega}$$

* $\frac{R_F}{R_1} = 1$ let $\boxed{\begin{matrix} R_1 = 10k\Omega \\ R_F = 10k\Omega \end{matrix}}$

Example (3):

Design the KHN Filter to realize a LPF with a cut-off frequency 10KHz(Use C of 1 nF).



Solution:

* $f_o = 10^4 = \frac{1}{2\pi RC}$

$10^4 = \frac{1}{2\pi R(1\times10^{-9})}$

$\therefore \boxed{R = 15.92\ k\Omega}$

* $\frac{R_F}{R_1} = 1$

Let $\boxed{\begin{matrix} R_1 = 10\ k\Omega \\ R_f = 10\ k\Omega \end{matrix}}$

* $Q = \frac{1}{\sqrt{2}} = 0.707$

* $k = 2 - \frac{1}{Q} = 0.586$

* $\frac{R_3}{R_2} = 2Q - 1 = 0.414$

Let $\boxed{\begin{matrix} R_2 = 10\ k\Omega \\ R_3 = 4.14\ k\Omega \end{matrix}}$

Maximum gain

$$V_{Lp} = \frac{\omega_o^2}{s^2} V_{hp}$$

$$V_{Lp} = \frac{\omega_o^2}{s^2} \cdot \frac{k s^2}{s^2 + (\frac{\omega_o}{Q}) s + \omega_o^2}$$

$$T_{Lp} = \frac{V_{Lp}}{V_{in}} = \frac{k\omega_o^2}{s^2 + (\frac{\omega_o}{Q}) s + \omega_o^2}$$

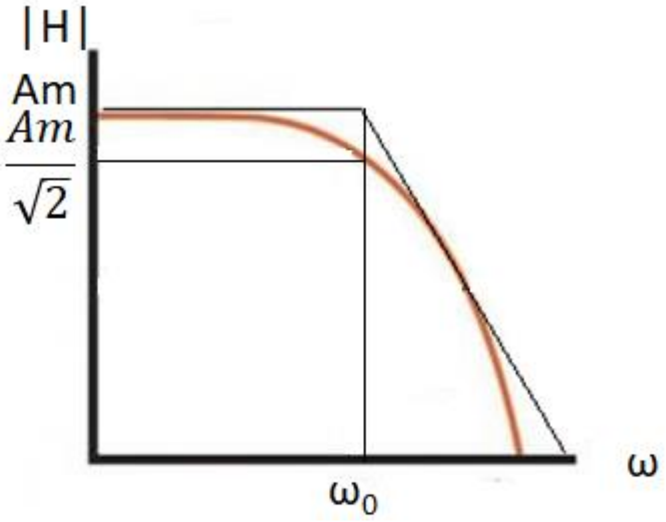
Max. gain at $f = 0 \therefore \omega = 0$

$\therefore s = 0$

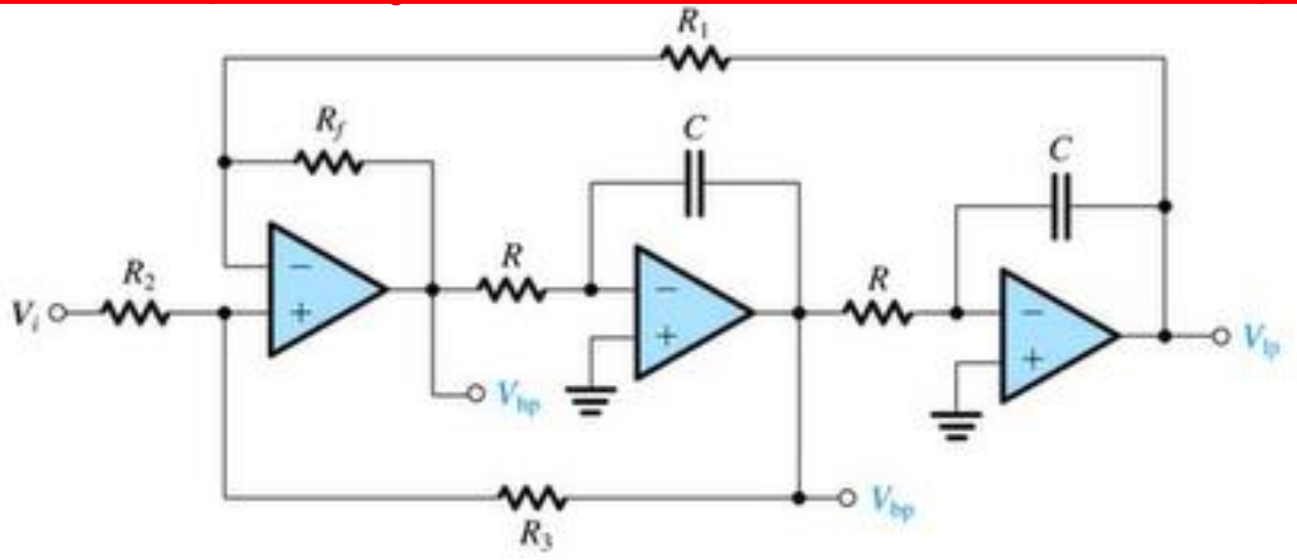
$$|T_{Lp}| = \frac{k\omega_o^2}{\omega_o^2} = k$$

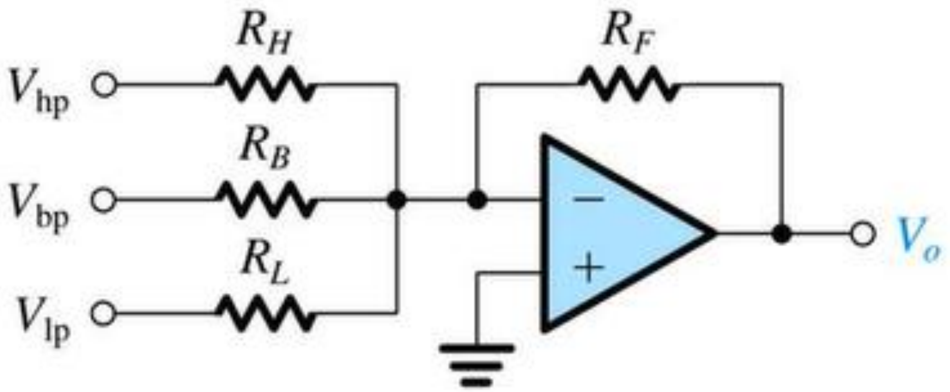
$$A_m = k = 0.586$$

$$H_{LP} = \frac{a_o}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$



(KHN) Biquad All Pass Filter (APF)





$$T_{bp} = \frac{V_{bp}}{V_i}$$

$$T_{lp} = \frac{V_{lp}}{V_i}$$

$$T_{hp} = \frac{V_{hp}}{V_i}$$

$$V_o = -\left(\frac{R_F}{R_H}V_{hp} + \frac{R_F}{R_B}V_{bp} + \frac{R_F}{R_L}V_{lp}\right)$$

$$= -V_i\left(\frac{R_F}{R_H}T_{hp} + \frac{R_F}{R_B}T_{bp} + \frac{R_F}{R_L}T_{lp}\right)$$

(KHN) Biquad All Pass Filter (APF)

$$V_o = -V_i\left(\frac{R_F}{R_H}T_{\rm bp} + \frac{R_F}{R_B}T_{\rm bp} + \frac{R_F}{R_L}T_{\rm lp}\right)$$

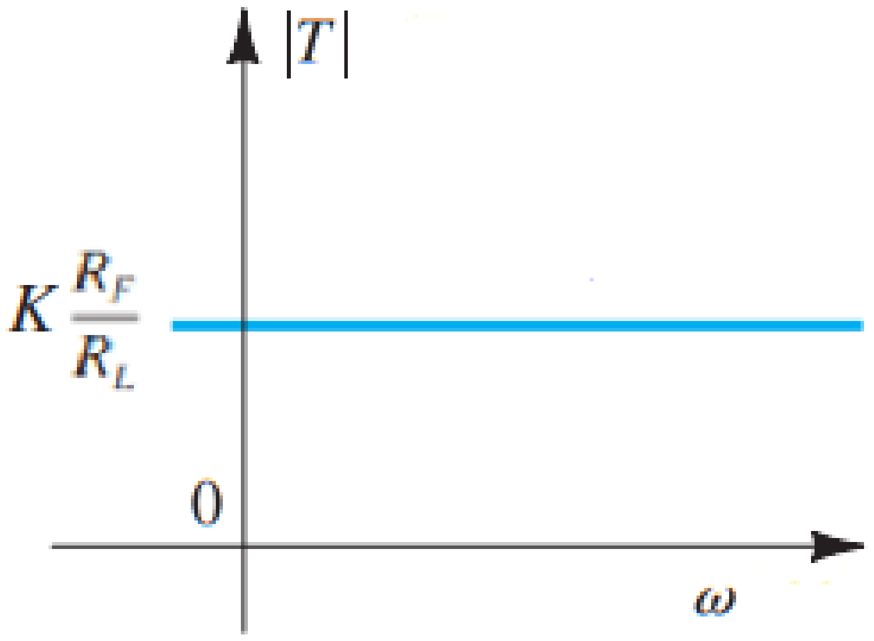
$$T_{\rm bp} = \frac{V_{\rm bp}}{V_i} = -\frac{K\omega_0 s}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

$$T_{\rm lp} = \frac{V_{\rm lp}}{V_i} = \frac{K\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

$$T_{\rm hp} = \frac{V_{\rm hp}}{V_i} = \frac{Ks^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

(KHN) Biquad All Pass Filter (APF)



Flat Gain =Am= $|T_{AP}| = K\,\frac{R_F}{R_L}$

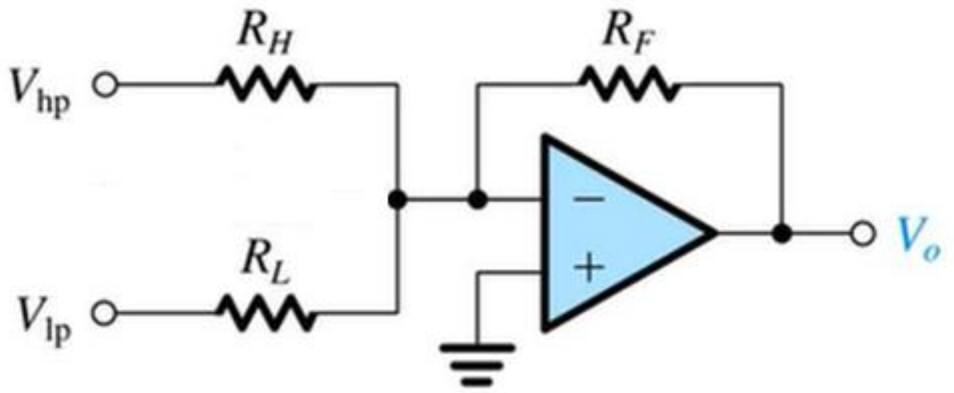
KHN Biquad Notch Filter (BSF with narrow B.W)

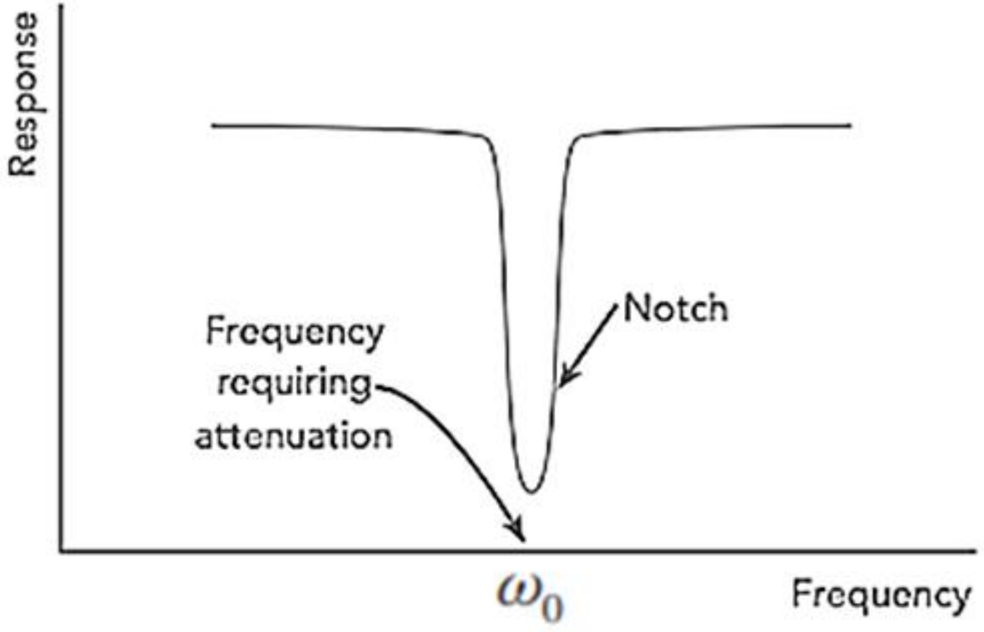
- KHN Biquad Notch Filter is obtained by:

$$R_B = \infty$$

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2$$

$$T_{NF} = \frac{V_o}{V_i} = -K\,\frac{(R_F/R_H)s^2 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

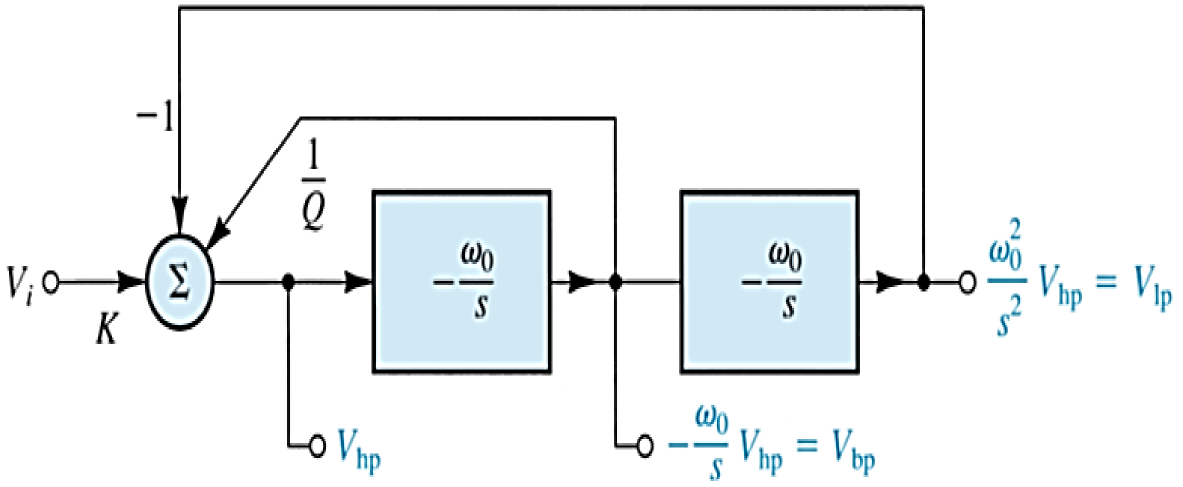




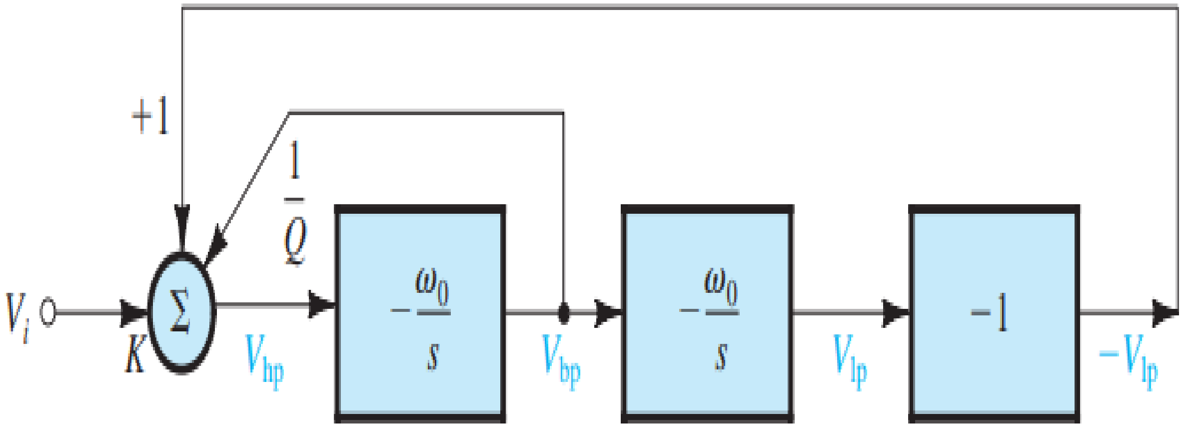
Two-Thomas Biquad Filter

Two-Thomas Biquad Filter

- An alternative two-integrator-loop biquad circuit in which all three op amps are used in a single-ended mode can be developed as follows: Rather than using the input summer to add signals with positive and negative coefficients, we can introduce an additional inverter, as shown.
- Now all the coefficients of the summer have the same sign, and we can dispense with the summing amplifier altogether and perform the summation at the virtual-ground input of the first integrator.



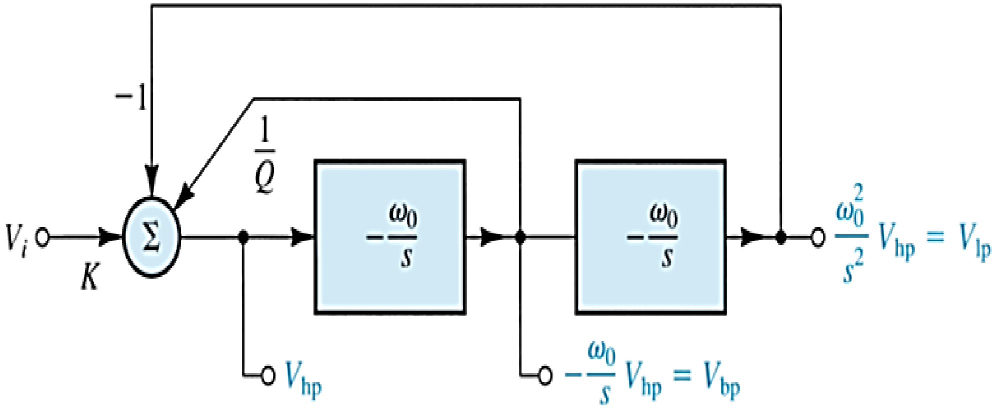
KHN Biquad Filter

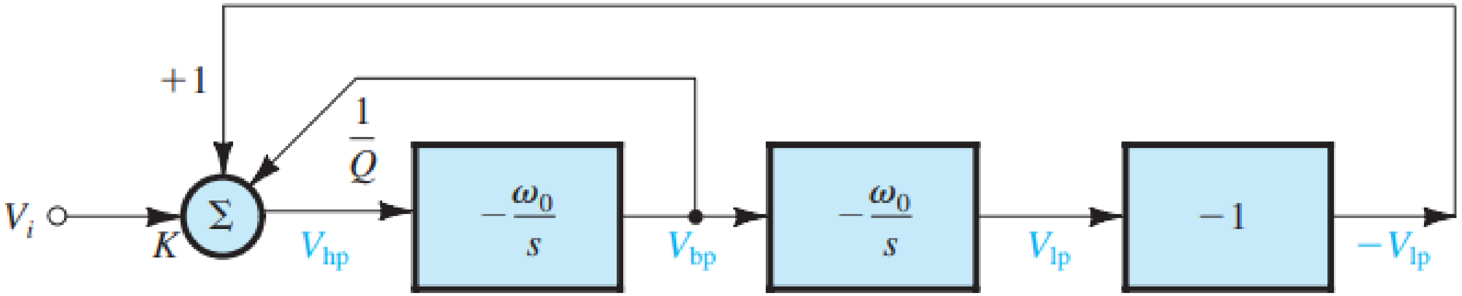


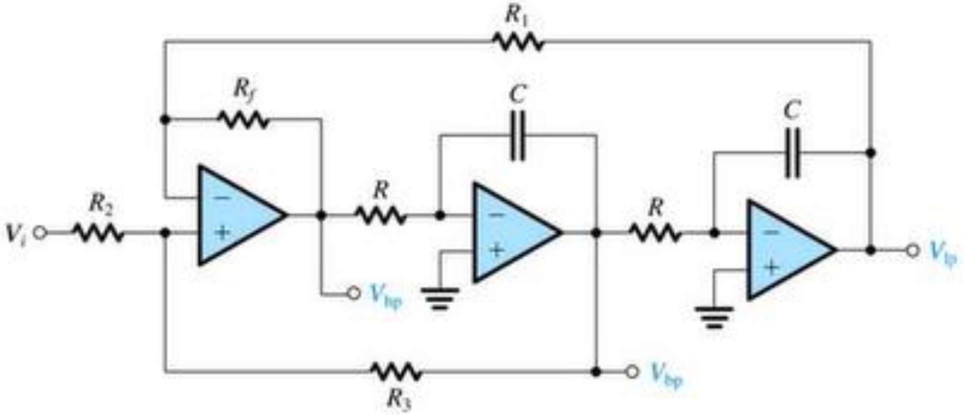
Two-Thomas Biquad Filter

Two-Thomas Biquad Filter

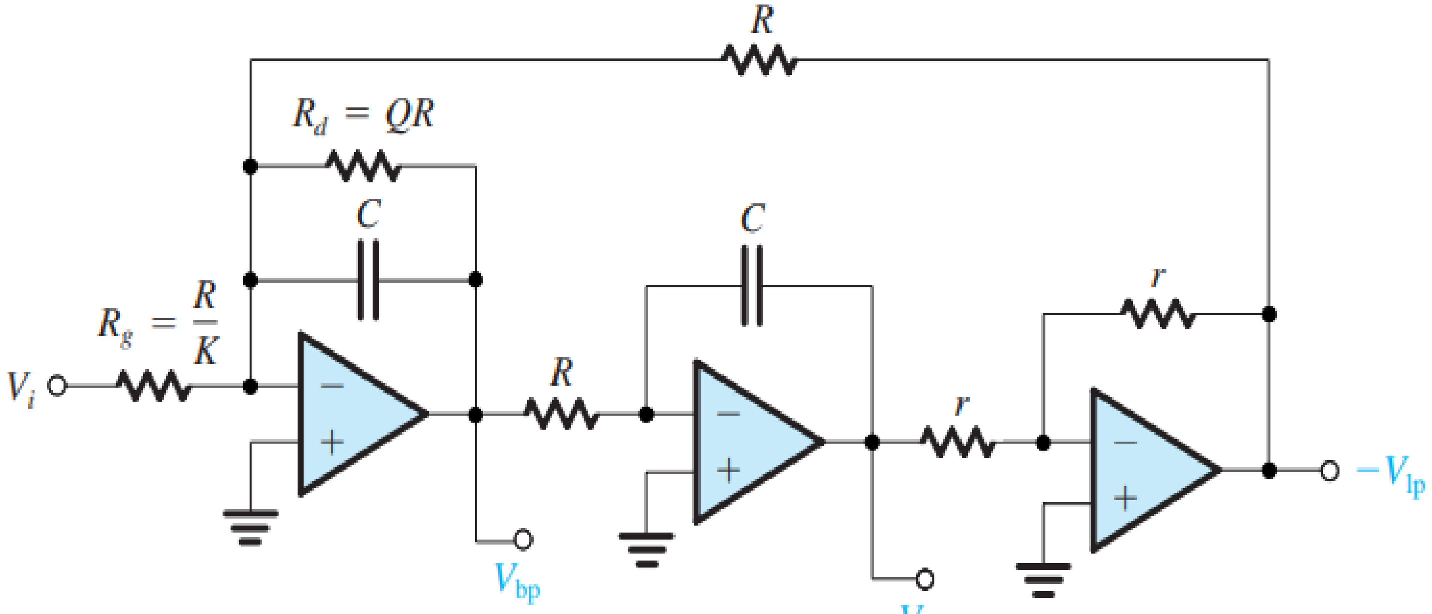
Observe that the summing weights of 1, 1/Q, and K are realized by using resistances of R, QR, and R/K, respectively. we observe that the high-pass function is no longer available!. The circuit is known as the Tow–Thomas biquad, after its originators.





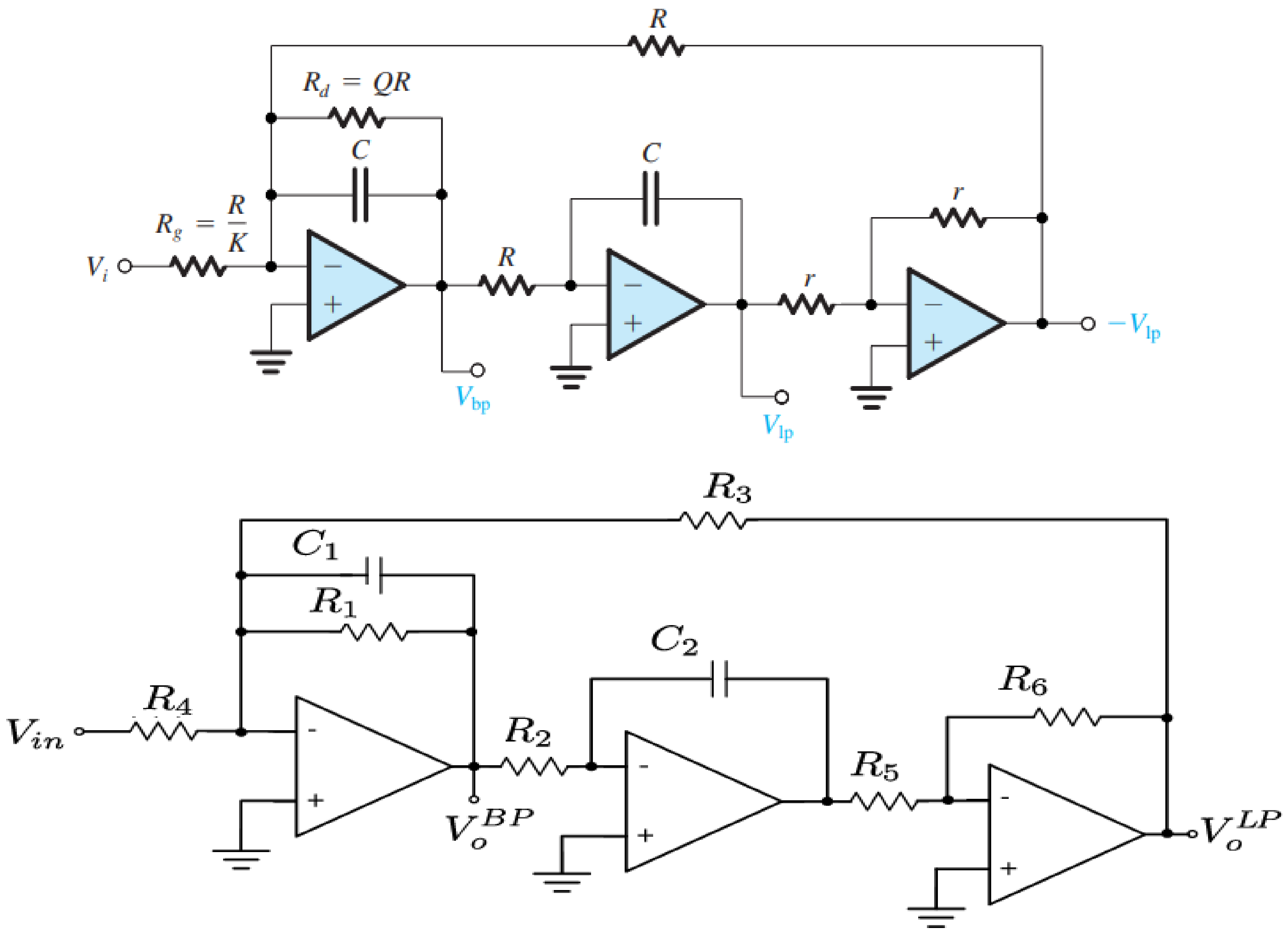


KHN Biquad Filter

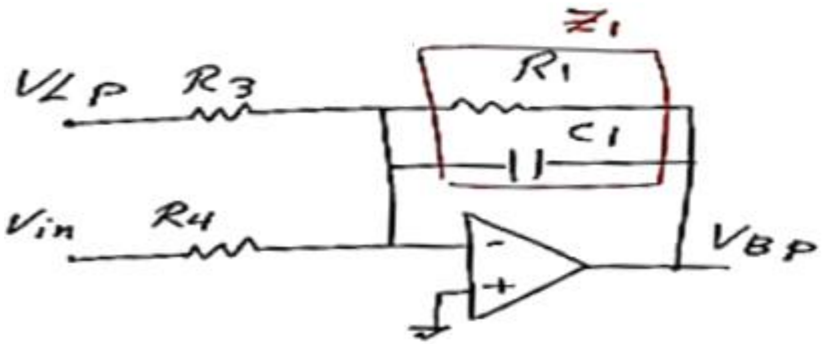


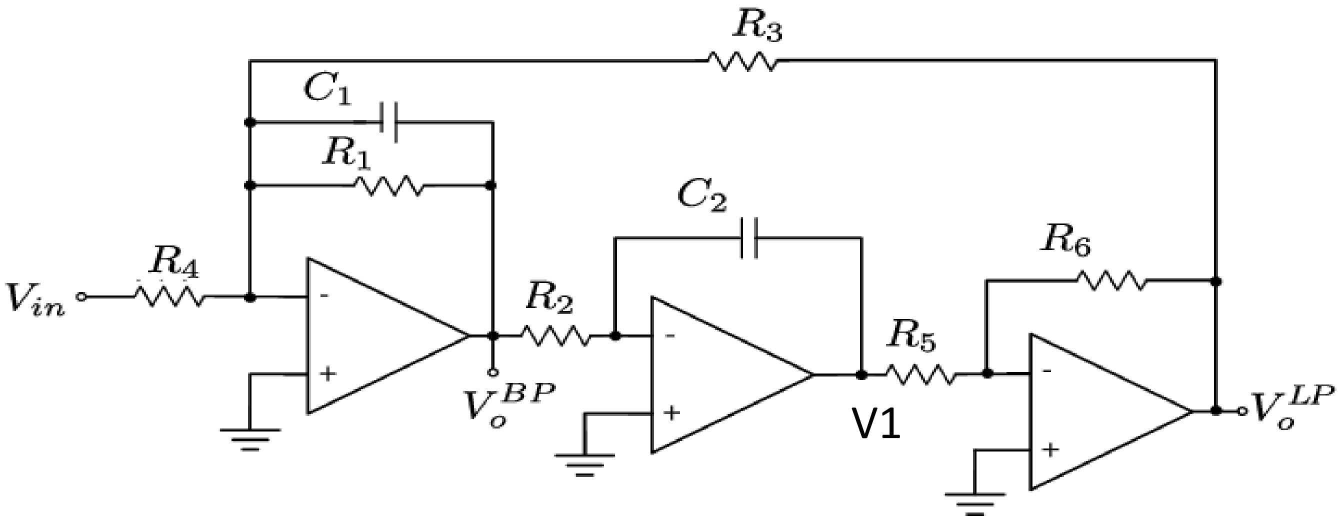
Two-Thomas Biquad Filter

Two-Thomas Biquad Filter



Two-Thomas Biquad Band Pass Filter





$$* \; Z_1 = \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{sC_1R_1 + 1}$$

1 * Band Pass Filter output (BPF)

$$\boxed{V_{BP} = -\frac{Z_1}{R_3} V_{LP} - \frac{Z_1}{R_4} V_{in}} \longrightarrow (1)$$

but $V_1 = -\frac{1/sC_2}{R_2} V_{BP} = -\frac{1}{sC_2R_2} V_{BP}$

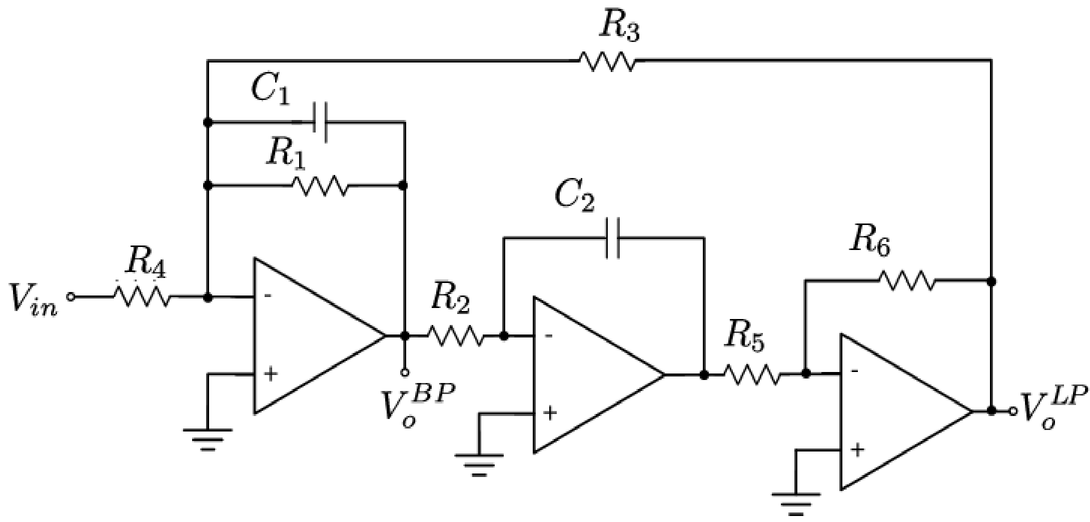
and $V_{LP} = -\frac{R_6}{R_5} V_1 = -\frac{R_6}{R_5} \left[-\frac{1}{sC_2R_2} V_{BP} \right]$

$$\therefore \boxed{V_{LP} = \frac{R_6}{R_5} \frac{1}{sC_2R_2} V_{BP}} \quad (2)$$

Two-Thomas Biquad Band Pass Filter

sub. into ①

$$\therefore V_{BP} = -\frac{Z_1}{R_3}\left[\frac{R_6}{R_5}\frac{1}{sC_2R_2}V_{BP}\right] - \frac{Z_1}{R_4}V_i$$



$$V_{BP}\left[1+\frac{Z_1}{R_3}\frac{R_6}{R_5}\frac{1}{sC_2R_2}\right] = -\frac{Z_1}{R_4}V_{in}$$

$$V_{BP}\left[1+\left(\frac{R_1}{sC_1R_1+1}\right)\frac{1}{R_3}\frac{R_6}{R_5}\frac{1}{sC_2R_2}\right] = -\frac{1}{R_4}\frac{R_1}{sC_1R_1+1}V_{in}$$

$$* \; Z_1 = \frac{R_1\,\frac{1}{sC_1}}{R_1+\frac{1}{sC_1}} = \frac{R_1}{sC_1R_1+1}$$

$$V_{BP}\left[\frac{(sC_1R_1+1)R_3R_5\,sC_2R_2 + R_1R_6}{(\cancel{sC_1R_1+1})R_3R_5\,sC_2R_2}\right] = \frac{-R_1}{R_4(\cancel{sC_1R_1+1})}V_{in}$$

Two-Thomas Biquad Band Pass Filter

$$\therefore \frac{V_{BP}}{V_{in}} = T_{BP} = \frac{-\frac{R_1}{R_4} R_3 R_5 s C_2 R_2}{(sC_1R_1+1) R_3 R_5 s C_2 R_2 + R_1 R_6}$$

$$T_{BP} = \frac{-\frac{R_1}{R_4} R_3 R_5 s C_2 R_2}{s^2 R_1 R_2 C_1 C_2 R_3 R_5 + s R_2 R_3 R_5 C_2 + R_1 R_6}$$

$$T_{BP} = \frac{-\frac{\cancel{R_1}}{R_4} \cancel{R_3} \cancel{R_5} s \cancel{C_2} \cancel{R_2}}{\cancel{R_1} \cancel{R_2} C_1 \cancel{C_2} \cancel{R_3} \cancel{R_5} \left[s^2 + \frac{R_2 R_3 R_5 C_2}{R_1 R_2 C_1 C_2 R_3 R_5} s + \frac{R_1 R_6}{R_1 R_2 C_1 C_2 R_3 R_5} \right]}$$

$$T_{BP} = \frac{-\frac{1}{C_1 R_4} s}{s^2 + \frac{1}{R_1 C_1} s + \frac{(R_6/R_5)}{R_2 R_3 C_1 C_2}} \quad \text{(I)} = \frac{V_{BP}}{V_{in}}$$

$$T_{BP} = \frac{-a s}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2} \quad \text{(II)}$$

$Q \rightarrow$ quality-Factor

Two-Thomas Biquad Band Pass Filter

Comparing (I) and (II)

$$\omega_o^2 = \frac{R_6/R_5}{R_2 R_3 C_1 C_2}$$

$$\therefore \boxed{\omega_o = \sqrt{\frac{R_6/R_5}{R_2 R_3 C_1 C_2}}}$$

and

$$f_o = \frac{1}{2\pi}\sqrt{\frac{R_6/R_5}{R_2 R_3 C_1 C_2}}$$

→ Center Frequency

Also,

$$\boxed{\frac{\omega_o}{Q} = \frac{1}{R_1 C_1}} \quad \text{III}$$

$$\frac{\omega_o^2}{Q^2} = \frac{1}{R_1^2 C_1^2}$$

$$\therefore Q^2 = \omega_o^2 R_1^2 C_1^2$$

$$Q^2 = \frac{R_6/R_5}{R_2 R_3 C_1 C_2} \cdot R_1^2 C_1^2$$

$$Q^2 = \frac{R_1^2}{R_2 R_3} \frac{R_6}{R_5} \cdot \frac{C_1}{C_2}$$

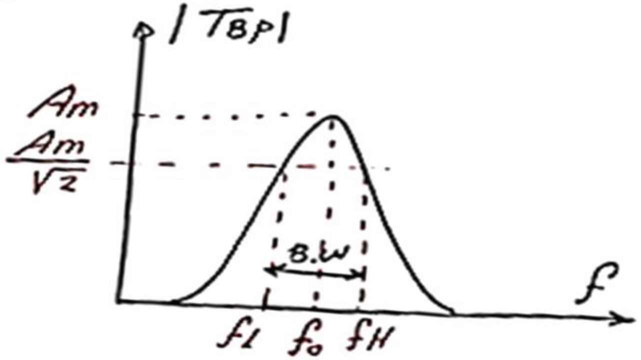
$$\therefore Q = \frac{R_1}{\sqrt{R_2 R_3}} \cdot \sqrt{\frac{R_6}{R_5} \cdot \frac{C_1}{C_2}}$$

and the Band-width is

$$B.W = \frac{f_o}{Q}$$

$$f_L = f_o - \frac{B.W}{2}$$

$$f_H = f_o + \frac{B.W}{2}$$



Am = center Frequency gain

Two-Thomas Biquad Band Pass Filter

The Center Frequency gain (A_m)

$$T_{BP} = \frac{-\frac{1}{C_1 R_4} S}{S^2 + \frac{1}{R_1 C_1} S + \frac{R_6 R_5}{R_2 R_3 C_1 C_2}}$$

$$T_{BP} = \frac{-a\, S}{S^2 + \left(\frac{\omega_o}{Q}\right) S + \omega_o^2}$$

$$T_{BP}(j\omega) = \frac{-j\omega a}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right) j\omega + \omega_o^2}$$

$$T_{BP}(j\omega) = \frac{-j\omega a}{[\omega_o^2 - \omega^2] + j\left(\frac{\omega_o \omega}{Q}\right)}$$

$$|T_{BP}| = \frac{+a\omega}{\sqrt{[\omega_o^2 - \omega^2]^2 + \left(\frac{\omega_o \omega}{Q}\right)^2}}$$

at $\omega = \omega_o \Rightarrow |T_{BP}| = A_m$

$$\therefore A_m = \frac{a\,\omega_o}{\sqrt{[\cancel{\omega_o^2 - \omega_o^2}]^2 + \left(\frac{\omega_o \omega_o}{Q}\right)^2}} = \frac{a.\omega_o}{\frac{\omega_o^2}{Q}}$$

$$\boxed{A_m = \frac{a}{\omega_o} . Q} \quad (4) , \quad \boxed{a = \frac{1}{C_1 R_4}} \quad (5)$$

but From III

$$\frac{\omega_o}{Q} = \frac{1}{R_1 C_1} \Rightarrow \therefore \frac{Q}{\omega_o} = R_1 C_1$$

sub. into (4)

$$\therefore A_m = a . \frac{Q}{\omega_o} = \frac{1}{R_4 C_1} . R_1 C_1$$

$$\therefore \boxed{A_m = \frac{R_1}{R_4}}$$

$\longrightarrow$ Center Frequency gain

Two-Thomas Biquad Band Pass Filter

Special case:-

For $\boxed{R_5 = R_6}$ and $\boxed{C_1 = C_2 = C}$

* $$f_0 = \frac{1}{2\pi C\sqrt{R_2 R_3}}$$

* $$Q = \frac{R_1}{\sqrt{R_2 R_3}}$$

* $$B.W = \frac{f_0}{Q}$$

* $$A_m = \frac{R_1}{R_4}$$

Two-Thomas Biquad Band Pass Filter

Example 1:

Design a Two-Thomas Biquad Filter to realize a BPF with center frequency of 10 KHz and bandwidth of 100Hz. The center frequency gain (Am) is required to be 200.

Hint: use 1 nF capacitor.

Two-Thomas Biquad Band Pass Filter

Solution:

* Assuming $\rightarrow$ $\boxed{R_5 = R_6 = 10\,k\Omega}$

$\boxed{C_1 = C_2 = C = 1\,nF}$

* $\therefore f_o = \dfrac{1}{2\pi C\sqrt{R_2 R_3}} = 10^4\ Hz$

$\therefore (10^4)^2 = \dfrac{1}{(2\pi C)^2 R_2 R_3}$

Assuming $\boxed{R_2 = 10\,k\Omega}$

$\therefore (10^4)^2 = \dfrac{1}{[2\pi \times 1 \times 10^{-9}]^2 (10 \times 10^3) R_3}$

$\therefore R_3 = 25330.3\,\Omega \cong 25.33\,k\Omega$

* $Q = \dfrac{f_o}{B.W} = \dfrac{10^4}{100}$

$\therefore \boxed{Q = 100} = \dfrac{R_1}{\sqrt{R_2 R_3}}$

$\therefore R_1 = Q\sqrt{R_2 R_3} = 100\sqrt{10 \times 25.33}$

$\therefore R_1 = 1591.54\,k\Omega \cong 1.59\,M\Omega$

* Center Frequency gain (A_m)

$A_m = \dfrac{R_1}{R_4} = 200$

$\therefore R_4 = \dfrac{R_1}{200} = \dfrac{1591.54}{200}$

$R_4 = 7.9577\,k\Omega$

Two-Thomas Biquad Low Pass Filter

[2] Low Pass Filter output: (LPF)

From ②

$$V_{LP} = \frac{R_6}{R_5} \frac{1}{S C_2 R_2} V_{BP}$$

$$\therefore V_{LP} = \frac{R_6}{R_5} \frac{1}{\cancel{S} C_2 R_2} \left\{ \frac{-\frac{1}{C_1 R_4} \cancel{S}}{S^2 + \frac{1}{R_1 C_1} S + \frac{R_6/R_5}{R_2 R_3 C_1 C_2}} V_{in} \right\}$$

$$\therefore T_{LP} = \frac{V_{LP}}{V_{in}} = \frac{-\left(\frac{R_6/R_5}{R_2 R_4 C_1 C_2}\right)}{S^2 + \frac{1}{R_1 C_1} S + \frac{R_6/R_5}{R_2 R_3 C_1 C_2}}$$

but

$$T_{LP} = \frac{-a}{S^2 + \left(\frac{\omega_o}{Q}\right) S + \omega_o^2}$$

$$a = \frac{R_6/R_5}{R_2 R_4 C_1 C_2}$$

$$\omega_o = \sqrt{\frac{R_6/R_5}{R_2 R_3 C_1 C_2}}$$

$$\therefore f_o = \frac{1}{2\pi} \sqrt{\frac{R_6/R_5}{R_2 R_3 C_1 C_2}}$$

$$\frac{\omega_o}{Q} = \frac{1}{R_1 C_1}$$

$$\therefore Q = \frac{R_1}{\sqrt{R_2 R_3}} \sqrt{\frac{R_6}{R_5} \cdot \frac{C_1}{C_2}}$$

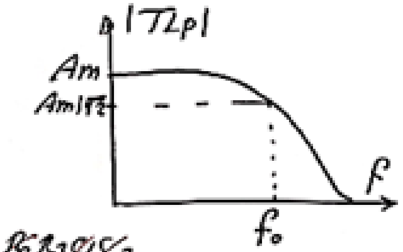
Max. gain (D.C gain)

put $S = 0$

$$|T_{LP}| = A_m = \frac{a}{\omega_o^2}$$

$$A_m = \frac{\cancel{R_6}}{\cancel{R_5}} \cdot \frac{1}{\cancel{R_2} R_4 \cancel{C_1} \cancel{C_2}} \cdot \frac{\cancel{R_2} R_3 \cancel{C_1} \cancel{C_2}}{\cancel{R_6/R_5}}$$

$$\boxed{A_m = \frac{R_3}{R_4}}$$



Two-Thomas Biquad Low Pass Filter

Example 2:

Design Two-Thomas Biquad Filter to realize a LPF with a cut-off Frequency 10KHz. (use 1 nF capacitors) The D.C gain is 100.

Two-Thomas Biquad Low Pass Filter

Solution:

* Assuming

$$\boxed{R_5 = R_6 = 10\,k\Omega}$$

$$\boxed{C_1 = C_2 = C = 1\,nF}$$

* $f_0 = \frac{1}{2\pi C}\frac{1}{\sqrt{R_2 R_3}}$

$$(10^4) = \frac{1}{2\pi(1\times10^{-9})\sqrt{R_2R_3}}$$

Assuming $\boxed{R_2 = 10\,k\Omega}$

$$\therefore \boxed{R_3 = 25.33\,k\Omega}$$

* $Q = \frac{1}{\sqrt{2}}$ (LPF)

$$Q = 0.707 = \frac{R_1}{\sqrt{R_2R_3}}$$

$$\therefore R_1 = Q\sqrt{R_2R_3}$$

$$R_1 = 0.707\sqrt{10\times25.33}$$

$$\boxed{R_1 = 11.254\,k\Omega}$$

* D.C gain (Am)

$$Am = \frac{R_3}{R_4}$$

$$100 = \frac{25.33\,k\Omega}{R_4}$$

$$\therefore R_4 = 0.2533\,k\Omega = 253.3\,\Omega$$