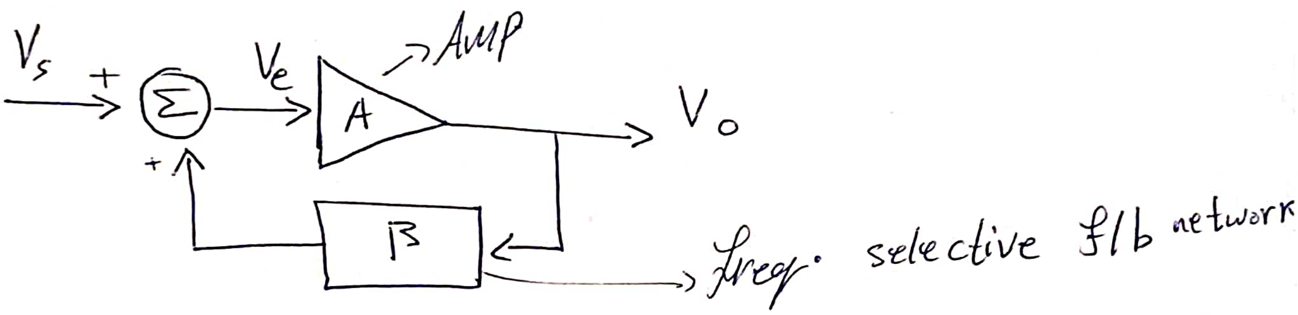
# Oscillators

\# oscillators are ckts that produces periodic waveform without having an i/p.

⇒ For a system to oscillate, it must satisfy the Barkhausen criterion.

\# Barkhausen criterion :




$$\Rightarrow V_o = [V_s + \beta V_o] A$$

$$\Rightarrow V_o [1 - A\beta] = V_s \cdot A$$

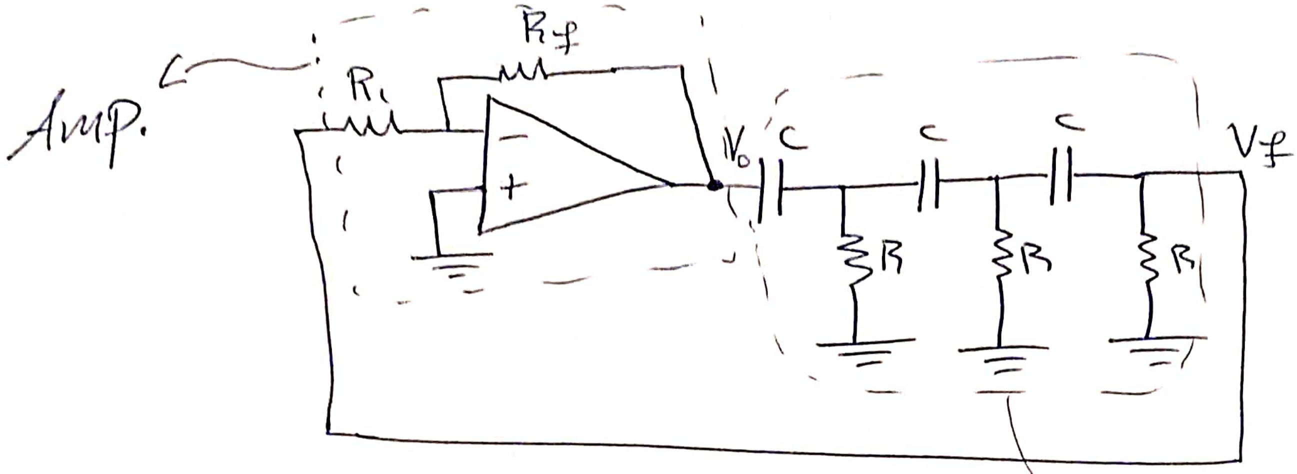
$$\Rightarrow \boxed{\frac{V_o}{V_s} = \frac{A}{1 - A\beta}}$$ → closed loop gain

$\boxed{A\beta}$ → open loop gain

$$\Rightarrow \boxed{\begin{array}{c} |A\beta| = 1 \\ \angle A\beta = 0^\circ \text{ or } 360^\circ \end{array}}$$

↳ so if we are using an inverting Amplifier ⇒ gain = $-A$ → $180^\circ$ phase shift
↳ we need β → to add $180^\circ$ phase shift

# $\boxed{1}$ RC phase-shift oscillator :




$$* \; A_V = -\frac{R_f}{R_1} \Rightarrow \angle A_V = 180^\circ$$

$\beta$

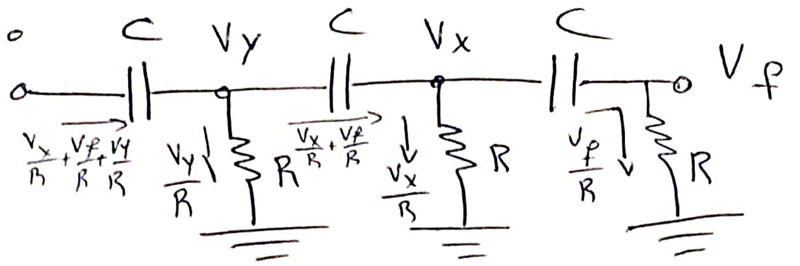
$\Rightarrow$ we need to choose the values of R & C that make $\angle \beta = 180^\circ$ at $\underset{\sim}{\omega_o}$.

$\hookrightarrow$ desired freq. of oscillation

$$\Rightarrow A = -\frac{R_f}{R_1} \Rightarrow \text{Real}$$

$\because$ oscillation condition : $A\beta = 1$

$\Rightarrow$ imaginary part $= 0$ at $\omega_o$




$$\Rightarrow V_x = V_f + \frac{V_f}{R}\cdot\frac{1}{sC}$$

$$V_x = V_f\left[1+\frac{1}{sRC}\right]$$

$$\Rightarrow V_y = V_x + \left[\frac{V_x}{R}+\frac{V_f}{R}\right]\frac{1}{sC}$$

$$V_y = \frac{V_f}{sRC} + V_x\left[1+\frac{1}{sRC}\right] = \frac{V_f}{sRC} + V_f\left[1+\frac{1}{sRC}\right]^2 = V_f\left[\frac{1}{sRC}+\left(1+\frac{1}{sRC}\right)^2\right]$$

$$\Rightarrow V_o = V_y + \left[\frac{V_x}{R}+\frac{V_f}{R}+\frac{V_y}{R}\right]\frac{1}{sC}$$

$$\Rightarrow V_o = V_y\left[1+\frac{1}{sRC}\right] + \frac{V_x}{sRC} + \frac{V_f}{sRC}$$

$$\Rightarrow V_o = \left[V_f\left(\frac{1}{sRC}+\left[1+\frac{1}{sRC}\right]^2\right)\right]\left[1+\frac{1}{sRC}\right] + \frac{V_f}{sRC}\left[1+\frac{1}{sRC}\right] + \frac{V_f}{sRC}$$

$$\Rightarrow \beta = \frac{V_f}{V_o} = \frac{1}{\left(\frac{1}{sRC}+\frac{1}{s^2R^2C^2}\right)+\left(1+\frac{1}{sRC}\right)^3+\left(\frac{1}{sRC}+\frac{1}{s^2R^2C^2}\right)+\frac{1}{sRC}}$$

$$\left(1+\frac{1}{sRC}\right)^3 = \left(1+\frac{3}{s^2R^2C^2}+\frac{3}{sRC}+\frac{1}{s^3R^3C^3}\right)$$

at $s = j\omega_o$

$$\Rightarrow \beta = \frac{1}{\left(\frac{1}{j\omega_oRC}-\frac{1}{\omega_o^2R^2C^2}\right)+\left(1-\frac{3}{\omega_o^2R^2C^2}+\frac{3}{j\omega_oRC}-\frac{1}{j\omega_o^3R^3C^3}\right)+\left(\frac{1}{j\omega_oRC}-\frac{1}{\omega_o^2R^2C^2}\right)+\frac{1}{j\omega_oRC}}$$

∵ $A \rightarrow$ pure Real $\Rightarrow \beta \xrightarrow[\text{at } \omega_o]{} \text{Real} \rightarrow$ imaginary part $= 0$

$$\Rightarrow \frac{6}{\omega_oRC} - \frac{1}{\omega_o^3R^3C^3} = 0 \quad \Rightarrow 6\omega_o^2R^2C^2 = 1$$

$$\Rightarrow \boxed{\omega_o = \frac{1}{\sqrt{6}\,RC}}$$

$$Re\{\beta\} = \frac{1}{-\frac{1}{\omega_o R^2 C^2} + 1 - \frac{3}{\omega_o^2 R^2 C^2} - \frac{1}{\omega_o^2 R^2 C^2}}$$

$$= \frac{\omega_o^2 R^2 C^2}{-1 + \omega_o R^2 C^2 - 3} \qquad \because \omega_o^2 = \frac{1}{6R^2C^2}$$

$$\Rightarrow \quad \beta = \frac{\frac{1}{\cancel{6R^2C^2}} \cancel{R^2C^2}}{-5 + \frac{1}{\cancel{6R^2C^2}} \cancel{R^2C^2}} = \frac{\frac{1}{6}}{-5+\frac{1}{6}} = -\frac{1}{29}$$

$$\Rightarrow \quad |A\beta| = 1 \quad \Rightarrow \beta = \frac{1}{A}$$

$$\Rightarrow \boxed{A = 29}$$

gain must be at least 29 for the phase-shift oscillator.

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※ You may do the same analysis using Mesh analysis

# In an RC phase shift oscillator, if $R_1 = R_2 = R_3 = 200\,K\Omega$ & $C_1 = C_2 = C_3 = 100\,PF$. Find the frequency of of oscillation

sol:

$$\because f_o = \frac{1}{2\pi RC\sqrt{6}}$$

$$\Rightarrow f_o = \frac{1}{2\pi (200*10^3)(100*10^{-12}) * \sqrt{6}}$$

$$\Rightarrow f_o = 3.248\ KHz$$

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# Given a BJT-based RC phase shift oscillator, if $R = 10\,K\Omega$, $C = 0.01\,\mu F$ & $R_C = 2.2\,K\Omega$. Find the frequency of oscillation & the minimum current gain ($\beta$) needed to achieve the oscillation condition.

sol:

$$\Rightarrow f_o = \frac{1}{2\pi RC\sqrt{6 + 4\frac{R_c}{R}}}$$

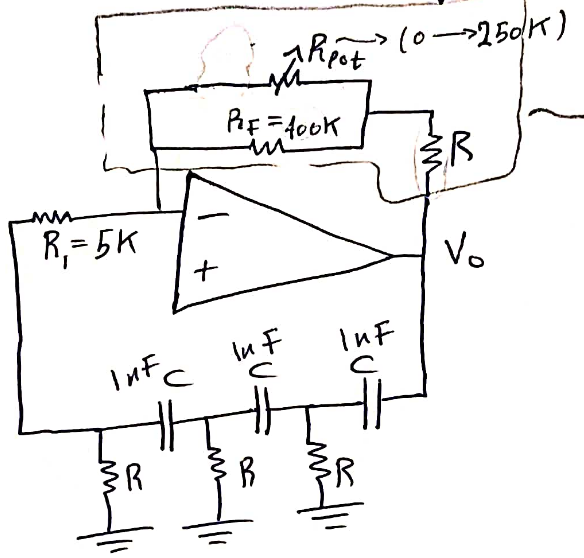
$$= \frac{1}{2\pi (10*10^3)(0.01*10^{-6})\sqrt{6+4\left(\frac{2.2}{10}\right)}}$$

$$= 606.7\ HZ$$

$$\Rightarrow \beta_{min} = 23 + 4\frac{R_c}{R} - \frac{29R}{R_c}$$

$$= 23 + 4\left(\frac{2.2}{10}\right) - 29\left(\frac{10}{2.2}\right) = 155.7$$

# For the given RC-phase shift oscillator, find the value of R & $R_{pot}$ that results in the minimum frequency of oscillation (note that $R_1 \geq 10R$ must be satisfied), then find the value of this frequency.




$\to R_f$

$\Rightarrow A = -\dfrac{R_f}{R_1}$

## Sol:

$\because R_1 \geq 10R$

$\Rightarrow R_{max} = \dfrac{R_1}{10} = 500\ \Omega$

$\because f_o = \dfrac{1}{2\pi\sqrt{6}\,RC} \Rightarrow f_{o_{min}} \xrightarrow{\text{for}} R_{max}$ ✓

$\Rightarrow$ for phase-shift oscillator: $A \geq 29$

$\Rightarrow \dfrac{(R + R_{pot} \parallel R_F)}{R_1} \geq 29$

$\rightarrow \quad 500 + \left(\dfrac{R_{pot}\,(400K)}{R_{pot} + 400K}\right) = 29\,(5K)$

$\Rightarrow \quad 500\,(R_{pot} + 400K) + R_{pot}\,(400K) = 29\,(5K)\,R_{pot} + (400K)(29)(5K)$

$\Rightarrow \quad R_{pot} = 226.2\ K\Omega$

$\Rightarrow f_{o_{min}} = \dfrac{1}{2\pi\,\underbrace{(5 * 29 * 10^3)}_{R_{Feedback\,(Total)}}(1 * 10^{-9})\sqrt{6}} = 448.1\ Hz$