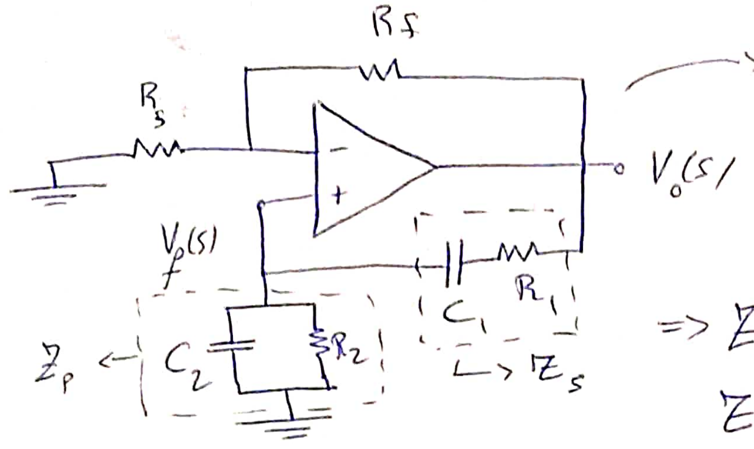
The wien-Bridge oscillator :



→ non-inverting Amp. :

$$\Rightarrow A = 1 + \frac{R_F}{R}$$

$$\Rightarrow Z_s = R_1 - jX_{c_1}$$

$$Z_p = [R_2 // (-jX_{c2})] = \frac{-jR_2X_{c2}}{R_2 - jX_{c2}}$$

$$\Rightarrow V_f = \frac{V_o}{Z_p + Z_s} \cdot Z_p$$

$$\Rightarrow \beta = \frac{V_f}{V_o} = \frac{Z_p}{Z_p + Z_s}$$

$$\Rightarrow \beta = \frac{\frac{-jR_2X_{c2}}{R_2 - jX_{c2}}}{R_1 - jX_{c_1} + \frac{(-jR_2X_{c2})}{R_2 - jX_{c2}}}$$

$$= \frac{-jR_2X_{c2}}{(R_1 - jX_{c_1})(R_2 - jX_{c_2}) - jR_2X_{c_2}}$$

$$= \frac{-jR_2X_{c2}}{R_1R_2 - jR_1X_{c2} - jR_2X_{c1} - X_{c_1}X_{c_2} - jR_2X_{c2}}$$

$$= \frac{R_2X_{c2}}{j[R_1R_2 - jR_1X_{c2} - jR_2X_{c1} - X_{c_1}X_{c_2} - jR_2X_{c2}]}$$

$$\Rightarrow \boxed{\beta = \frac{R_2X_{c2}}{R_1X_{c_2} + R_2X_{c_1} + R_2X_{c_2} + j(R_1R_2 - X_{c_1}X_{c2})}}$$

$\because A \rightarrow$ pure Real & +ve $\Rightarrow$ we need β to have 0 phase shift. $\Rightarrow$ pure Real

$\hookrightarrow$ imaginary part $= 0$

$$\Rightarrow R_1 R_2 - X_{c_1} X_{c_2} = 0$$

$$\Rightarrow R_1 R_2 - \frac{1}{\omega_o C_1} \cdot \frac{1}{\omega_o C_2} = 0$$

$$\Rightarrow \frac{1}{\omega_o^2 C_1 C_2} = R_1 R_2$$

$$\Rightarrow \omega_o^2 = \frac{1}{R_1 R_2 C_1 C_2} \Rightarrow \boxed{\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$$

if $R_1 = R_2 = R$ & $C_1 = C_2 = C \Rightarrow \omega_o = \frac{1}{RC}$

$$\because |A\beta| = 1$$

at ω_o :

$$\Rightarrow \beta = \frac{R_2 X_{c_2}}{R_1 X_{c_2} + R_2 X_{c_1} + R_2 X_{c_2}}$$

for $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$\Rightarrow \beta = \frac{\cancel{R} \cancel{X_c}}{3 \cancel{R} \cancel{X_c}} = \frac{1}{3}$$

$$\Rightarrow A = 3 \Rightarrow 1 + \frac{R_f}{R_3} = 3$$

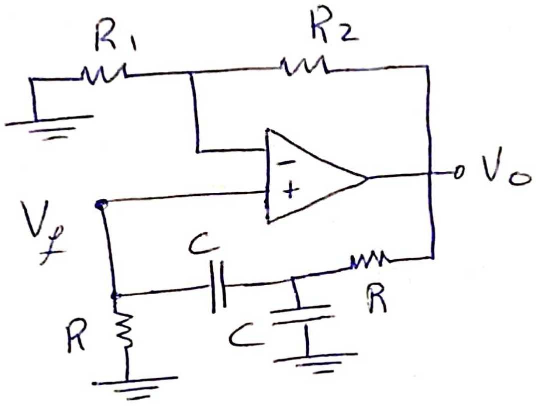
$$\Rightarrow \boxed{\frac{R_f}{R_3} = 2}$$

1 [1] For the shown circuit:

a) Find the frequency of oscillation.

b) Define the condition of oscillation.

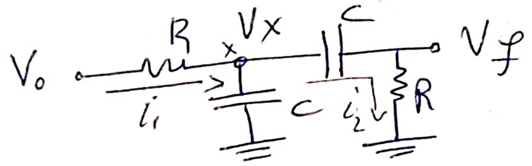
c) what is the type of oscillator.



Sol:

$$A = 1 + \frac{R_2}{R_1} \quad \text{(non-inverting Amplifier)}$$

$$\beta = \frac{V_f}{V_o}$$



Applying KCL at node x:

$$\Rightarrow \frac{V_o - V_x}{R} = \frac{V_x}{(-jX_c)} + \frac{V_x - V_f}{-jX_c} \qquad \Rightarrow i_1 = \frac{V_o - V_x}{R}$$

$$\therefore \frac{V_f}{R} = \frac{V_x - V_f}{-jX_c} = i_2 \Rightarrow \frac{V_x}{-jX_c} = V_f\left[-\frac{1}{jX_c} + \frac{1}{R}\right]$$

$$\Rightarrow \boxed{V_x = V_f\left[1 - \frac{jX_c}{R}\right]}$$

$$\rightarrow \frac{V_o}{R} = V_x\left[\frac{1}{R} + \frac{2}{-jX_c}\right] - \frac{V_f}{-jX_c}$$

$$\frac{V_o}{R} = V_f\left\{\left[1 - \frac{jX_c}{R}\right]\left[\frac{1}{R} + \frac{2}{-jX_c}\right] - \frac{1}{-jX_c}\right\}$$

$$\Rightarrow \beta = \frac{V_f}{V_o} = \frac{1}{R\left[\left(\frac{1}{R} - \frac{2}{jX_c} - \frac{jX_c}{R^2} + \frac{2}{R}\right) + \frac{1}{jX_c}\right]}$$

$$\Rightarrow \beta = \frac{1}{R\left[\frac{3}{R} - \frac{1}{jX_c} - \frac{jX_c}{R^2}\right]} = \frac{1}{3 + j\left[\frac{R}{X_c} - \frac{X_c}{R}\right]}$$

* oscillation condition : $|A\beta| = 1$
$\angle A\beta = 0$

$\because$ $A \rightarrow$ pure Real
$\rightarrow$ 0° shift

$\Rightarrow \beta \rightarrow$ must be pure Real $\xrightarrow[\text{achieve}]{\text{to}}$ $\angle A\beta = 0$

$$\Rightarrow \frac{R}{X_c} - \frac{X_c}{R} = 0$$

$$\Rightarrow \frac{R}{X_c} = \frac{X_c}{R}$$

$$\Rightarrow R(w_o C) = \frac{1}{w_o R C}$$

a) $\Rightarrow w_o^2 = \frac{1}{R^2 C^2} \Rightarrow \boxed{w_o = \frac{1}{RC}}$

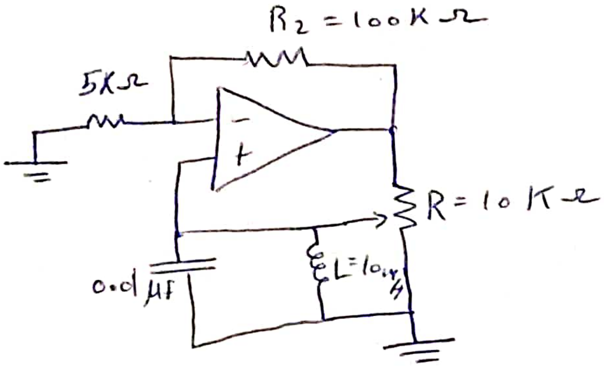
at w_o $\Rightarrow \beta = \frac{1}{3}$ $\because |A\beta| = 1$

$\Rightarrow A = 3 \Rightarrow 1 + \frac{R_2}{R_1} = 3$

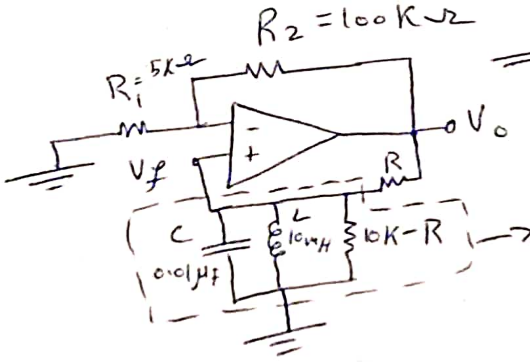
b) $\Rightarrow \boxed{\frac{R_2}{R_1} = 2}$ $\Rightarrow$ oscillation condition

c) $\Rightarrow$ This is a wien Bridge oscillator

2 For the shown oscillator circuit, find the frequency of oscillation & min. value of R



Sol:



$\Rightarrow A = 1 + \frac{R_2}{R_1}$

$\Rightarrow \frac{1}{Z} = j\omega C + \frac{1}{j\omega L} + 10K$

$$\Rightarrow \beta = \frac{V_f}{V_o} = \frac{Z}{R+Z}$$

$\Rightarrow$ oscillation condition:

$\therefore A\beta = 1$

$$\Rightarrow A = \frac{1}{\beta} = \frac{R+Z}{Z}$$

$$\Rightarrow \cancel{1} + \frac{R_2}{R_1} = \frac{R}{Z} + \cancel{1}$$

$$\Rightarrow \frac{R_2}{R_1} = \frac{R}{Z}$$

$$\Rightarrow \frac{R_2}{R_1} = R\left[j\omega_o C + \frac{1}{j\omega_o L} + \frac{1}{R}\right]$$

$$\frac{R_2}{R_1} = R\left[\frac{1}{10K-R} + j\left[\omega_o C - \frac{1}{\omega_o L}\right]\right]$$

* By comparing the imaginary parts in both sides $\Rightarrow$ $\omega_o C - \frac{1}{\omega_o L} = 0 \Rightarrow \omega_o^2 = \frac{1}{LC} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \boxed{\frac{R_2}{R_1} = \frac{R}{10K-R}}$$ $\Rightarrow$ oscillation condition

$$\because \frac{R_2}{R_1} = \frac{\overset{20}{\cancel{100K}}}{\cancel{5K_1}} = \frac{R}{10K - R}$$

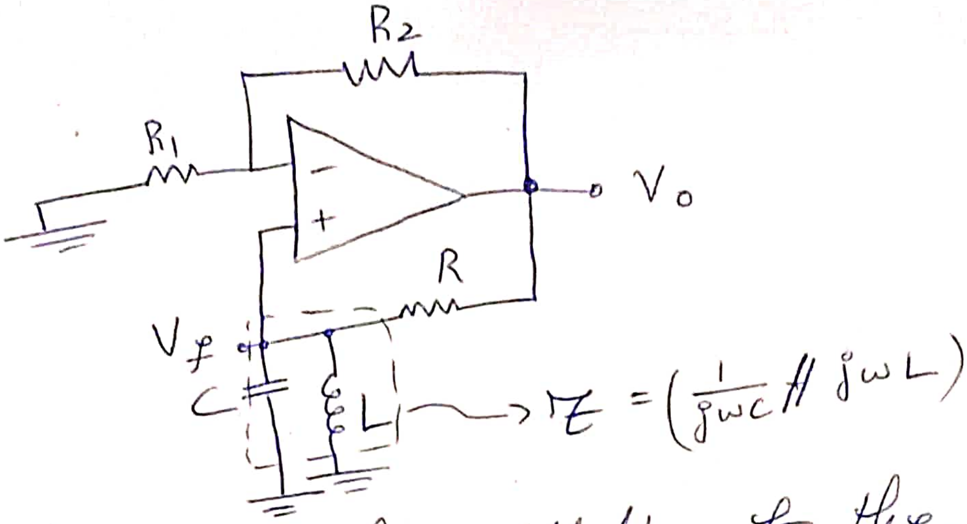
$$\Rightarrow \quad 20(10K - R) = R$$

$$\Rightarrow \quad 200K - 20R = R$$

$$\Rightarrow \quad \boxed{R = \frac{200K}{21} = 9.523\,K\Omega} \rightarrow \text{min Val. of } R$$

to sustain oscillation

3



- find the frequency of oscillation & the condition of oscillation.

sol:

$$\Rightarrow \frac{1}{Z} = \frac{1}{j\omega L} + j\omega C$$

$$A = 1 + \frac{R_2}{R_1}$$

$$\beta = \frac{V_f}{V_o} = \frac{Z}{R+Z}$$

$\Rightarrow$ oscillation condition:

$\therefore \; A\beta = 1$

$$\Rightarrow A = \frac{1}{\beta} \quad \Rightarrow 1 + \frac{R_2}{R_1} = \frac{R+Z}{Z}$$

$$\Rightarrow \cancel{1} + \frac{R_2}{R_1} = \frac{R}{Z} + \cancel{1}$$

$$\Rightarrow \frac{R_2}{R_1} = R\left[j\omega C + \frac{1}{j\omega L}\right]$$

* equating: Re = Re & imag = imag. for both sides

$$= jR\left[\omega C - \frac{1}{\omega L}\right]$$

$$\Rightarrow \omega C - \frac{1}{\omega L} = 0 \quad \Rightarrow \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \boxed{\frac{R_2}{R_1} = 0}$$

$\Rightarrow$ This means we only need $A = 1$

↳ since we don't have any Resistance in the LC Tank → which is an Ideal case