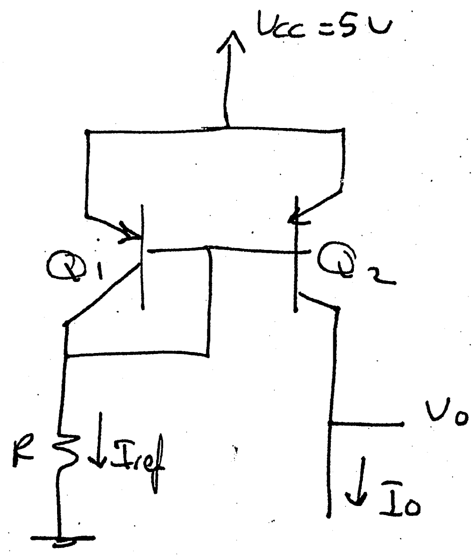
1- The Current Source shown, Trs are matched having $I_S = 10^{-15}$ A $\beta = 50$ and $|V_A| = 50$ V.

Design the circuit for $I_o = 1$ mA at $V_o = 2$ V. what I_{ref} and R needed.

what change in I_o corresponding to V_o change from max +ve to -5 V



عندما نأخذ في اعتبارنا الـ early voltage

∴ يصبح القانون لـ Mirror CS

$$I_o = \frac{I_{ref}}{1+2/\beta}\left(1+\frac{V_o - V_{EB}}{V_A}\right)$$

$$V_{EB_1} = V_{EB_2} = V_T \ln\frac{I_C}{I_S}$$

$$= V_T \ln\frac{I_o}{I_S}$$

$$\therefore V_{EB_1} = 0.025 \ln\left(\frac{10^{-3}}{10^{-15}}\right) = 0.69 \text{ volt}$$

$$\therefore I_o = \frac{I_{ref}}{1+2/\beta}\left(1+\frac{V_o - V_{EB}}{V_A}\right)$$

$$10^{-3} = \frac{I_{ref}}{1+2/50}\left(1+\frac{2-0.69}{50}\right)$$

$$\boxed{I_{ref} = 1.013 \text{ mA}} \quad , \quad R = \frac{V_{CC} - V_{EB}}{I_{ref}}$$

$$R = \frac{5 - 0.69}{1.013} = 4.25 \text{ k}\Omega$$

$$\boxed{R = 4.25 \text{ k}\Omega}$$

for $V_o = -5\,V$

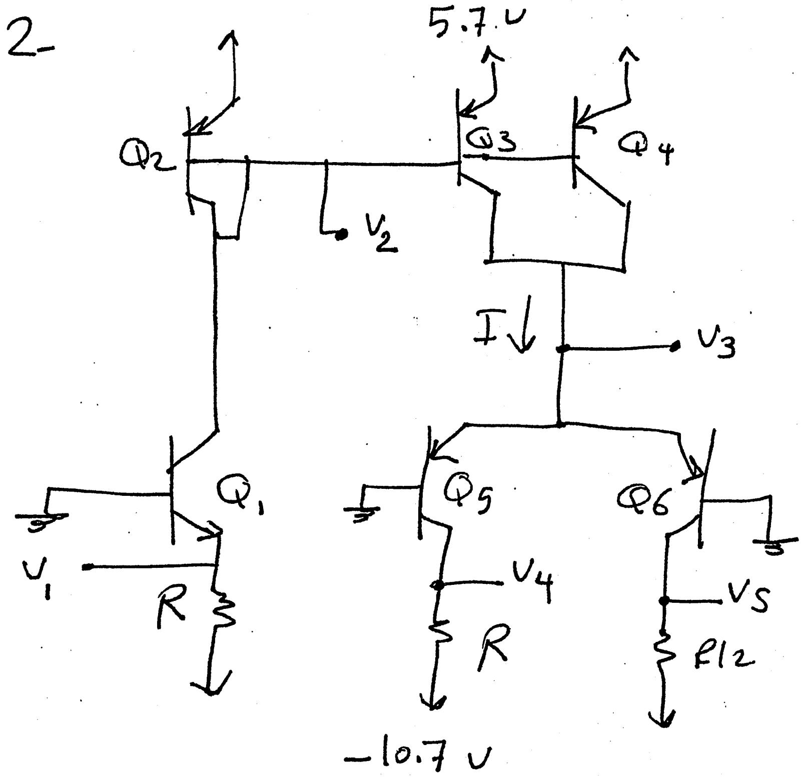
$$I_o = \frac{I_{ref}}{1 + 2/\beta}\left(1 + \frac{V_o - V_{EB}}{V_A}\right)$$

$$= \frac{1.013 \times 10^{-3}}{1 + 2/50}\left(1 + \frac{-5 - 0.69}{50}\right)$$

$$I_o = 0.86\ mA$$

$\therefore$ I_o changes from $1\,mA$ to $0.86\,mA$

2-



For the circuit shown $|V_{BE}| = 0.7$

$\beta = \infty$, find I, V_1, V_2, V_3, V_4, V_5

for $R = 10 k\Omega$, $R = 100 k\Omega$

For $\underline{R = 10k\Omega}$

$$V_1 = -0.7\ V$$

$$I_{C1} = \frac{-0.7 - (-10.7)}{10k\Omega} = 1mA$$

$$V_2 = 5.7 - 0.7 = 5\ V$$

$$I = I_{C3} + I_{C4} \quad , \quad I_{C3} = I_{C4} = I_{C1}$$

$$I = 2 \times 1 = 2mA$$

$$V_3 = V_{EB5} = V_{EB6} = 0.7V$$

$$V_4 = -10.7 + 1 \times 10 = -0.7\ V$$

$$V_5 = -10.7 + 1 \times \frac{10}{2} = -5.7\ V$$

for $R = 100\,k\Omega$

$V_1 = -0.7\,V$

$$I_{C1} = \frac{-0.7 + 10.7}{100k} = 0.1\,mA$$

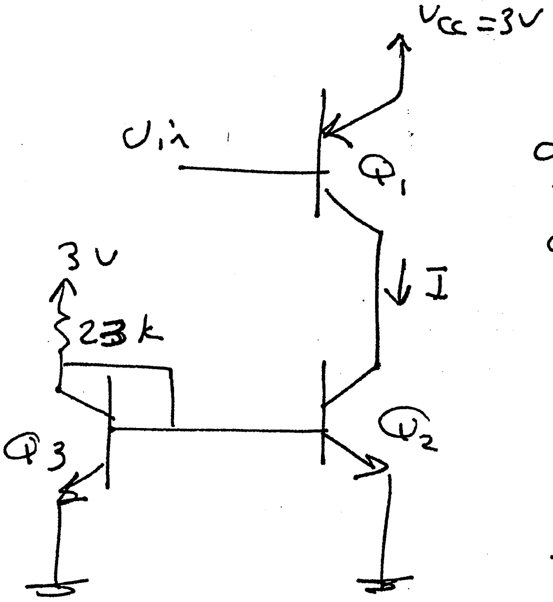
$I = 2 I_{C1} = 0.2\,mA$

$V_3 = 0.7\,V$, $V_2 = 5.7 - 0.7 = 5\,V$

$$V_4 = -10.7 + \frac{1 \times 100}{10} = -0.7\,V$$

$$V_5 = -10.7 + 0.1 \times \frac{100}{2} = -5.7\,V$$

3 -



a] neglect I_B of Q_2, Q_3 and let $V_{BE} = 0.7$ V

Q_2 has 5-area times area of Q_3.

- find I

b] If Q_1, Q_2 have $V_A = 50$ V, find r_{o1}, and r_{o2} and total r_o at Collector of Q_1

c] find $r_{\pi 1}$, g_{m1} assuming $\beta = 50$.

a) $\because$ Neglect I_B

$\therefore \quad I_{ref} \simeq I_{E_3} \simeq I_{C_3}$

$$I_{ref} = \frac{3 - 0.7}{23} = 0.1\,mA$$

$\therefore \quad \boxed{I_{C_3} = 0.1\,mA}$

$$\frac{I_{C_2}}{I_{C_3}} = \frac{\text{Area of } Q_2}{\text{Area of } Q_3} = 5$$

$\therefore \quad I_{C_2} = I = 5\,I_{C_3} = 0.5\,mA$

$\therefore \boxed{I_{C_2} = 0.5\,mA}$

b) $|V_A| = 50$

$$r_{o1} = \frac{|V_A|}{I_{C_1}} = \frac{|V_A|}{I} = \frac{50}{0.5\,mA}$$

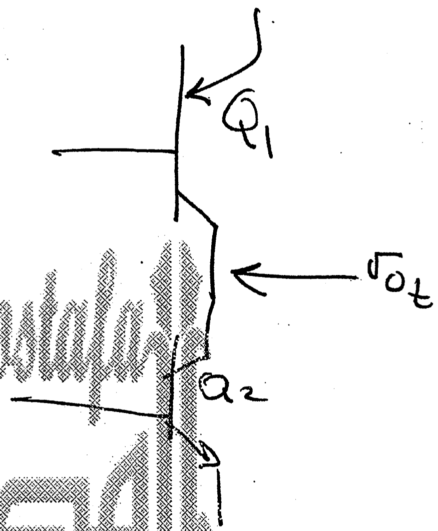
$$\boxed{r_{o1} = 100\,k\Omega}$$

$$r_{o2} = \frac{|V_A|}{I_{C_2}} = \frac{|V_A|}{I} = \frac{50}{0.5\,mA}$$

$$\therefore \boxed{r_{o2} = 100\,k\Omega}$$

at Collector of Q_1

$$r_{ot} = r_{o1} \parallel r_{o2}$$



$$\therefore r_{o\,total} = \frac{100\,k\Omega}{2}$$

$$\boxed{r_{o\,total} = 50\,k\Omega}$$

C] $\beta = 50$

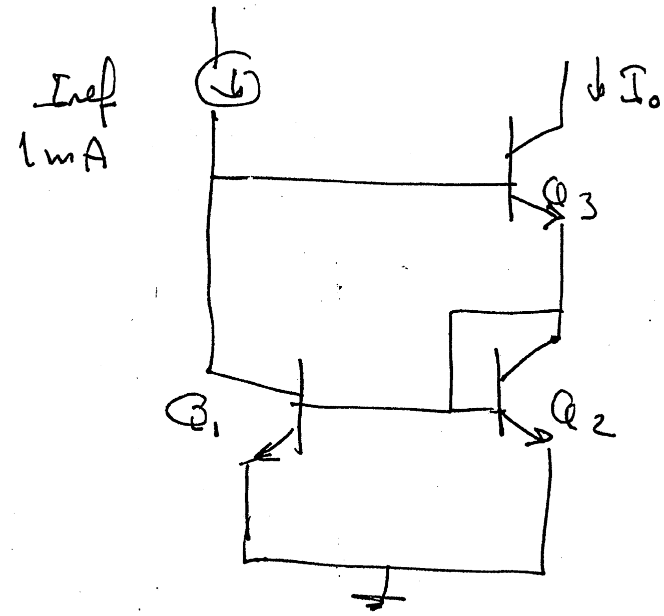
$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.5}{0.025}$$

$$g_{m1} = 20\,mA/V$$

$$r_{\pi 1} = \frac{\beta}{g_m} = \frac{50}{20} = 2.5\ k\Omega$$

$$r_{\pi 1} = 2.5\ k\Omega \equiv R_{in}$$

4- for wilson C.S shown $I_{ref} = 1mA$ what change in I_O due to change of +10V at collector of Q_3. Let $\beta = 100$, $V_A = 100$ V



Hint:

output resistance of wilson C.S = $\frac{\beta r_o}{2}$

for wilson Current source

$$I_o \simeq \frac{1}{1+\frac{2}{\beta^2+2\beta}} I_{ref}$$

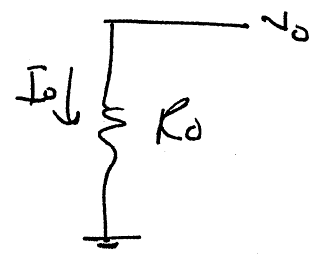
for $\beta = 100$

$$\boxed{\therefore I_o = 0.999 \text{ mA}}$$

for wilson current source $R_o = \dfrac{\beta r_o}{2}$

$$\therefore \Delta I_o = \frac{\Delta v_o}{R_o}$$

$R_o \rightarrow$ ثابتة



$\Delta v_o = 10$ volt

$$R_o = \frac{\beta r_o}{2} = \frac{\beta}{2}\left(\frac{V_A}{I_c}\right)$$

$I_c = I_o$

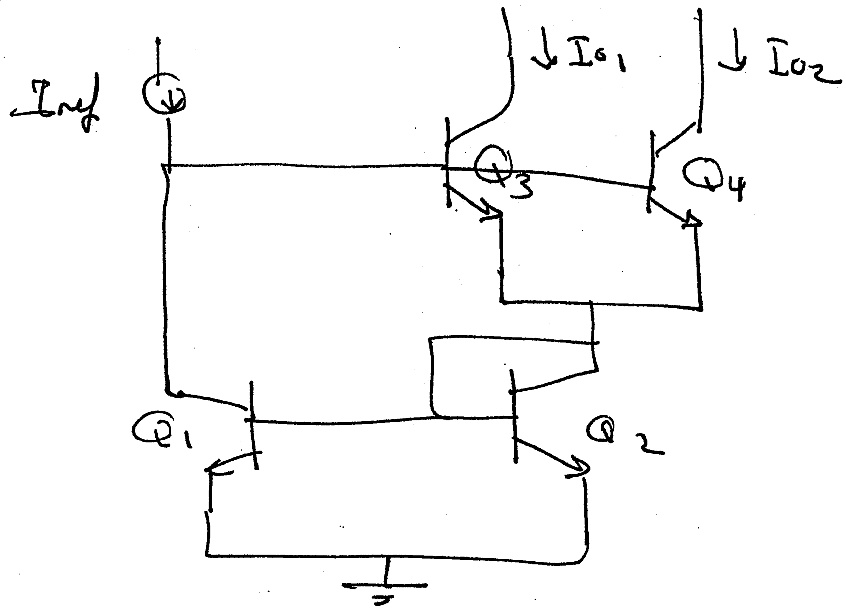
$$= \frac{100}{2} * \frac{100}{0.999}$$

$$\therefore \boxed{R_o = 5.005 \text{ M}\Omega}$$

$$\therefore \Delta I_o = \frac{10}{5.005 \text{ M}\Omega} = 1.998 * 10^{-6}$$

$$\simeq 2 \mu A$$

5- for circuit shown

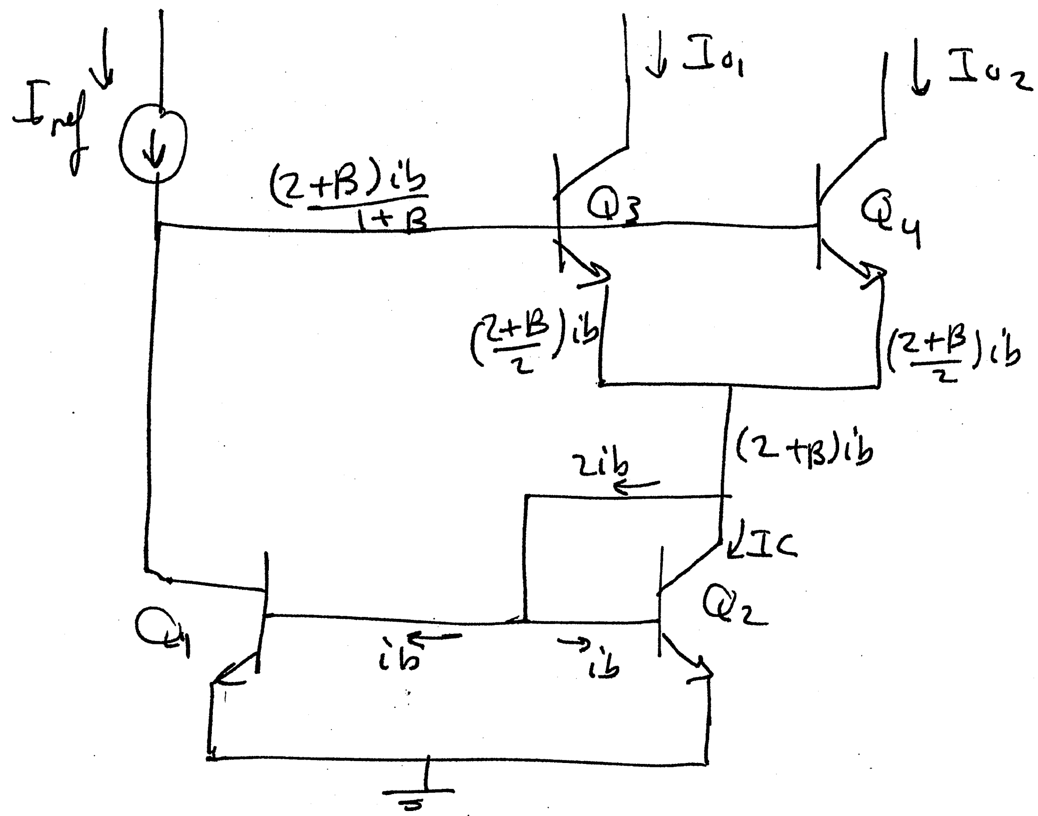


Q_3, Q_4 Matched find I_{o1}, I_{o2}

- in terms of I_{ref} assum all Tr are matched

- use above idea to design a circuit to generate 1mA, 2mA and 4mA using I_{ref} = [illegible] mA

- what actual of Currents for $\beta = 50$.



$$I_{o1} = \beta\left[\frac{(2+\beta)i_b}{2(1+\beta)}\right] = I_{o2} \longrightarrow (1)$$

$$I_{ref} = \frac{(2+\beta)i_b}{1+\beta} + \beta i_b$$

$$I_{ref} = \left[\frac{2+\beta}{1+\beta} + \beta\right] i_b \longrightarrow (2)$$

from ①, ②

$$\therefore\ I_{o1} = \frac{\beta(2+\beta)}{2(1+\beta)} \cdot \frac{1}{\frac{2+\beta}{1+\beta} + \beta}\ I_{ref}$$

$$= \frac{\beta(2+\beta)}{2\left[2+\beta + \beta(1+\beta)\right]}\ I_{ref}$$

$$I_{o1} = \frac{\beta(2+\beta)}{2\left[\beta^2 + 2\beta + 2\right]}\ I_{ref}$$

$$= \frac{1}{2} * \frac{\beta^2 + 2\beta}{\beta^2 + 2\beta + 2}\ I_{ref}$$

$\beta^2 \gg 2\beta$

$$= \frac{1}{2} \frac{\beta^2}{\beta^2 + 2\beta + 2}$$

$$= \frac{1}{2} \frac{1}{1 + 2/\beta + 2/\beta^2}\ I_{ref}$$

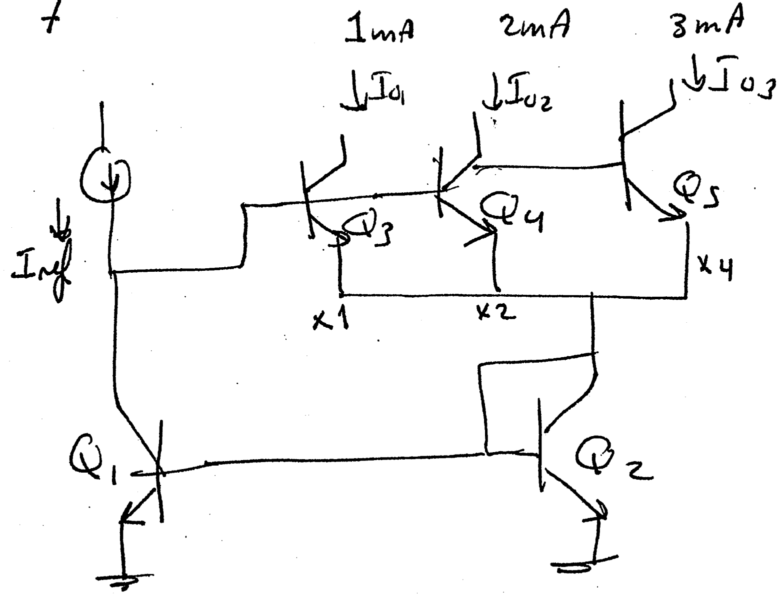
$$\boxed{I_{o1} \simeq \frac{1}{2}\left(\frac{1}{1 + 2/\beta^2}\right) I_{ref}}$$

$I_{o1} = I_{o2} =$ نفس المعادلة

b) using 3 transistors in parallel having relative area ratios of 1, 2, 4 as shown below

$$I_{o1} = \frac{1}{7} I_{ref}$$

$\therefore$ Area of $Q_3 = \frac{1}{7}$ Area of Q_1



$\therefore I_{o2} = \frac{2}{7} I_{ref}$ Area of Q_4 = 2 [Area of Q_3]

$I_{o3} = \frac{4}{7} I_{ref}$ Area of Q_5 = 4 [Area of Q_3]

$$\boxed{\text{Area of } Q_5 = 2\left[\text{Area of } Q_4\right] = 4\left[\text{Area of } Q_3\right]}$$

for $\beta = 50$

Actual value of I_{o1}, I_{o2}, I_{o3}

$$I_{o1} = \frac{1}{7}\left(\frac{I_{ref}}{1+2/\beta^2}\right)$$

$$= \frac{1}{7}\left(\frac{1}{1+2/\beta^2}\right) = \frac{1}{7}\left(\frac{1}{1+2/50^2}\right)$$

$$\boxed{I_{o1} = 0.998\,mA}$$ (1mA ideally)

$$I_{o2} = \frac{2}{7} I_{ref}$$

$$\boxed{I_{o2} = 1.996\,mA}$$ (2mA ideally)

$$I_{o3} = \frac{4}{7} I_{ref}$$ (4mA ideally)

$$\boxed{I_{o3} = 3.992\,mA}$$

Note:

سواء استخدمت Wilson C.S

$$I_o = \frac{1}{1+\frac{2}{\beta^2+2\beta}}$$

أو

$$I_o = \frac{1}{1+2/\beta^2}$$

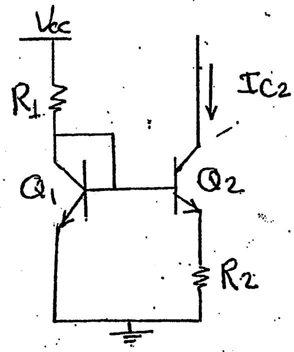
مش هتفرق في الناتج النهائي بنسبة كبيرة

Sheet # 1

Electronics (3)

Sheet#1

Current Source , Active loads , Output stages:



1-For the shown Widler current source :

-Derive an expression for the output current .

-compare this circuit with the mirror current source.

-find the output impedance of the shown widler current source.

Neglecting base Currents we Can write

$$I_{E_2} \simeq I_{C_2} \simeq I_o$$

$$I_{ref} \simeq I_{C_1} \simeq I_{E_1}$$

from KVL

$$V_{BE_1} = V_{BE_2} + I_{E_2} R_2 \longrightarrow (1)$$

$$\therefore \; I_C = I_S \left(e^{V_{BE}/V_T} \right)$$

where $I_S \rightarrow$ Reverse Saturation Current

2

$$I_{C_1} \approx I_{ref}$$
$$= I_{S_1} \left(e^{V_{BE1}/V_T} \right) \rightarrow (1)$$

$$I_{C_2} \approx I_o$$
$$= I_{S_2} \left(e^{V_{BE2}/V_T} \right) \rightarrow (2)$$

for identical transistors $\therefore I_{S1} = I_{S2}$

from (1)

$$V_{BE_1} = V_T \ln \left(\frac{I_{ref}}{I_{S_1}} \right)$$

from (2)

$$V_{BE_2} = V_T \ln \left(\frac{I_o}{I_{S_2}} \right)$$

$$V_{BE_1} - V_{BE_2} = V_T \ln \left(\frac{I_{ref}}{I_o} \right)$$

from Eq (1")

$$V_{BE_1} - V_{BE_2} = I_o R_e$$
$$= V_T \ln \left(\frac{I_{ref}}{I_o} \right)$$

So,

$$I_o R_e = V_T \ln\left(\frac{I_{ref}}{I_o}\right)$$

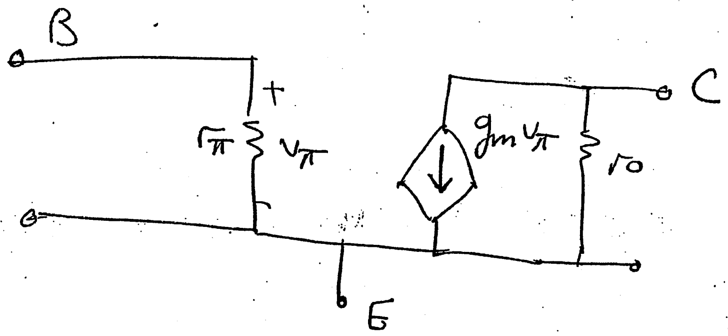
Note
use Try and Error to solve this Equation.

- Compare This Circuit with Mirror C.S
  Widlar differs from the basic Current Mirror Circuit in The Resistor (R_E) in the Emitter of Q_2.
- And Widlar Current source has an output resistance higher Than Mirror.
- Widlar Circuit allows The generation of a small Constant Current using relatively small Resistors. (Saving in Chip Area).

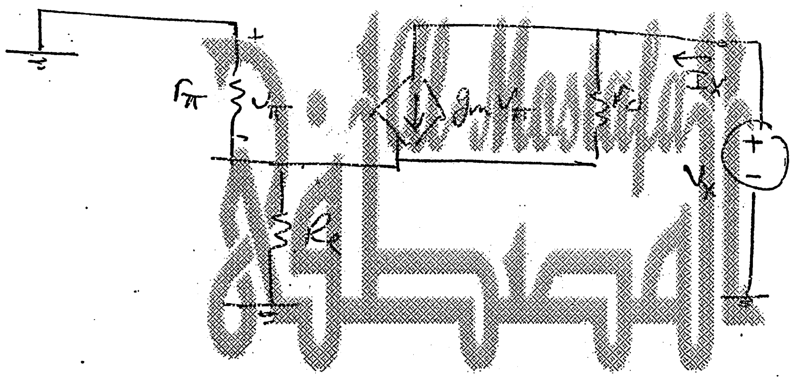
- To find the output impedance of widlar current source.

use the π-Model of $\underline{Q_2}$

Replace the BJT with its low-frequency hybrid-π Model.



To find R_o, apply a Test voltage $\underline{V_x}$ to the Collector



5

Note

R_E parallel with r_π

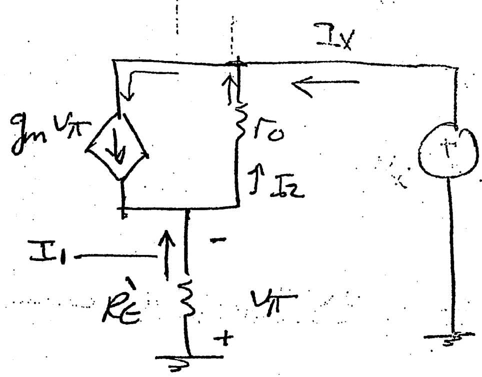
$$R_E' = R_E \,//\, r_\pi = \frac{R_E * r_\pi}{R_E + r_\pi}$$

From The Model

$$I_1 = \frac{V_\pi}{R_E'}$$

$$I_1 = -I_X$$

$$\therefore \; I_X = \frac{-V_\pi}{R_E'}$$



Note

$$I_X = g_m V_\pi + I_2$$

So, $$I_2 = g_m V_\pi - I_X$$

$$I_2 = g_m V_\pi + \frac{V_\pi}{R_E'} = V_\pi\left(g_m + \frac{1}{R_E'}\right)$$

$$R_o = \frac{V_X}{I_X} = \frac{-\left[r_o\left(V_\pi\left(g_m + \frac{1}{R_E'}\right)\right) + V_\pi\right]}{-V_\pi / R_E'}$$

6

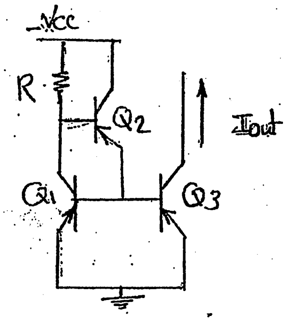
$$R_o = \frac{r_o\left(g_m + \frac{1}{R_E'}\right) + 1}{1/R_E'}$$

$$R_o = \frac{r_o(g_m R_E' + 1) + R_E'}{1}$$

$\therefore$ $\boxed{\text{The output Resistance} = r_o(g_m R_E' + 1) + R_E'}$

$\boxed{7}$

2-Determine the value of the output current
if β of the PNP transistor is in the rang of 2-10.
Compare this circuit with the simple (mirror) current source.



Note: All Transistor pnp

For matching between (Q_1, Q_3)

$\therefore \; I_{b1} = I_{b3}$

And The two operate in Active Mode

$\therefore \; V_{BE_1} = V_{BE_3} = 0.7 \text{ V}$

From (KCL)

$$I_{ref} = \frac{-1.4 + V_{CC}}{R}$$

$$I_{E_2} = I_{b1} + I_{b3}$$
$$= 2 I_b$$

$$I_{ref} = I_{b1} + I_{C_1}$$
$$= \frac{I_{E_2}}{(1+\beta)} + \beta I_b$$
$$= \frac{2 I_b}{(1+\beta)} + \beta I_b = I_b\left(\beta + \frac{2}{1+\beta}\right)$$

8

$$I_{ref} = I_b \left(\beta + \frac{2}{1+\beta}\right) \rightarrow (1)$$

$$I_o \simeq I_{C3}$$

$$= \beta I_b$$

$$= \beta \frac{I_{ref}}{\left(\beta + \frac{2}{1+\beta}\right)}$$

$$\boxed{I_o \simeq \frac{I_{ref}}{1 + \frac{2}{(1+\beta)\beta}}}$$ #

for β in range 2-10 (small)

The greater value (β^2)

So $\boxed{I_o \simeq I_{ref}}$

* This circuit give I_o approximately equal I_{ref} for small value of β, which that not satisfied in Mirror Current Source.

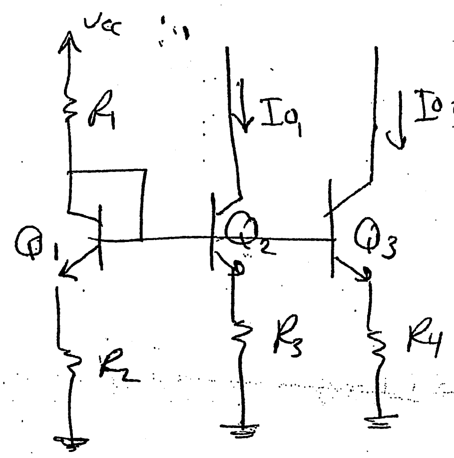
(T1)

Current Source Examples

Example:

For The Current Source shown in fig(1).

a) Derive an approximate expression for I_{o1}, I_{o2} as a function of V_{cc} [assume identical transistors]



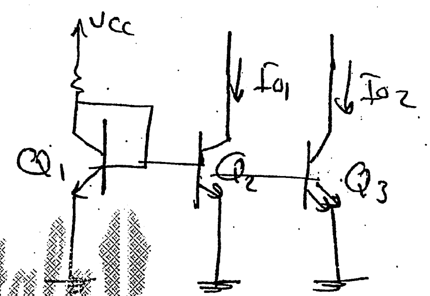
Fig(1)

b) Find I_{o1}, I_{o2}

If $V_{cc} = 10V$ $R_1 = 1k\Omega$

$R_2 = R_3 = 50\,\Omega$ $R_4 = 100\,\Omega$

c) If The Circuit of fig(1) IS Replaced by the Current source Shown in fig(2), find The new Values of I_{o1}, I_{o2}. [assume identical transistors]



Fig(2)

d) which of The two Circuits do you prefer from the point of view

i) I.C fabrication

ii) Current Source performance.

(2)

Solution

* $R_2 = R_3$ for (Q_1, Q_2)

So Q_1, Q_2 matched (Mirror C.S)

$I_{o_1} \simeq I_{ref}$

$$\boxed{I_{ref} = \frac{V_{CC} - V_{BE_1}}{R_1 + R_2} = I_{o_1}}$$

$$I_{ref} = \frac{10 - 0.7}{1k + 50\Omega} = 9.25\ mA$$

* for Q_1, Q_3

$R_2 \neq R_4$ ∴ not matched (Not Mirror C.S) [Widlar C.S]

from KVL

$$V_{BE_1} + R_2 I_{ref} = V_{BE_3} + R_4 I_{o_2}$$

$$V_{BE_1} - V_{BE_3} = R_4 I_{o_2} - R_2 I_{ref}$$

(3)

$$\therefore \quad V_{BE_1} - V_{BE_2} = V_T \ln \frac{I_{ref}}{I_{o2}}$$

$$\therefore \quad \boxed{V_T \ln \frac{I_{ref}}{I_{o2}} = R_4 I_{o2} - R_2 I_{ref}}$$

$I_{ref} \simeq 9\ mA$ $\qquad R_2 = 50\ \Omega$

$V_T \simeq 25\ mV$ $\qquad R_4 = 100\ \Omega$

So, we use try and Error to find I_{o2} using Calculator in this equation

So, $\boxed{I_{o2} = 18\ mA}$

c) for Circuit in fig (2)

Q_1, Q_2 matched

$$\therefore \quad I_{o1} \simeq I_{ref} = \frac{V_{CC} - V_{BE_1}}{R_1} = 9.3\ mA$$

Q_1, Q_3

$$I_{o2} \simeq 2 I_{ref} = 2(9.3) = 18.6\ mA$$

multi-Emitter (Q_3)

4

d) For I.C fabrication

Circuit in fig(2) Contain less Resistors
So, it prefer for IC fabrication
(less Area Needed)

For performance in Current Source
→ Circuit in fig(1) Widlar Current Source
for Q_3 which derive Small Current.

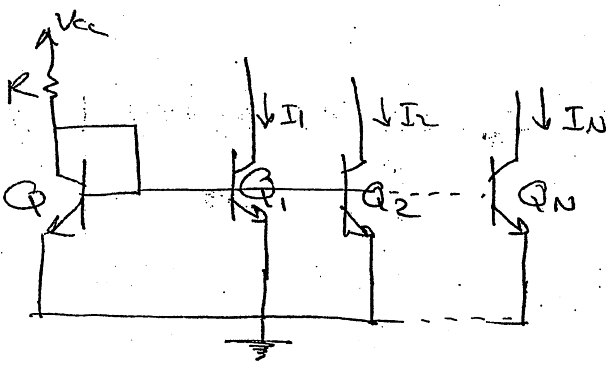
example:

For shown figure

a) prove that $I_1 = I_2 = \dots = I_N = \dfrac{I_{ref}}{1 + (N+1)/\beta}$

Assuming that all transistors are matched and have finite β

b) for $\beta = 100$, find the maximum number of outputs for an Error not exceeding 10%.



Sol:

a) for matched transistors

$\therefore V_{BE_1} = V_{BE_2} = V_{BE_3} \dots = V_{BE_N} = V_{BE}$

for (active) $V_{BE} = 0.7$ Volt

This is Mirror C.S

So $I_1 = I_2 = \dots = I_N \simeq I_{ref}$

$$\frac{1}{1+\frac{(N+1)}{\beta}} \geq 0.9$$

$\frac{1+\left(\frac{N+1}{\beta}\right)}{0.9}$ بالضرب الطرفين ×

$$\frac{1}{0.9} \geq 1+\frac{(N+1)}{\beta} \rightarrow 100$$

$$N \leq 10.11$$

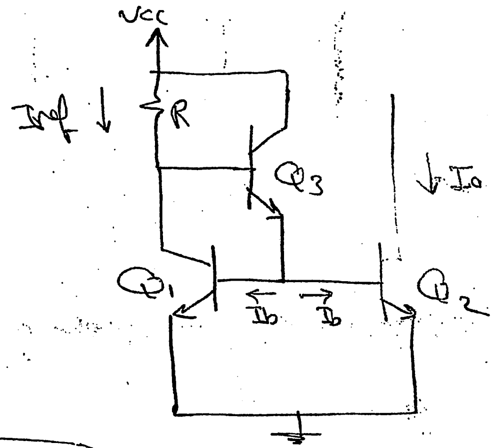
$$\therefore \boxed{N_{max} = 10} \quad \#$$

Example:

For the shown Circuit find the smallest value of β so that $I_o \geq 0.99\, I_{ref}$

Assume identical transistors.

This is improved Mirror Current Source.



$$\boxed{I_o \simeq \frac{1}{1+\frac{2}{\beta^2+\beta}}\, I_{ref}}$$

$$\frac{1}{1+\frac{2}{\beta^2+\beta}} \geq 0.99$$

$$1+\frac{2}{\beta^2+\beta} \leq \frac{1}{0.99}$$

$$\beta^2+\beta+2 \leq \frac{1}{0.99}(\beta^2+\beta)$$

$$\beta^2 + \beta + 2 \leq 1.01\beta^2 + 1.01\beta$$

$$(0.01\beta^2 + 0.01\beta - 2) \geq 0$$

$$\beta = 13.6$$

$$\therefore \quad \beta \geq 13.6$$

So the smallest value of $\boxed{\beta = 13.6}$

<u>If you use approximation</u>

$$\beta^2 \gg \beta$$

$$\therefore \quad I_o \simeq \frac{1}{1 + \frac{2}{\beta^2}} I_{ref}$$

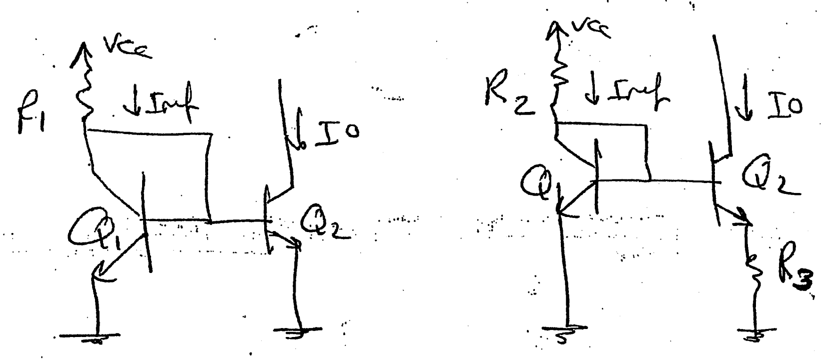
$$1 + \frac{2}{\beta^2} \leq \frac{1}{0.99}$$

$$\therefore \quad \frac{2}{\beta^2} \leq 0.01$$

$$\boxed{\beta_{min} = 14}$$

Example:

The two circuits for generating a Constant Current $I_o = 10\mu A$. operate from a 10volt Supply. Determine the required resistors, Assuming $V_{BE} = 0.7\ V$ at a Current of 1mA.



(a) (b)

Solution

for Circuit in fig(a)

we Choose The value of $I_{ref} = 10\mu A$

at This current, the voltage drop accross Q_1

will be.

$V_{BE} = 0.7$ at $1mA$

$V_{BE} = ??$ at $10\mu A$

we know

$$\therefore \quad \boxed{V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)}$$

$$\therefore \quad V_{BE_1} - V_{BE_2} = V_T \ln\left(\frac{I_1}{I_2}\right)$$

$$0.7 - V_{BE_2} = \underbrace{V_T}_{0.025} \ln\left(\frac{1\,mA}{10\,\mu A}\right)$$

$$\therefore \quad \boxed{V_{BE_2} = 0,584 \text{ volt}}$$

So, $I_{ref} = \dfrac{\overset{10\,volt}{V_{cc}} - \overset{0.584}{V_{BE}}}{R_1} = 10\,\mu A$

$$\therefore \quad \boxed{R_1 = 942\,k\Omega}$$ which is large (for IC fabrication)

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for The Circuit in fig (b)

$I_o = 10\,\mu A$

في حالة وجود قيمة لل R_3

$\therefore I_{ref} > I_o$

$\therefore$ هنا نختار قيمة ال $\boxed{I_{ref} = 1mA}$

For The widlar Circuit, we must first decide On The Suitable value of I_{ref}

If we select

$I_{ref} = 1mA$ then $V_{BE_1} = 0.7$

$$\therefore R_2 = \frac{V_{CC} - 0.7}{I_{ref}}$$

$$= \frac{10 - 0.7}{1mA} = 9.3\,k\Omega$$

The value of R_3 Can be determined from This Equation

$$I_O R_3 = V_T \ln\left(\frac{I_{ref}}{I_O}\right)$$

$I_O = 10\mu A$ $\quad I_{ref} = 1mA$

$V_T = 25mV$

$$\therefore R_3 = \frac{0.025 \ln\left(\frac{1mA}{10\mu A}\right)}{1\mu A}$$

$$\therefore \boxed{R_3 = 11.5\,k\Omega}$$