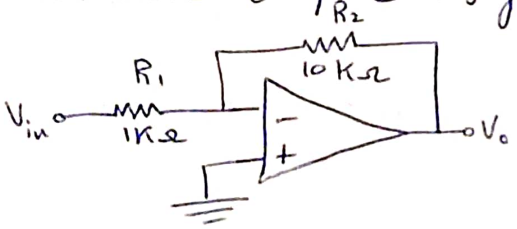
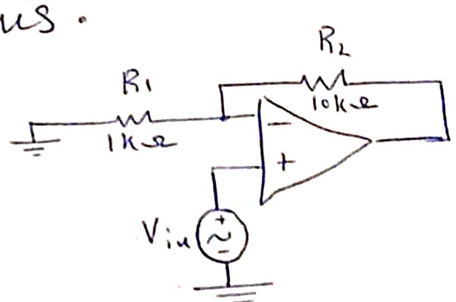
Active Filters

sheet 1

1. The circuits shown represent an op-amp inverting & non-inverting amplifiers respectively. op-amp finite open-loop DC gain $A_o = 10^5$ & open-loop Band width $\omega_b = 10$ rad/s

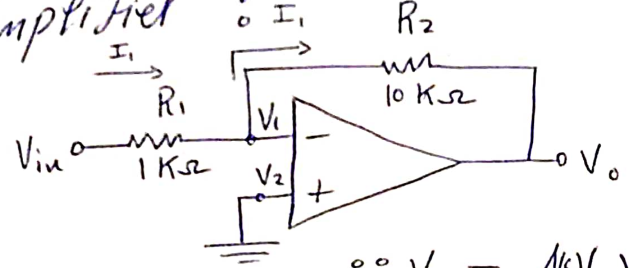
- Derive an expression for the closed-loop Band-width ω_c.
- Calculate the closed loop DC gain A_m & closed-loop Bandwidth ω_c.
- Calculate the Gain-BW product (GBP) for open & closed loop configurations.





Sol :-

for inverting Amplifier :



$\because V_o = A(s)(V_2 - V_1)$

but, $V_2 = 0$

$\Rightarrow \boxed{V_o = -A(s)\,V_1}$ (1)

$$\Rightarrow I_1 = \frac{V_{in} - V_1}{R_1} = \frac{V_1 - V_o}{R_2}$$

$$\Rightarrow \frac{R_2}{R_1} V_{in} = V_1\left(1 + \frac{R_2}{R_1}\right) - V_o$$

* sub. by V_1 from (1) :

$$\Rightarrow \frac{R_2}{R_1} V_{in} = \left[1 + \frac{R_2}{R_1}\right]\left(-\frac{V_o}{A(s)}\right) - V_o$$

$$\Rightarrow \frac{R_2}{R_1} V_{in} = \left(\left[1 + \frac{R_2}{R_1}\right] \cdot \frac{1}{A_1} + 1\right)(-V_o)$$

$$\Rightarrow H(s) = \frac{V_o}{V_{in}} = \frac{-\frac{R_2}{R_1}}{1 + \left(\frac{1 + \frac{R_2}{R_1}}{A(s)}\right)}$$

$$\because A(s) = \frac{A_0}{1 + \frac{s}{\omega_b}}$$

$$\Rightarrow H(s) = \frac{V_o}{V_{in}} = \frac{-\frac{R_2}{R_1}}{1 + \left[\frac{1 + \frac{R_2}{R_1}}{\frac{A_0}{1+\frac{s}{\omega_b}}}\right]} = \frac{-R_2/R_1}{1 + \frac{\left(1+\frac{R_2}{R_1}\right)\left(1+\frac{s}{\omega_b}\right)}{A_0}}$$

$$= \frac{-R_2/R_1}{1 + \underbrace{\cancel{\frac{\left(1+\frac{R_2}{R_1}\right)}{A_0}}}_{\substack{\text{negligable} \\ \left(A_0 >> 1+\frac{R_2}{R_1}\right)}} + \frac{\left(1+\frac{R_2}{R_1}\right)\left(\frac{s}{\omega_b}\right)}{A_0}}$$

$$\Rightarrow H(s) \simeq \frac{-R_2/R_1}{1 + \frac{\left(1+\frac{R_2}{R_1}\right)\left(\frac{s}{\omega_b}\right)}{A_0}}$$

$$= \frac{-R_2/R_1}{1 + \frac{s}{\frac{A_0(\omega_b)}{\left(1+\frac{R_2}{R_1}\right)}}} \xrightarrow[\text{with}]{\text{compare}} \underbrace{\frac{A_m}{1 + \frac{s}{\omega_c}}}$$

↳ standard form for a simple single pole system.

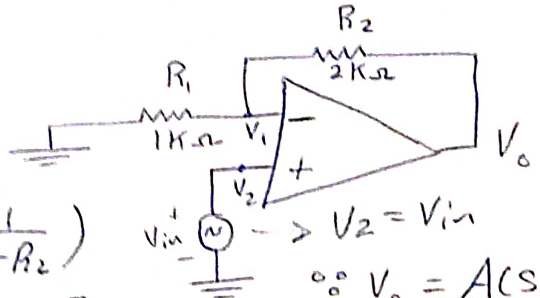
$$\Rightarrow A_m = \frac{R_2}{R_1} = 10$$

$$\Rightarrow \omega_c = \frac{A_0 \omega_b}{1 + \frac{R_2}{R_1}} = \frac{(10^5)(10)}{1 + 10} = 90909.09 \text{ rad/sec}$$

$$\Rightarrow GBP|_{\text{open loop}} = A_0 \omega_b = (10^5)(10) = 10^6 \text{ rad/sec}$$

$$\Rightarrow GBP|_{\text{closed loop}} = A_m \omega_c = (10)(90909.09) = 909090.9 \text{ rad/sec}$$

For the non-inverting Amplifier :



$\rightarrow V_2 = V_{in}$

$\therefore V_o = A(s)\,(V_2 - V_1)$

$$\Rightarrow V_1 = V_o \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow V_o = A(s)\left(V_{in} - V_o \frac{R_1}{R_1 + R_2}\right)$$

$$\Rightarrow A(s)V_{in} = V_o\left[1 + \frac{A(s)R_1}{R_1 + R_2}\right]$$

$$\Rightarrow H(s) = \frac{V_o}{V_{in}} = \frac{A(s)}{1 + A(s)\frac{R_1}{R_1 + R_2}}$$

$$\Rightarrow H(s) = \frac{A(s)}{1 + \frac{A(s)}{1 + R_2/R_1}} = \frac{1 + R_2/R_1}{\frac{(1 + R_2/R_1)}{A(s)} + 1}$$

$$\therefore A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}}$$

$$\Rightarrow H(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1 + R_2/R_1}{\left(\frac{A_o}{1 + \frac{s}{\omega_b}}\right)}} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{(1 + \frac{R_2}{R_1})(1 + \frac{s}{\omega_b})}{A_o}}$$

$$\Rightarrow H(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \underbrace{\cancel{\frac{1 + \frac{R_2}{R_1}}{A_o}}}_{\text{negligable}} + \frac{(1 + \frac{R_2}{R_1})\frac{s}{\omega_b}}{A_o}} \simeq \frac{1 + \frac{R_2}{R_1}}{1 + \frac{(1 + \frac{R_2}{R_1})s}{\omega_b A_o}}$$

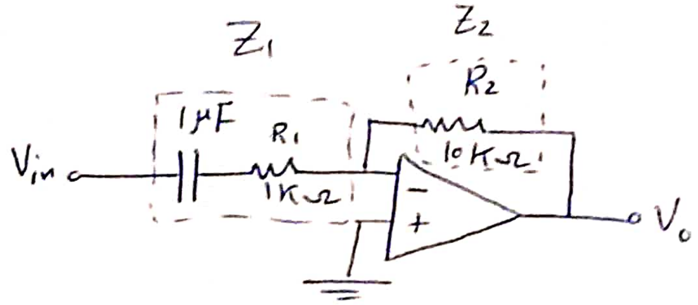
$$\Rightarrow A_m = 1 + \frac{R_2}{R_1}, \quad \omega_c = \frac{\omega_b A_o}{1 + \frac{R_2}{R_1}}$$

2. Analyze the circuits using ideal op-amp :

- Drive an expression for the closed loop gain (A_v)
- Find Max DC gain (A_m) & cut-off freq. (f_c).
- Find the unity gain freq. (f_T).
- Sketch the freq. response Magnitude.

Sol:

for the HPF :



$$\Rightarrow A_V = -\frac{Z_2}{Z_1}$$

$$Z_2 = R_2 \quad , \quad Z_1 = \frac{1}{SC} + R_1$$

$$\Rightarrow A_V = -\frac{R_2}{R_1 + \frac{1}{SC}} = \frac{-R_2/R_1}{1 + \frac{1}{SR_1C}} = \frac{-R_2/R_1}{1 + \frac{(1/R_1C)}{S}} = \frac{A_m}{1 + \frac{\omega_c}{S}}$$

$$\Rightarrow A_m = \frac{R_2}{R_1} = 10 \, , \quad \omega_c = \frac{1}{R_1 C} = \frac{1}{(10^3)(10^{-6})} = 1000 \qquad = 1$$

$$\Rightarrow f_c = \frac{1}{2\pi R_1 C} = 159.15 \text{ Hz}$$

* to get unity gain freq. :

$$\Rightarrow \text{at } |A_m| = 1 = \left| \frac{10}{1 + \frac{1000}{j\omega}} \right|$$

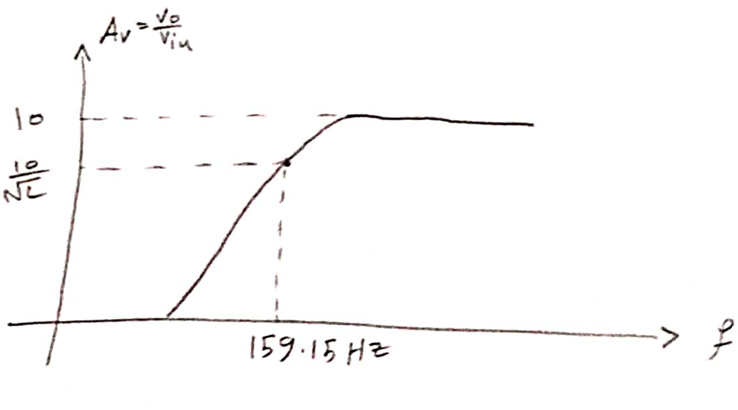
$$1 = \frac{10}{\sqrt{1 + \left(\frac{1000}{\omega}\right)^2}}$$

$$\Rightarrow 1 + \left(\frac{1000}{\omega}\right)^2 = (10)^2$$

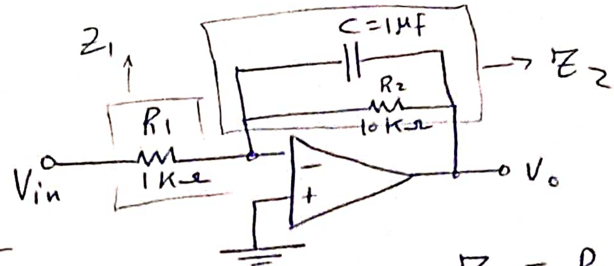
$$\Rightarrow \omega^2 + (1000)^2 = 100\,\omega^2$$

$$\Rightarrow \omega_t^2 = \frac{10^6}{99} \Rightarrow \omega_t = 100.5 \text{ Hz}$$

$$\Rightarrow f_t = \frac{\omega_t}{2\pi} = 16 \text{ Hz}$$



for the LPF :



$$\Rightarrow A_V = \frac{V_o}{V_{in}} = \frac{-Z_2}{Z_1}$$

$$Z_1 = R_1$$

$$Z_2 = \frac{1}{sC} // R_2 = \frac{\frac{R_2}{sC}}{R_2 + \frac{1}{sC}} = \frac{R_2}{1 + R_2 sC}$$

$$\Rightarrow A_V = - \frac{\frac{R_2}{1 + R_2 sC}}{R_1} = - \frac{R_2 / R_1}{(1 + R_2 sC)} = - \frac{R_2/R_1}{1 + \frac{s}{\left(\frac{1}{R_2 C}\right)}}$$

$$\Rightarrow A_m = \frac{R_2}{R_1} = 10 \quad , \quad \omega_c = \frac{1}{R_2 C} = \frac{1}{10 \times (10^3)(10^{-6})} = 10^2 \text{ rad/sec}$$

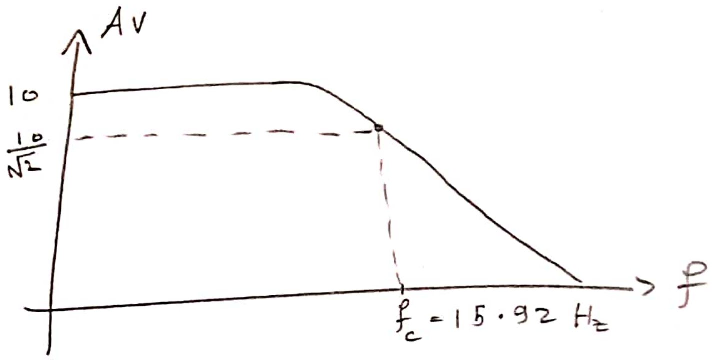
$$\Rightarrow f_c = \frac{\omega_c}{2\pi} = 15.915 \text{ Hz}$$

$$\Rightarrow |A_m| = 1 = \left| \frac{10}{1 + \frac{j\omega_t}{10^2}} \right| = \frac{10}{\sqrt{1 + \frac{\omega_t^2}{10^4}}}$$

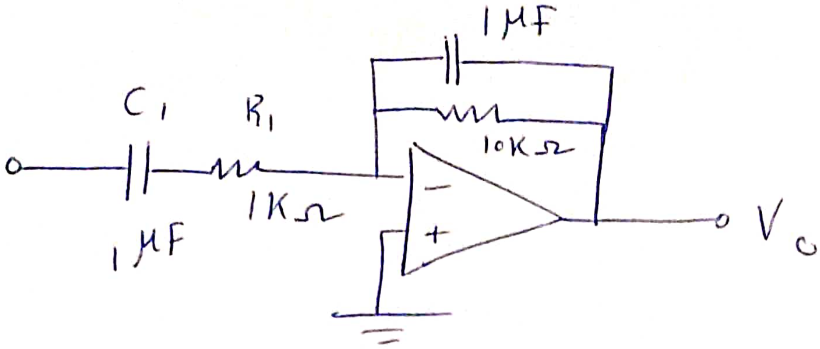
$$\Rightarrow 1 + \frac{\omega_t^2}{10^4} = 10^2$$

$$\Rightarrow \omega_t = 994.98 \text{ rad/sec}$$

$$\Rightarrow f_t = 158.357 \text{ Hz}$$ $\Rightarrow$ unity gain frequency



3.



$$\Rightarrow A_V = -\frac{Z_2}{Z_1} \quad , \quad Z_2 = \frac{1}{sC_2} \parallel R_2 = \frac{R_2}{1+R_2 s C_2}$$

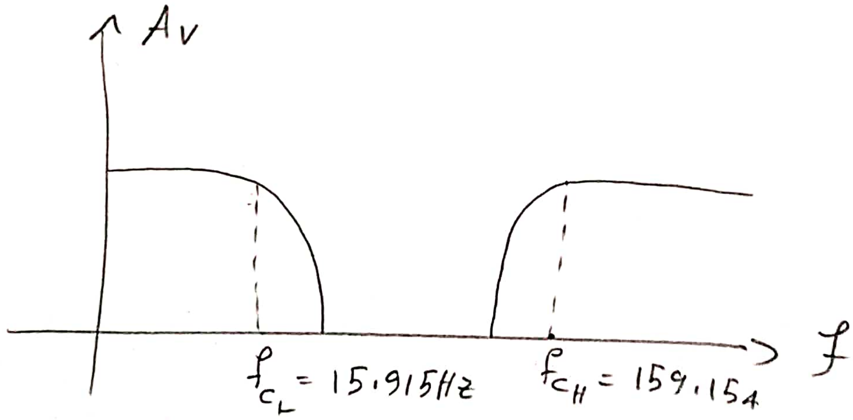
$$Z_1 = R_1 + \frac{1}{sC_1}$$

$$A_V = \frac{-\frac{R_2}{1+R_2 s C_2}}{R_1 + \frac{1}{sC_1}} = -\frac{R_2}{(1+R_2 s C_2)(R_1 + \frac{1}{sC_1})}$$

$$= -\frac{R_2/R_1}{\underbrace{(1+R_2 s C_2)}_{LPF}\underbrace{(1+\frac{1}{R_1 s C_1})}_{HPF}}$$

$$\Rightarrow f_{C_L} = \frac{1}{2\pi(10\times10^3)(10^{-6})} = 15.915\ Hz$$

$$\Rightarrow f_{C_H} = \frac{1}{2\pi(10^3)(10^{-6})} = 159.154$$



↳ BSF

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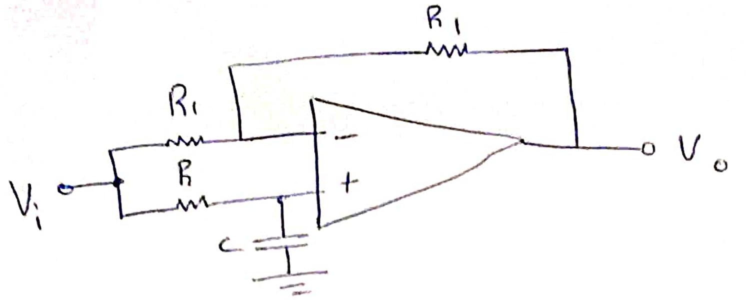
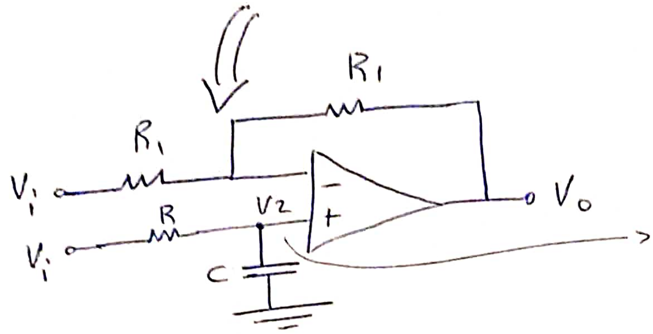


Sol:



$$V_2 = V_i\left(\frac{\frac{1}{sC}}{R+\frac{1}{sC}}\right)$$

$$\Rightarrow V_o = V_i\left(-\frac{\cancel{R_1}}{\cancel{R_1}}\right) + V_i\left(\frac{\frac{1}{sC}}{R+\frac{1}{sC}}\right)\left(1+\frac{\cancel{R_1}}{\cancel{R_1}}\right)$$

$$= -V_i + V_i\left(\frac{1}{1+RSC}\right)(2) = \left(-1+\frac{2}{1+RSC}\right)V_i$$

$$\Rightarrow A_v = \frac{V_o}{V_i} = \frac{2}{1+RSC} - 1 = \frac{2-(1+RSC)}{1+RSC}$$

$$\Rightarrow A_v = \frac{+1-RSC}{1+RSC} = -\left[\frac{S-\frac{1}{RC}}{S+\frac{1}{RC}}\right]$$

$$\Rightarrow |A_v| = \left[\frac{\sqrt{\omega^2+\frac{1}{R^2C^2}}}{\sqrt{\omega^2+\frac{1}{R^2C^2}}}\right] = 1$$

$\Rightarrow$ gain is always equal to one for all ω.

$\Downarrow$

all-pass filter

$\Downarrow$

used only to add phase shift