Electronic Systems

Active Filters

Lecture 2

Dr. Roaa Mubarak

1. Inverting Amplifier (Finite-Gain): Closed Loop

Where

Am = The closed loop maximum gain

Ao = The Open Loop maximum gain

Wc = The Closed Loop Band Width

Wb = The Open Loop Band Width

$Am = -\frac{R2}{R1}$ and **Wc** = $\frac{AoW_b}{(1+\frac{R2}{R1})}$

Closed Loop Gain-Band width product:

GBP(Closed-Loop) = Am Wc = $(\frac{R2}{R1})\ (\frac{AoW_b}{(1+\frac{R2}{R1})}) = AoW_b$ **= GBP(Open-Loop)**

Since $\frac{R2}{R1} >> 1$

2. Non-Inverting Amplifier (Finite-Gain): Closed Loop

Where

Am = The closed loop maximum gain

Ao =The Open Loop maximum gain

Wc = The Closed Loop Band Width

Wb = The Open Loop Band Width

$Am = 1 + \frac{R2}{R1}$ and Wc = $\frac{AoW_b}{(1+\frac{R2}{R1})}$

Closed Loop Gain-Band width product:

GBP(Closed-Loop) = Am Wc = $\left(1 + \frac{R2}{R1}\right)\left(\frac{AoW_b}{(1+\frac{R2}{R1})}\right) = AoW_b$ = GBP(Open-Loop)

Example 1

Analyze the circuit shown in Figure :

(1) Derive an expression for the closed loop gain (H(s) =Vo/Vin) in terms of the finite open loop gain (A).

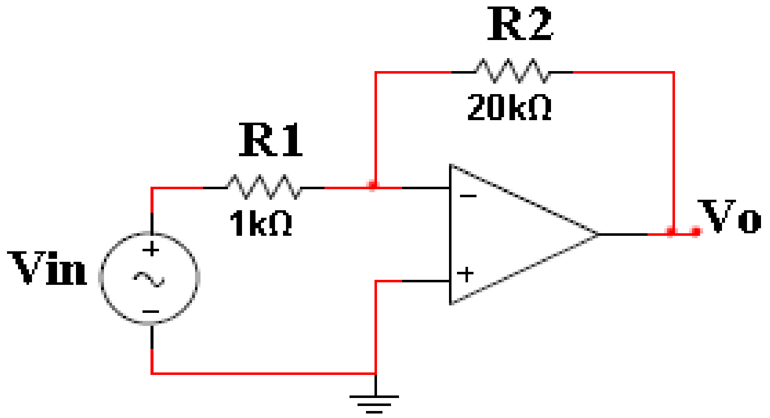
(2) If the open loop gain is given by:

$$A(S) = \frac{Ao}{1 + \frac{S}{\omega b}}$$

Where: $A_o = 10^4$, and $\omega_b = 10$ rad/sec.

Derive H(s) and sketch the closed loop frequency response.

(3) Calculate the Gain-Band-width product (GBP) for open and closed loop cases..

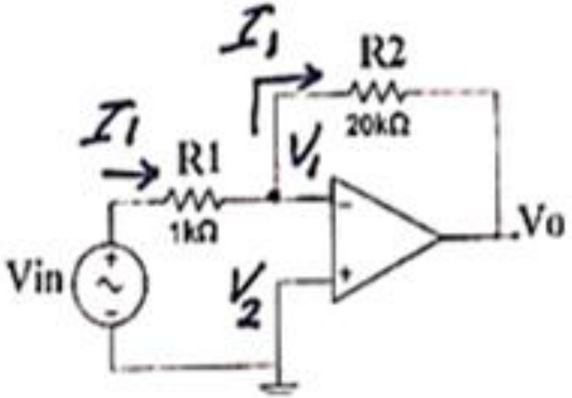


Example 1

$*\ V_o = A(V_2 - V_1)\ , V_2 = 0$

$\therefore\ V_o = -A V_1$

$\therefore \boxed{V_1 = -\frac{V_o}{A}}\ \boxed{1}$



$*\ I_1 = \frac{V_{in} - V_1}{R_1} = \frac{V_1 - V_o}{R_2}$

$\therefore\ \frac{V_{in}}{R_1} - \frac{V_1}{R_1} = \frac{V_1}{R_2} - \frac{V_o}{R_2} \qquad \times R_2$

$\frac{R_2}{R_1} V_{in} - \frac{R_2}{R_1} V_1 = V_1 - V_o$

$\frac{R_2}{R_1} V_{in} = [1 + \frac{R_2}{R_1}] V_1 - V_o \qquad$ sub. From $\boxed{1}$

$\therefore\ \frac{R_2}{R_1} V_{in} = [1 + \frac{R_2}{R_1}] \frac{-V_o}{A} - V_o$

$\frac{R_2}{R_1} V_{in} = \{[1 + \frac{R_2}{R_1}]/A + 1\}(-V_o)$

$\therefore\ H(s) = \frac{V_o}{V_{in}} = \frac{-(\frac{R_2}{R_1})}{1 + (\frac{1 + \frac{R_2}{R_1}}{A})}$

$\therefore\ H(s) = \frac{V_o}{V_{in}} = \frac{-(\frac{R_2}{R_1})}{1 + (\frac{1 + \frac{R_2}{R_1}}{A})}$

$$\boxed{H(s) = \frac{-(R_2/R_1)}{1 + \frac{1 + (R_2/R_1)}{A(s)}}}$$

Example 1

ii- Derive H(s) and sketch the closed loop frequency response.

$$H(s) = \frac{-(R_2/R_1)}{1 + \dfrac{1+R_2/R_1}{\left(\dfrac{A_0}{1+\frac{s}{\omega_b}}\right)}} = \frac{-(R_2/R_1)}{1 + \underbrace{\dfrac{1+R_2/R_1}{A_0}}_{\to 0} + \dfrac{(1+R_2/R_1)\,s}{A_0\,\omega_b}}$$

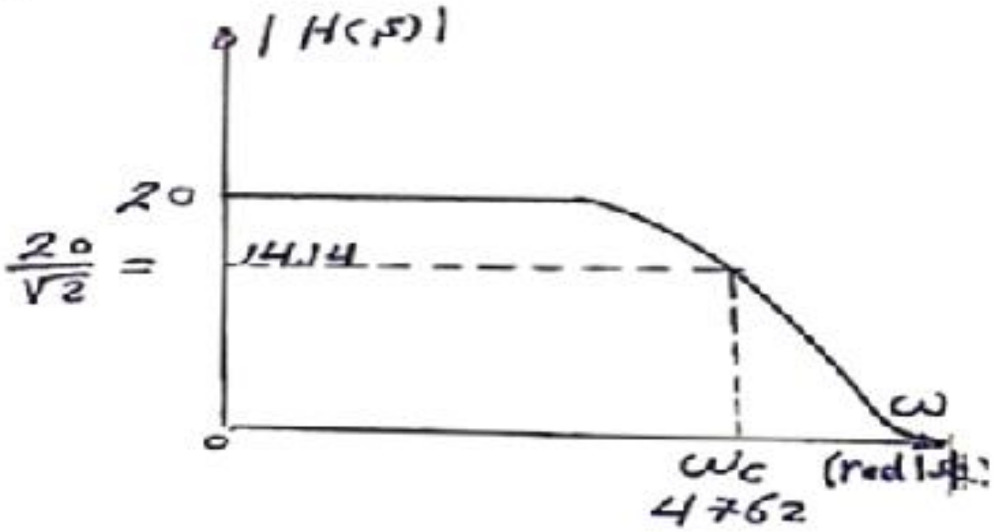
$$H(s) \cong \frac{-(R_2/R_1)}{1 + \dfrac{s}{\left(\dfrac{A_0\,\omega_b}{1+\frac{R_2}{R_1}}\right)}} = \frac{-A_m}{1+\dfrac{s}{\omega_c}}$$

$$\boxed{A_m = \frac{R_2}{R_1} = 20}$$

$$\omega_c = \frac{A_0\,\omega_b}{1+\frac{R_2}{R_1}}$$

$$\boxed{\omega_c = 4761.9 \ \text{rad/sec}}$$

$$H(s) \cong \frac{-20}{1+\dfrac{s}{4762}}$$



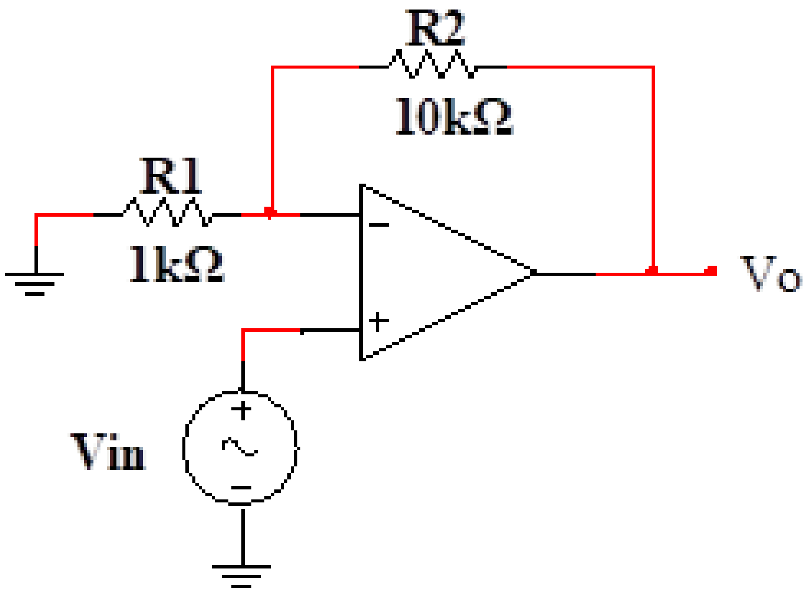
Example 1

iii-Calculate the Gain-Band-width product (GBP) for open and closed loop cases.

- GBP(Open-Loop) = $A_o\ \omega_b = 10^5$ rad/sec = 100000 rad/Sec
- GBP(Closed-Loop) = $A_m\ \omega_c$ = 20 x 4762 = 95240 rad/Sec

Example 2

(a)Derive an expression for the closed loop gain (H(s) =Vo/Vin) in terms of the finite open loop gain (A).



(b) If the open loop gain is given by:

$$A(S) = \frac{A_o}{1 + \frac{S}{\omega_b}}$$

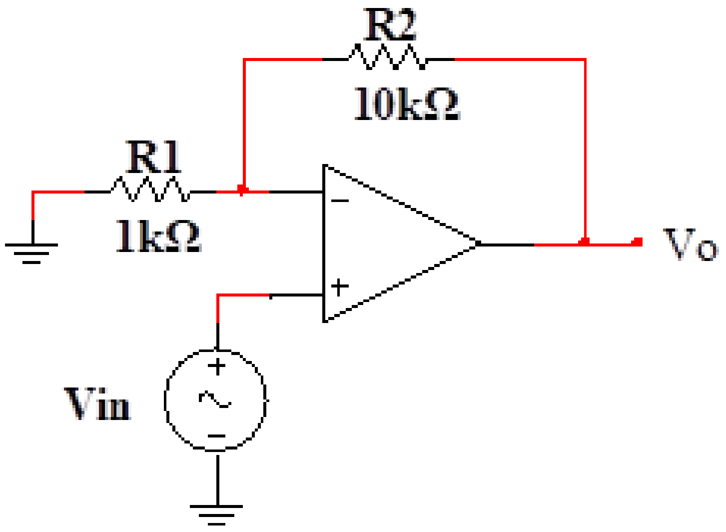
Where: $A_o = 10^5$, and $\omega_b = 10$ rad/sec.

Derive H(s) and sketch the closed loop frequency response.

(c) Calculate the Gain-Band-width product(GBP).

Example 2

(a)Derive an expression for the closed loop gain (H(s) =Vo/Vin) in terms of the finite open loop gain (A).



$* \; V_o = (V_2 - V_1).A$

$$V_o = \left[V_{in} - \frac{V_o.R_1}{R_1 + R_2}\right]A$$

$$V_o = A\,V_{in} - \frac{A\,R_1}{R_1 + R_2}V_o$$

$$V_o\left[1 + \frac{A}{1 + \frac{R_2}{R_1}}\right] = A.V_{in}$$

$$\therefore \; H(s) = \frac{V_o}{V_{in}} = \frac{A}{1 + \frac{A}{1+\frac{R_2}{R_1}}} \qquad \times \frac{(1 + \frac{R_2}{R_1})}{A}$$

$$H(s) = \frac{(1+\frac{R_2}{R_1})}{1 + \frac{(1+\frac{R_2}{R_1})}{A}} = \frac{K}{1 + \frac{K}{A}}$$

where: $K = 1 + \frac{R_2}{R_1}$

Example 2

Derive H(s) and sketch the closed loop frequency response.

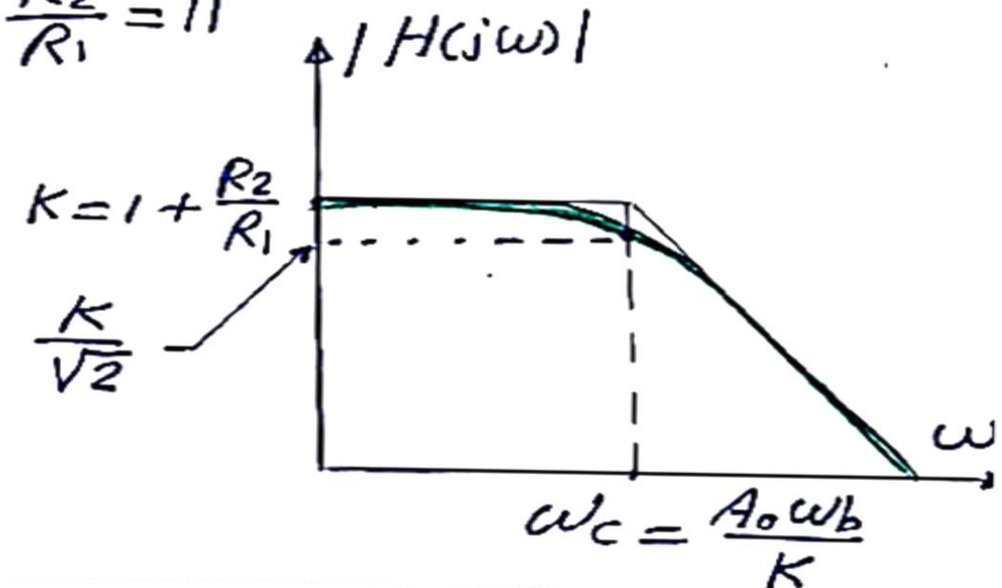
$$H(s) = \frac{K}{1+\frac{K}{A_0}\left(1+\frac{s}{\omega_b}\right)} = \frac{K}{1+\underbrace{\frac{K}{A_0}}_{\to 0}+\frac{Ks}{A_0\,\omega_b}}$$

$$\boxed{H(s) = \frac{K}{1+\frac{s}{\left(\frac{A_0\,\omega_b}{K}\right)}} = \frac{K}{1+\frac{s}{\omega_c}}}$$

* New d.c gain $= K = 1 + \frac{R_2}{R_1} = 11$

* New cut-off Frequency

$$\boxed{\omega_c = \frac{A_0\,\omega_b}{K}} = 90.91\times10^{3} \text{ rad/sec.}$$



Example 2

(c) Calculate the Gain-Band-width product(GBP).

- GBP(Open-Loop) = $A_o\ \omega_b = 10^6$ rad/sec

- GBP(Closed-Loop) $= K\ \omega_c = K\ \dfrac{A_o\ \omega_b}{K} = A_o\ \omega_b = 10^6$ rad/Sec

- GBP = Constant

Types of Filters

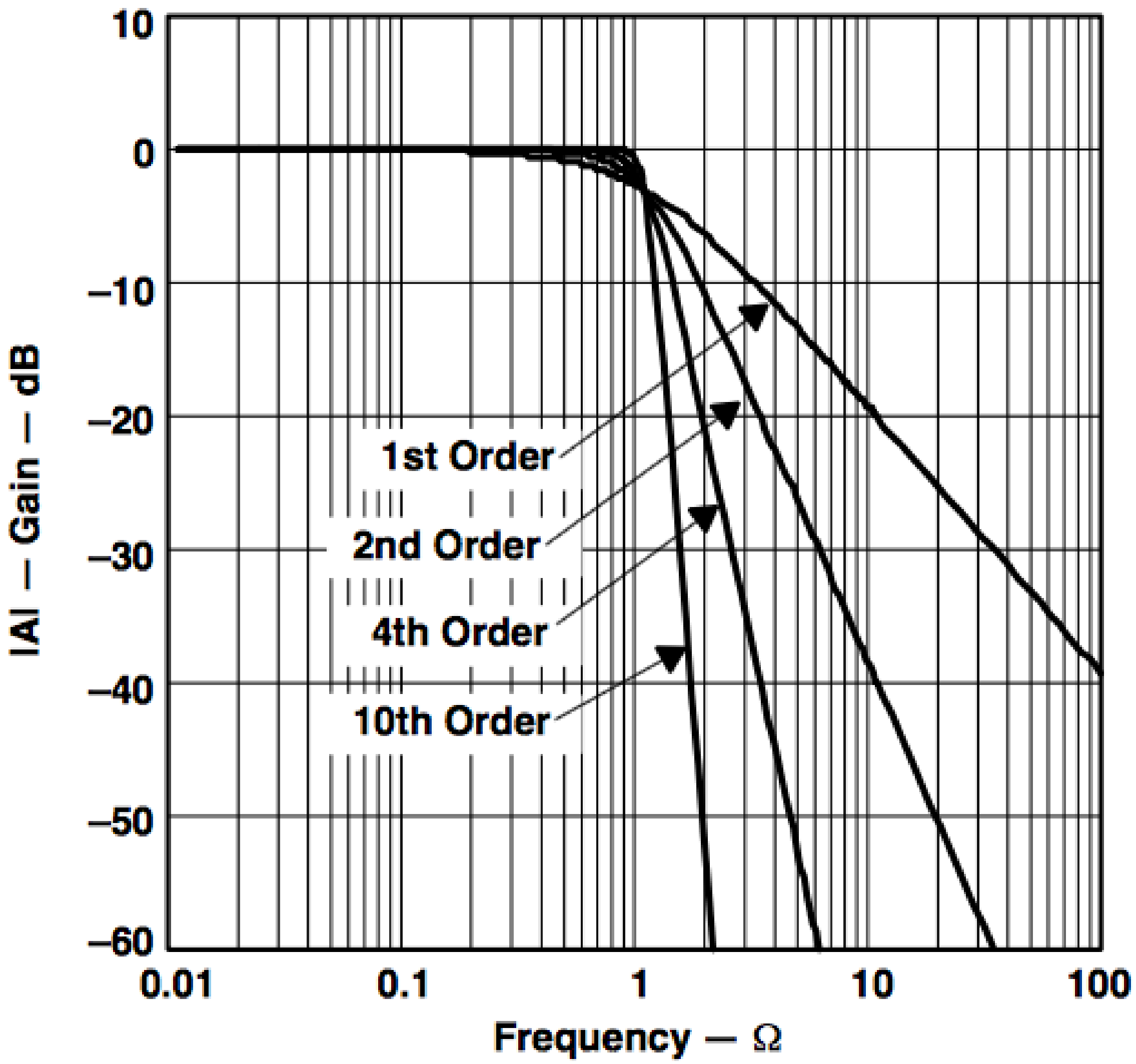
Types of Filters

- The four primary types of filters include:

1. The Low-pass filter.
2. The High-pass filter.
3. The Band-pass filter.
4. The Notch filter (band-reject or band-stop filter).

- The terms "low" and "high" do not refer to any absolute values of frequency, but rather they are relative values with respect to the cutoff frequency.

Impact of High Order Filters



First Order Filters: Low pass filter (LPF)

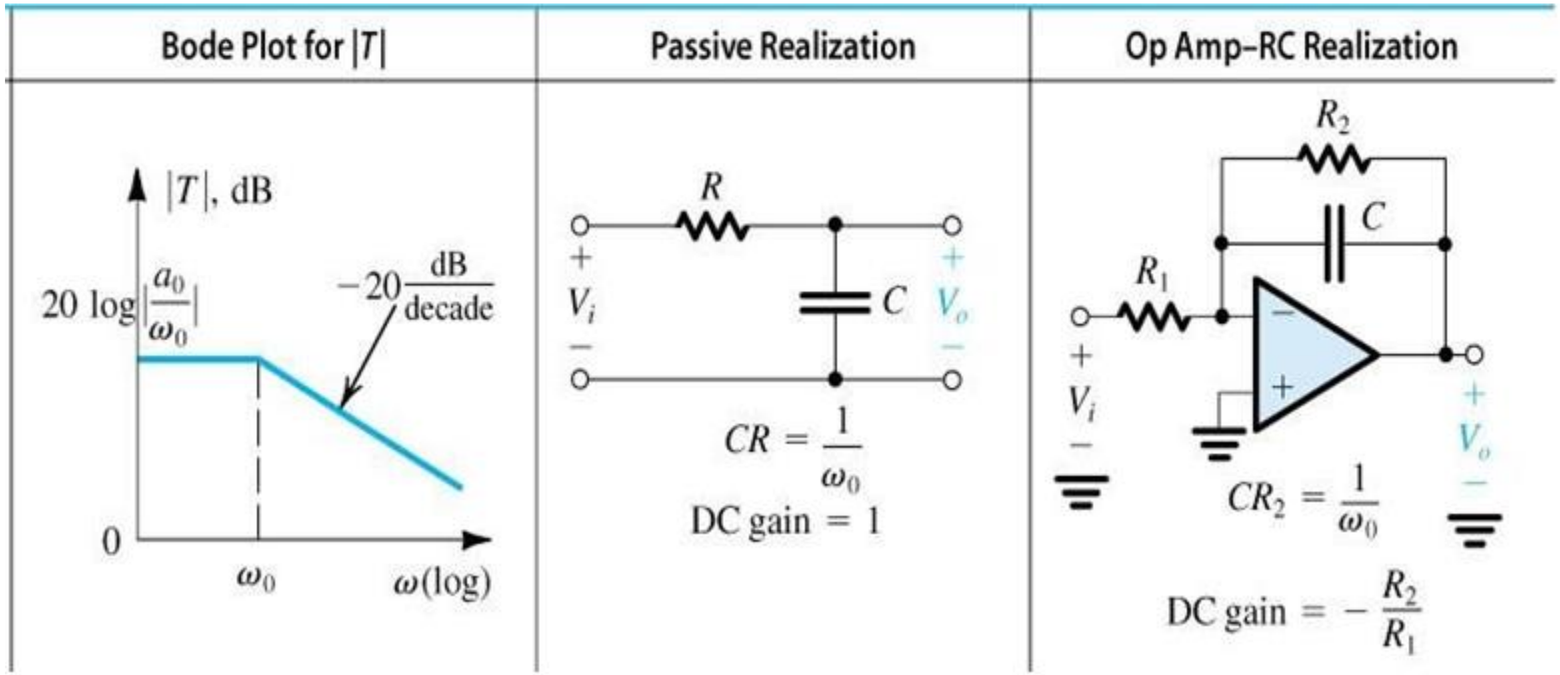
$$A(S) = \frac{A_m}{1 + \frac{S}{W_c}}$$

A_m → is the maximum gain (DC Gain),

Wc → is the Cut-Off Frequency rad/Sec

$W_C = 2\pi f_C$

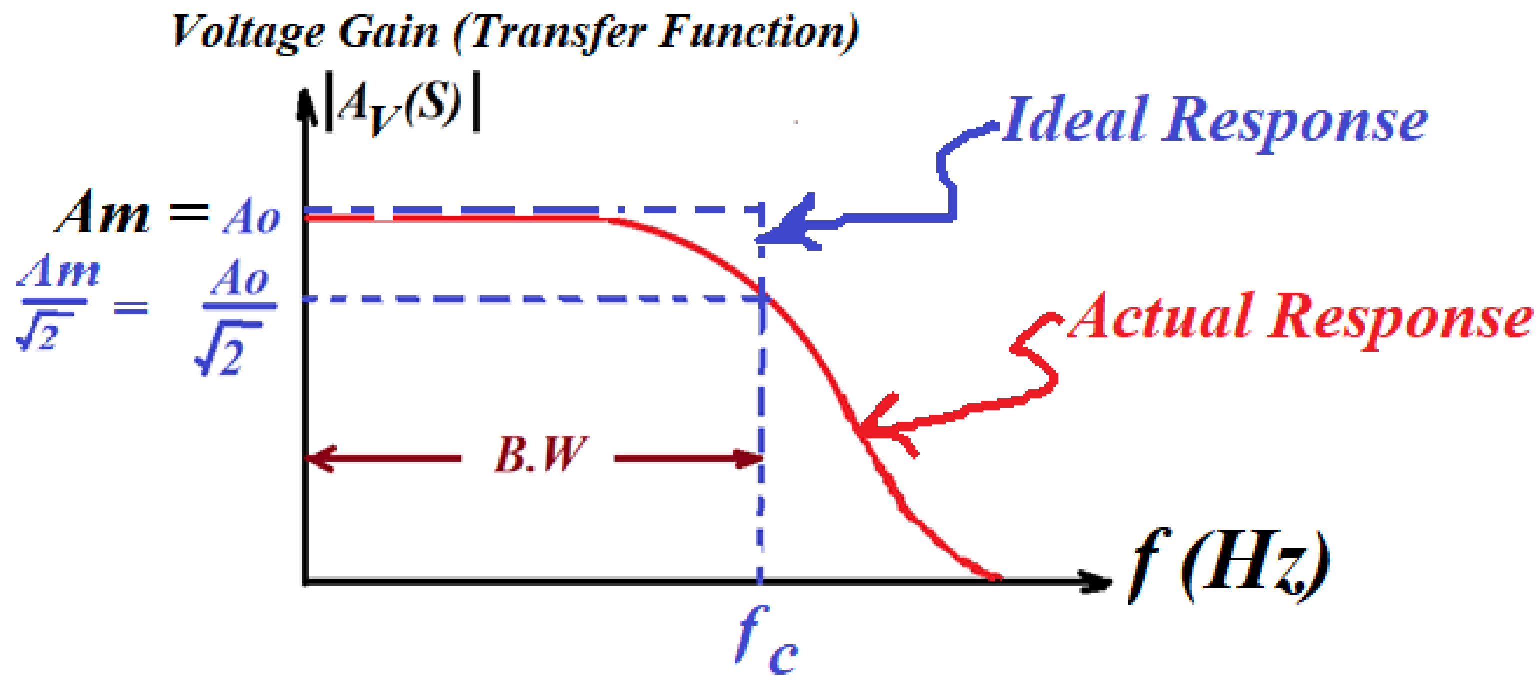
f_C → is the Cut-Off Frequency Hz



$$A(s) = -\frac{R_2 || \frac{1}{sC}}{R_1} = -\frac{1}{R_1} \times \frac{R_2 \times \frac{1}{sC}}{R_2 + \frac{1}{sC}}$$

$$= -\frac{R_2}{R_1} \times \frac{1}{1 + sCR_2}$$

First Order Filters: Low pass filter (LPF)

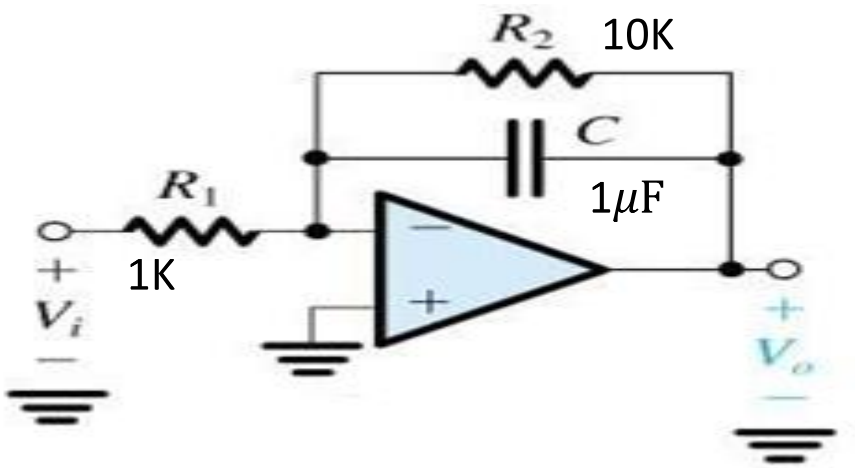


fc = The Cut-Off Frequency

Am = The Maximum (DC)Gain

First Order Filters: Low pass filter (LPF)

Example 1



For the circuit shown, Assuming ideal Op-Amp :

1) Derive the transfer function H(S) (Vo/Vin).

2) Calculate the DC gain (Am) and the Cut-off Frequency (fc).

3) Calculate the unity gain frequency (f_T)

1-Circuit Transfer Function Av or H(s)

$$A_v = \frac{V_o}{V_{in}} = -\frac{Z_2}{Z_1}$$

$$* \; Z_2 = \frac{R_2 \frac{1}{sC}}{R_2 + \frac{1}{sC}} = \frac{R_2}{sCR_2 + 1}$$

$$* \; Z_1 = R_1$$

$$\therefore A_v = -\frac{(R_2 / R_1)}{1 + s R_2 C}$$

$$A_v = -\frac{\left(\frac{R_2}{R_1}\right)}{1 + \frac{s}{\left(\frac{1}{R_2 C}\right)}} = -\frac{A_m}{1 + \frac{s}{\omega_c}}$$

$$= -\frac{10}{1 + \frac{s}{100}}$$

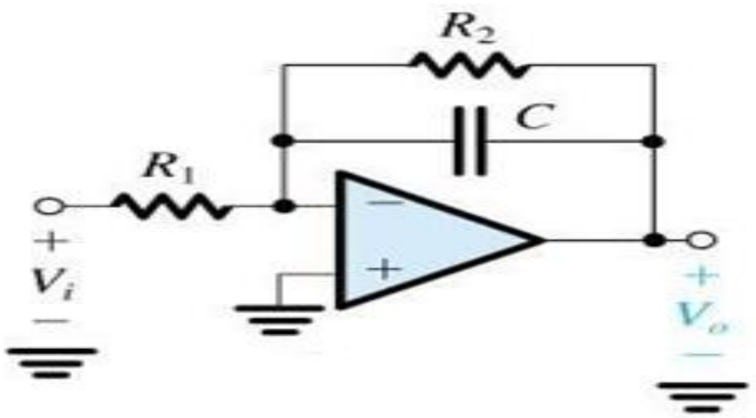
2- Calculate the DC gain and the cutoff frequency

$$A_m = \frac{R_2}{R_1} = 10$$

cut-off frequency

$$\omega_c = \frac{1}{R_2 C} = 100 \text{ rad/sec.}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{100}{2\pi} = 15.92 \text{ Hz}$$



3- Calculate the unity gain frequency F_T

$$A_v = \frac{A_m}{1 + \frac{S}{\omega_c}}$$

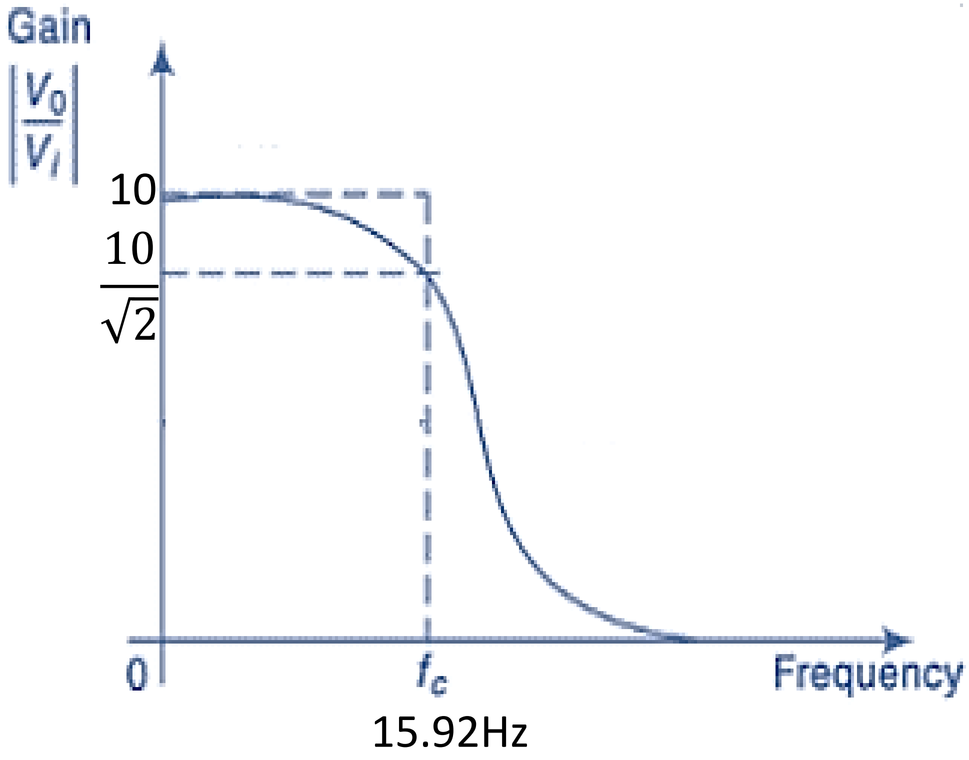
$$A_v(j\omega) = \frac{A_m}{1 + j\frac{\omega}{\omega_c}}$$

$$|A_v| = \frac{A_m}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

$$|A_v| = \frac{A_m}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$

$$|A_v| = \frac{10}{\sqrt{1 + \left(\frac{f}{15.92}\right)^2}} = 1$$

$$f_T \cong 158.4\ \text{Hz}$$



First Order Filters: High pass filter (HPF)

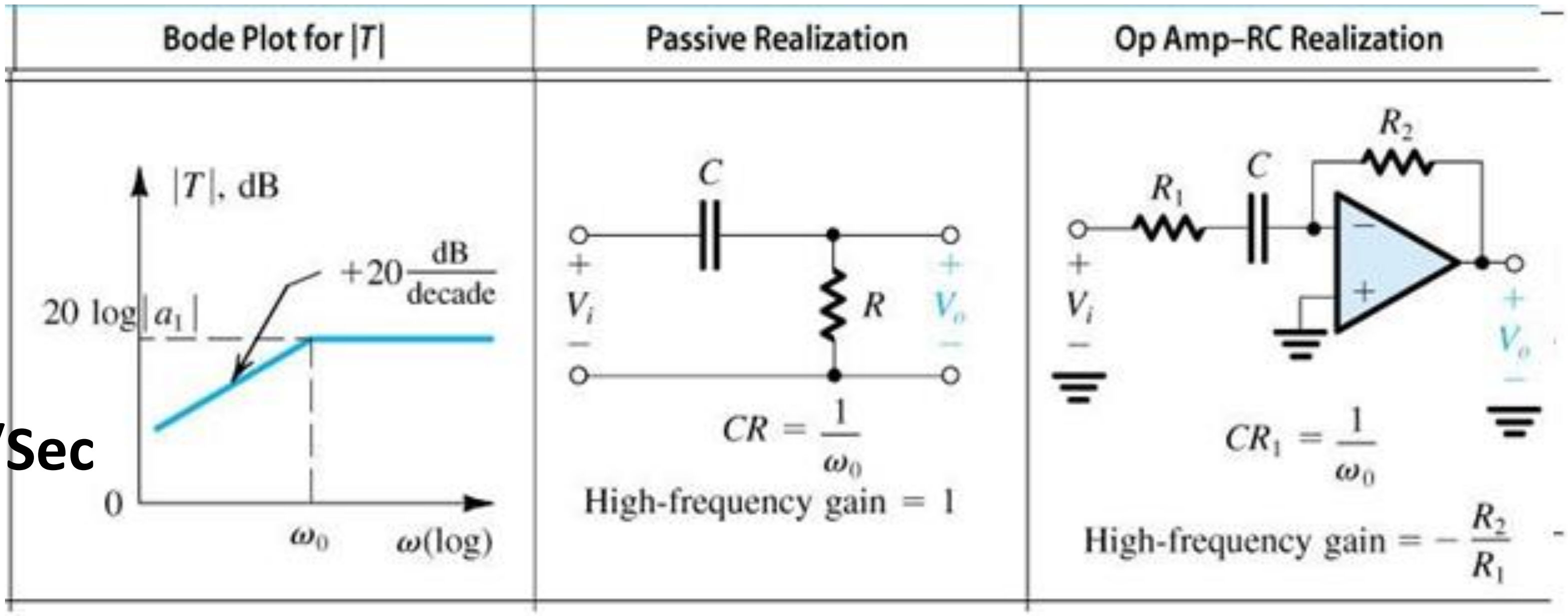
$$A(S) = \frac{A_m}{1 + \frac{W_c}{S}}$$

A_m → is the maximum gain (High Frequency Gain),

Wc → is the Cut-Off Frequency rad/Sec

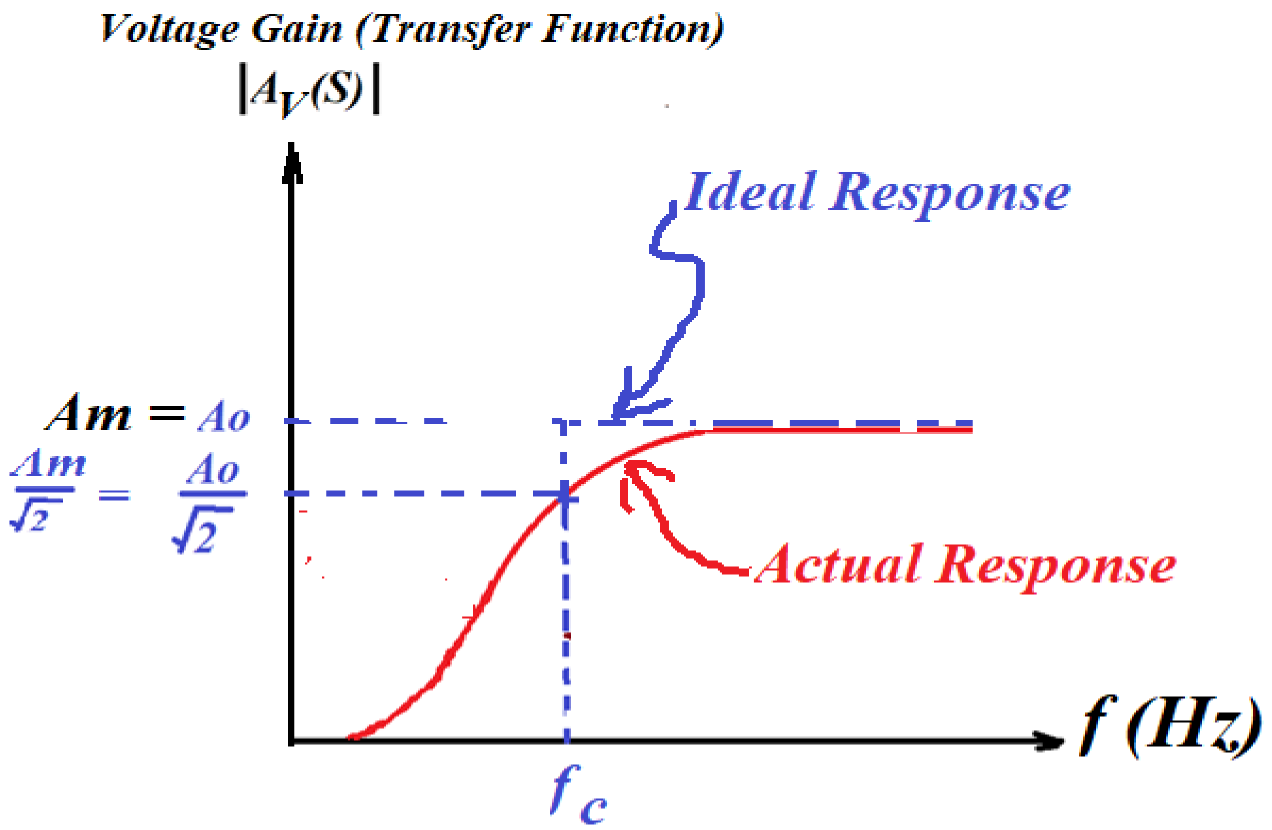
$W_C = 2\pi f_C$

f_C → is the Cut-Off Frequency Hz



$$A(s) = -\frac{R_2}{R_1 + \frac{1}{sC}} = -\frac{R_2}{R_1} \times \frac{1}{1 + \frac{1}{sCR_1}}$$

First Order Filters: High pass filter (HPF)

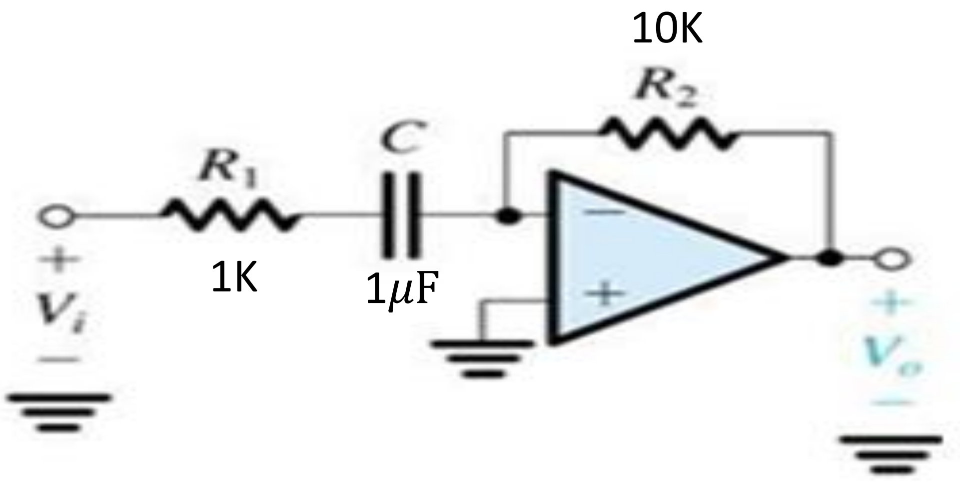


fc = The Cut-Off Frequency

Am = The Maximum (High-frequency) Gain

First Order Filters: High pass filter (HPF)

Example 2



For the circuit shown, Assuming ideal Op-Amp :
1) Derive the transfer function H(S) (Vo/Vin).
2) Calculate the DC gain (Am) and the Cut-off Frequency (fc).
3) Calculate the unity gain frequency (f_T)

1-Circuit Transfer Function Av or H(s)

Sol.

ⓐ Inverting Amplifier

$$A_v = \frac{V_o}{V_{in}} = -\frac{Z_2}{Z_1}$$

* $Z_2 = R_2$
* $Z_1 = R_1 + \frac{1}{sC}$

$$A_v = \frac{V_o}{V_{in}} = -\frac{R_2}{R_1 + \frac{1}{sC}}$$

$$A_v = -\frac{(R_2/R_1)}{1 + \frac{1}{sCR_1}} = -\frac{(R_2/R_1)}{1 + \frac{(1/R_1C)}{s}}$$

First Order Filters: High pass filter (HPF)

$$A_v = -\frac{(R_2/R_1)}{1+\frac{1/R_1C}{s}}$$

$$A_v = \frac{A_m}{1+\frac{\omega_c}{s}}$$

$$A_v = -\frac{10}{1+\frac{1000}{s}}$$

(b) Max. gain

$$\boxed{A_m = \frac{R_2}{R_1} = 10}$$

$$\omega_c = \frac{1}{R_1C} = 1000 \text{ rad/sec}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1000}{2\pi} = 159.2 \text{ Hz}$$

(c) $$A_v = -\frac{10}{1+\frac{1000}{s}}$$

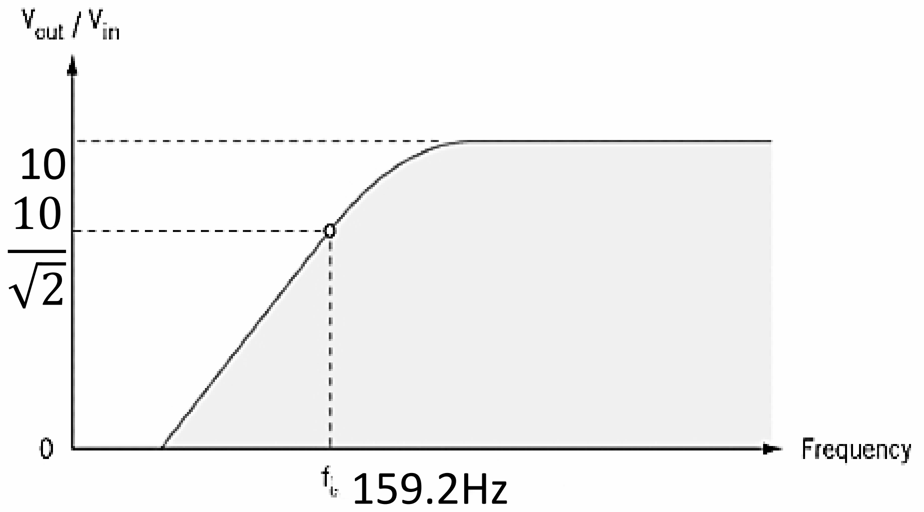
$$A_v = \frac{-10}{1+\frac{1000}{j\omega}}$$

$$A_v = -\frac{10}{1-j\frac{1000}{\omega}}$$

$$|A_v| = \frac{10}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{10}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}}$$

$$|A_v| = \frac{10}{\sqrt{1+\left(\frac{159.2}{f}\right)^2}} = 1$$

Solving $f_T \cong 16 \text{ Hz}$



First Order Filters: Band pass filter (BPF)

$$Z_1 = R_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = R_2 || \frac{1}{j\omega C_2}$$

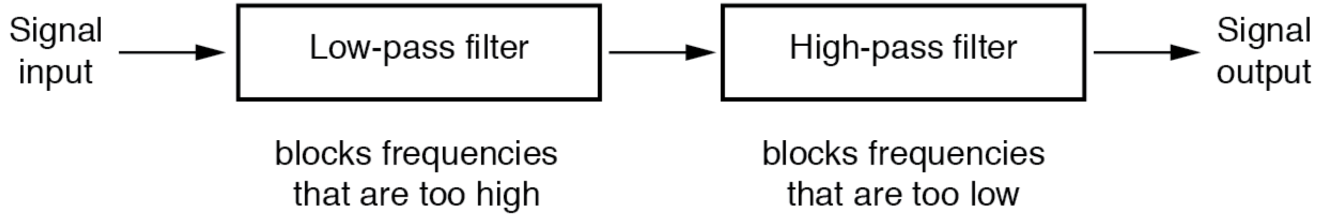
$$Z_2 = \frac{R_2 \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}$$

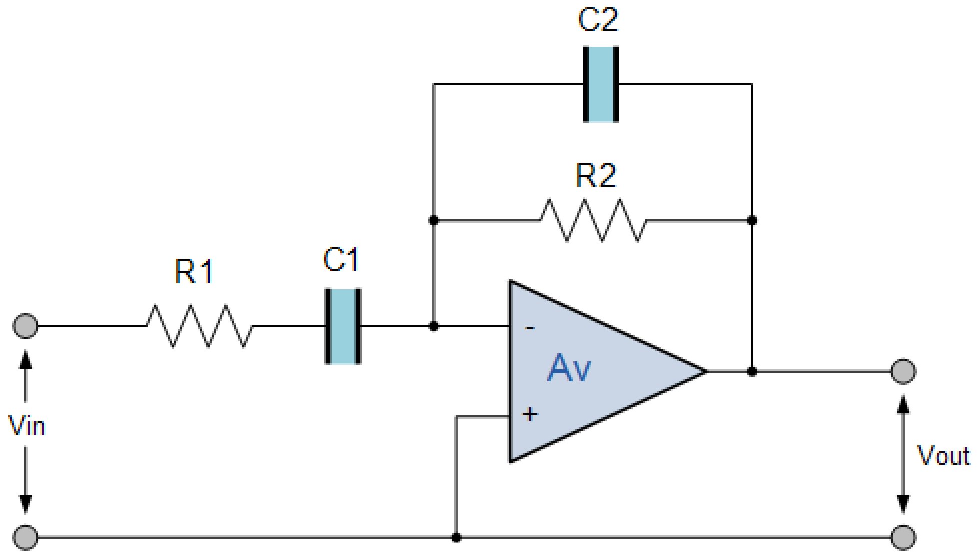
$$H(j\omega) = -\frac{Z_2}{Z_1}$$

$$= -\frac{\frac{R_2}{j\omega C_2}}{(R_2 + \frac{1}{j\omega C_2})(R_1 + \frac{1}{j\omega C_1})}$$

$$= -\frac{\frac{R_2}{j\omega C_2}}{(\frac{R_2 j\omega C_2 + 1}{J\omega C_2})(\frac{R_1 j\omega C_1 + 1}{j\omega C_1})}$$

$$= -\frac{jR_2\omega C_1}{(1 + j\omega C_2 R_2)(1 + j\omega C_1 R_1)}$$



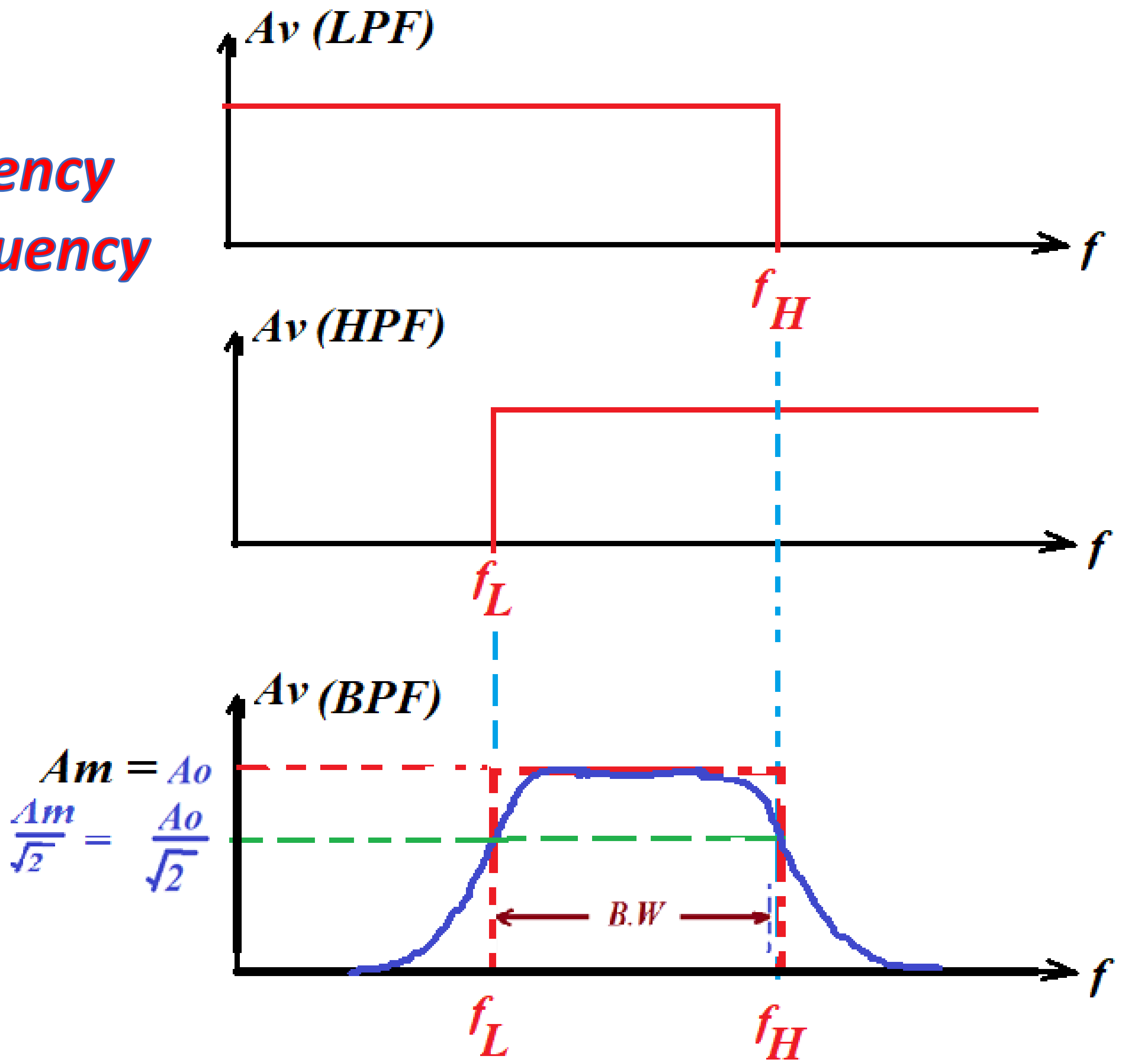


$$\text{Voltage Gain} = -\frac{R_2}{R_1}$$

$$fc_1 = \frac{1}{2\pi R_1 C_1}, \quad fc_2 = \frac{1}{2\pi R_2 C_2}$$

First Order Filters: Band pass filter (BPF)

- f_L is the lower Cut-off Frequency
- f_H is the Higher Cut-off Frequency
- B.W is the Band-Width
- $B.W = f_H - f_L$



Tuned Filter (Narrow B.W Band pass filter)

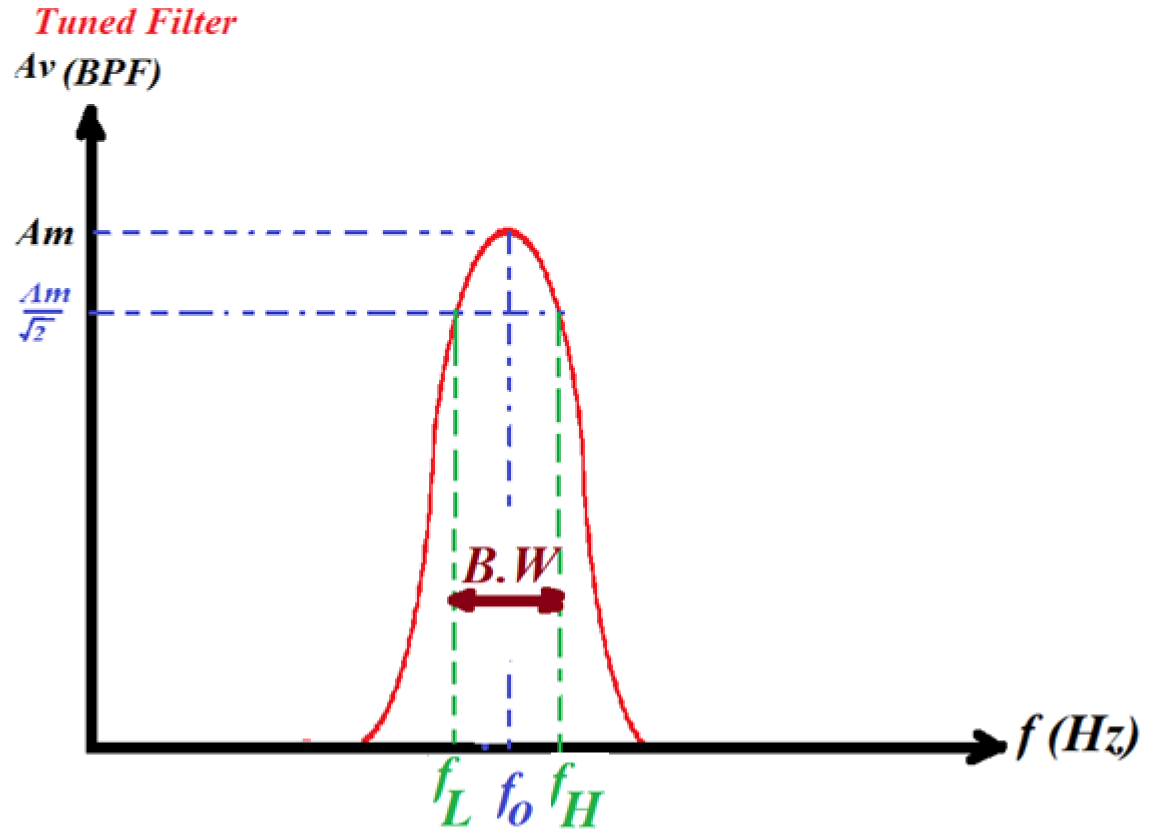
The Center Frequency (Tuned Freq.)

$$\boxed{f_o = \sqrt{f_L \cdot f_H}}$$

The Quality Factor

(How Sharp is the response)

$$\boxed{Q = \frac{f_o}{B.W}}$$

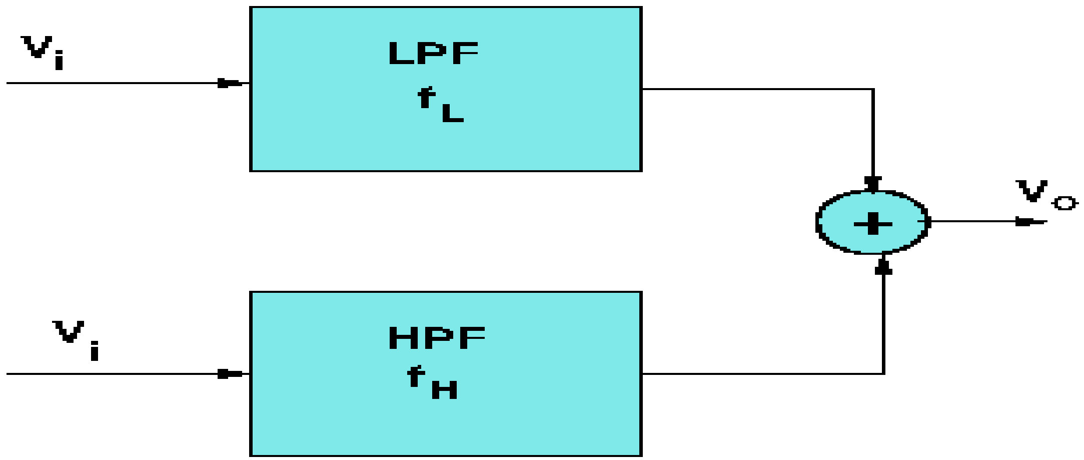


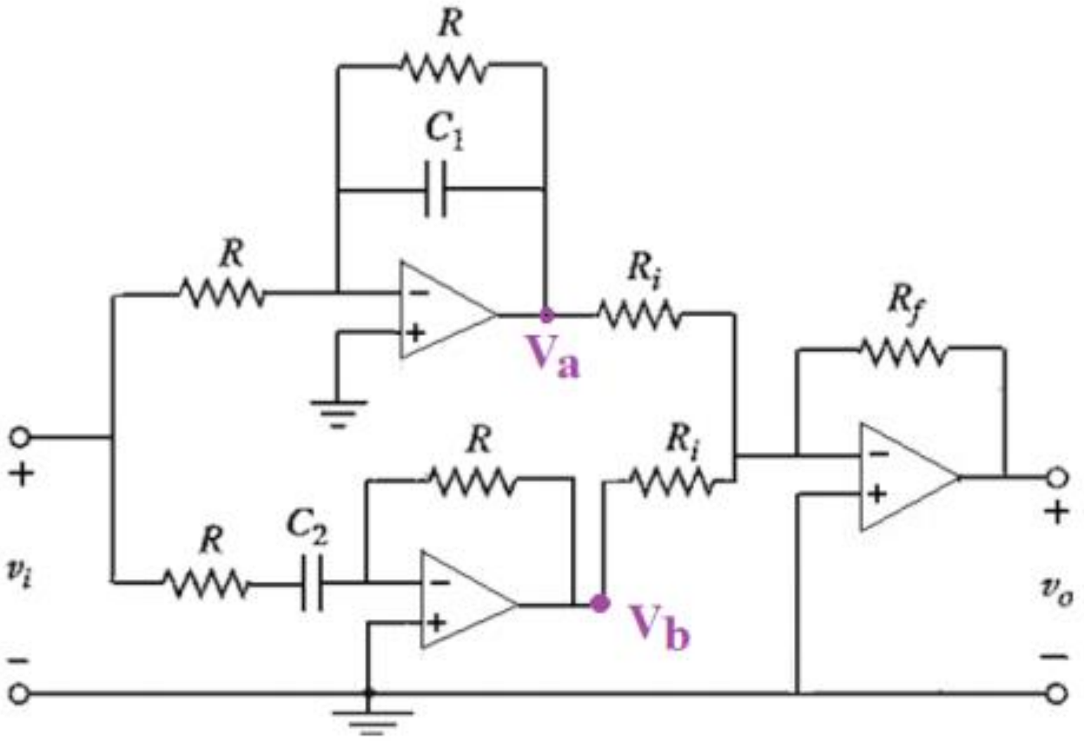
f_o *is the Center Frequency*

f_L *is the Lower Cut-Off frequency*

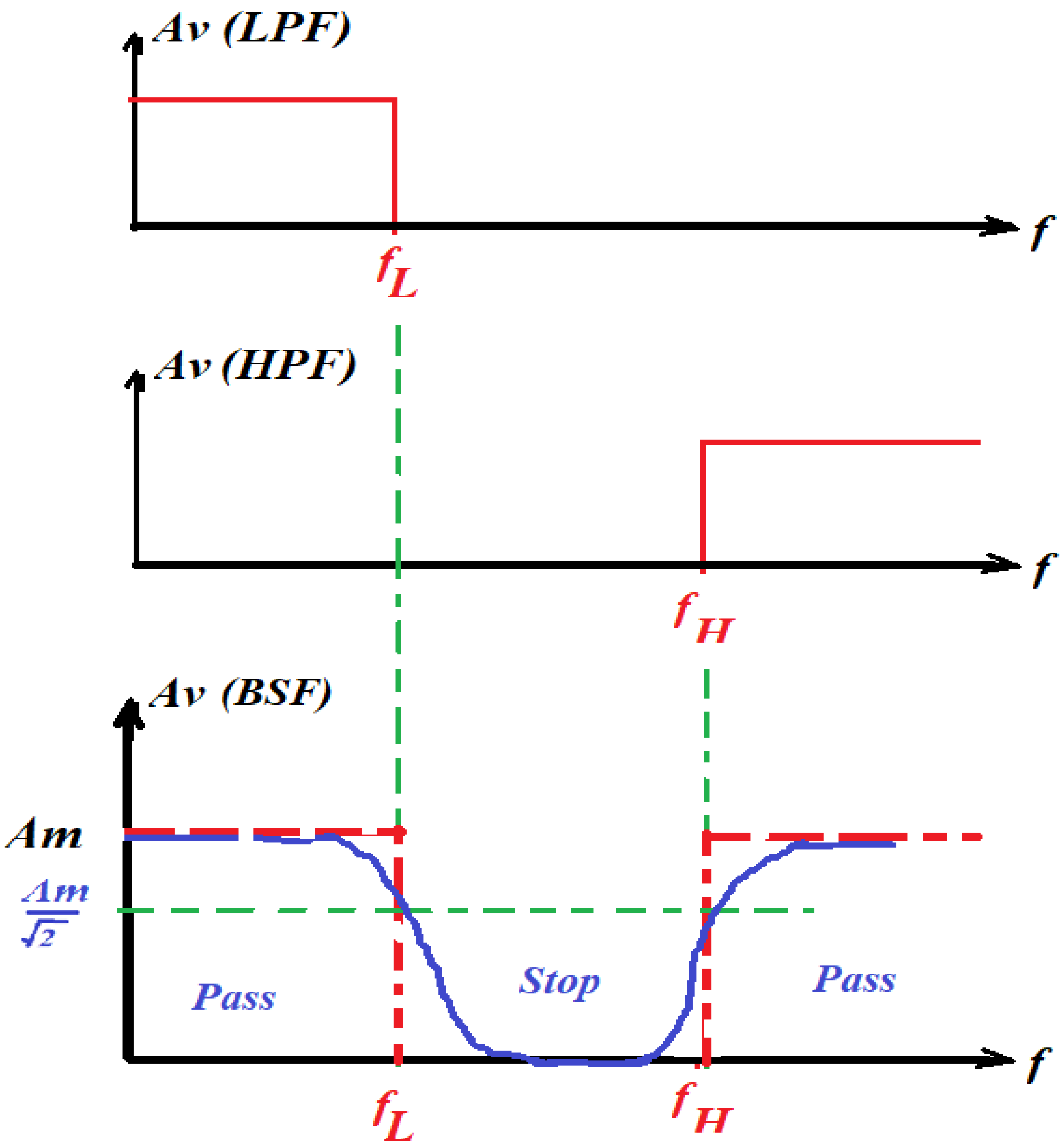
f_H *is the Higher Cut-Off frequency*

Band Stop filter (BSF)





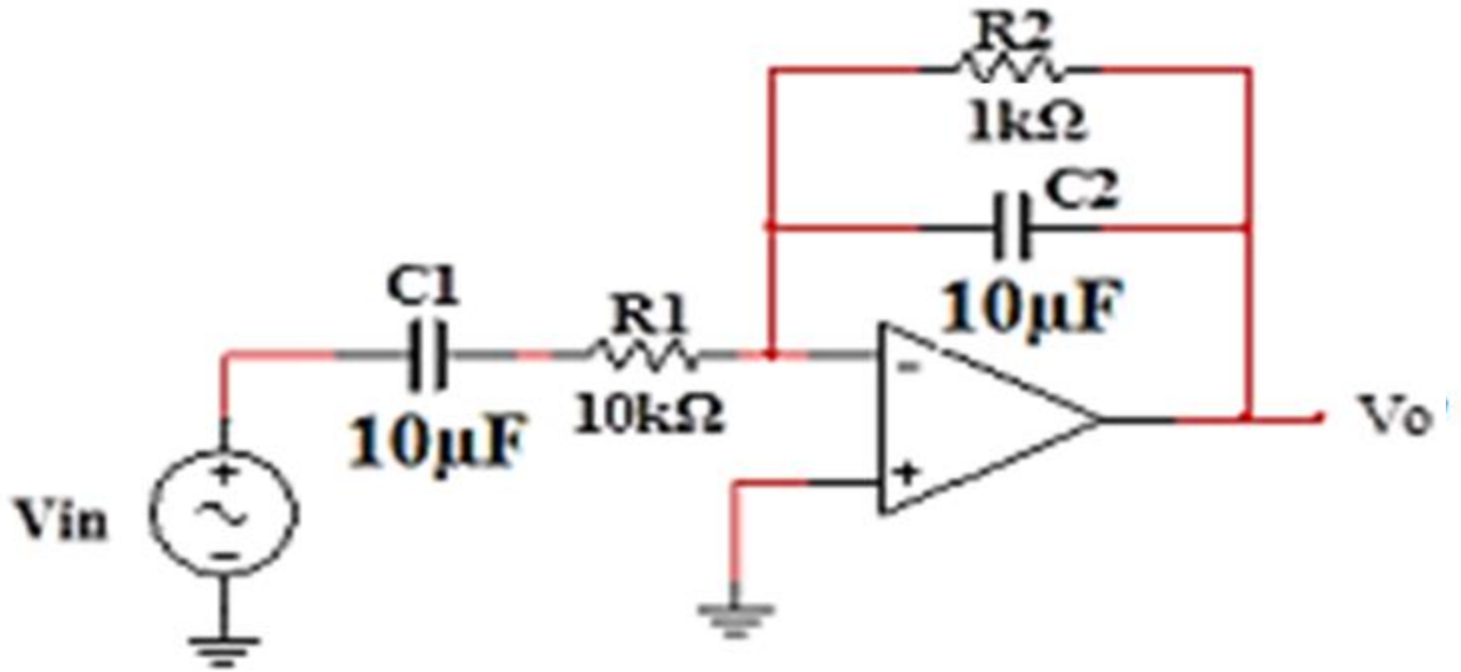
Band Stop filter (BSF)



Example 3

Analyze the circuit shown in Figure :

(a) Derive an expression for the circuit transfer function H(s).
(b) Calculate the lower and upper cut-off frequencies (f_L and f_H).
(c) Sketch the frequency response



Example 3

1-Circuit Transfer Function Av or H(s)

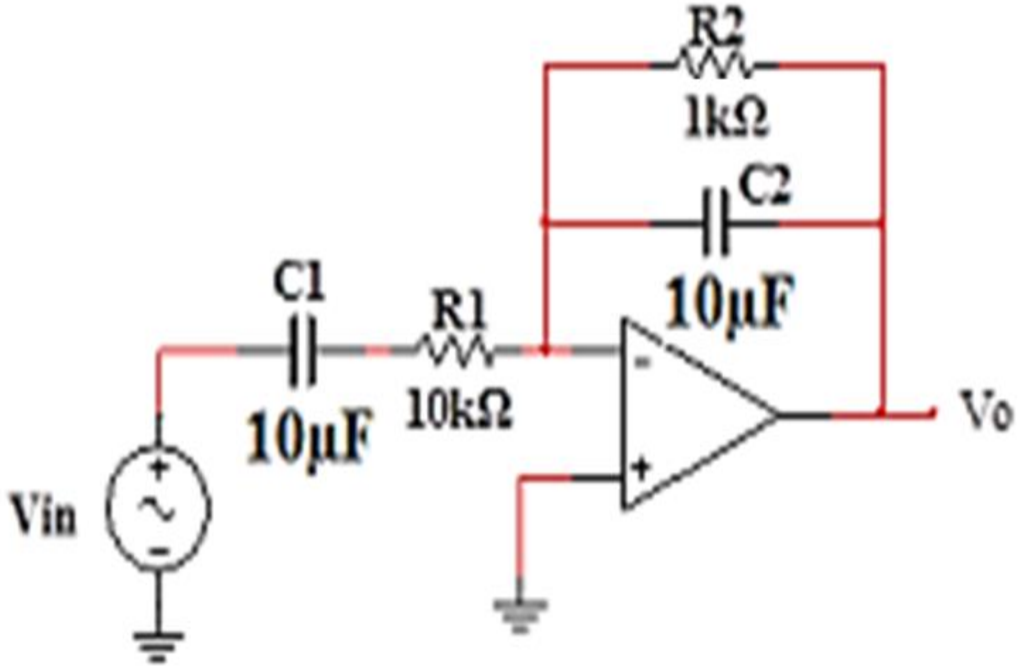
$$H(s) = -\frac{Z_2}{Z_1}$$

$$Z_2 = \frac{R_2 \cdot \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{sC_2R_2+1}$$

$$Z_1 = R_1 + \frac{1}{sC_1}$$

$$\therefore H(s) = -\frac{R_2}{(sC_2R_2+1)} \frac{1}{R_1 + \frac{1}{sC_1}}$$

$$H(s) = -\frac{(R_2/R_1)}{[sC_2R_2+1]\left[1 + \frac{1}{sC_1R_1}\right]}$$



Example 3

$$H(s) = -\frac{(R_2/R_1)}{[sC_2R_2+1]\left[1+\frac{1}{sC_1R_1}\right]}$$

$$H(s) = -\frac{(R_2/R_1)}{\left[1+\frac{s}{1/R_2C_2}\right]\left[1+\frac{1/R_1C_1}{s}\right]}$$

$$H(s) = -\frac{(R_2/R_1)}{\left[1+\frac{s}{\omega_2}\right]\left[1+\frac{\omega_1}{s}\right]} = -\frac{10}{\underbrace{\left[1+\frac{s}{100}\right]}_{LPF}\underbrace{\left[1+\frac{10}{s}\right]}_{HPF}}$$

$$H(s) = \frac{-10}{\left[1+\frac{s}{100}\right]\left[1+\frac{10}{s}\right]}$$

$\omega_2 = 100$ rad/sec

$\omega_1 = 10$ rad/sec.

Example 3

2- Calculate the lower and upper cutoff frequencies

$* \omega_L = 10 \text{ rad/sec.} = 2\pi f_L$

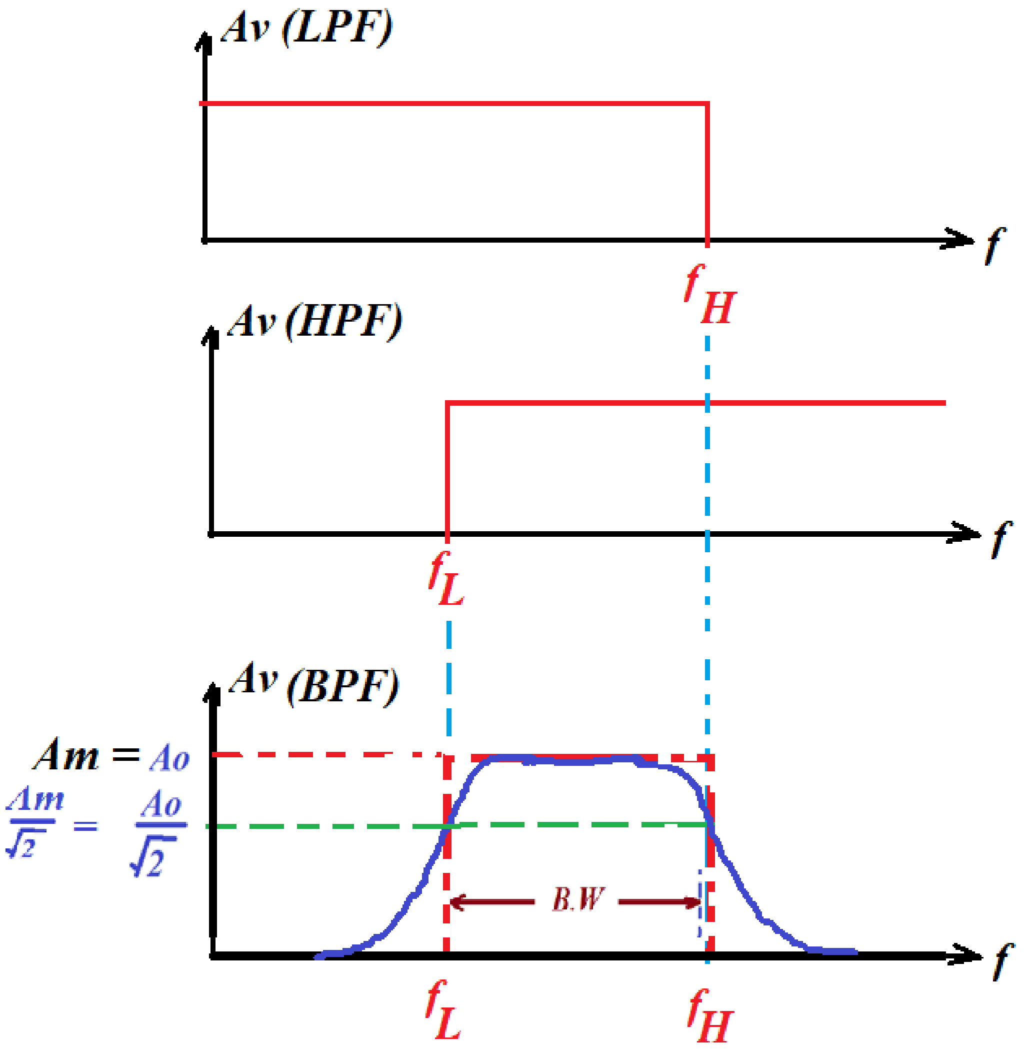
$\therefore f_L = \frac{10}{2\pi}$

$f_L = 1.592 \text{ Hz}$

$* \omega_H = 100 \text{ rad/sec} = 2\pi f_H$

$\therefore f_H = \frac{100}{2\pi}$

$f_H = 15.92 \text{ Hz}$



Example 3

3- Sketch the frequency response