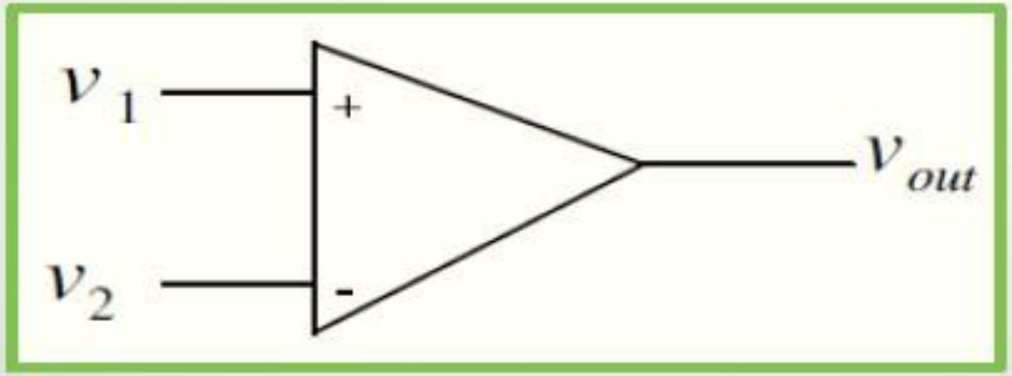
Introduction to Differential Am

- Differential Amplifiers are used extensively in IC
    - They are more immune to Noise and Interference Signals
    - No need for Bypass and coupling capacitors
- Example: the Voltage op-amp is a differential amplifier



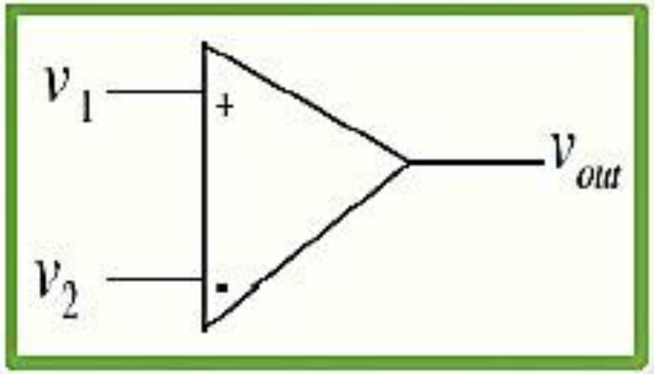
$$v_{out} = A_V\,(v_1 - v_2)$$

to There are two reasons for using differential in pre◻ference
:single-ended amplifiers

First, differential circuits are much less sensitive to noise and
.interference than single-ended circuits

The second reason for preferring differential amplifiers is that the ❖
couple differential configuration enables us to bias the amplifier and to
amplifier stages together without the need for bypass and coupling
capacitors such as those utilized in the design of discrete-circuit
amplifiers

- The Differential Amplifier amplifies the difference between its input terminals and rejects their average (Common Mode)
  - Let's define the following Input Signals:



$$v_{ID} = v_1 - v_2 \qquad v_{CM} = \frac{v_1 + v_2}{2}$$

  - Consequently, the two input signals can be replaced with the following:

$$v_1 = \frac{v_{ID}}{2} + v_{CM} \qquad v_2 = -\frac{v_{ID}}{2} + v_{CM}$$

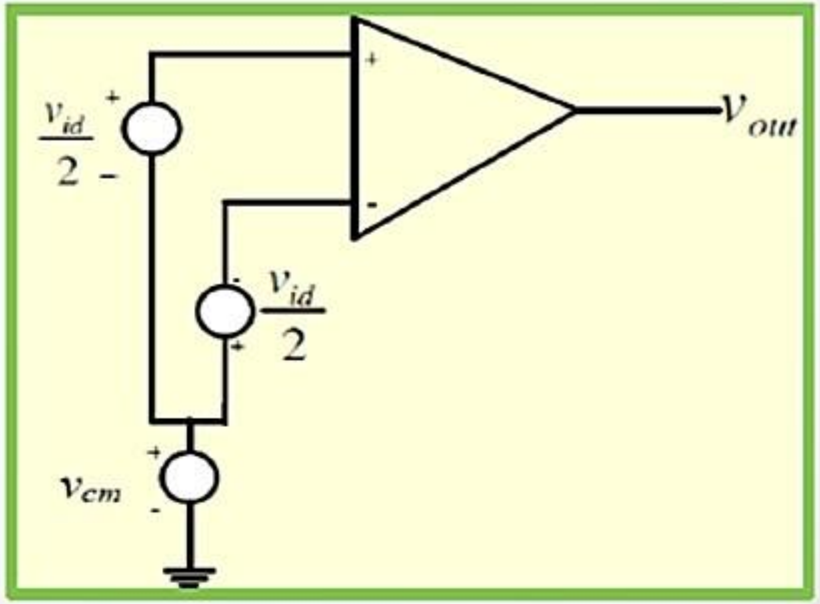
Objective

- Calculation of the Differential Mode Gain of the amplifier <u>(ideally tends to infinity)</u> **(Signal Gain)**

- Calculation of the Common Mode Gain of the Amplifier <u>(ideally tends to ZERO)</u>

  (Noise Gain)

- Calculation of the Common Mode Rejection Ratio (CMRR) <u>(ideally tends to infinity)</u> **(The ability of the amplifier to reject a common signal at its inputs**



$$A_d = \frac{v_{out}}{v_{id}}$$

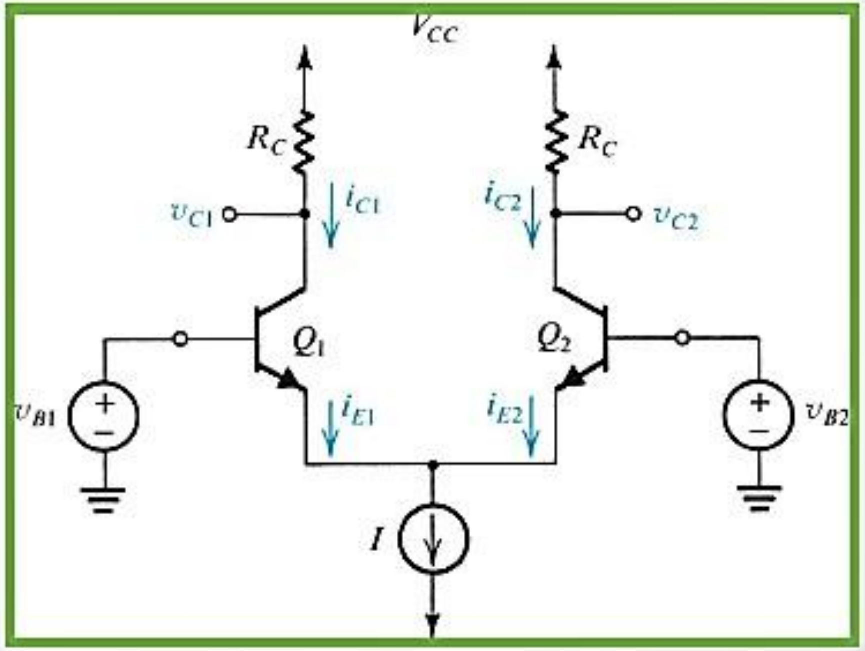
$$A_{cm} = \frac{v_{out}}{v_{cm}}$$

$$CMRR = \left|\frac{A_d}{A_{cm}}\right|$$

BJT Differential Pair

BJT Differential Amplifier/Pair

- Emitter Coupled Circuit:
  - Two Identical BJTs **(Same β and I_s)**
  - Their Emitters are connected together
  - The input signals are connecter to the Q's Bases
  - A Constant DC biasing Current 'I' is used to set the DC operating point.



$$i_{E1} + i_{E2} = I$$

BJT Differential Amplifier/Pair

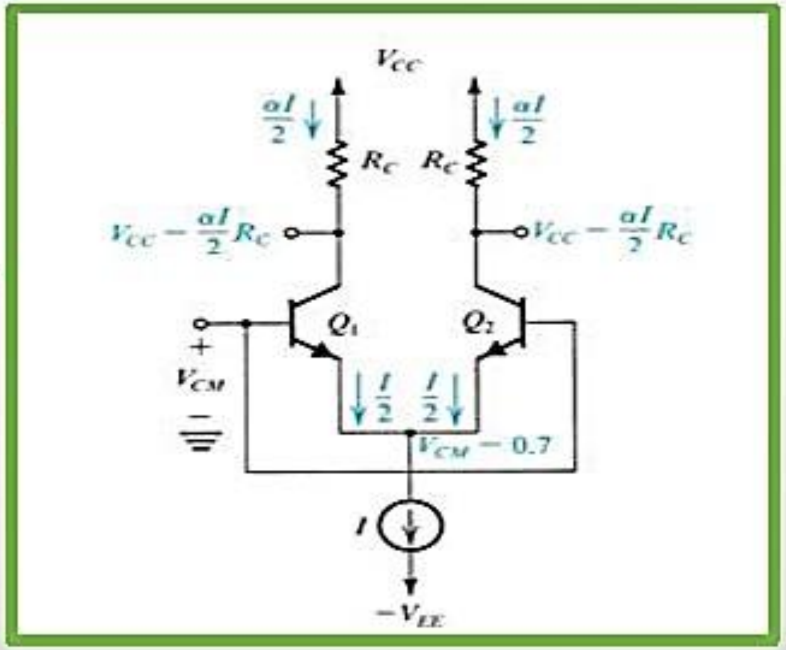
- Emitter Coupled Circuit Large Signal Analysis (**Active Mode**)
  - If Base 1 and Base 2 are connected together (Common mode signal), then the two emitter currents will be equal

$$v_{BE1} = v_{BE2} = v_{CM}$$

$$i_{E1} = i_{E2} = \frac{I}{2}$$

$$i_{C1} = i_{C2} = \alpha\frac{I}{2}$$

$$v_{C1} = v_{C2} = V_{CC} - \alpha\frac{I}{2}R_C$$



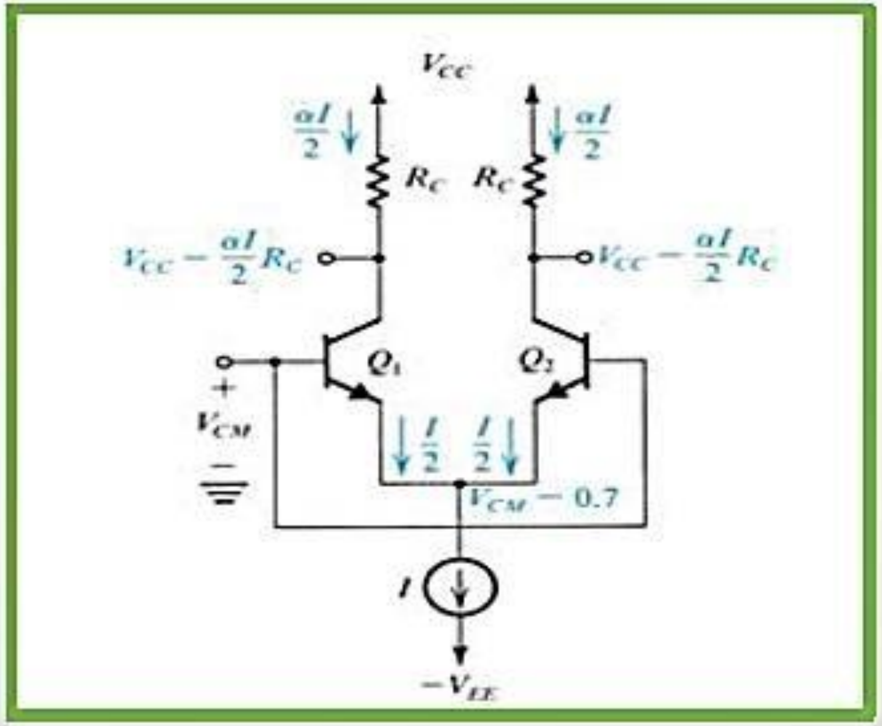
BJT Differential Amplifier/Pair

- Emitter Coupled Circuit Large Signal Analysis (Cont.):
  - Changing the value of the common mode signal <u>will not</u> change the transistors currents

$$v_{BE1} = v_{BE2} = v_{CM}$$

$$v_{C1} = v_{C2} = V_{CC} - \alpha \frac{I}{2} R_C$$

The DA rejects the common mode Signal



BJT Differential Amplifier/Pair

- Emitter Coupled Circuit Large Signal Analysis:
  - If Base 1 and Base 2 are not connected together (**Differential signal**), then the two emitter currents **won't be equal**

$v_{BE1} - v_{BE2} = v_{ID}$

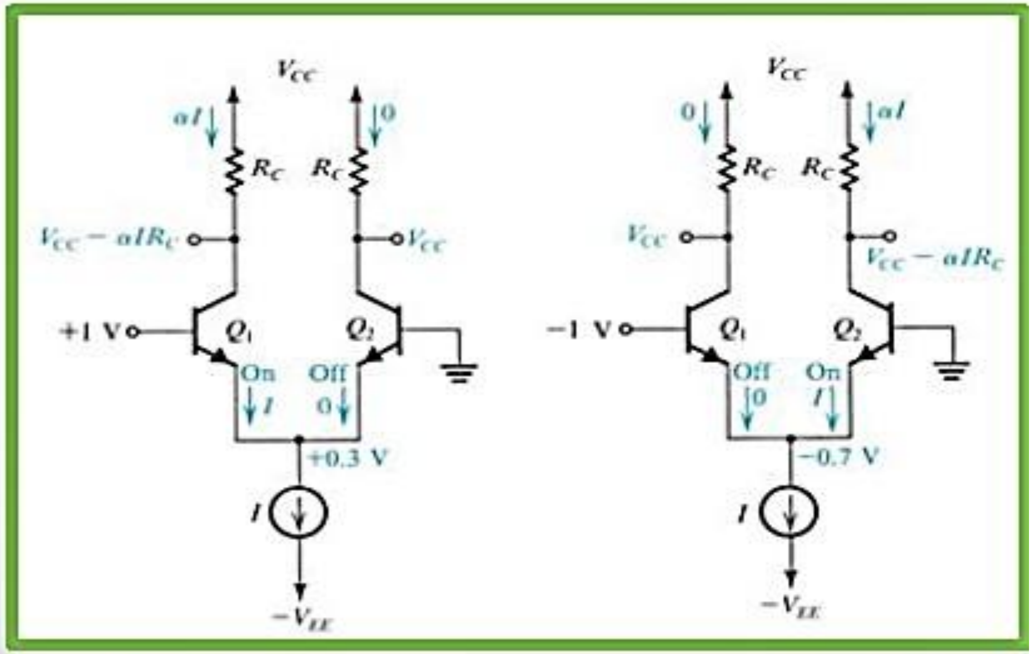
$i_{E1} \neq i_{E2}$

If $v_{ID} >> 0$ such that Q1 is Sat and Q2 is Off

$i_{E1} = I \ \& \ i_{E2} = 0$

If $v_{ID} << 0$ such that Q1 is OFF and Q2 is Sat

$i_{E1} = 0 \ \& \ i_{E2} = I$



The DA responds to any change in the Differential mode Signal

BJT Differential Amplifier/Pair

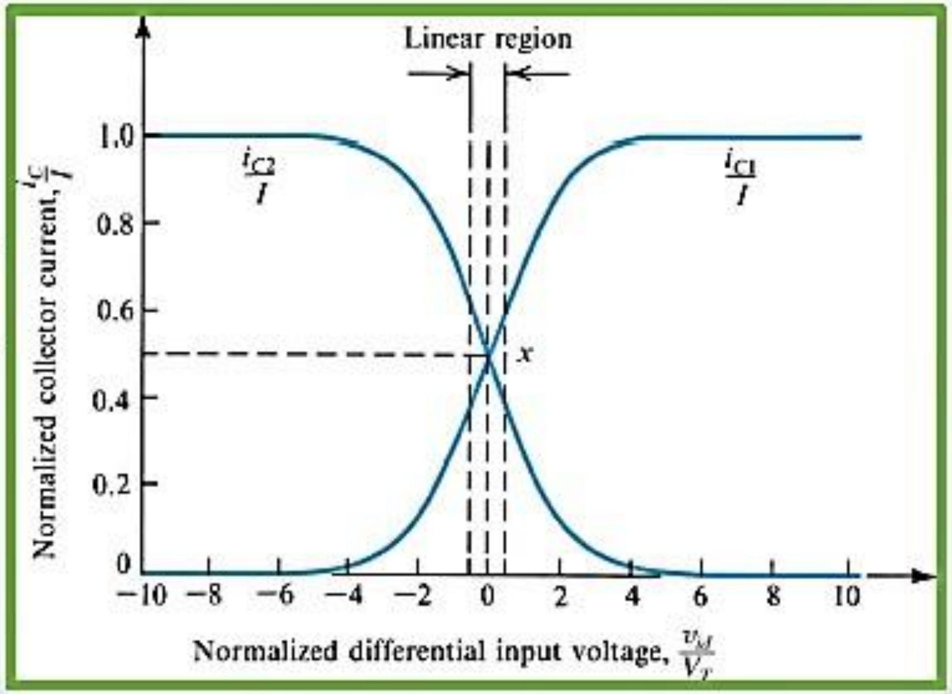
- Emitter Coupled Circuit Large Signal Analysis:
  - Q1 and Q2 are active and Identical

$$i_{E1} = \frac{I_S}{\alpha}\exp\left(\frac{v_{BE1}}{V_T}\right) \qquad i_{E2} = \frac{I_S}{\alpha}\exp\left(\frac{v_{BE2}}{V_T}\right)$$

$$\frac{i_{E1}}{i_{E2}} = \exp\left(\frac{v_{ID}}{V_T}\right) \qquad i_{E1} + i_{E2} = I$$

$$\frac{i_{E1}}{i_{E1}+i_{E2}} = \frac{1}{1+\exp\left(\frac{-v_{ID}}{V_T}\right)}$$

$$\frac{i_{E2}}{i_{E1}+i_{E2}} = \frac{1}{1+\exp\left(\frac{v_{ID}}{V_T}\right)}$$



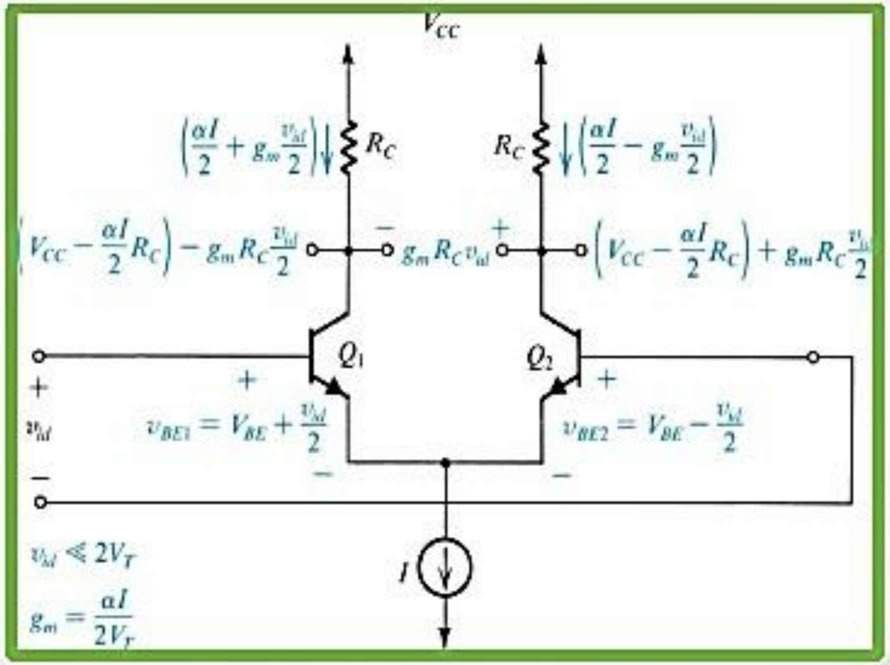
$$i_{E1} = \frac{I}{1+\exp\left(\frac{-v_{ID}}{V_T}\right)} \qquad i_{E2} = \frac{I}{1+\exp\left(\frac{v_{ID}}{V_T}\right)}$$

BJT Differential Amplifier/Pair

- **Small Signal Analysis**
  Differential Mode Gain
- DC Operating point is adjusted such that:

$$I_{c2} = I_{c1} = \alpha\frac{I}{2} \approx \frac{I}{2}$$

- The circuit is like two Common Emitters with the output taken as a difference between the two Collectors

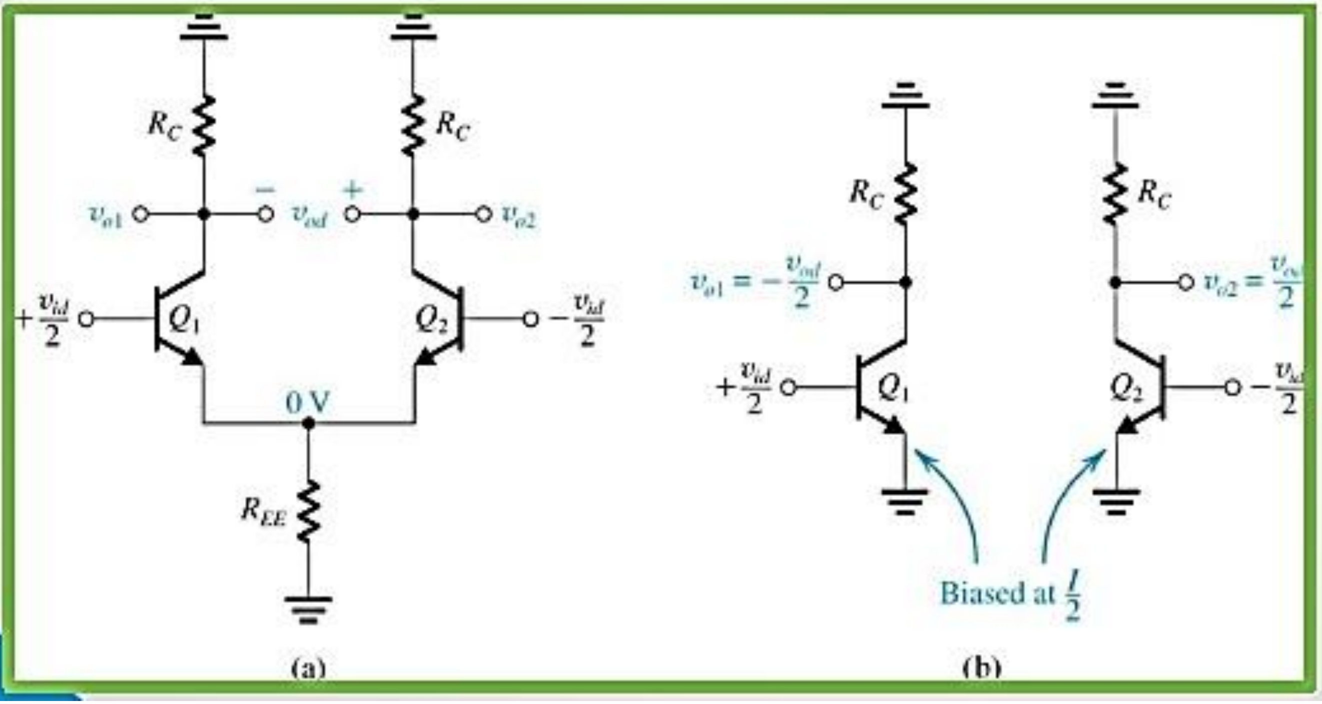


$$g_{m2} = g_{m1} \qquad r_{\pi 1} = r_{\pi 2} \qquad r_{o1} = r_{o2}$$

$$\frac{v_{c2} - v_{c1}}{v_{id}} = g_m R_C$$

BJT Differential Amplifier/Pair

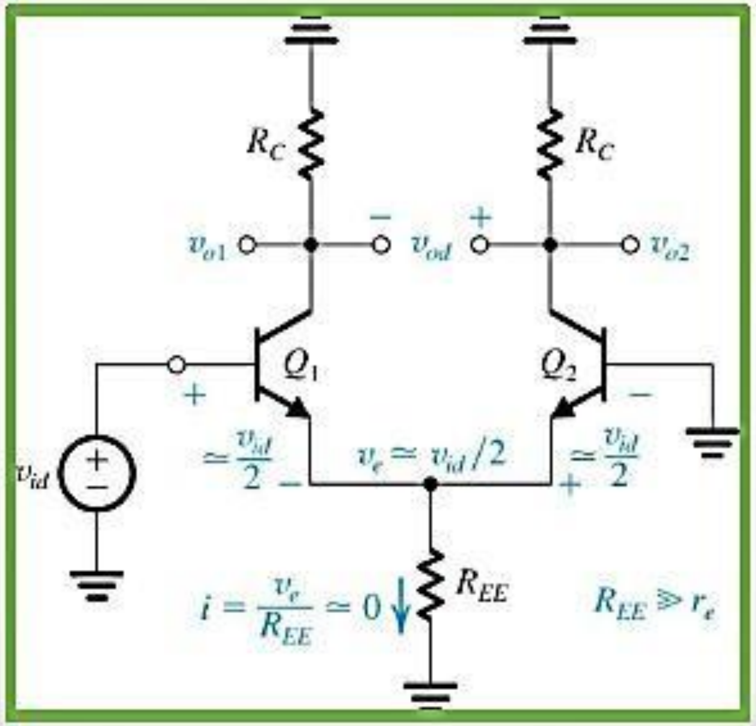
- **<u>Small Signal Analysis</u> Differential Mode Gain**
- We can use half Circuit Concept to calculate the gain
- R_{EE} is the Current Source Resistance (v_e is Zero, why?)



$$\frac{v_{od}}{v_{id}} = \frac{v_{c2} - v_{c1}}{v_{id}} = g_m R_C$$

BJT Differential Amplifier/Pair

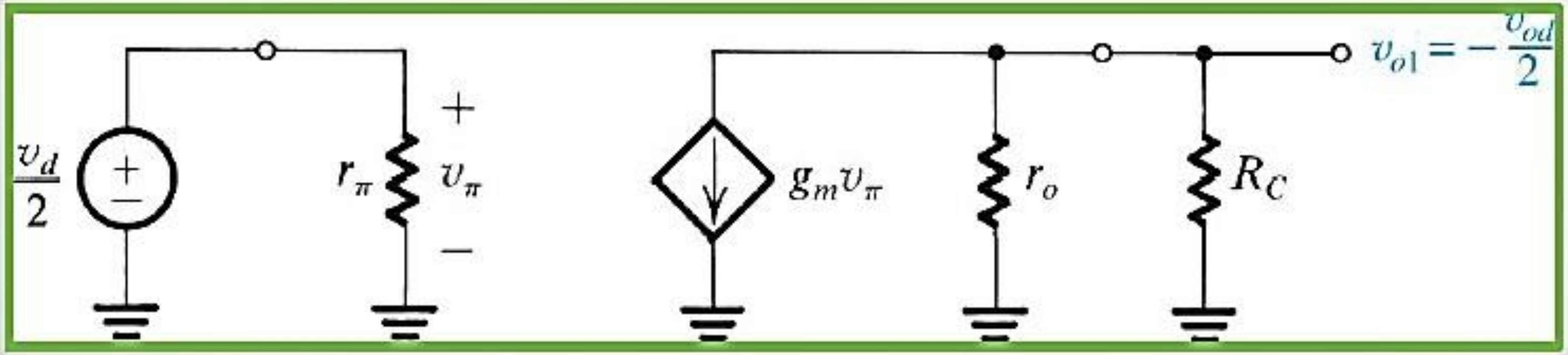
- **Small Signal Analysis** **Differential Mode Gain**
- Notes: What if the signal is not applied fully differential? **(Derive!)**



BJT Differential Amplifier/Pair

- **Small Signal Analysis** **Differential Mode Gain**
- Notes: if r_o is taken into Consideration, then:

$$\frac{v_{od}}{v_{id}} = \frac{v_{c2} - v_{c1}}{v_{id}} = g_m(R_C \,//\, r_o)$$



$$R_{in} = 2r_\pi$$

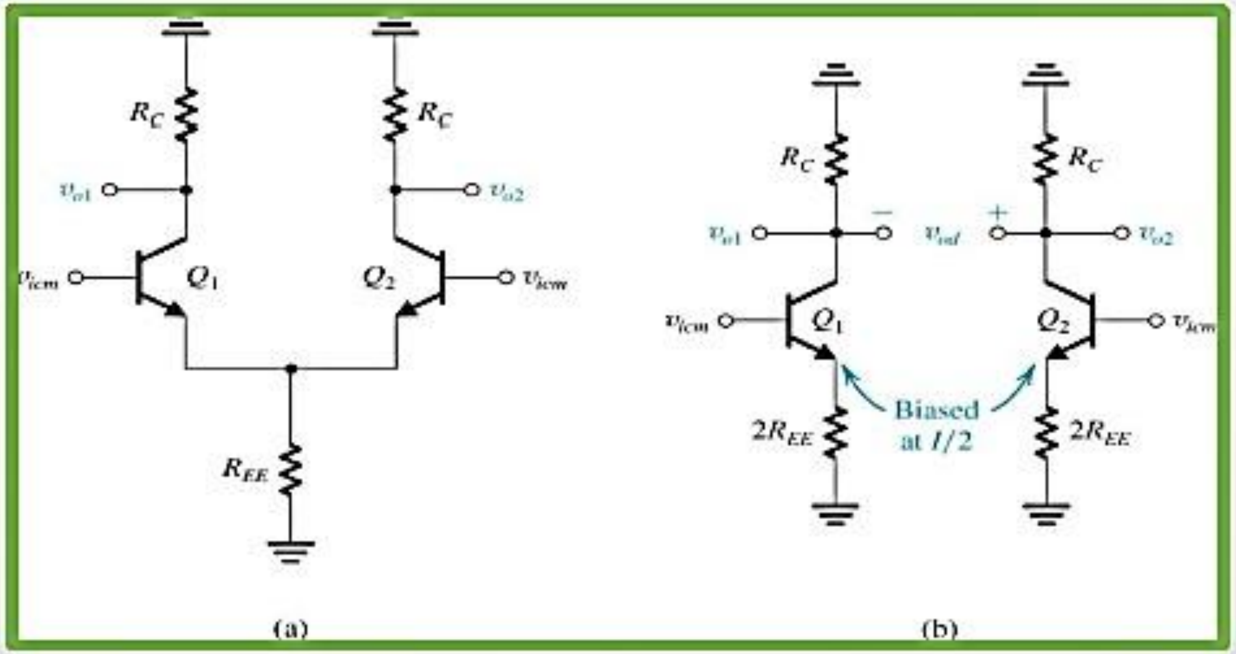
$$R_{out} = R_C \,//\, r_o$$

BJT Differential Amplifier/Pair

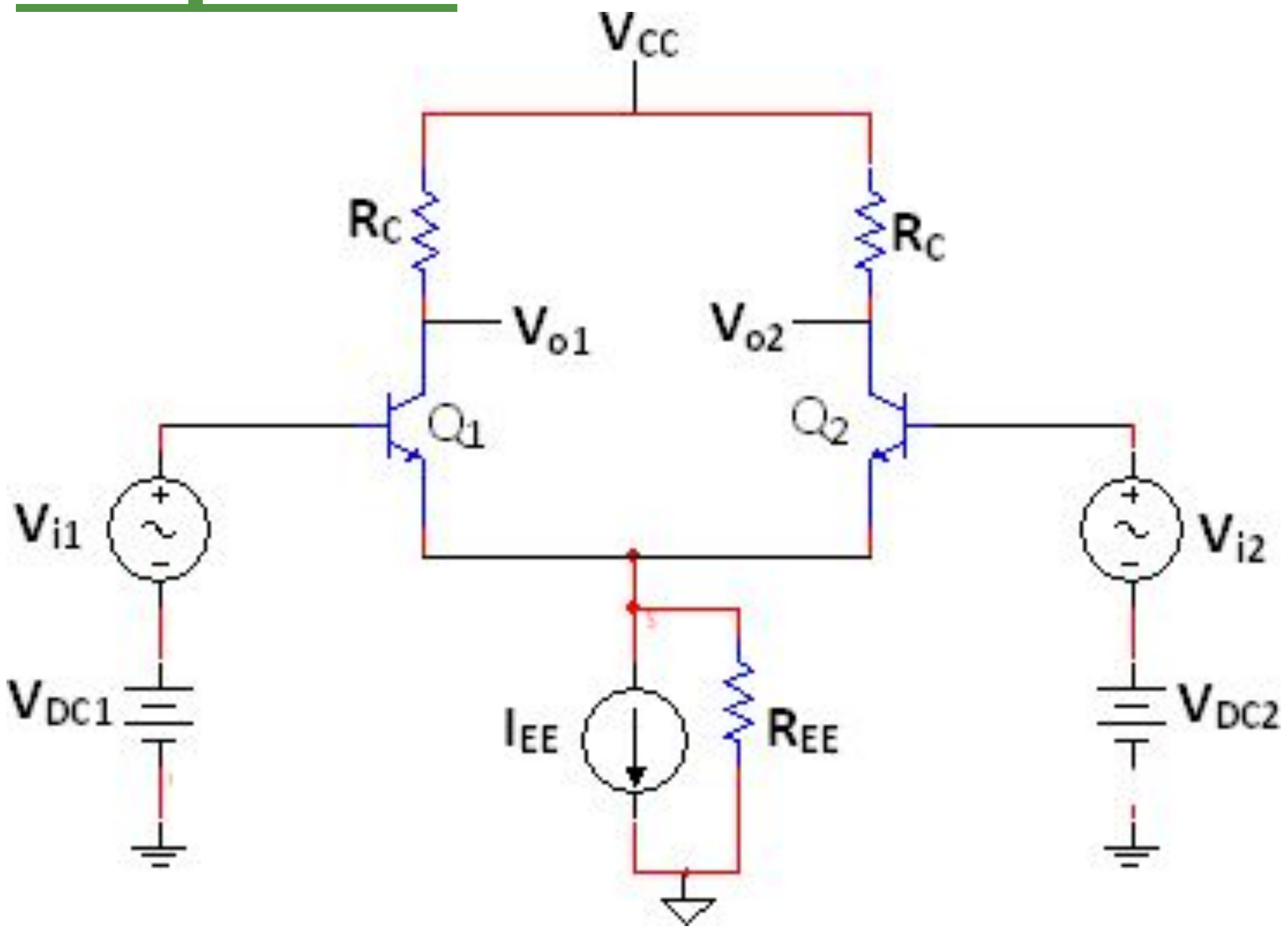
- **Small Signal Analysis** **Common Mode Gain**
- R_{EE} is the Current Source Resistance (v_e is not Zero in the common mode Case, why?)

$$\frac{v_{ocm}}{v_{id}} = \frac{v_{c2}}{v_{id}} \cong \frac{-g_m R_C}{1 + 2g_m R_{EE}}$$

$$|CMRR| \cong 1 + 2g_m\ R_{EE}$$



Summary- BJT Differential Amplifier



REE Current Source output resistance

IEE Current source value

Vi1 = Vic + (Vid/2)

Vi2 = Vic - (Vid/2)

Vic input common signal (Noise(

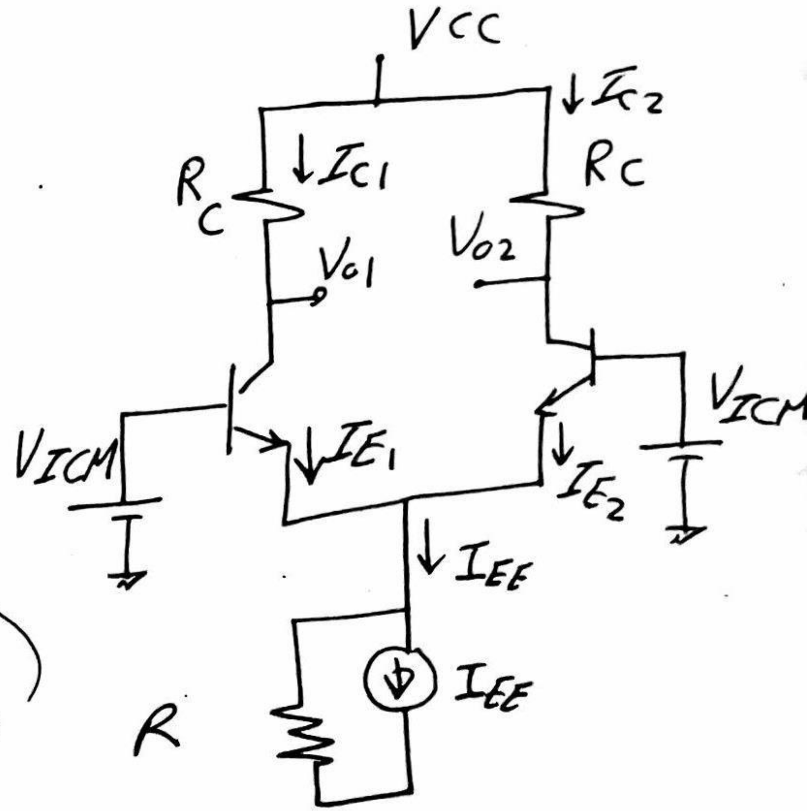
Vid differential input signal

1. D.C Analysis ($V_{i1} \to$ short , $V_{i2} \to$ short)

* $I_{E1} = I_{E2} = \frac{I}{2}$
* $I_C = \alpha I_E$

$\therefore$ $\boxed{I_{C1} = I_{C2} = \frac{\alpha I}{2}}$

$$V_{o1} = V_{o2} = V_{CC} - \frac{\alpha I}{2} R_C$$



$\boxed{V_{od} = 0}$ differential output.

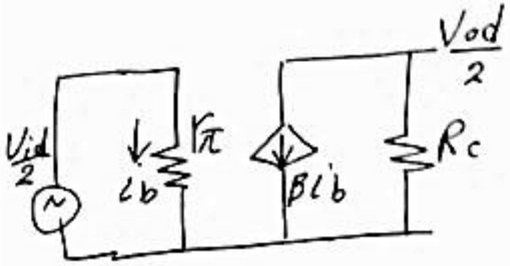
$\boxed{2}$ A.C Analysis ($V_{ICM} = 0$ short)
($V_{cc} = 0$)

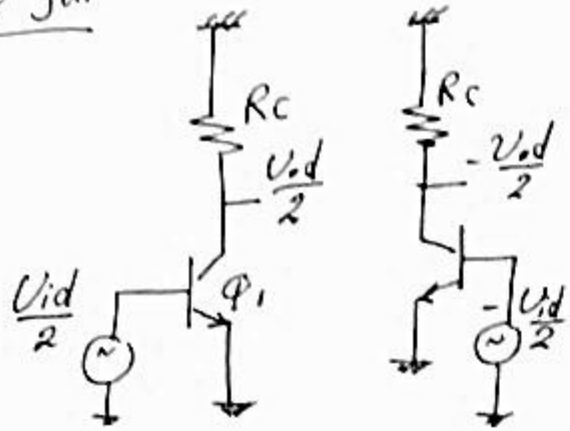
$$v_{i1} = v_{ic} + \frac{v_{id}}{2}$$

$$v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

using half-Circuit Concept

$\boxed{a}$ Differential-mode gain

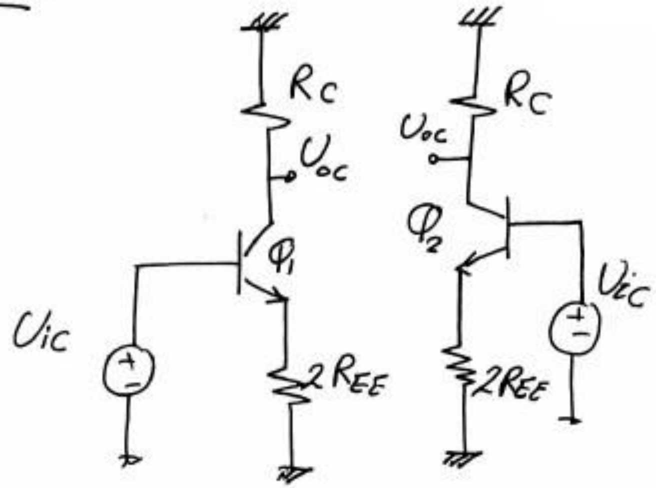




$$A_{DM} = \frac{V_{od}/2}{V_{id}/2} = \frac{-\beta i_b R_c}{i_b r_\pi} \quad , \beta = g_m r_\pi$$

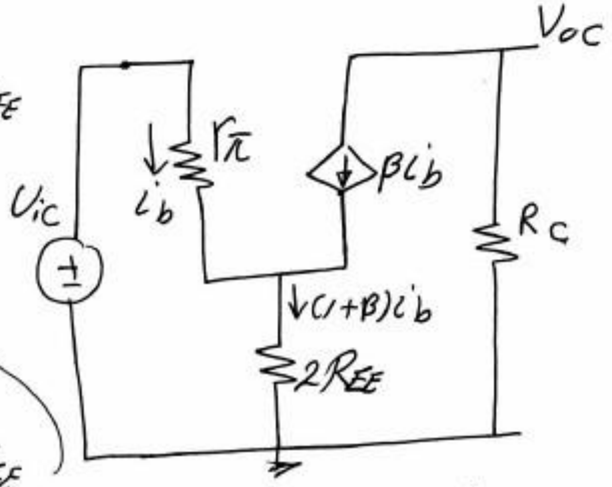
$$A_{DM} = -\beta \frac{R_c}{r_\pi} = -g_m R_c$$ differential mode gain

b] Common-mode gain



$$A_{CM} = \frac{V_{oc}}{V_{ic}}$$

$$= \frac{-\beta \cancel{i_b} R_C}{\cancel{i_b} r_\pi + (1+\beta) \cancel{i_b}\, 2R_{EE}}$$



$$A_{CM} = \frac{-\beta R_C}{r_\pi + 2(1+\beta) R_{EE}}$$

$$\beta = g_m r_\pi$$

$$1+\beta \cong \beta$$

$$A_{CM} \cong \frac{-g_m r_\pi R_C}{r_\pi + 2(g_m r_\pi) R_{EE}}$$

$$A_{CM} \cong \frac{-g_m R_C}{1+2 g_m R_{EE}}$$

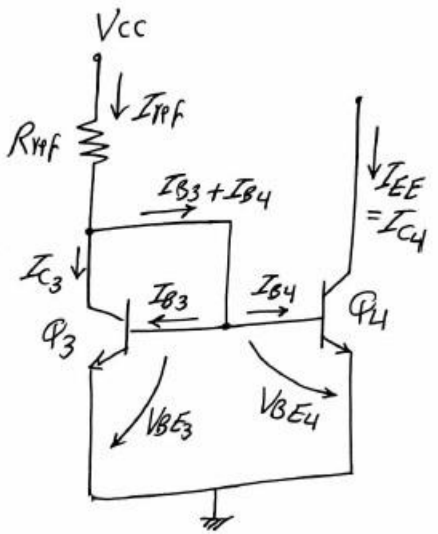
Common Mode Rejection Ratio (CMRR)

$$CMRR = \frac{A_{DM}}{A_{CM}} \cong \frac{-g_m R_C}{-g_m R_C}(1 + 2g_m R_{EE})$$

$$CMRR = 1 + 2g_m R_{EE}$$

$$CMRR|_{dB} = 20 \log(1 + 2g_m R_{EE})$$

Current Mirror Current Source



$$\boxed{I_{C3} = I_{Co3}\, e^{\frac{V_{BE3}}{V_T}}}$$

$$\boxed{I_{C4} = I_{Co4}\, e^{\frac{V_{BE4}}{V_T}}}$$

But $V_{BE3} = V_{BE4}$

$$\therefore \frac{I_{C3}}{I_{C4}} = \frac{I_{Co3}}{I_{Co4}} \quad \#$$

* IF Two Matched BJT's

$I_{Co3} = I_{Co4}$, $I_{B3} = I_{B4} = \frac{I_E}{1+\beta} \simeq \frac{I_C}{\beta}$

$$\therefore I_{C3} = I_{C4} = I_{EE}$$

$$\therefore I_{ref} = I_{C3} + I_{B3} + I_{B4}$$

$$= I_{EE} + \frac{I_{C3}}{\beta} + \frac{I_{C4}}{\beta}$$

$$\therefore I_{ref} = I_{EE} + 2\frac{I_{C3}}{\beta} = I_{EE} + 2\frac{I_{EE}}{\beta}$$

$$I_{ref} = I_{EE}\left[1 + \frac{2}{\beta}\right]$$

$$\therefore I_{EE} = \frac{I_{ref}}{1 + \frac{2}{\beta}} \quad \#$$

$$* \; I_{ref} = \frac{V_{CC} - V_{BE3}}{R_{ref}} = \frac{V_{CC} - 0.7}{R_{ref}}$$

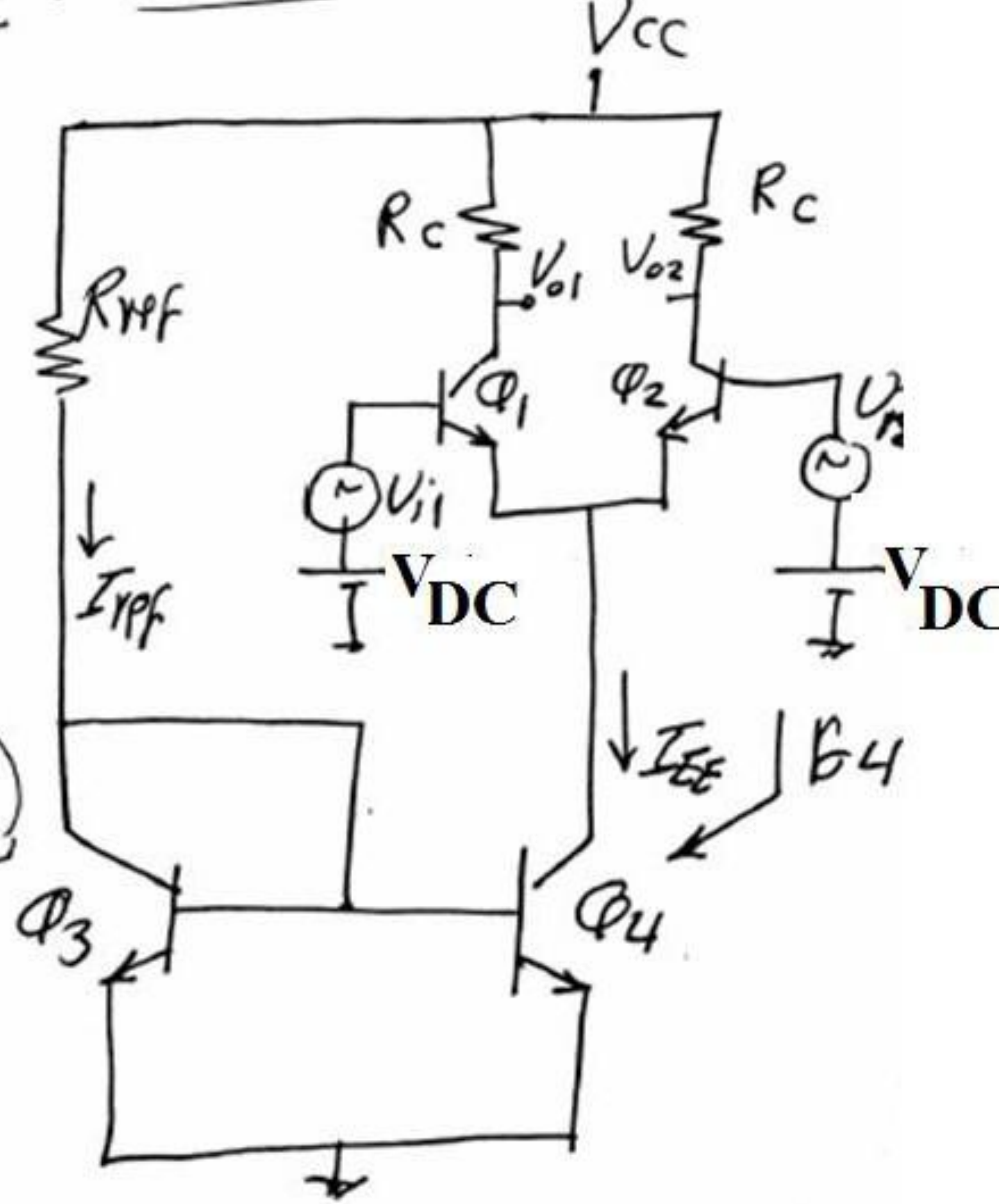
Differential Amplifier with current mirror

$$\boxed{I_{EE} = \frac{I_{ref}}{1+\frac{2}{\beta}}}$$

$$\boxed{I_{ref} = \frac{V_{CC}-0.7}{R_{ref}}}$$

$$R_{EE} = r_{o4} = \frac{V_A}{I_{C4}} = \frac{V_A}{I_{EE}}$$

$\Downarrow$ output resistance of current source



Example 1:

For the BJT differential pair shown in Figure, Calculate:

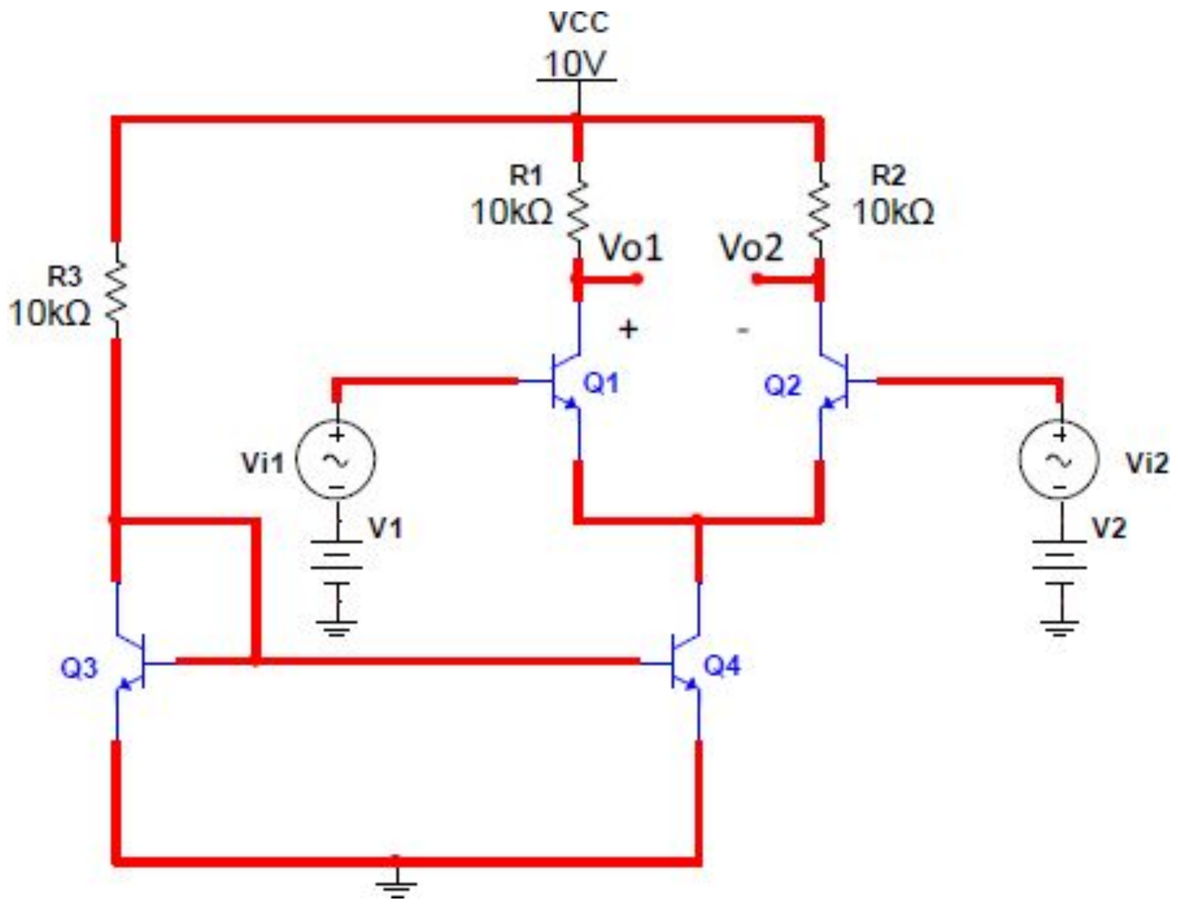
(a) All DC collector currents.

(b) The differential mode gain (Adm).

(c) The common mode gain (Acm).

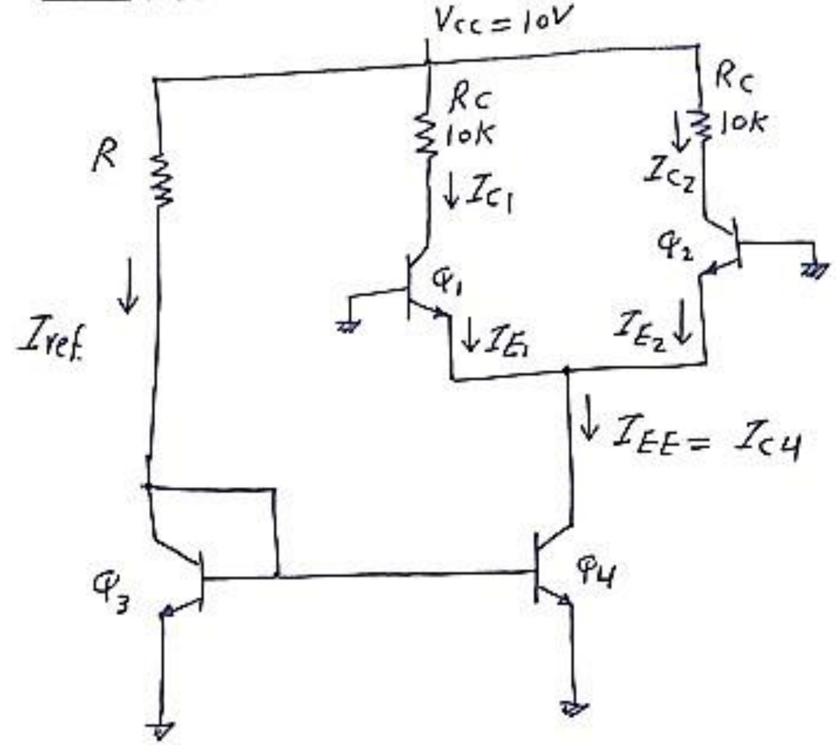
(d) The CMRR in dB.

B = 100, VT = 0.025 V, VA4 = 100 V.



Solution:

(a) D.C Analysis:-



* $I_{ref} = \frac{V_{CC} - V_{BE_3}}{R} = \frac{10 - 0.7}{10} = 0.93\,mA$

* $I_{C4} = I_{EE} = \frac{I_{ref}}{1 + \frac{2}{\beta}} = \frac{0.93}{1 + \frac{2}{100}} \cong 0.912\,mA$

* $I_{E_1} = I_{E_2} = \frac{I_{EE}}{2} = 0.456\,mA$

* $I_{C_1} = I_{C_2} = \frac{\beta}{1+\beta} I_{E_1} = 0.451\,mA$

* $R_{EE} = r_{o4} = \frac{V_{A4}}{I_{C4}} = \frac{100V}{0.912mA} = 109.65\,k\Omega$

* $r_{\pi_1} = r_{\pi_2} = \beta \frac{V_T}{I_{C_1}} = 100 \times \frac{0.025}{0.451} = 5.54\,k\Omega$
$= r_\pi$

(b) The differential Mode gain: (A_{dm})

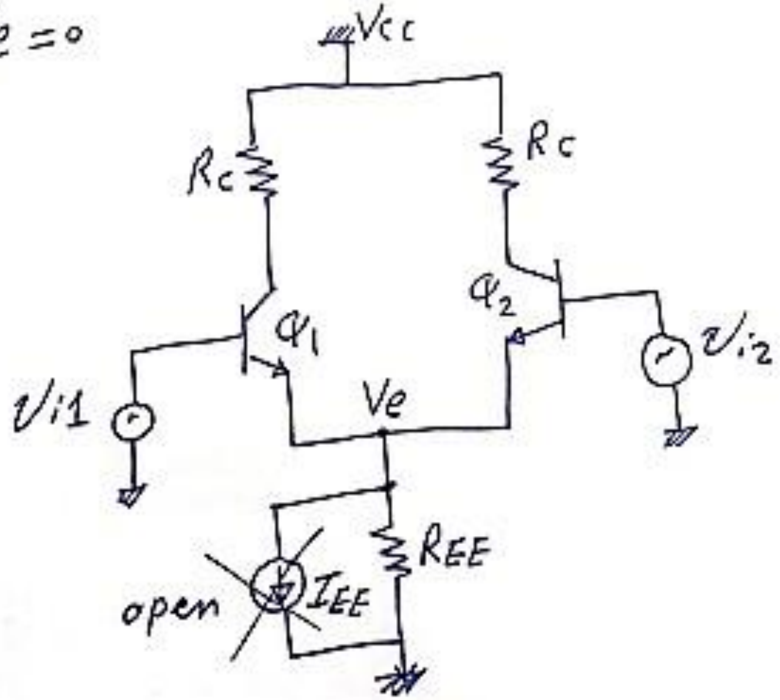
In general, $v_{i1} = v_{ic} + \frac{v_{id}}{2}$

$v_{i2} = v_{ic} - \frac{v_{id}}{2}$

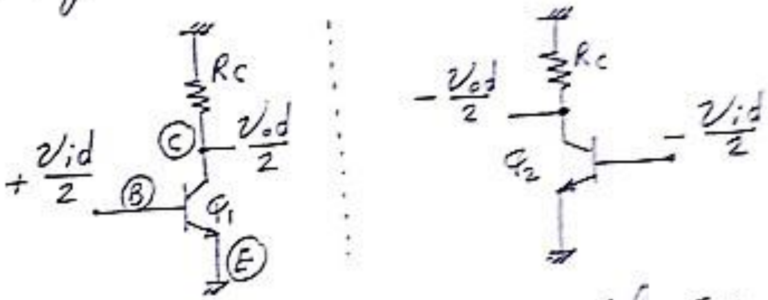
For Differential input,

$v_{i1} = +\frac{v_{id}}{2}$, $v_{i2} = -\frac{v_{id}}{2}$

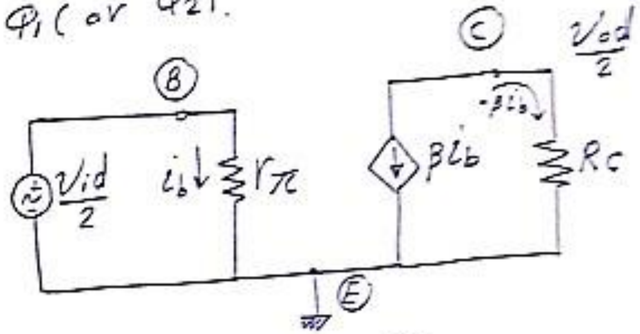
$\therefore V_e = 0$



Using the Half-Circuit Concept



Draw the small-signal π-model for Q_1 (or Q_2).



Differential mode gain is:

$$A_{dm} = \frac{v_{od}}{v_{id}} = \frac{(v_{od}/2)}{(v_{id}/2)} = \frac{(-\beta i_b) R_C}{i_b \cdot r_\pi}$$

$$\boxed{A_{dm} = -\beta \frac{R_C}{r_\pi} = -g_m R_C} = -100 \times \frac{1k}{5.54}$$

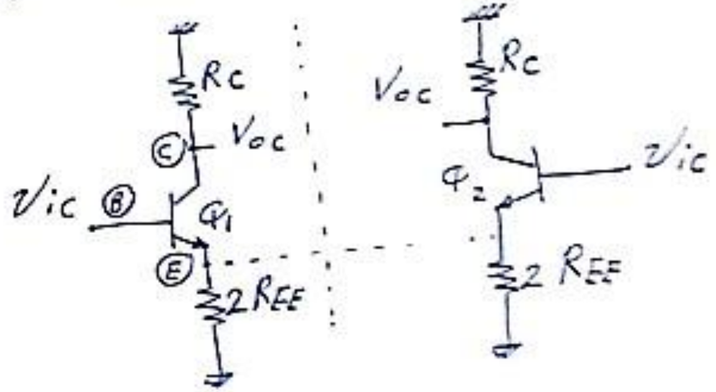
$A_{dm} = -180.51$ (signal gain)

(c) Common Mode gain (A_{cm}):

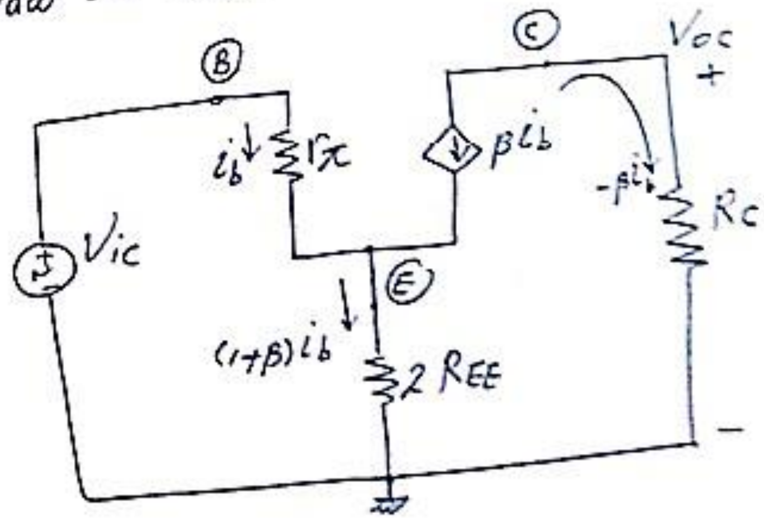
$v_{i1} = v_{ic}$, $v_{i2} = v_{ic}$

then, $V_e \neq 0$

The half-circuit then will be:



Draw the small-signal π-model.



$$A_{cm} = \frac{v_{oc}}{v_{ic}} = \frac{(-\beta i_b) \cdot R_C}{i_b \cdot r_\pi + (1+\beta) i_b \cdot 2R_{EE}}$$

$$\boxed{A_{cm} = \frac{-\beta R_C}{r_\pi + (1+\beta) 2R_{EE}}} = \frac{-100 \times 10}{5.54 + 101 \times 2 \times 109.6}$$

$$A_{cm} = -0.04514$$

(d) Common-Mode-Rejection-Ratio (CMRR)

$$CMRR = \left| \frac{A_{dm}}{A_{cm}} \right| = \left| \frac{-180.51}{-0.04514} \right|$$

$$\boxed{CMRR = 3998.9 \cong 4000}$$

In decible (dB)

$$CMRR(dB) = 20 \log \left| \frac{A_{dm}}{A_{cm}} \right| = 20 \log 3998.9$$

$$\boxed{CMRR(dB) \cong 72 \ dB}$$

Example 2:

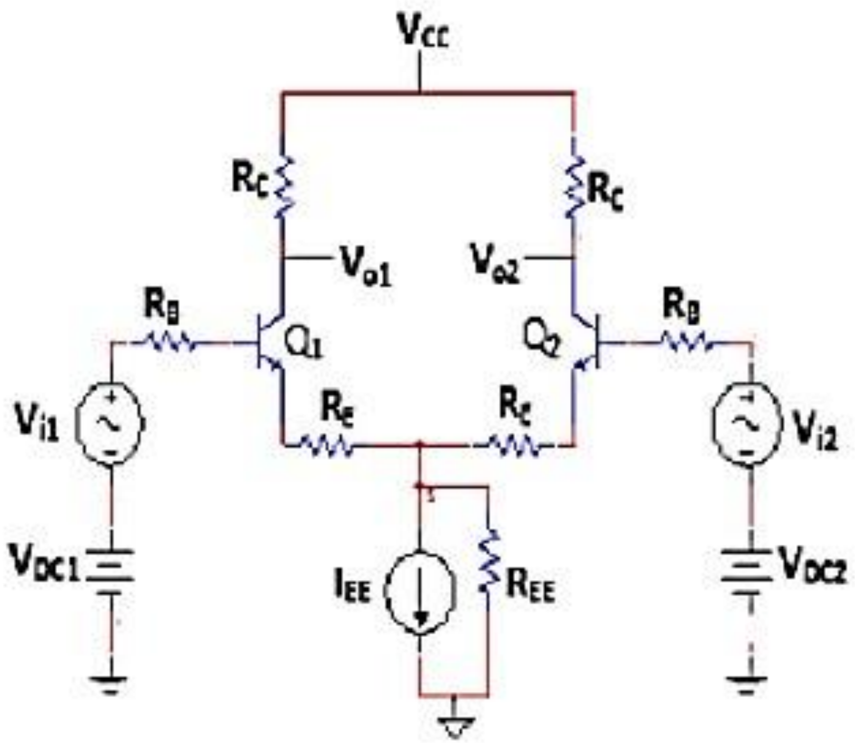
Analyze the differential amplifier circuit shown in Figure given:

$R_c = 10\ k\Omega$, $R_B = 1\ k\Omega$, $R_E = 0.5\ k\Omega$, $R_{EE} = 100\ k\Omega$ $I_{EE} = 2mA$, $\beta = 200$, Calculate:

(a) The differential mode gain.

(b) The common mode gain.

(c) The CMRR in dB.



Solution:

(a) The differential mode gain.

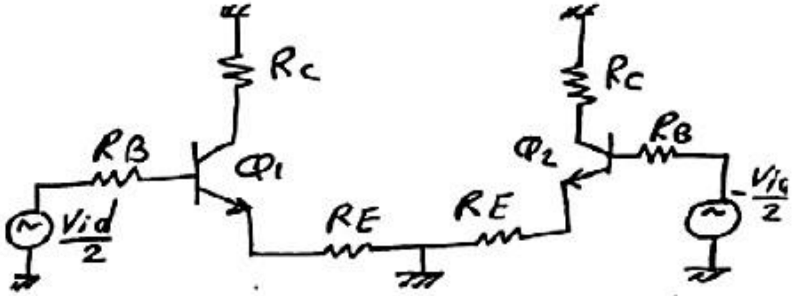
$$v_{i1} = v_{ic} + \frac{v_{id}}{2}$$

$$v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

Differential Mode gain

$$v_{i1} = \frac{v_{id}}{2}$$

$$v_{i2} = -\frac{v_{id}}{2}$$



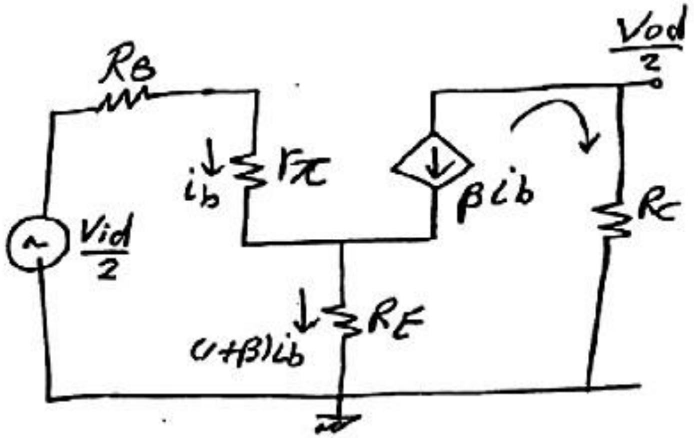
$$A_d = \frac{V_{od}/2}{V_{id}/2} = \frac{V_{od}}{V_{id}} = \frac{(-\beta i_b) R_C}{i_b R_B + i_b r_\pi + (1+\beta) i_b R_E}$$

$$A_d = \frac{-\beta R_C}{R_B + r_\pi + (1+\beta) R_E}$$

* $I_{E1} = I_{E2} = \frac{I_{EE}}{2}$

$I_{E1} = I_{E1} = 1mA$

* $I_{C1} \cong I_{E1} = 1mA$
* $I_{C2} \cong I_{E2} = 1mA$



$$A_d = \frac{-200(10)}{1 + 5 + (201 \times 0.5)}$$

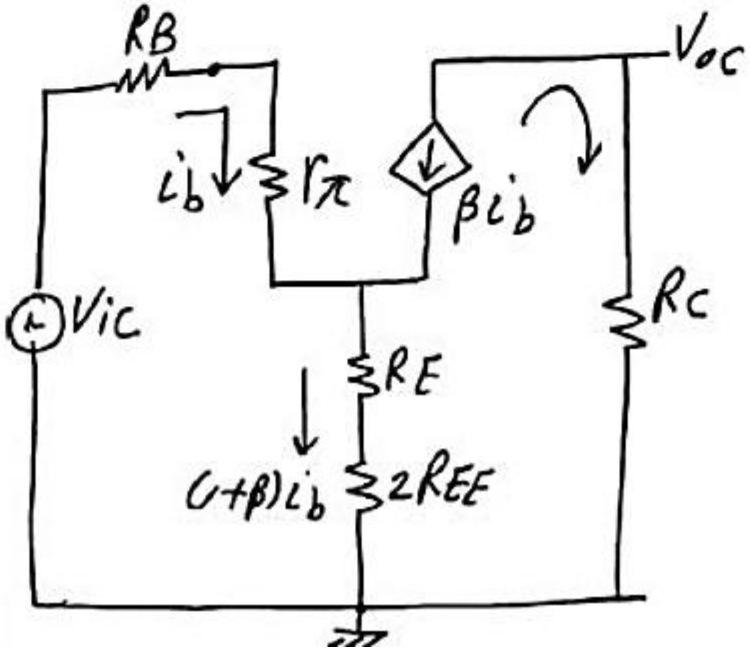
$$A_d = -18.78$$

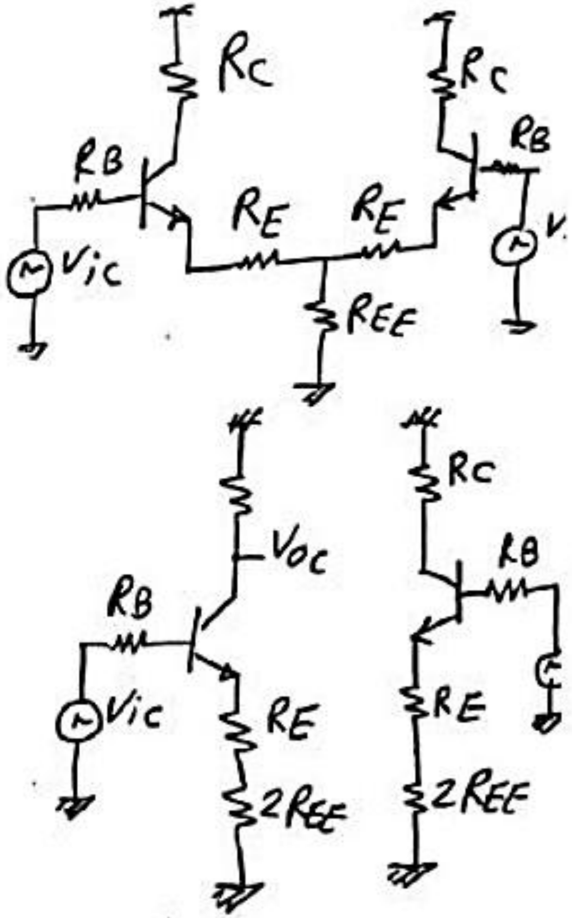
$$r_{\pi1} = r_{\pi2} = \beta \frac{V_T}{I_C} = 200 \times \frac{0.025V}{1mA} = r_\pi$$

$$r_\pi = 5 K\Omega$$

(b) The common mode gain.

$$v_{i1} = v_{ic}$$
$$v_{i2} = v_{ic}$$





$$* A_c = \frac{V_{oc}}{V_{ic}} = \frac{(-\beta i_b)\, R_C}{i_b R_B + i_b r_\pi + (1+\beta) i_b [R_E + 2R_{EE}}$$

$$\boxed{A_c = \frac{-\beta R_C}{R_B + r_\pi + (1+\beta)(R_E + 2R_{EE})}}$$

$$A_c = \frac{-200(10)}{1 + 5 + 201(0.5 + 200)}$$

$$\boxed{A_c = -0.04962}$$

(c) The CMRR in dB.

$$CMRR = \frac{Ad}{Ac} = \frac{-18.78}{-0.04962}$$

$$CMRR = 378.478$$

$$CMRR(dB) = 20\log(378.478) = 51.561\ dB$$