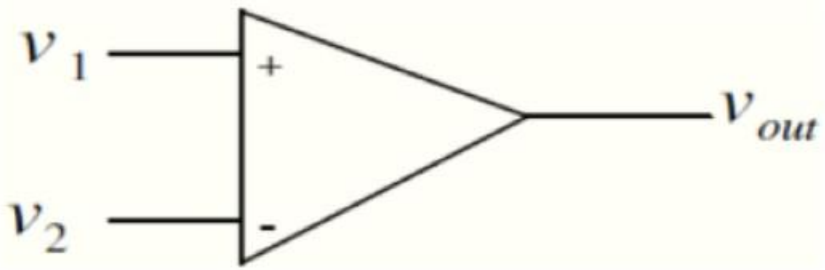
The Differential Amplifier

Introduction

- Differential Amplifiers are used extensively in IC
  - They are more immune to Noise and Interference Signals
  - No need for Bypass and coupling capacitors
    - Example: the Voltage op-amp is a differential amplifier

$$\boxed{V_{Out} = A_V(V_1 - V_2)}$$



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- The Differential Amplifier amplifies the difference between its input terminals and rejects their average (Common Mode).
- The differential input signal v_{Id} is simply the difference between the two input signals v_1 and v_2; that is,

$$\boxed{v_{Id} = (v_1 - v_2)}$$

- The common-mode input signal v_{Icm} is the average of the two input signals v_1 and v_2; namely,

$$\boxed{v_{Icm} = \frac{(v_1 + v_2)}{2}}$$

- The previous equations can be used to express the input signals v_1 and v_2 in terms of their differential and common-mode components as follows:

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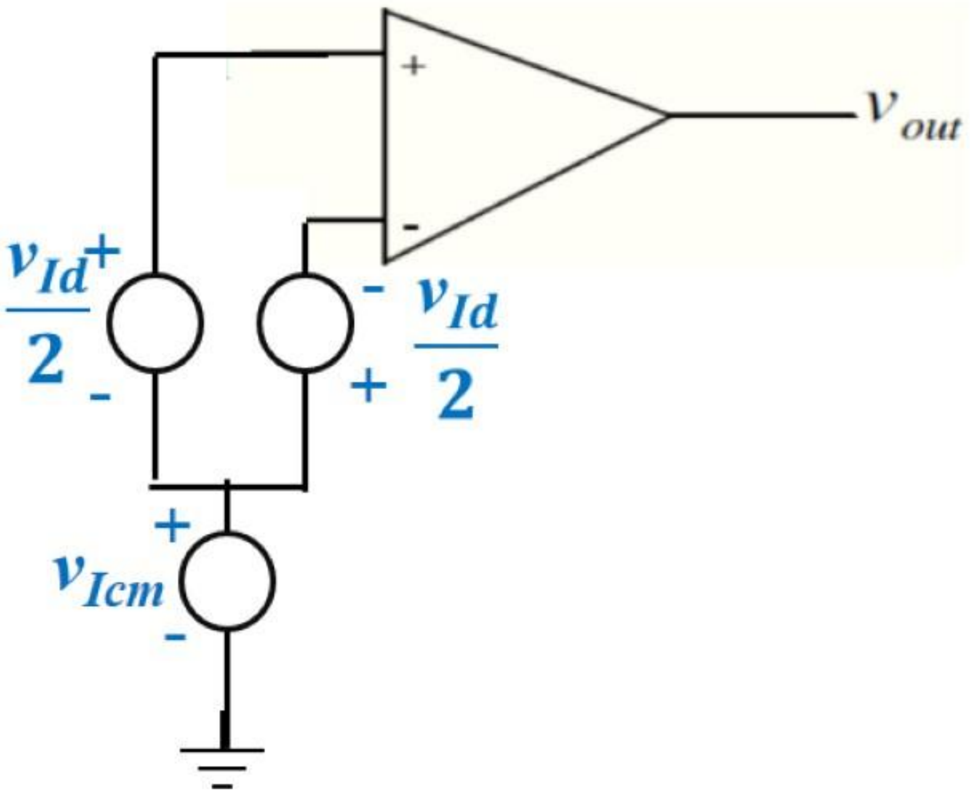
$$\boxed{v_1 = \frac{v_{Id}}{2} + v_{Icm}}$$

$$\boxed{v_2 = -\frac{v_{Id}}{2} + v_{Icm}}$$



$$\boxed{v_{out} = A_d v_{Id} + A_{cm} v_{Icm}}$$

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Objective

- Calculation of the Differential Mode Gain of the amplifier **(ideally tends to infinity)**
- Calculation of the Common Mode Gain of the amplifier **(ideally tends to ZERO)**
- Calculation of the Common Mode Rejection Ratio (CMRR) **(ideally tends to infinity)**

$$\boxed{A_d = \frac{v_{out}}{v_{Id}}}$$

$$\boxed{A_{cm} = \frac{v_{out}}{v_{Icm}}}$$

$$\boxed{CMRR = \left| \frac{A_d}{A_{cm}} \right|}$$

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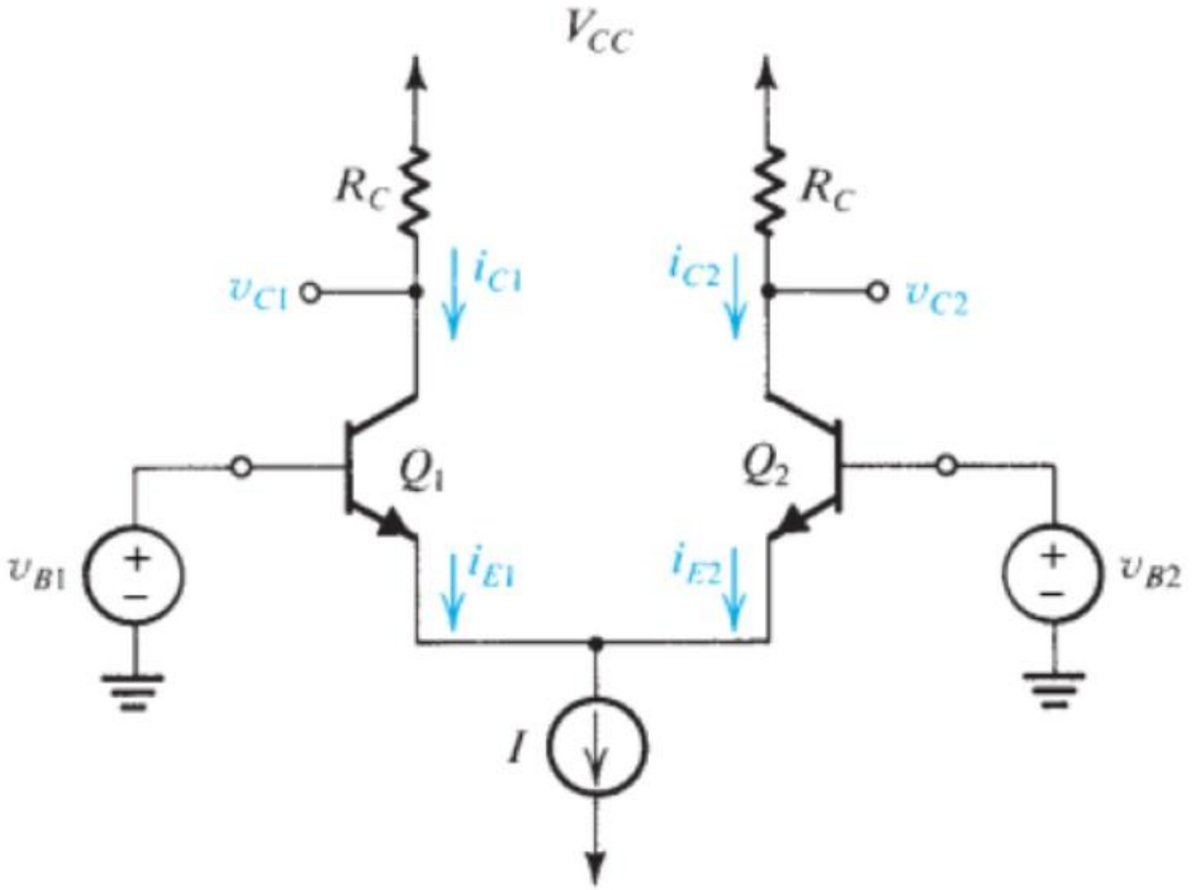
The BJT Differential Pair



The basic BJT differential-pair configuration.

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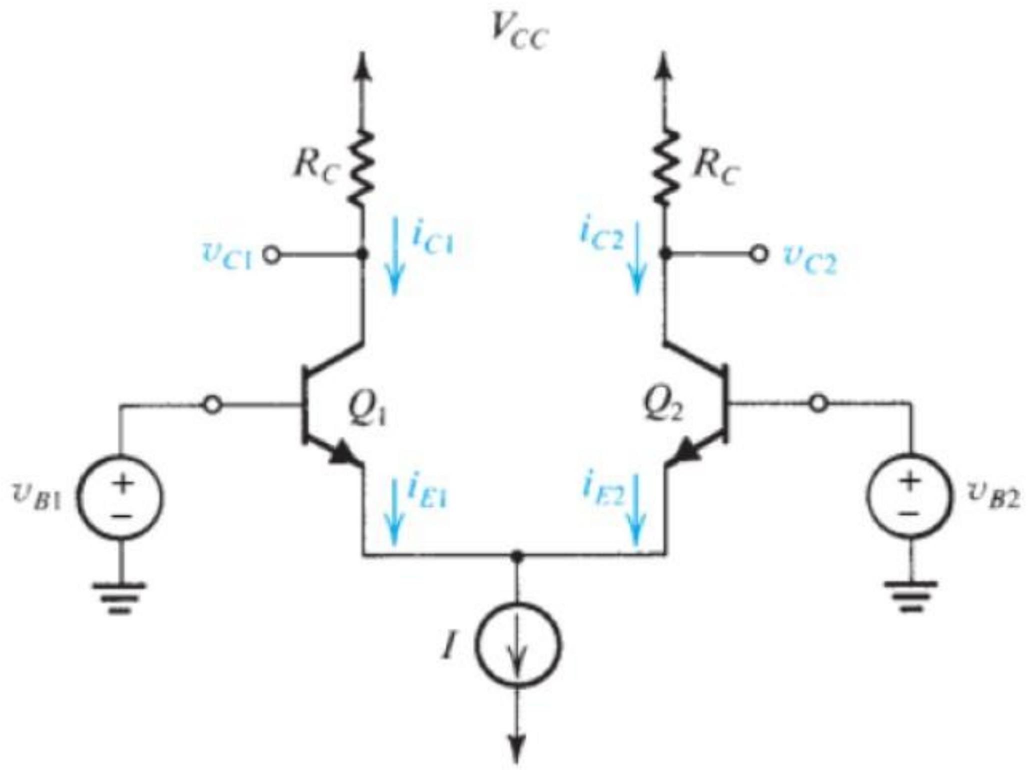
Large Signal Analysis for Differential Amplifier

Basic Operation

BJT Differential Amplifier/Pair

Emitter Coupled Circuit:

- Two Identical BJTs (Same β and Is).
- Their Emitters are connected together.
- The input signals are connecter to the Q's Bases.
- A Constant DC biasing Current 'I' is used to set the DC operating point.



$$\boxed{i_{E1} + i_{E2} = I}$$

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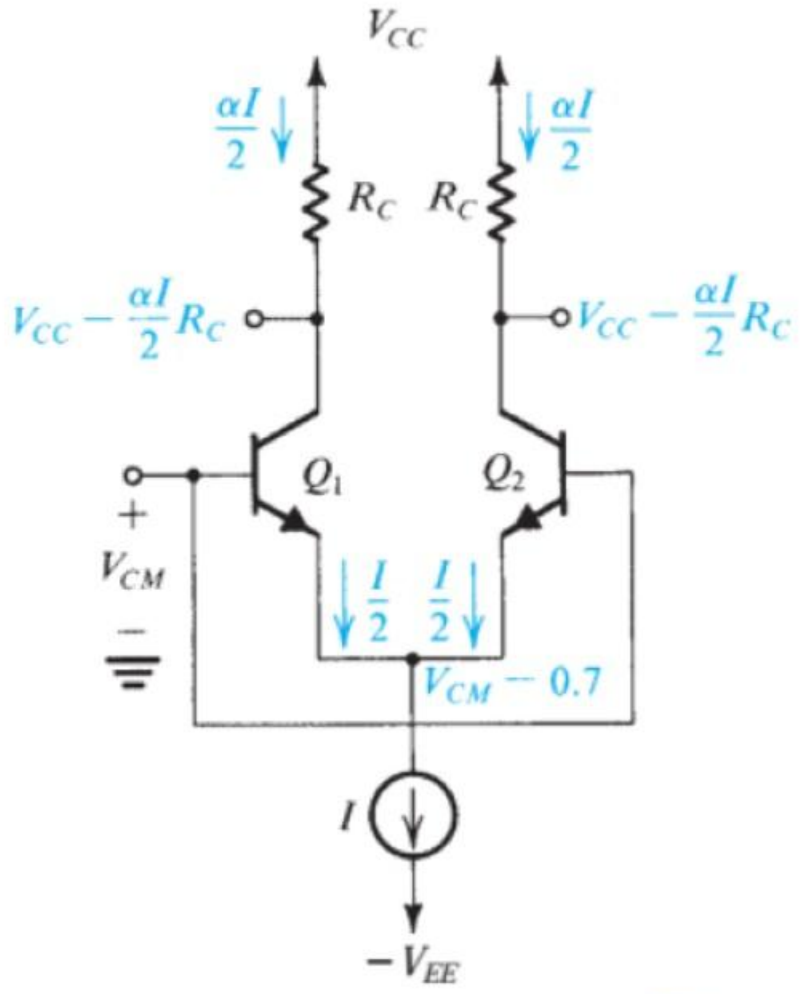
BJT Differential Amplifier/Pair

- Emitter Coupled Circuit Large Signal Analysis (**Active Mode**)
- If Base 1 and Base 2 are connected together (Common mode signal), then the two emitter currents will be equal

$$\boxed{v_{B1} = v_{B2} = v_{CM}}$$

$$\boxed{i_{E1} = i_{E2} = \frac{I}{2}}$$

$$\boxed{i_{C1} = i_{C2} = \frac{\alpha I}{2}}$$



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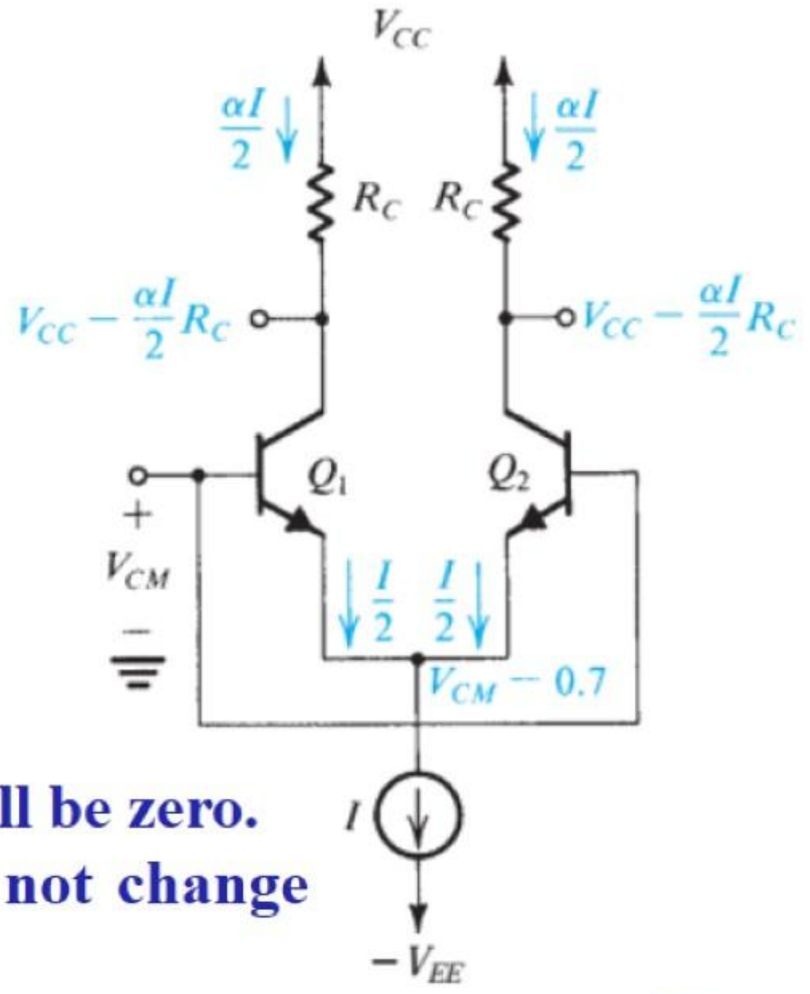
BJT Differential Amplifier/Pair

- Emitter Coupled Circuit Large Signal Analysis (**Active Mode**)
- If Base 1 and Base 2 are connected together (Common mode signal), then the two emitter currents will be equal

$$\boxed{v_{C1} = v_{C2} = v_{CC} - \frac{\alpha I}{2} R_C}$$



- **The difference in voltage between the two collectors will be zero.**
- **Changing the value of the common mode signal will not change the transistors currents.**
- **Thus the differential pair does not respond to (i.e., it <u>rejects</u>) changes in the common-mode input voltage.**

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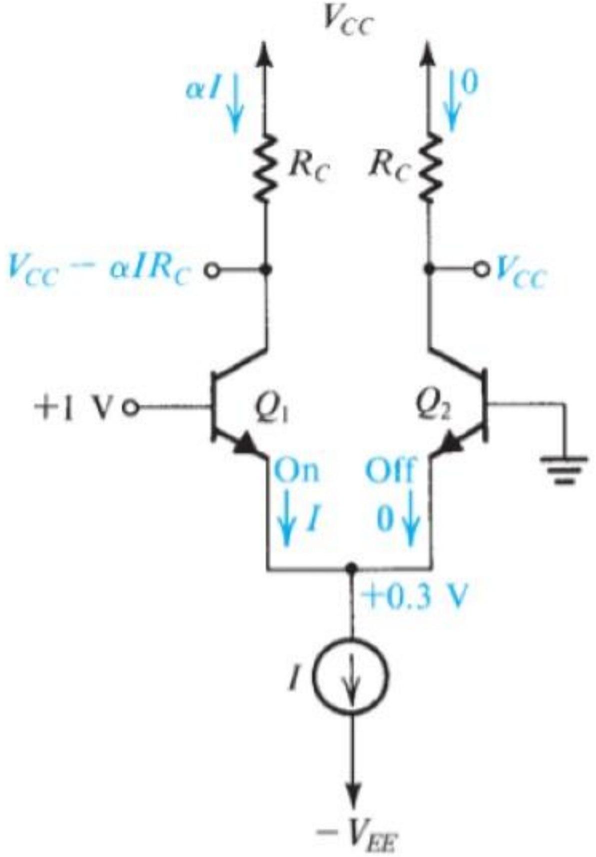
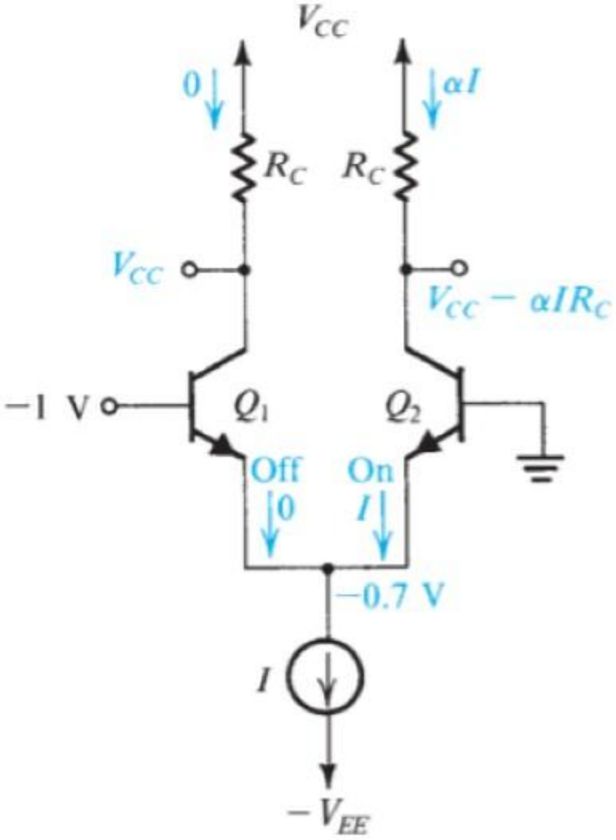
BJT Differential Amplifier/Pair

- Emitter Coupled Circuit Large Signal Analysis:
- If Base 1 and Base 2 are not connected together (**Differential signal**), then the two emitter currents will be equal.





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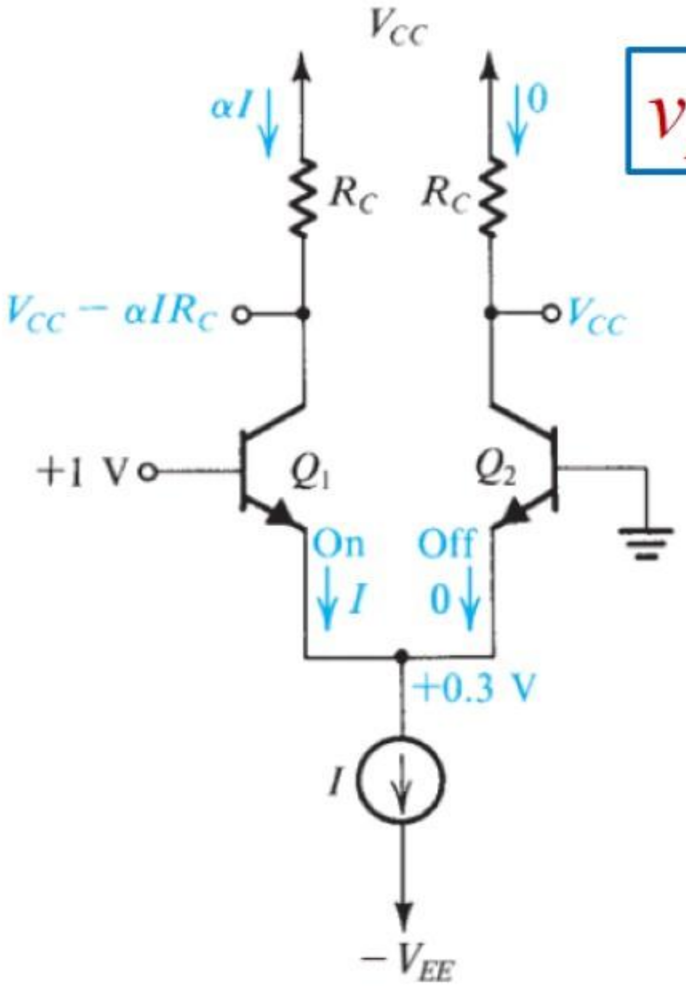
BJT Differential Amplifier/Pair

- let the voltage v_{B2} be set to a constant value, say, zero (by grounding B2), and let $v_{B1} = +1$ V. With a bit of reasoning it can be seen that Q_1 will be on and conducting all of the current I and that Q_2 will be off. For Q_1 to be on (with $V_{BE1} = 0.7$ V), the emitter has to be at approximately +0.3 V, which keeps the EBJ of Q_2 reverse-biased.



$$\boxed{v_{B1} - v_{B2} = v_{Id} \gg 0}$$

$$\boxed{i_{E1} \neq i_{E2}}$$

$$\boxed{i_{E1} = I}$$

$$\boxed{i_{E2} = 0}$$

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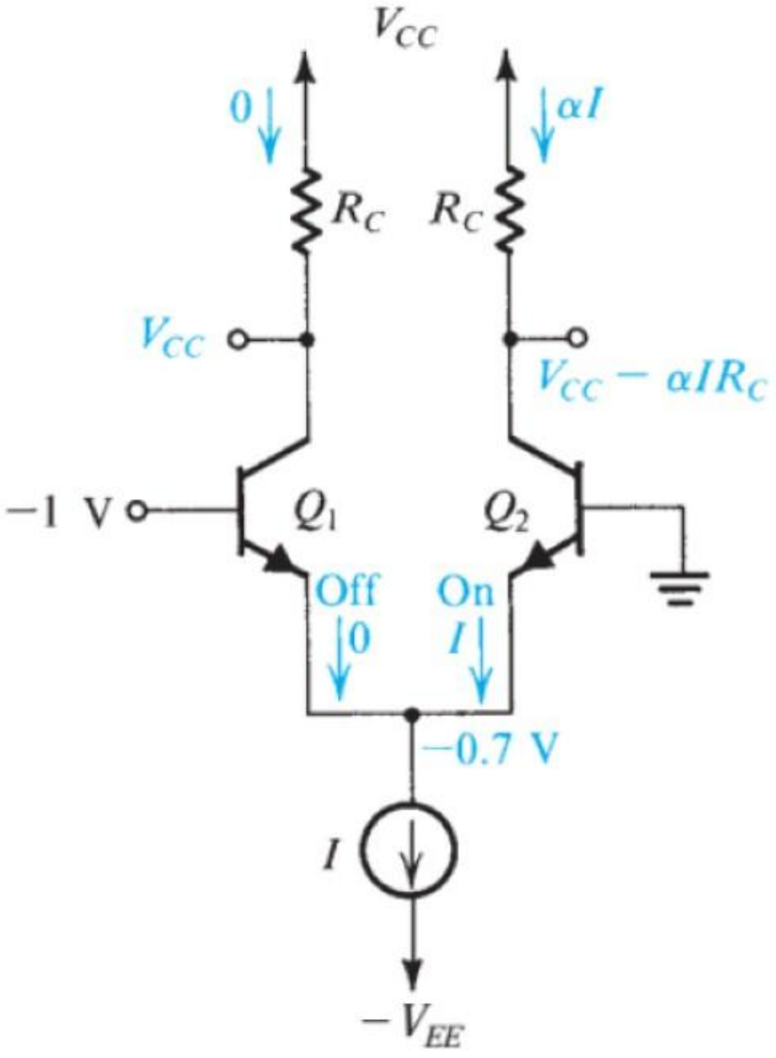
BJT Differential Amplifier/Pair



$$\boxed{v_{B1} - v_{B2} = v_{Id} \ll 0}$$

$$\boxed{i_{E1} \neq i_{E2}}$$

$$\boxed{i_{E1} = 0}$$

$$\boxed{i_{E2} = I}$$

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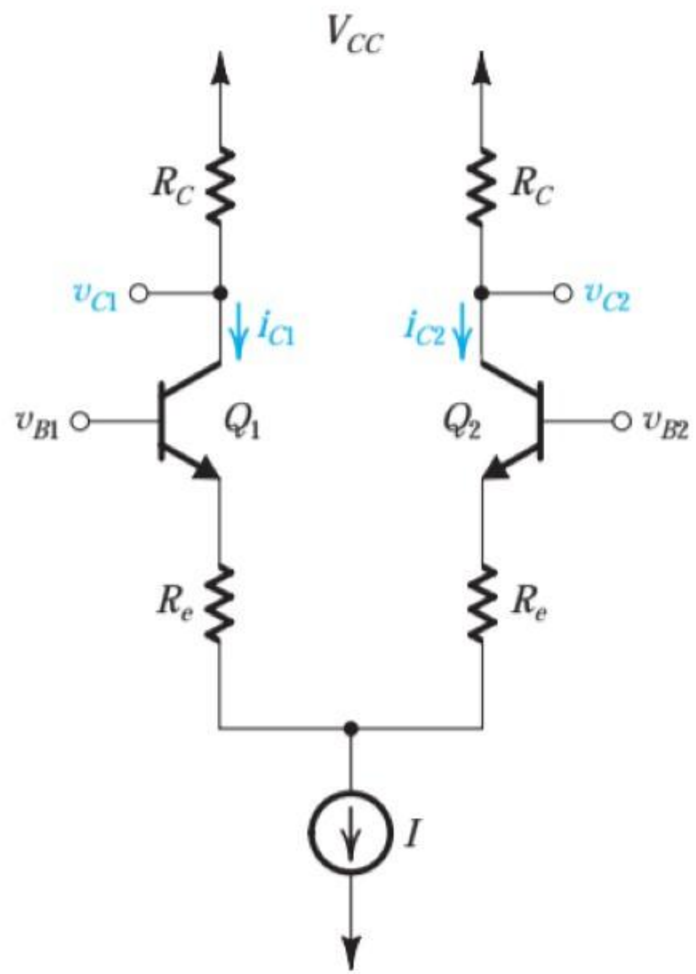
BJT Differential Amplifier/Pair

- Note: The linear range of operation can be extended by including resistances in the emitters.



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Small Signal Analysis for Differential Amplifier

Differential and Common Mode Gain for BJT

BJT Differential Amplifier/Pair

- We shall study the application of the BJT differential pair in small-signal amplification.
- The voltage at one of the input terminals will be $\frac{v_{id}}{2} + v_{cm}$ while that at the other input terminal will be $-\frac{v_{id}}{2} + v_{cm}$

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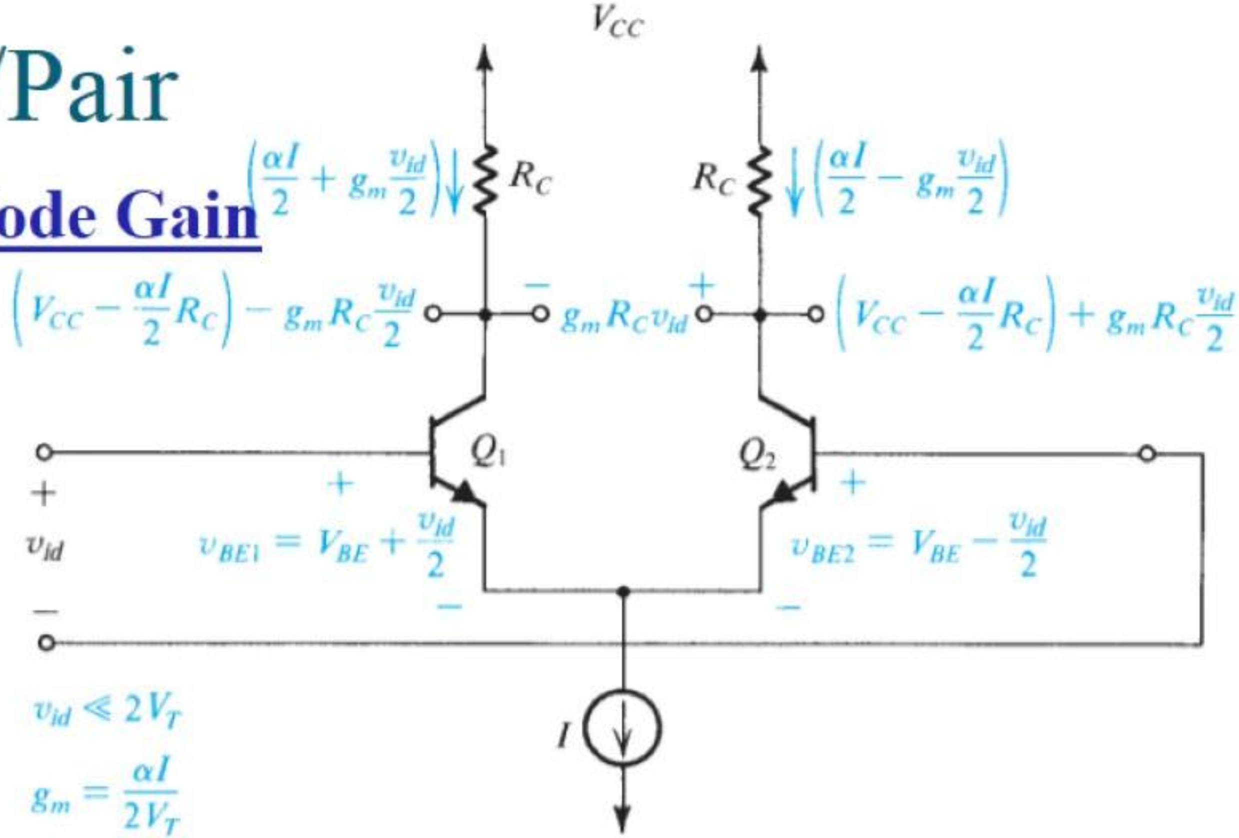
BJT Differential Amplifier/Pair

➢ **Small Signal Analysis Differential Mode Gain**

DC Operating point is adjusted such that:

$$\boxed{i_{C1} = i_{C2} = \frac{\alpha I}{2}}$$



➢ The circuit is like two Common Emitters with the output taken as a difference between the two Collectors.

$$\boxed{g_{m1} = g_{m2}} \qquad \boxed{r_{\pi 1} = r_{\pi 2}} \qquad \boxed{r_{o1} = r_{o2}}$$

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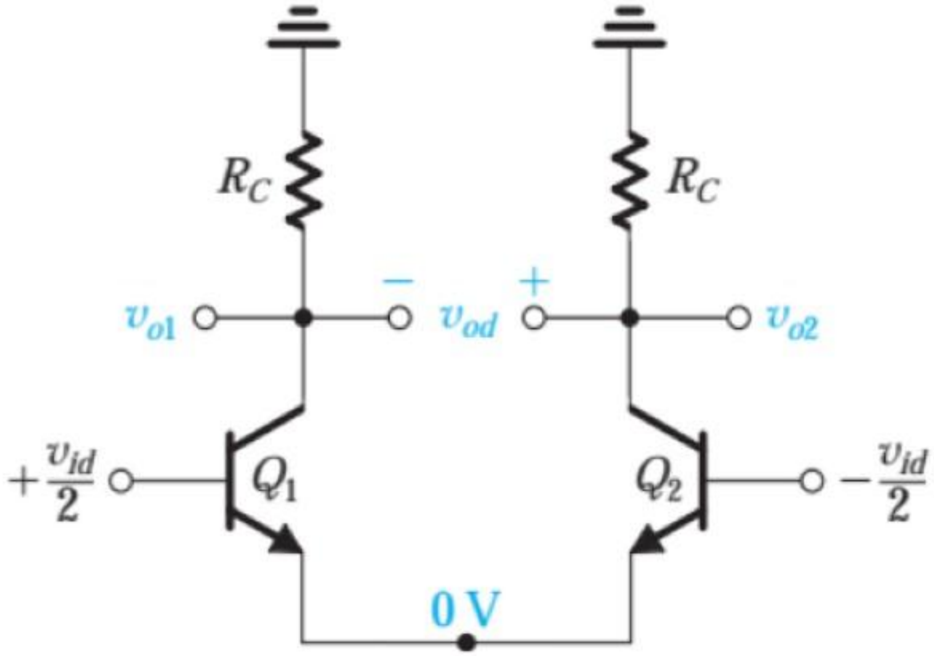
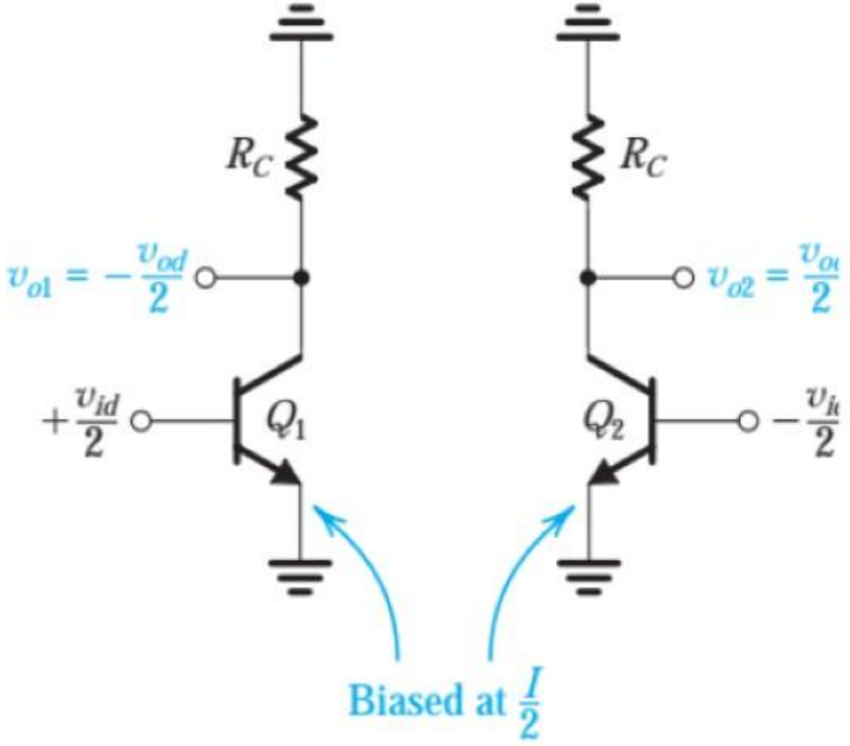
BJT Differential Amplifier/Pair

➢ Small Signal Analysis Differential Mode Gain

- We can use half Circuit Concept to calculate the gain





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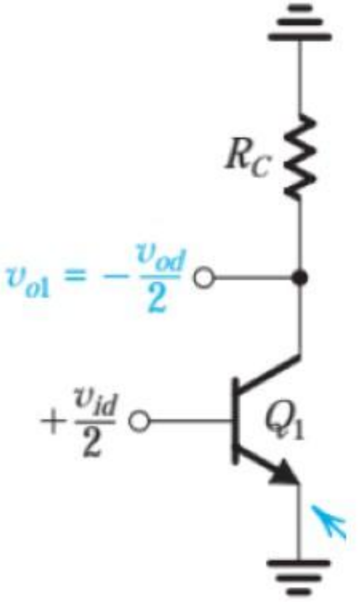
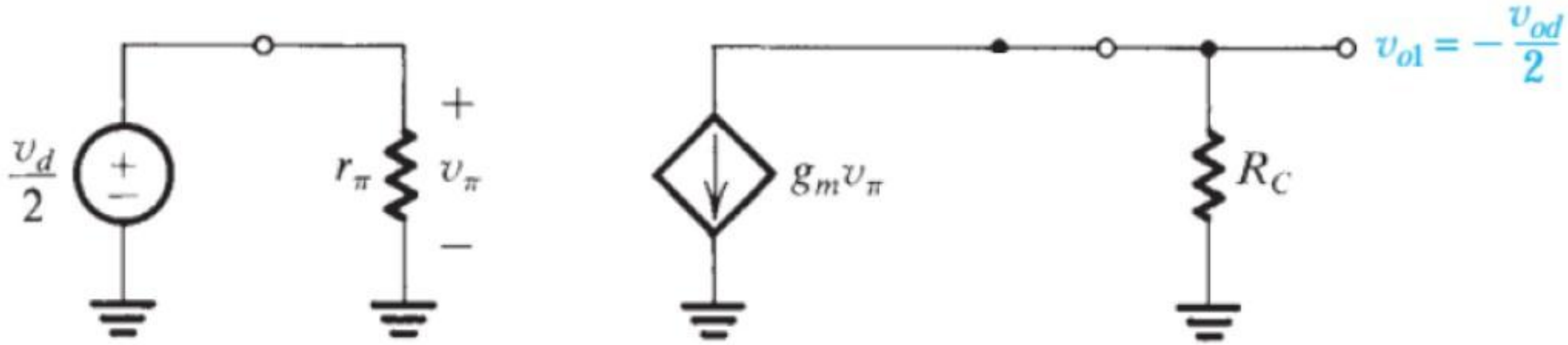
BJT Differential Amplifier/Pair

- **Small Signal Analysis Differential Mode Gain**

- We can use half Circuit Concept to calculate the gain





Equivalent-circuit model of the differential half-circuit formed by $Q1$

$$R_{in} = 2r_{\pi}$$

$$A_d = \frac{v_{od}}{vi_d} = \frac{v_{o2} - v_{o1}}{vi_d} = \frac{v_{C2} - v_{C1}}{vi_d} = g_m R_C$$

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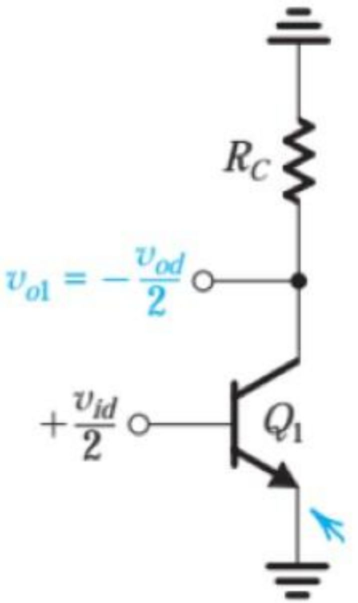
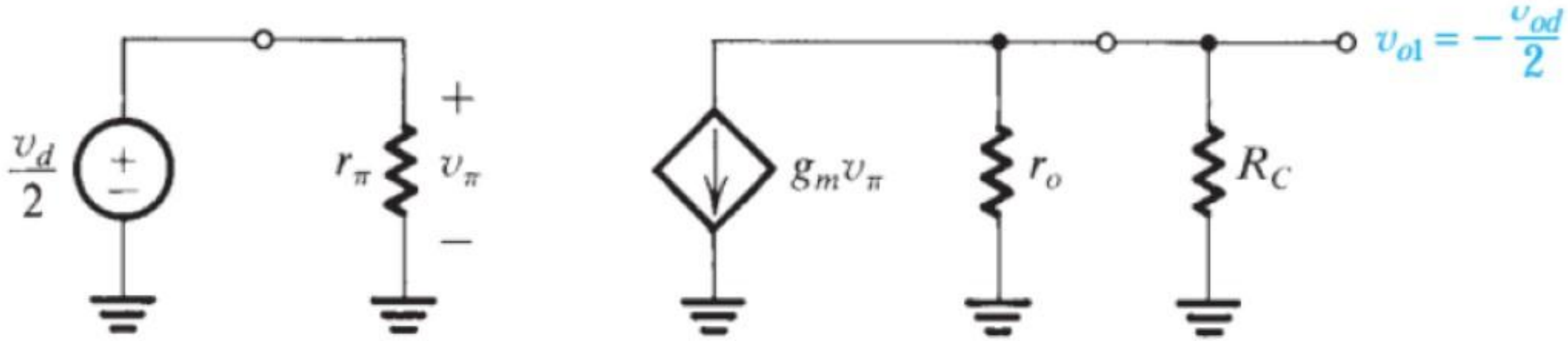
BJT Differential Amplifier/Pair

- **Small Signal Analysis Differential Mode Gain**
- We can use half Circuit Concept to calculate the gain
- **Notes: if r_o is taken into Consideration, then:**





Equivalent-circuit model of the differential half-circuit formed by $Q1$

$$\boxed{A_d = \frac{v_{od}}{vi_d} = \frac{v_{o2} - v_{o1}}{vi_d} = \frac{v_{C2} - v_{C1}}{vi_d} = g_m\,(r_o // R_C)}$$

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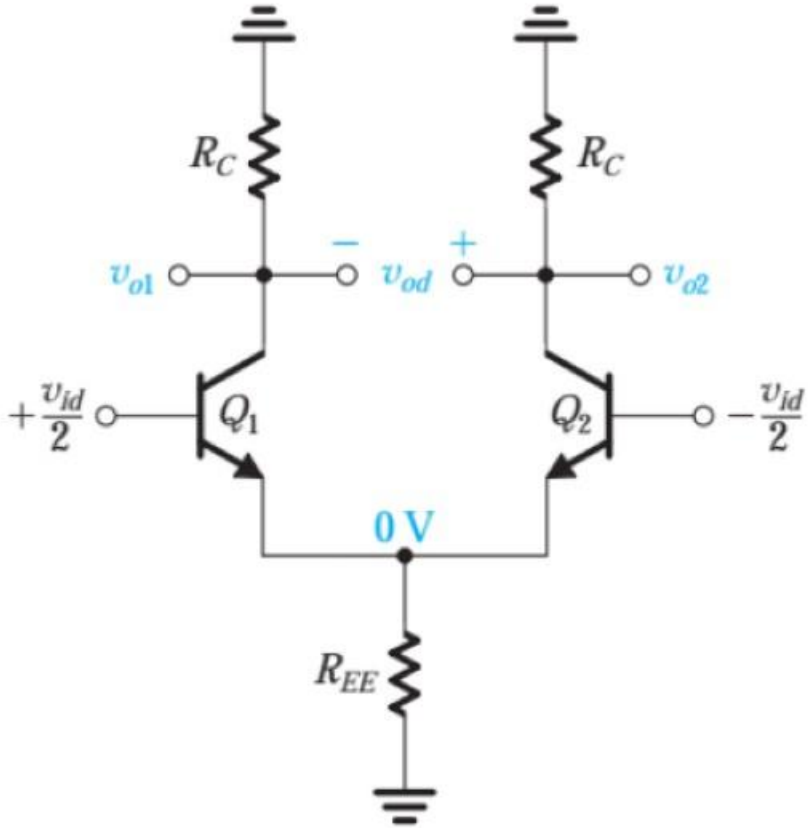
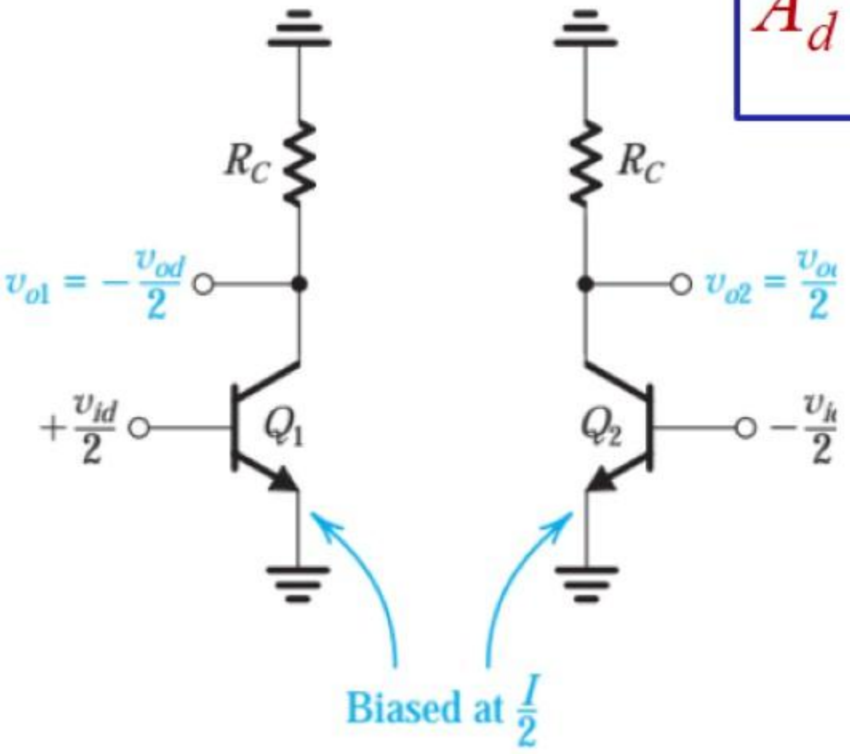
BJT Differential Amplifier/Pair

- **Small Signal Analysis Differential Mode Gain**
- ***R_{EE}* is the Current Source Resistance.**
- We can use half Circuit Concept to calculate the gain

$$\boxed{A_d = \frac{v_{od}}{vi_d} = g_m\,(r_o // R_C)}$$





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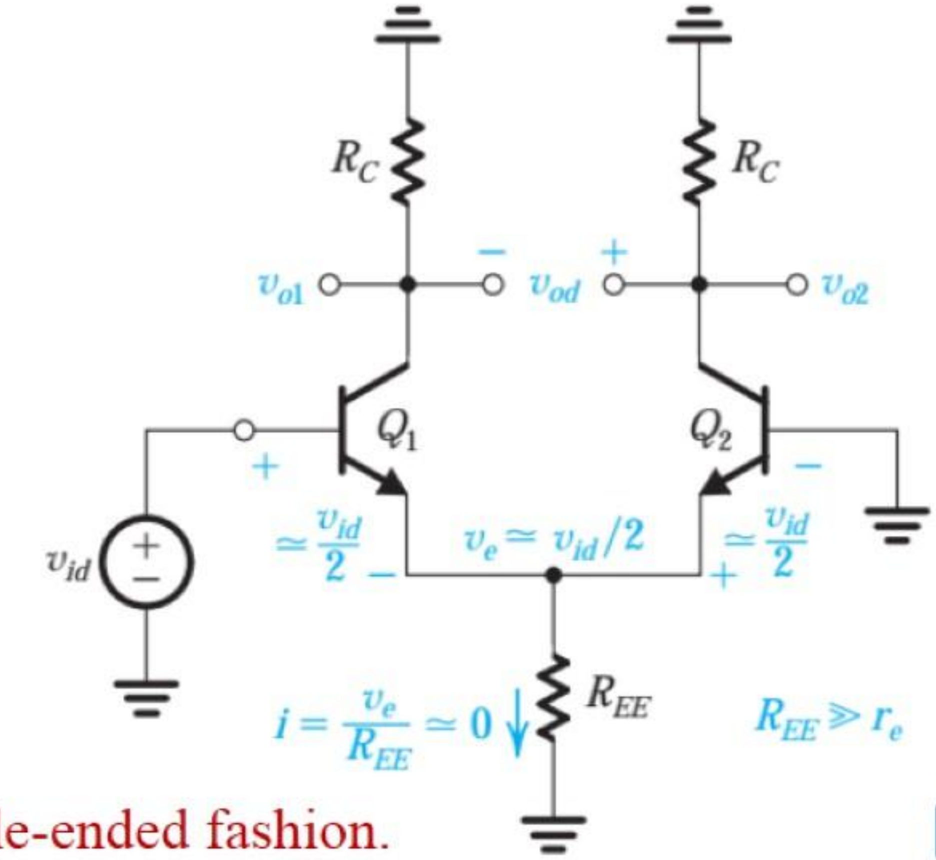
BJT Differential Amplifier/Pair

- **Small Signal Analysis Differential Mode Gain**
- **Notes: What if the signal is not applied fully differential? (Derive!)**



The differential amplifier fed in a single-ended fashion.

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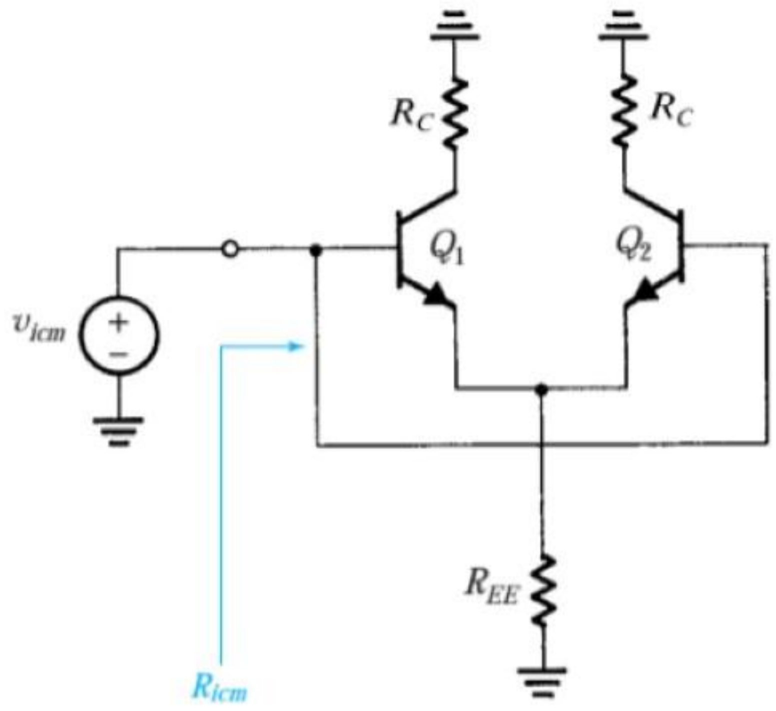
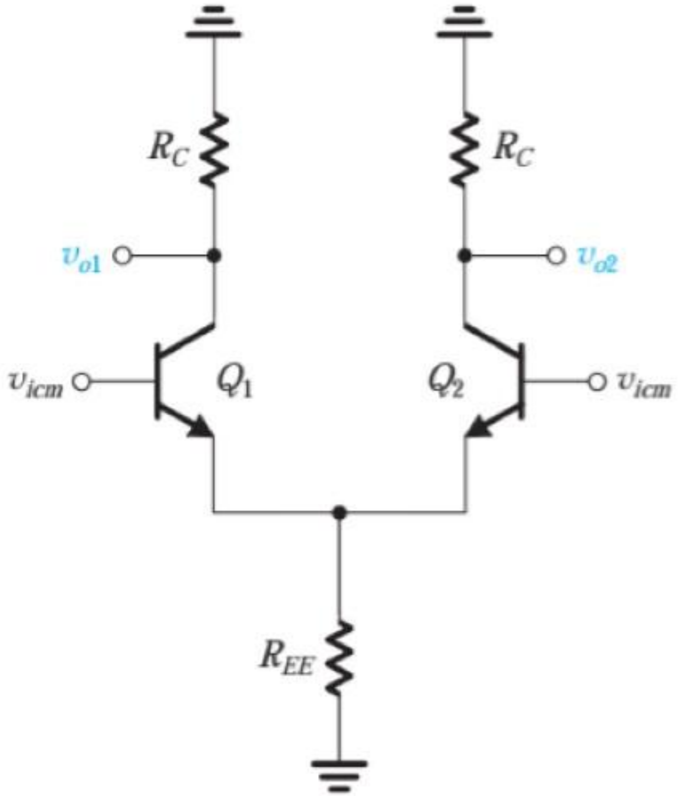
BJT Differential Amplifier/Pair

- **Small Signal Analysis Common Mode Gain**
- R_{EE} **is the Current Source Resistance (ve is not Zero in the common mode Case, why?)**





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BJT Differential Amplifier/Pair

➢ **Small Signal Analysis Common Mode Gain**

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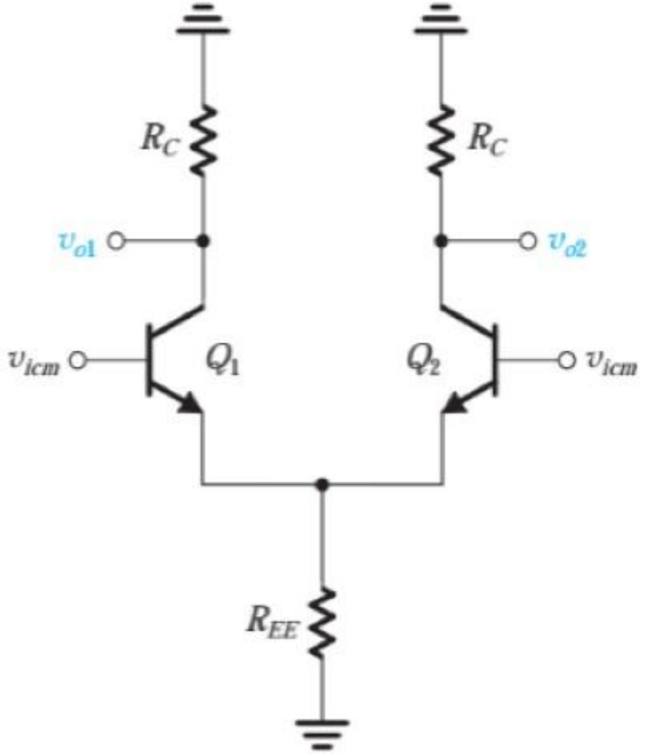

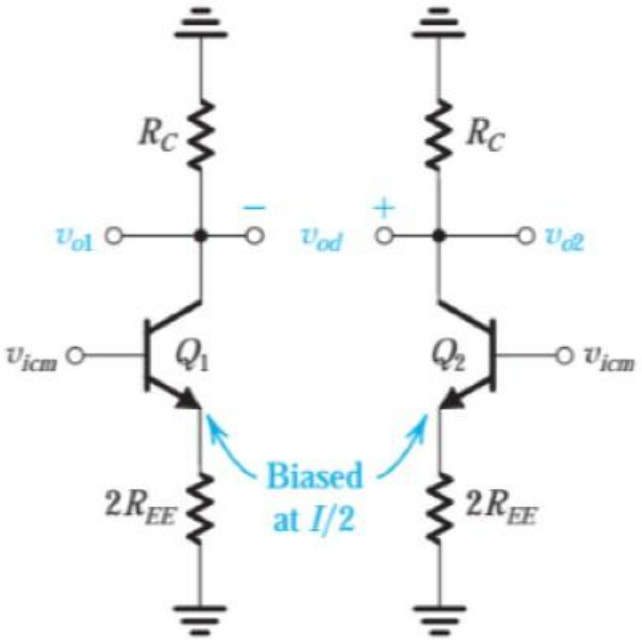

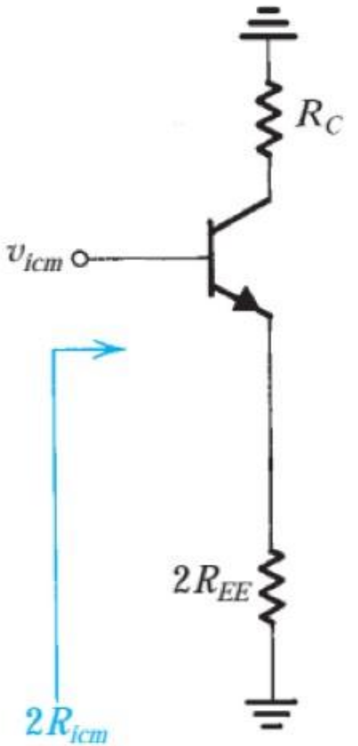

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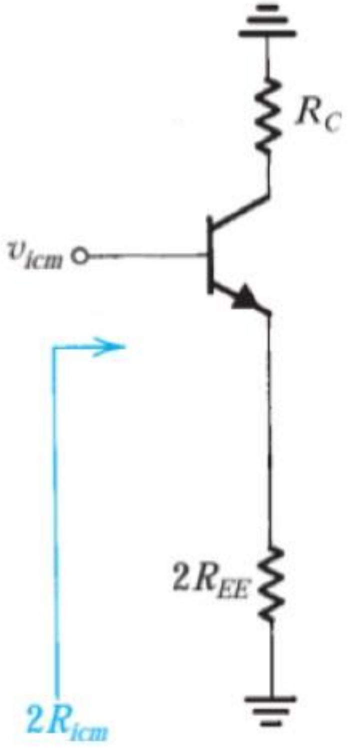
BJT Differential Amplifier/Pair

➢ **Small Signal Analysis Common Mode Gain**

➢ This condition, however, is based on the assumption of perfect matching between the two sides of the differential amplifier.



$$v_{o1} = -\frac{\alpha R_C}{r_e + 2R_{EE}} v_{icm}$$

$$v_{o2} = -\frac{\alpha R_C}{r_e + 2R_{EE}} v_{icm}$$

$$v_{od} = v_{o2} - v_{o1} = 0$$

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BJT Differential Amplifier/Pair

➢ Small Signal Analysis Common Mode Gain

➢ Any mismatch will result in v_{od} acquiring a component proportional to v_{Icm}. For example, consider the case of a mismatch ΔR_C between the two collector resistances: If the collector of has a collector resistance R_C ,

$$v_{o1} = -\frac{\alpha R_C}{2R_{EE} + r_e} v_{icm}$$

and the collector of has a collector resistance $(R_C + \Delta R_C)$

$$v_{o2} = -\frac{\alpha(R_C + \Delta R_C)}{2R_{EE} + r_e} v_{icm}$$

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BJT Differential Amplifier/Pair

➢ **Small Signal Analysis Common Mode Gain**

$$\begin{aligned} v_{od} &\equiv v_{o2} - v_{o1} \\ &= -\frac{\alpha \Delta R_C}{2R_{EE} + r_e} v_{icm} \end{aligned}$$

$$A_{cm} \equiv \frac{v_{od}}{v_{icm}} = -\frac{\alpha \Delta R_C}{2R_{EE} + r_e}$$

➢ Since $\alpha \approx 1$, $r_e << 2\,R_{EE}$, Eq. can be approximated and written in the form

$$A_{cm} \simeq -\left(\frac{R_C}{2R_{EE}}\right)\left(\frac{\Delta R_C}{R_C}\right)$$

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The common-mode rejection ratio can now be found from

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|}$$

$$\text{CMRR} = (2g_m R_{EE}) \Big/ \left(\frac{\Delta R_C}{R_C}\right)$$

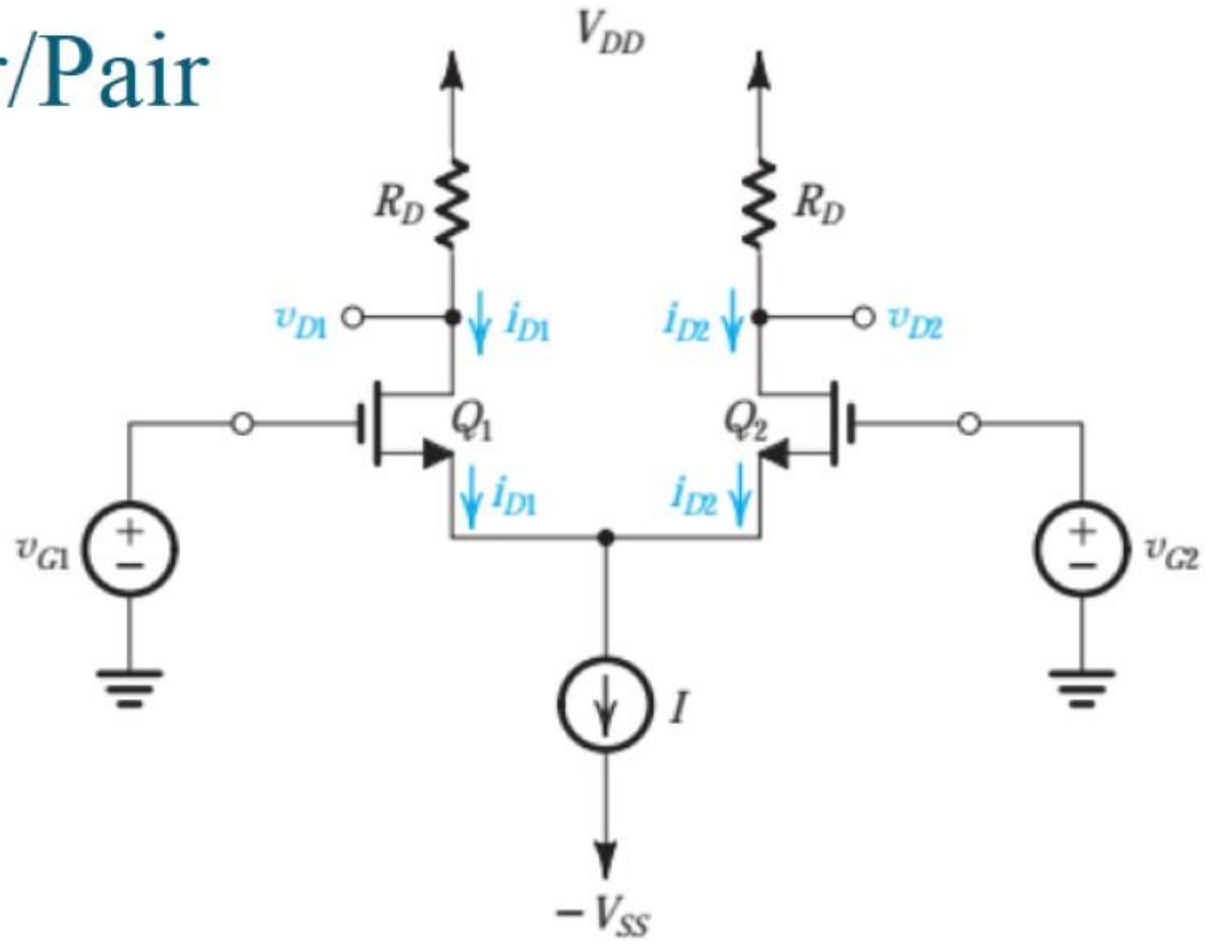
The MOS Differential Pair

MOS Differential Amplifier/Pair

Source Coupled Circuit:

- Two matched MOS-FET (Same K and V_T)
- Their Sources are connected together.
- The input signals are connected to the M's Gates
- A Constant DC biasing Current 'I' is used to set the DC operating point.

$$\boxed{i_{D1} + i_{D2} = I}$$



The basic MOS differential-pair configuration.

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MOS Differential Amplifier/Pair

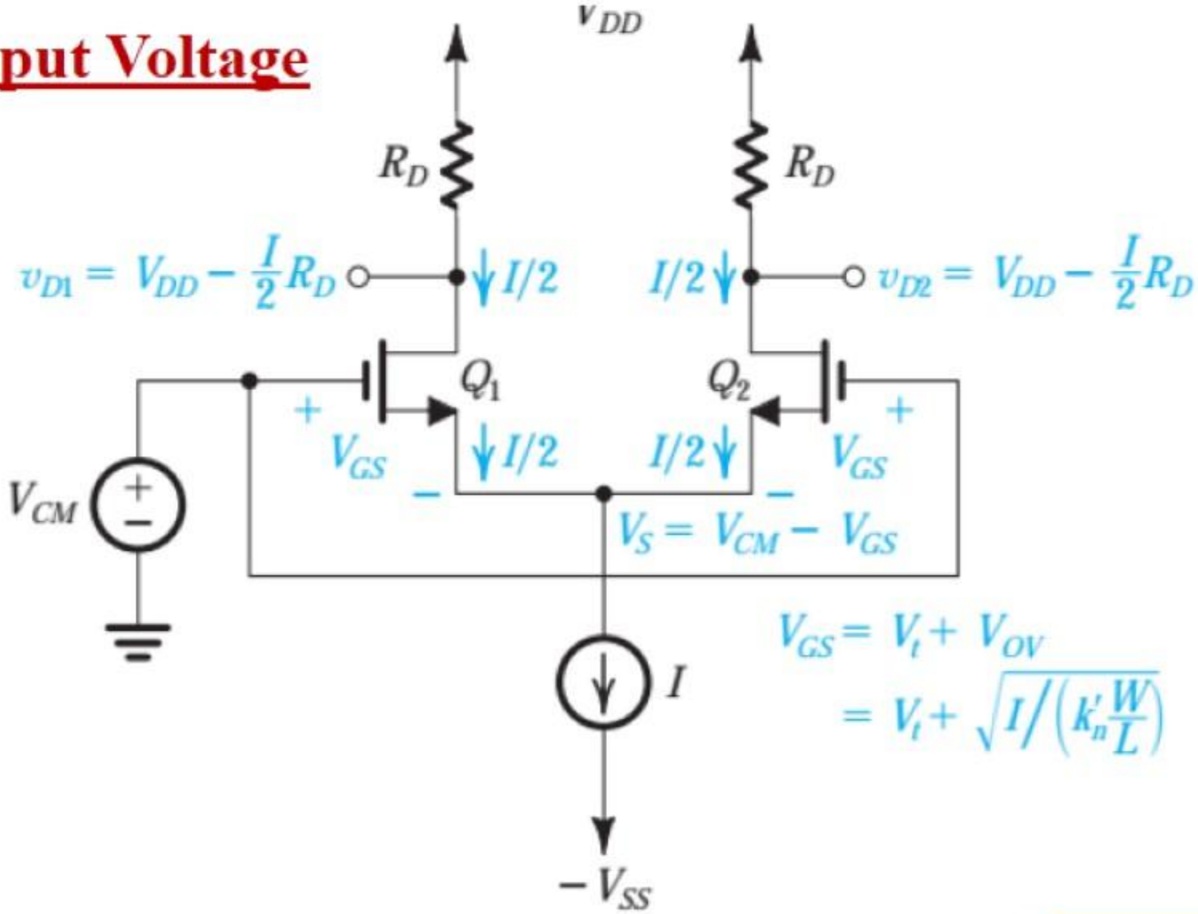
Operation with a Common-Mode Input Voltage

- To see how the differential pair works, consider first the case when the two gate terminals are joined together and connected to a voltage V_{cm}, called the common-mode voltage.

$$\boxed{v_{G1} = v_{G2} = v_{CM}}$$

$$\boxed{i_{D1} = i_{D2} = \frac{I}{2}}$$

$$\boxed{v_{D1} = v_{D2} = V_{DD} - \frac{I}{2}R_D}$$



The MOS differential pair with a common-mode input voltage V_{cm}.

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MOS Differential Amplifier/Pair

Operation with a Common-Mode Input Voltage

$$V_S = V_{CM} - V_{GS}$$

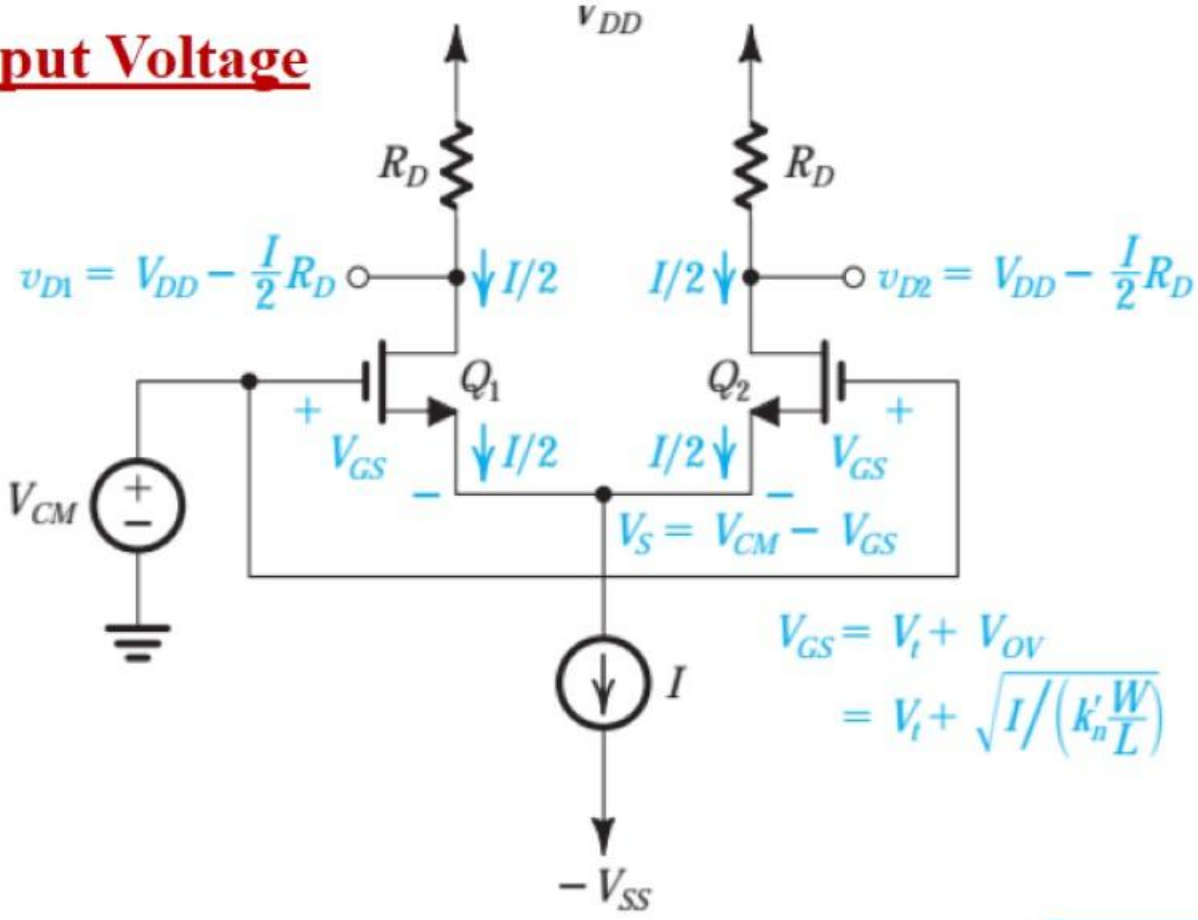
$$\frac{I}{2} = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

- The overdrive voltage

$$V_{OV} = V_{GS} - V_t$$

$$\frac{I}{2} = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$V_{OV} = \sqrt{I / k_n'(W/L)}$$



The MOS differential pair with a common-mode input voltage V_{cm}.

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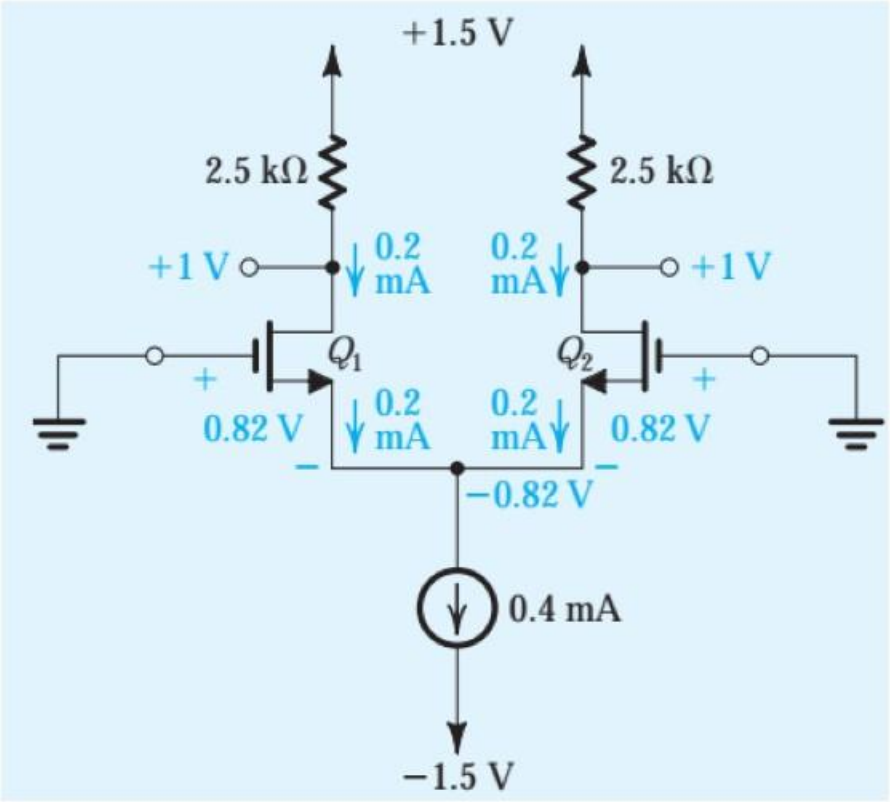
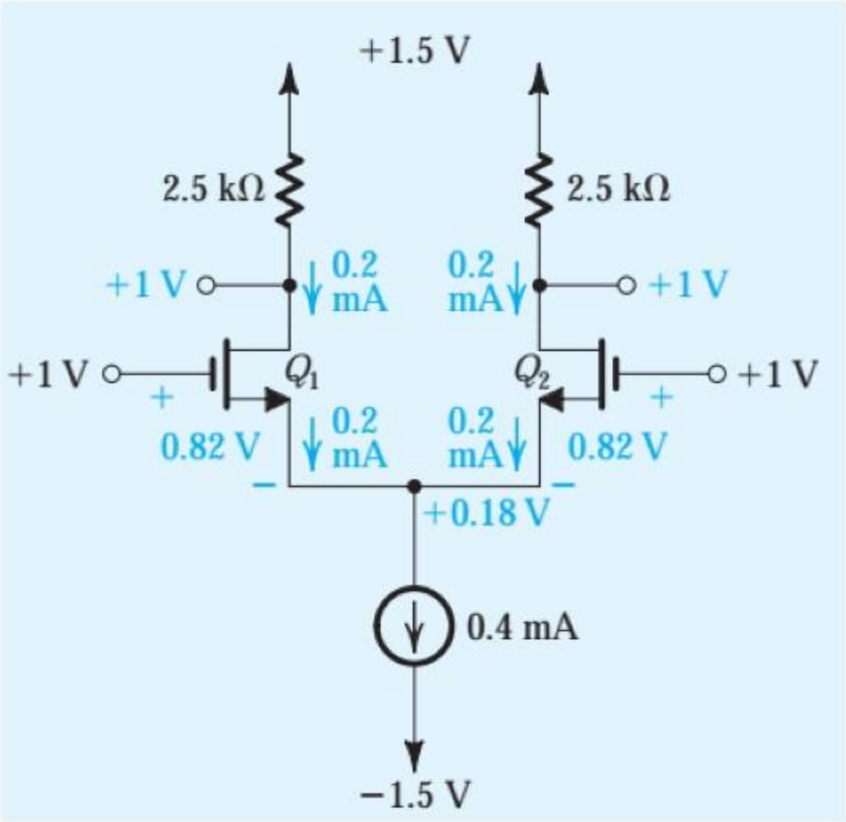
MOS Differential Amplifier/Pair

- Changing the value of the common mode signal **will not change** the transistors currents.





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MOS Differential Amplifier/Pair

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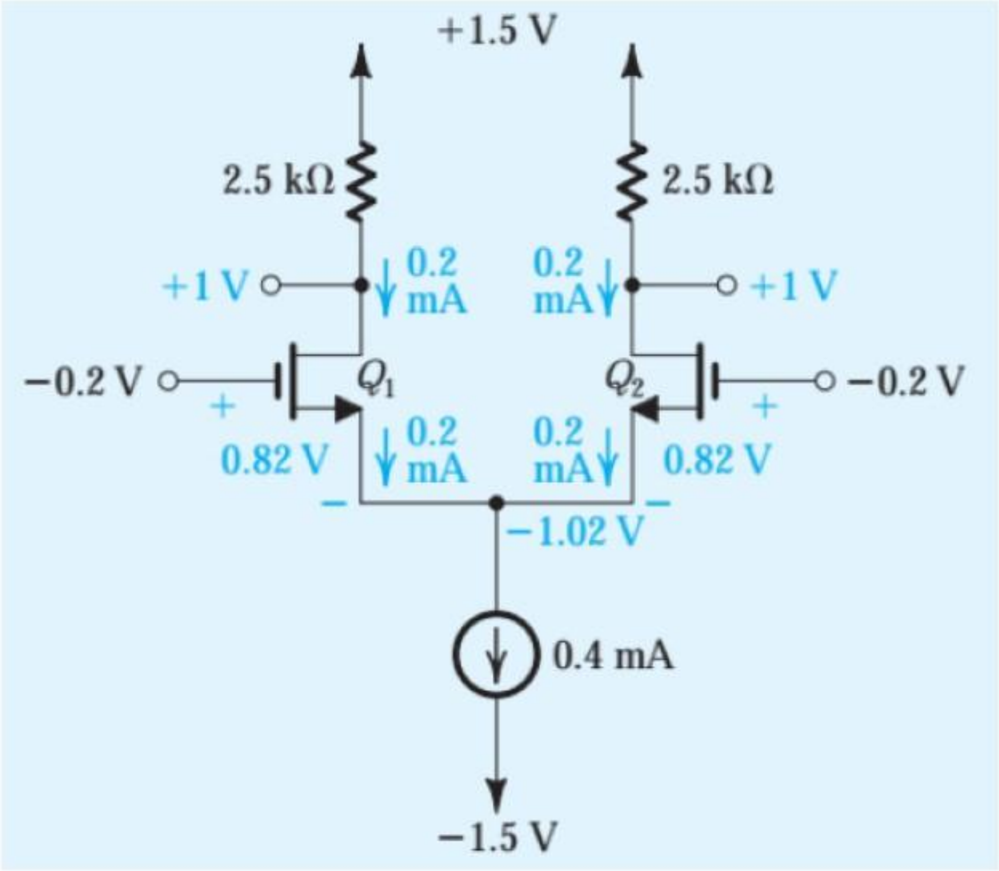

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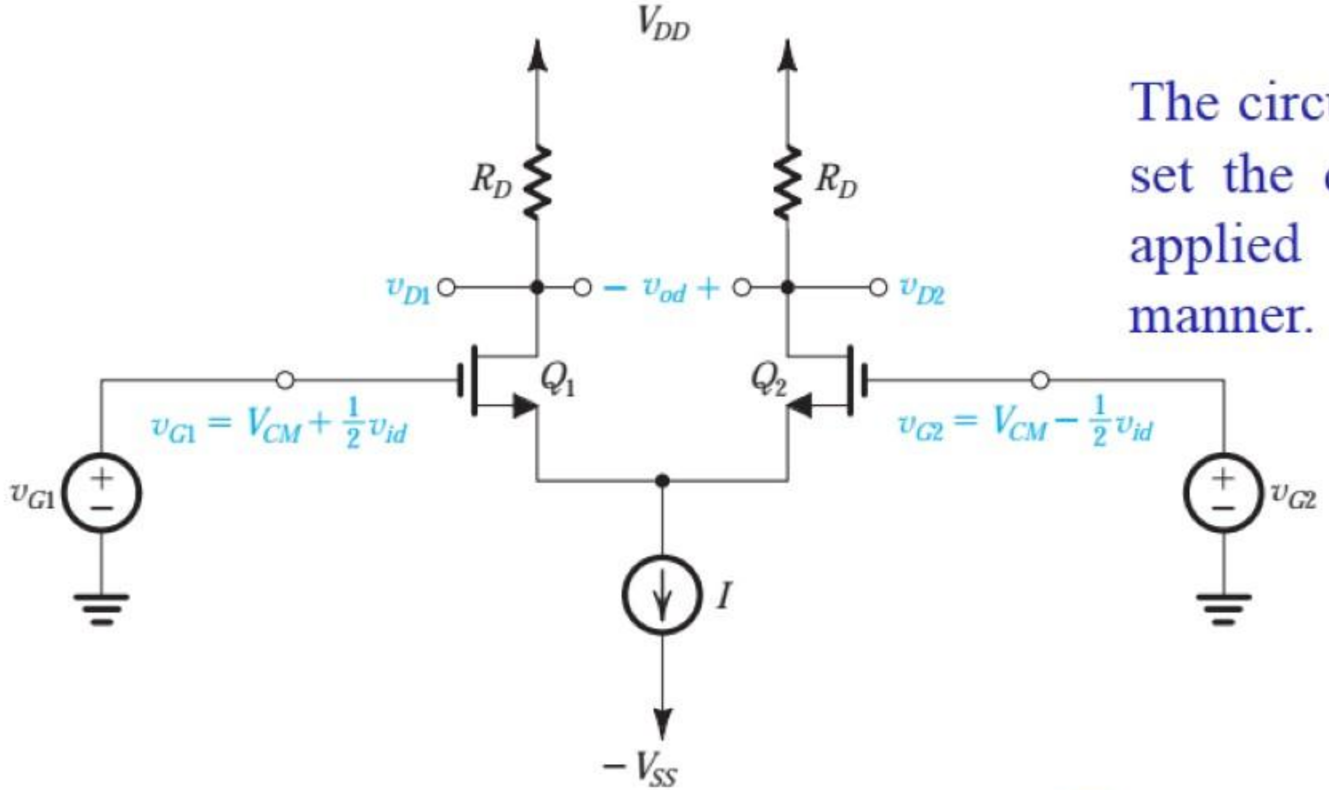
Small Signal Analysis for Differential Amplifier

Differential and Common Mode Gain for MOS-FET

MOSFET Differential Amplifier/Pair

- **Small Signal Analysis Differential Mode Gain**
- **The circuit is like two Common Source Amplifiers, where the output is taken as a difference between the two Drains.**



The circuit with a common-mode voltage applied to set the dc bias voltage at the gates and with *vid* applied in a complementary (or balanced) manner.

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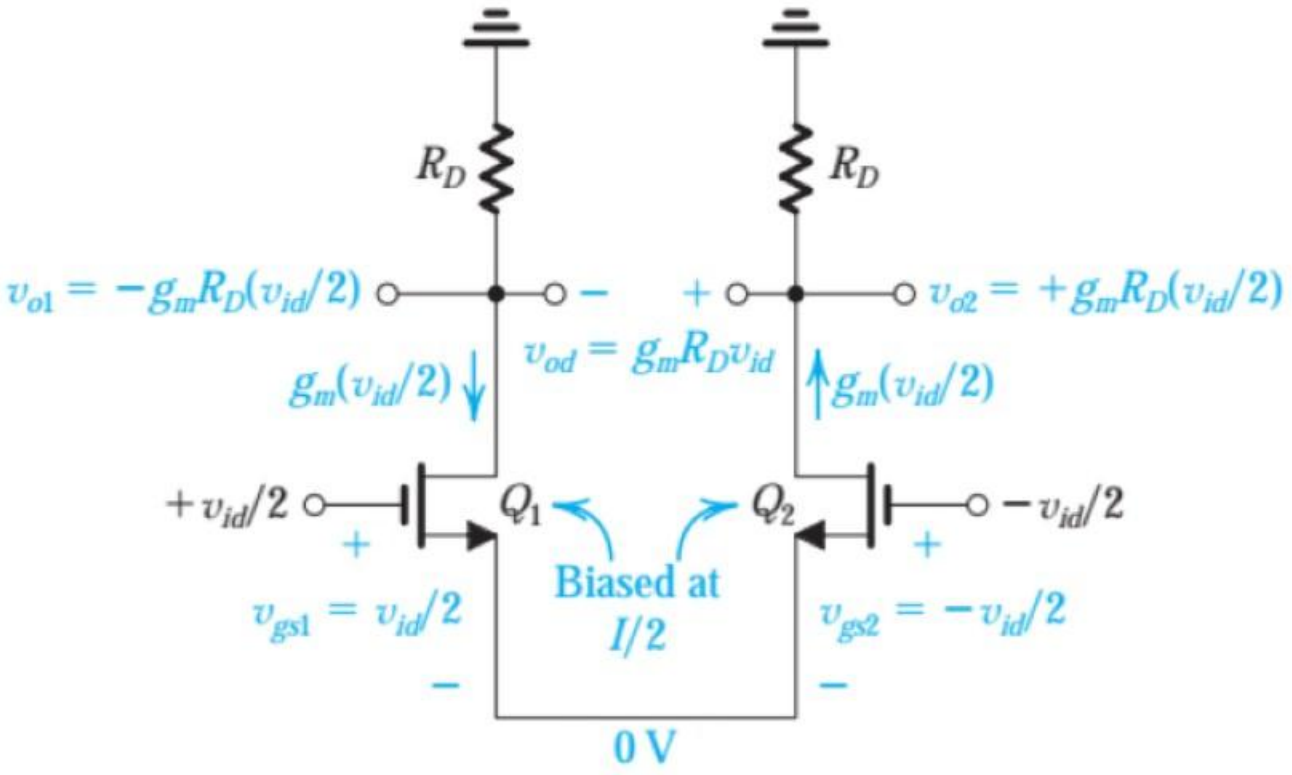
MOSFET Differential Amplifier/Pair

➤ **Small Signal Analysis Differential Mode Gain**



The circuit prepared for small-signal analysis.

$$\boxed{g_{m1} = g_{m2}}$$

$$\boxed{r_{o1} = r_{o2}}$$

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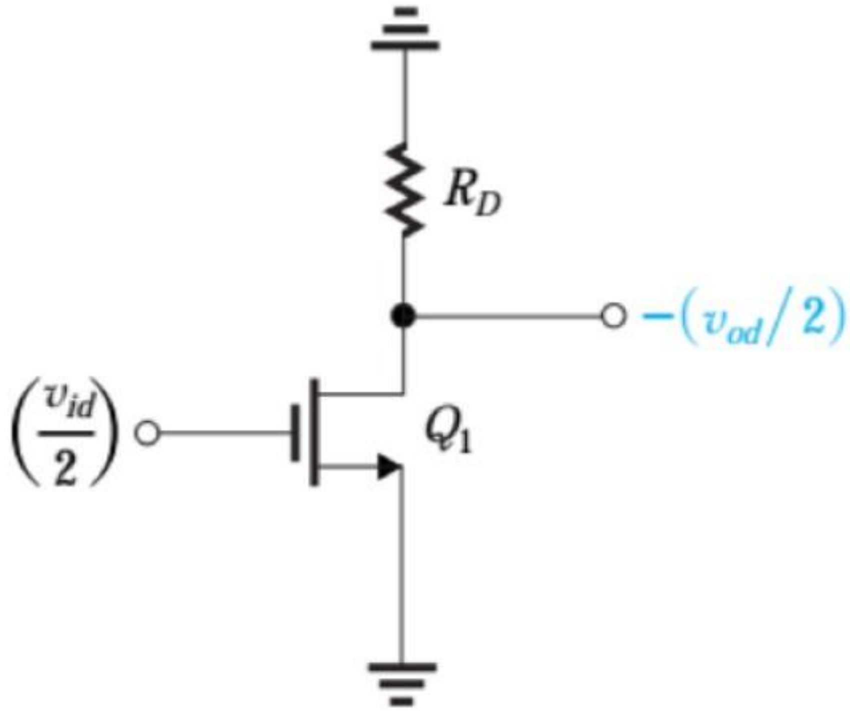
MOSFET Differential Amplifier/Pair

➢ Small Signal Analysis Differential Mode Gain

- ➢ We can use half Circuit Concept to calculate the gain



$$\boxed{v_{o1} = -g_m \frac{v_{id}}{2} R_D}$$

$$\boxed{v_{o2} = +g_m \frac{v_{id}}{2} R_D}$$

$$\boxed{\frac{v_{o1}}{v_{id}} = -\frac{1}{2} g_m R_D}$$

$$\boxed{\frac{v_{o2}}{v_{id}} = \frac{1}{2} g_m R_D}$$

$$\boxed{A_d = \frac{v_{od}}{vi_d} = \frac{v_{o2} - v_{o1}}{vi_d} = \frac{v_{D2} - v_{D1}}{vi_d} = g_m R_D}$$

$$\boxed{R_{in} = \infty}$$

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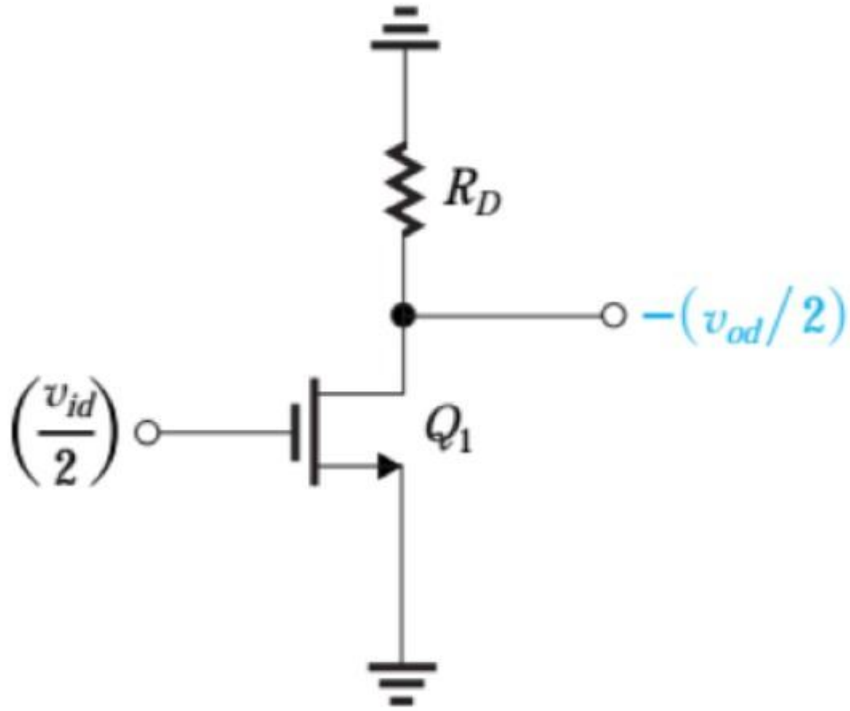
MOSFET Differential Amplifier/Pair

- **Small Signal Analysis Differential Mode Gain**
- We can use half Circuit Concept to calculate the gain
- **Notes: if r_o is taken into Consideration, then:**



$$\boxed{A_d = \frac{v_{od}}{vi_d} = \frac{v_{o2} - v_{o1}}{vi_d} = \frac{v_{D2} - v_{D1}}{vi_d} = g_m\,(r_o // R_D)}$$

$$\boxed{R_{in} = \infty}$$

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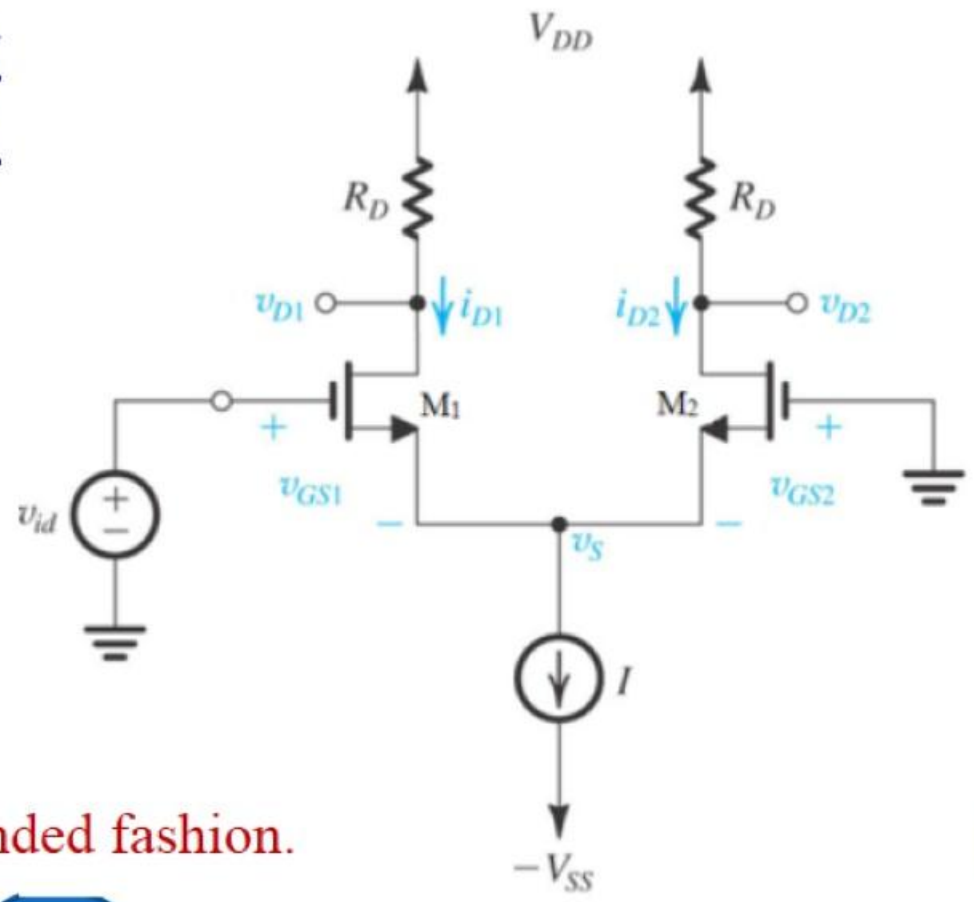
MOSFET Differential Amplifier/Pair

➢ **Small Signal Analysis Differential Mode Gain**

➢ **Notes: What if the signal is not applied fully differential? (Derive!)**



The differential amplifier fed in a single-ended fashion.

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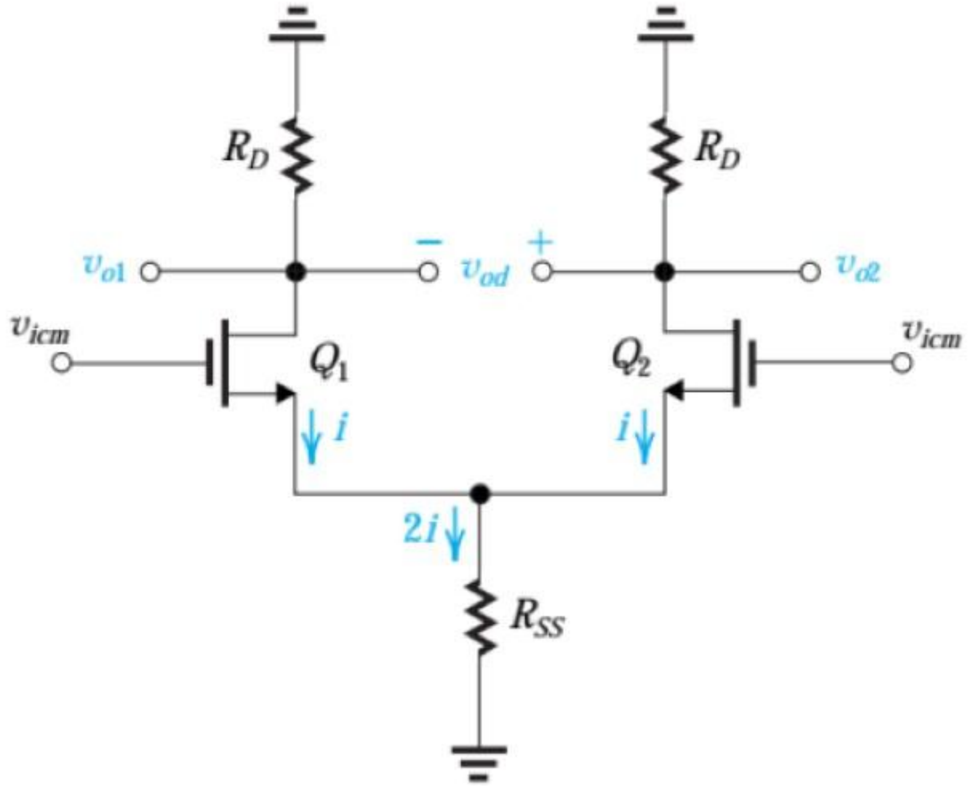
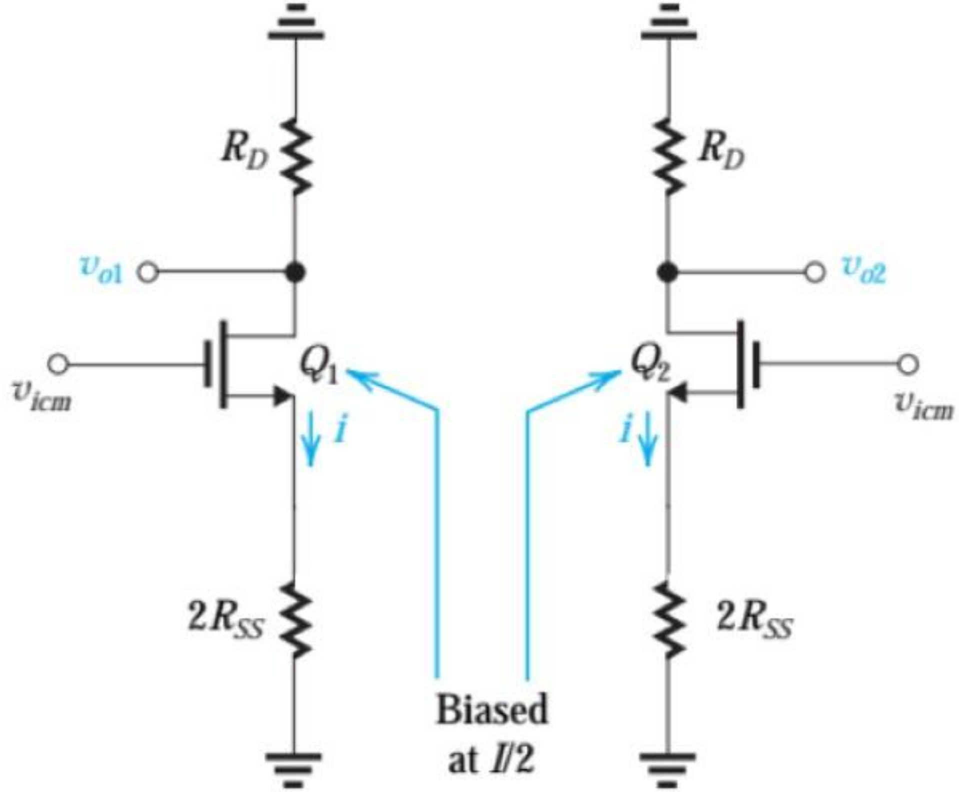
MOSFET Differential Amplifier/Pair

- **Small Signal Analysis Common Mode Gain**
- **R_{SS} is the Current Source Resistance (vs is not Zero in the common mode Case, why?)**





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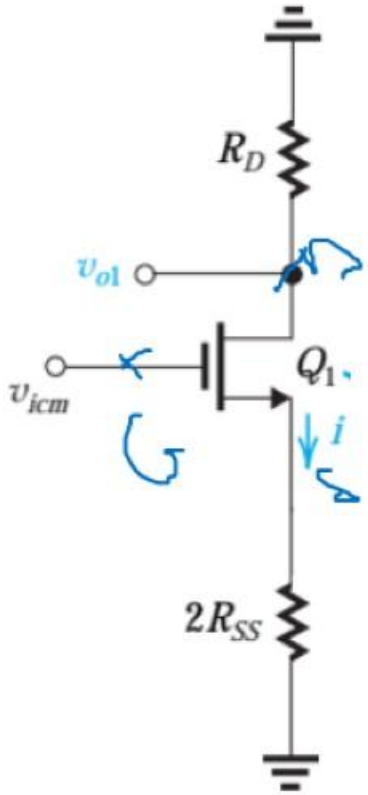
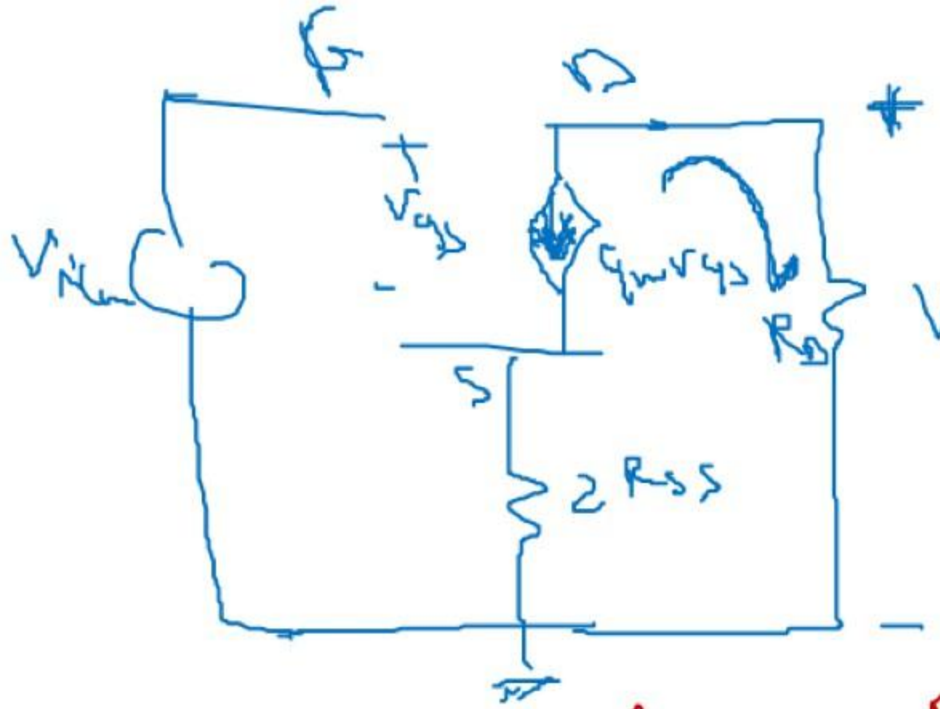
MOSFET Differential Amplifier/Pair

➢ Small Signal Analysis Common Mode Gain





$v_{o1} = -R_D g_m v_{gs}$

$v_{icm} = v_{gs} + 2R_{SS} g_m v_{gs}$

$= v_{gs}(1 + 2R_{SS} g_m)$

$$v_{o1} = v_{o2} = -\frac{R_D}{1/g_m + 2R_{SS}} v_{icm}$$

$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} = -\frac{R_D}{1/g_m + 2R_{SS}}$$

$$\frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} \simeq -\frac{R_D}{2R_{SS}}$$

$$v_{od} = v_{o2} - v_{o1} = 0$$

$$v_{o1} = -\frac{R_D \cancel{g_m} v_{icm} / g_m}{(1 + 2R_{SS}\cancel{g_m}) / g_m}$$

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MOSFET Differential Amplifier/Pair

- **Small Signal Analysis Common Mode Gain**

- **Effect of R_D Mismatch**

$$v_{o1} \simeq -\frac{R_D}{2R_{SS}} v_{icm}$$

$$v_{o2} \simeq -\frac{R_D + \Delta R_D}{2R_{SS}} v_{icm}$$

$$v_{od} = v_{o2} - v_{o1} = -\frac{\Delta R_D}{2R_{SS}} v_{icm}$$

$$A_{cm} \equiv \frac{v_{od}}{v_{icm}} = -\frac{\Delta R_D}{2R_{SS}}$$

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- It follows that a mismatch in the drain resistances causes the differential amplifier to have a finite common-mode gain.
- Thus, a portion of the interference or noise signal v_{icm} will appear as a component of v_{od}.
- A measure of the effectiveness of the differential amplifier in amplifying differential-mode signals and rejecting common-mode interference is the ratio of the magnitude of its differential gain $|A_d|$ to the magnitude of its common-mode gain $|A_{cm}|$.
- This ratio is termed common-mode rejection ratio (CMRR).

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➢ The common-mode rejection ratio can now be found from

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{g_m R_D}{\frac{\Delta R_D}{2R_{SS}}} = \frac{2R_{SS}\, g_m}{\frac{\Delta R_D}{R_D}}$$

$$\text{CMRR} = (2g_m R_{SS}) \Big/ (\Delta R_D / R_D)$$

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Thank You

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Have a Wonderful Semester