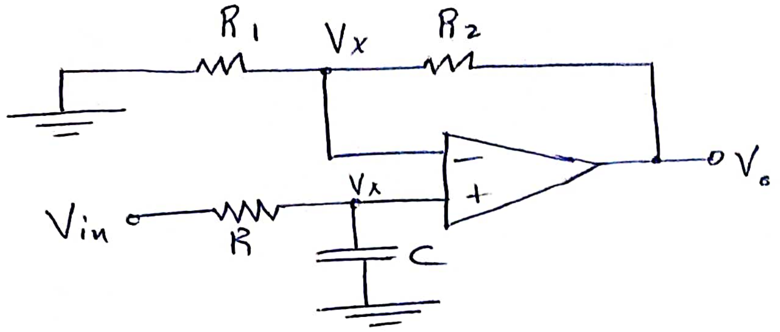
sheet 2

1.

a.



$$\Rightarrow V_X = V_o \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow \frac{V_o}{V_X} = \frac{R_1 + R_2}{R_1}$$

$$\Rightarrow \frac{V_o}{V_X} = 1 + \frac{R_2}{R_1}$$

$$\because V_X = \frac{V_{in}}{R + \frac{1}{SC}} \cdot \frac{1}{SC}$$

$$\Rightarrow \frac{V_X}{V_{in}} = \frac{1}{1 + RSC}$$

$$\Rightarrow \frac{V_o}{V_{in}} = A_V(S) = \frac{V_X}{V_{in}} \cdot \frac{V_o}{V_X} = \frac{1}{1+RSC} \cdot \left(1 + \frac{R_2}{R_1}\right)$$

$$\Rightarrow A_V(S) = \frac{1 + \frac{R_2}{R_1}}{1 + RSC} = \frac{A_m}{1 + \frac{S}{\omega_c}} \Rightarrow LPF$$

$$\Rightarrow A_m = 1 + \frac{R_2}{R_1}$$

$$\Rightarrow \omega_c = \frac{1}{RC} \Rightarrow f_c = \frac{1}{2\pi RC}$$

* unity gain frequency (f_T) :

$$\Rightarrow |A(j\omega)| = \frac{A_m}{\sqrt{1 + \omega^2 R^2 C^2}} = 1$$

$$\Rightarrow 1 + \omega_T^2 R^2 C^2 = \left(1 + \frac{R_2}{R_1}\right)^2$$

$$\Rightarrow \omega_T = \frac{\sqrt{\left(1 + \frac{R_2}{R_1}\right)^2 - 1}}{RC}$$

$$\Rightarrow f_T = \frac{\sqrt{A_m^2 - 1}}{2\pi RC}$$

- for $A_m = 20$ & $f_c = 20\ KHz$

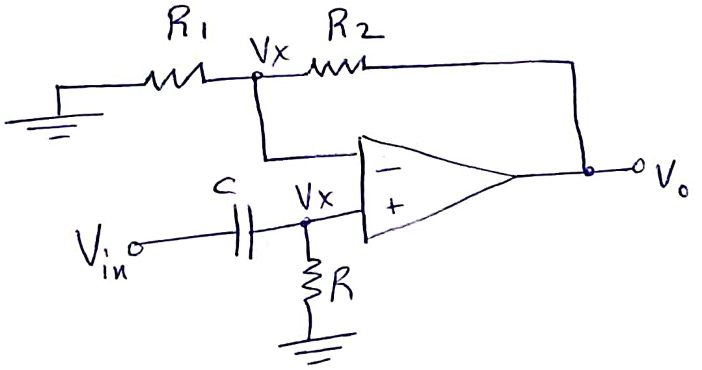
$\Rightarrow 1 + \frac{R_2}{R_1} = 20$, $f_c = 20 \times 10^3 = \frac{1}{2\pi R C}$

Let $R_1 = 1 K\Omega$

$\Rightarrow R_2 = 19 K\Omega$

Let $C = 0.1\ \mu F$ $\Rightarrow$ $R = \frac{1}{2\pi \times 20 \times 10^3 \times 0.1 \times 10^{-6}}$

b.



$$\Rightarrow V_x = \frac{V_o}{R_1 + R_2} \cdot R_1$$

$$\Rightarrow \frac{V_o}{V_x} = 1 + \frac{R_2}{R_1}$$

$$\Rightarrow V_x = \frac{V_{in}}{\frac{1}{SC} + R} \cdot R$$

$$\Rightarrow \frac{V_x}{V_{in}} = \frac{R}{(\frac{1}{SC} + R)} = \frac{1}{(\frac{1}{SRC} + 1)}$$

$$\Rightarrow A_v(s) = \frac{V_o}{V_{in}} = \frac{V_o}{V_x} \cdot \frac{V_x}{V_{in}} = \frac{(1 + \frac{R_2}{R_1})}{\frac{1}{SRC} + 1} = \frac{A_m}{1 + \frac{\omega_c}{s}}$$

$\Downarrow$

HPF

$$\Rightarrow A_m = 1 + \frac{R_2}{R_1}$$

$$\Rightarrow \omega_c = \frac{1}{RC} \Rightarrow f_c = \frac{1}{2\pi R C}$$

$$\Rightarrow |A_v(j\omega)| = \frac{A_m}{\sqrt{1+\frac{1}{\omega_T^2 R^2 C^2}}} = 1$$

$$1 + \frac{1}{\omega_T^2 R^2 C^2} = A_m^2$$

$$\Rightarrow \omega_T^2 R^2 C^2 = \frac{1}{A_m^2 - 1}$$

$$\Rightarrow \omega_T = \frac{1}{RC\sqrt{A_m^2 - 1}}$$

$$\Rightarrow f_T = \frac{1}{2\pi RC\sqrt{A_m^2 - 1}}$$

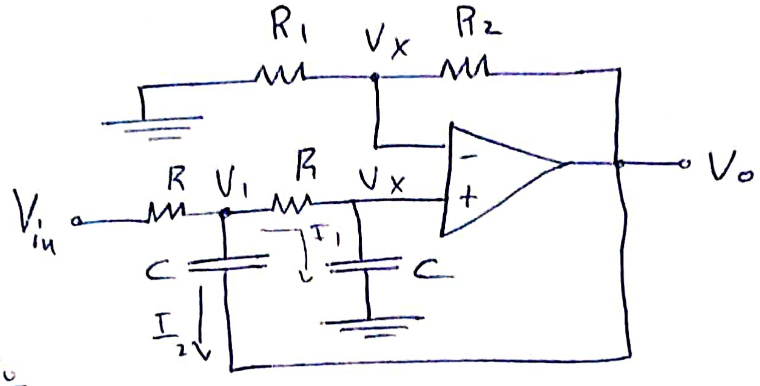
* For $A_m = 20$, $f_c = 20\ KHZ$:

$\Rightarrow 1 + \frac{R_2}{R_1} = 20$ Let $R_1 = 1\ K\Omega \Rightarrow R_2 = 19\ K\Omega$

Let ~~R~~ $C = 0.1\ \mu F \Rightarrow f_c = \frac{1}{2\pi RC} = 20 \times 10^3$

2.

a.



$I_1 = \frac{V_1}{R + \frac{1}{sc}}$

$I_2 = \frac{V_1 - V_0}{1/sc}$

$\Rightarrow V_x = \frac{V_0}{R_1 + R_2} \cdot R_1 \Rightarrow \boxed{\frac{V_0}{V_X} = 1 + \frac{R_2}{R_1}}$ (1)

$\Rightarrow V_X = \frac{V_1}{R + \frac{1}{SC}} \cdot \left(\frac{1}{SC}\right)$

$\Rightarrow \boxed{\frac{V_x}{V_1} = \frac{1}{RSC + 1}}$ (2)

$\Rightarrow V_{in} = V_1 + (I_1 + I_2) R$

$\Rightarrow V_{in} = V_1 + \left[\frac{V_1}{R + \frac{1}{sc}} + (V_1 - V_0) sc\right] R$

$\Rightarrow \boxed{V_{in} = V_1 \left[1 + \frac{SCR}{1 + SCR} + SCR\right] - V_0 SCR}$ (3)

* from (1) & (2):

$\Rightarrow \frac{V_0}{V_1} = \frac{V_0}{V_X} \cdot \frac{V_X}{V_1} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 + RSC} = \frac{A_m}{1 + RSC}$

$\Rightarrow \boxed{V_1 = V_0 \frac{(1 + RSC)}{A_m}}$ (4)

* sub. by (4) in (3):

$\Rightarrow V_{in} = V_0 \left(\frac{1 + RSC}{A_m}\right) \left(1 + \frac{SCR}{1 + SCR} + SCR\right) - V_0 SCR$

$\Rightarrow \frac{V_{in}}{V_0} = \frac{(1 + SCR)^2}{A_m} + \frac{SCR}{A_m} - SCR$

$$\Rightarrow \cancel{A_v(s)}\ \frac{V_{in}}{V_o} = \frac{(1+SCR)^2 + SCR - A_m(SCR)}{A_m}$$

$$\Rightarrow A_v(S) = \frac{V_o}{V_{in}} = \frac{A_m}{S^2C^2R^2 + (3-A_m)SCR + 1} = \frac{A_m}{\left(\frac{S}{\omega_c}\right)^2 + 2K\frac{S}{\omega_c} + 1}$$

$$\Rightarrow A_m = 1 + \frac{R_2}{R_1}$$

$$\Rightarrow \omega_c = \frac{1}{RC}$$

$$\Rightarrow f_c = \frac{1}{2\pi RC}$$

* for $n=2$ (Butterworth polynomial is $\underbrace{S^2+1.414S+1}_{\text{normalized } (\omega_c=1)}$)

& given $f_c = 10\,KHz$

$$\Rightarrow (3-A_m)\cancel{RC} = \frac{2K}{\cancel{\omega_c}}, \quad \omega_c = \frac{1}{RC}$$

$$\Rightarrow 3 - A_m = 2K$$

$$\Rightarrow 3 - 2K = A_m$$

$$\Rightarrow A_m = 3 - 1.414 = 1.586$$

$$\Rightarrow 1.586 = 1 + \frac{R_2}{R_1}$$

$$\Rightarrow \frac{R_2}{R_1} = 0.586$$

let $R_1 = 10\,K\Omega$ $\Rightarrow R_2 = 5.86\,K\Omega$

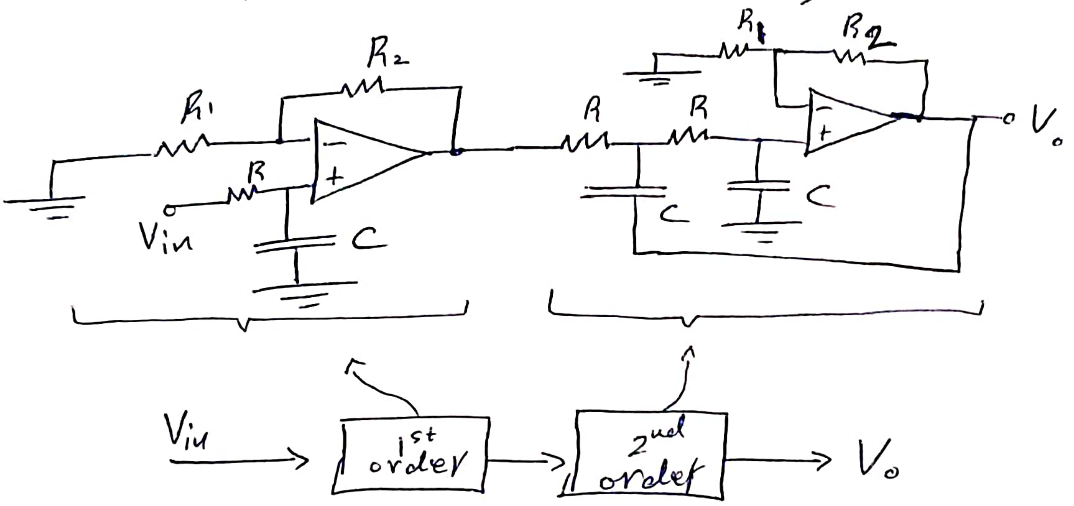
$\because f_c = \frac{1}{2\pi RC}$, let $C = 0.01\,\mu F$

$$\Rightarrow R = \frac{1}{2\pi * 0.01 * 10^{-6} * 10^4} = 1.59\,K\Omega$$

3. Design a 3rd order Butterworth LPF with $f_c = 30KHz$ (6)

for $n = 3$, $B_n(s) = (s+1)(s^2+s+1)$

Sol:



$\because f_c = \dfrac{1}{2\pi RC}$ (for both sections)

$\Rightarrow 10^4 = \dfrac{1}{2\pi RC}$ Let $C = 0.01 \mu F$

$\Rightarrow R = 1.59 K\Omega$

$\because B_n(s) = (s+1)(s^2+\cancel{3}+1)$

&

$A_v(s) = \dfrac{A_{m_1}}{1+RSC} = \dfrac{1+\frac{R_2}{R_1}}{1+RCS} \Rightarrow$ for 1st order

$A_v(s) = \dfrac{A_{m_2}}{S^2C^2R^2+(3-A_{m_2})SCR+1} = \dfrac{A_m'}{(\frac{s}{\omega_c})^2+2K\frac{s}{\omega_c}+1}$

$\Rightarrow 2K = 3 - A_{m_2}$

$\Rightarrow A_{m_2} = 3 - 1 = 2 \quad \Rightarrow 1 + \dfrac{R_2}{R_1} = 2$

Let $R_1 = R_2 = 10 K\Omega$