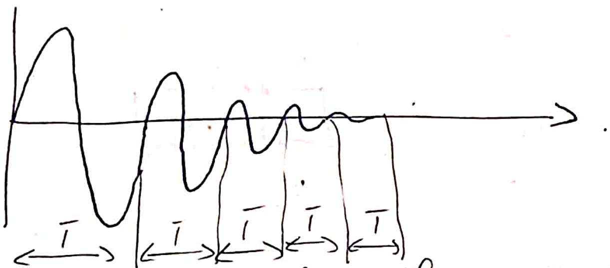
Q-factor :

- Quality factor is a dimensionless parameter that describes how "underdamped" an oscillator is.

$$Q = \frac{\overset{\text{Initial}}{\text{Energy}} \text{ stored in an resonator}}{\text{Energy lost in one radian}}$$

$$= 2\pi \, \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} = \omega * \frac{\text{Max. Energy stored}}{\text{power loss}}$$

Ex :

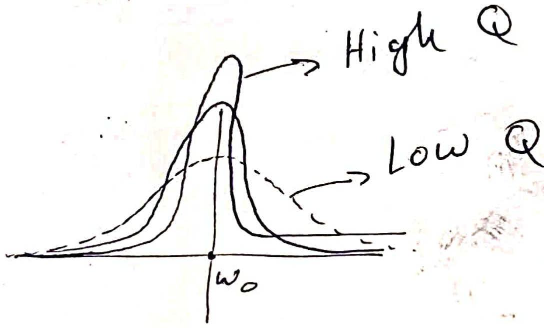


=> 5 cylces then the energy is completely dissipated

↳ $Q \simeq 5$

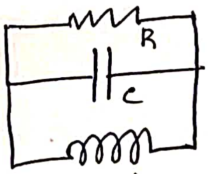
Another definition for Q :

$$Q = \frac{\omega_0}{BW}$$



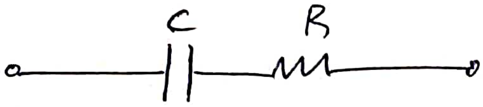
For an RLC-network :

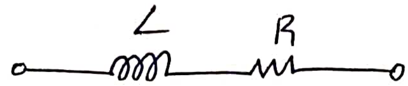
↳ $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$



↳ $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

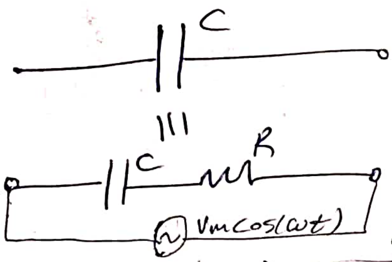
- The Capacitors & Inductors used are not ideal & can be modeled as :





→ so, the Resistance in series (L or C) used to Model the ohmic loss in the inductor turns/capacitor plates : → these losses limits the Quality factor when used in an RLC ct.

→ so, we define for an inductor/cap. : (Quality factor)



$\because X_C >> R$

$V_m \simeq \frac{I_m}{\omega C}$

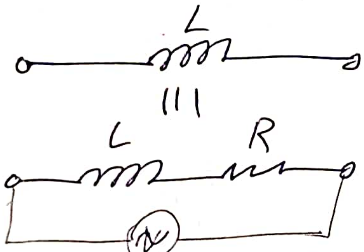
$$Q = \frac{\frac{1}{2} V_m^2 C (2\pi)}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R \cdot T} \simeq \frac{\frac{I_m^2}{\omega^2 C} \cdot \frac{1}{2} (2\pi)}{\frac{I_m^2}{2} R \cdot \left(\frac{1}{f}\right)}$$

$$\Rightarrow Q = \frac{\omega}{\omega^2 C (R)}$$

$$= \frac{1}{\omega C R}$$

$$\Rightarrow \boxed{Q_C = \frac{X_C}{R}}$$

capacitor Quality factor



$$Q_L = \frac{\text{Max. stored energy}}{\text{energy dissipated in one cycle}} * 2\pi$$

$$\Rightarrow Q_L = \frac{\frac{1}{2} L I_m^2}{\left(\frac{1}{\sqrt{2}} I_m\right)^2 R \cdot T} * 2\pi$$

max I

T → periodic time $= \frac{1}{f}$

$$= \frac{\frac{1}{2} L I_m^2 (2\pi)}{\frac{1}{2} I_m^2 R / f}$$

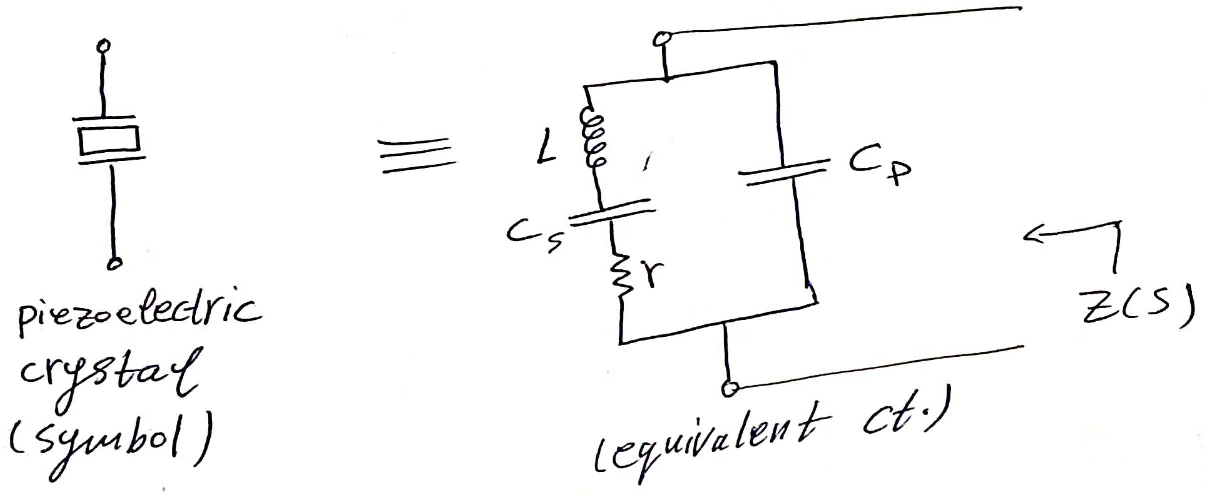
$$\Rightarrow Q_L = \frac{2\pi f L}{R} = \frac{\omega L}{R} = \frac{X_L}{R}$$

$$\Rightarrow \boxed{Q_L = \frac{X_L}{R}}$$

Inductor Quality factor

crystal oscillator :

piezoelectric crystals have electromechanical -resonance characteristics that are very stable (with time & temperature) & highly selective (have high Q).



$L \rightarrow$ typically hundreds of Henrys

$C_s \rightarrow$ very small (≈ 0.0005 PF) cap.

$r \rightarrow$ representing Q of inductor ($\frac{\omega_0 L}{r}$) (few hundred/thousands)

$C_p \rightarrow$ typically few pico farads.

$\because$ Q is very high $\rightarrow$ r is negligable
($X_L >>> r$)

$$\Rightarrow Z(S) = \frac{1}{sC_p} \,//\, \left(sL + \frac{1}{sC_s}\right)$$

$$= \frac{\frac{1}{sC_p}\left(sL + \frac{1}{sC_s}\right)}{\frac{1}{sC_p} + sL + \frac{1}{sC_s}}$$

$$= \frac{1}{sC_p} \; \frac{\left(sL + \frac{1}{sC_s}\right)}{\frac{1}{sC_p} + sL + \frac{1}{sC_s}}$$

$$\Rightarrow Z(s) = \frac{1}{sC_p} \cdot \frac{s^2LC_s + 1}{\frac{C_s}{C_p} + s^2LC_s + 1}$$

$$= \frac{1}{sC_p} \cdot \frac{\left(s^2 + \frac{1}{LC_s}\right)}{\frac{1}{LC_p} + s^2 + \frac{1}{LC_s}}$$

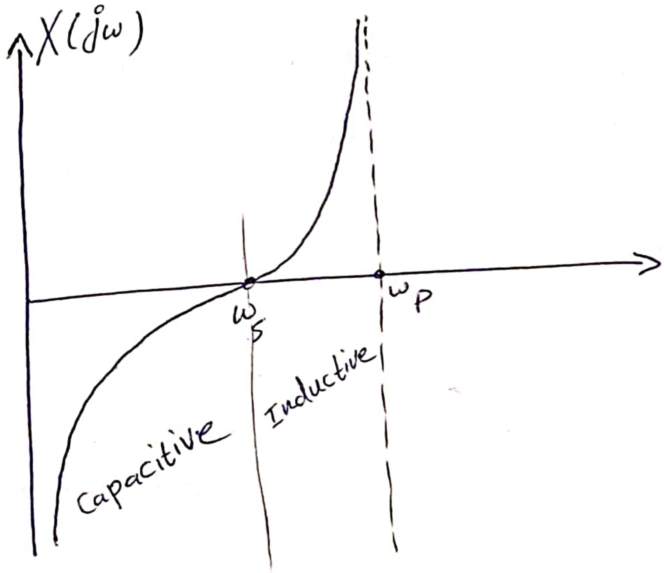
$$= \left(\frac{1}{sC_p}\right) \frac{\left[s^2 + \frac{1}{LC_s}\right]}{\left[s^2 + \left(\frac{C_s + C_p}{LC_sC_p}\right)\right]}$$

$$\Rightarrow Z(j\omega) = \left(\frac{1}{j\omega C_p}\right) \frac{-\omega^2 + \omega_s^2}{-\omega^2 + \omega_p^2}$$

where ; $\omega_s = \frac{1}{\sqrt{LC_s}}$, $\omega_p = \frac{1}{\sqrt{L\left(\frac{C_sC_p}{C_s+C_p}\right)}}$

$$\Rightarrow Z(j\omega) = -j\left(\frac{1}{\omega C_p}\right) \frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_p^2}$$

$\Rightarrow Z(j\omega) = jX(j\omega)$



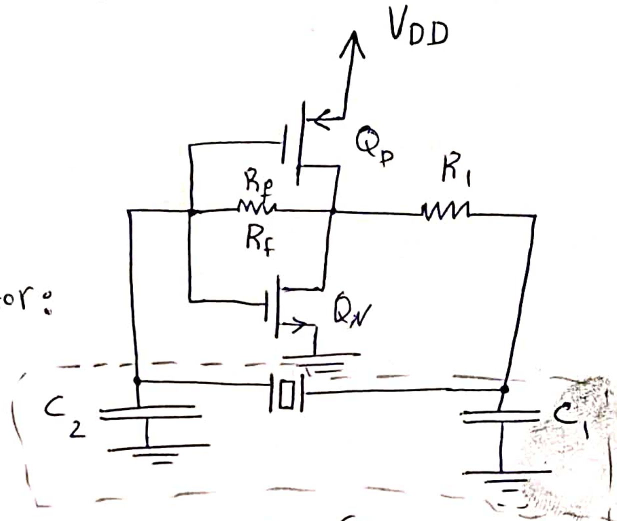
$\because$ $C_p \gg C_s \Rightarrow \omega_p \simeq \frac{1}{\sqrt{LC_s}}$

So, at ~~ω~~ $\omega_s < \omega < \omega_p$ $\rightarrow$ the crystal can be used as an inductor.

$\because$ $\omega_p \simeq \omega_s = \frac{1}{\sqrt{LC_s}}$

$\Rightarrow \omega_0 \simeq \frac{1}{\sqrt{LC_s}}$

Q1. For the shown pierce oscillator. Let C_1 be variable in range 1pF to 10pF & let C_2 be fixed at 10pF. Find the range over which the oscillation frequency can be tuned. [For the crystal: $L = 0.52H$, $C_s = 0.012pF$ & $C_p = 4pF$].

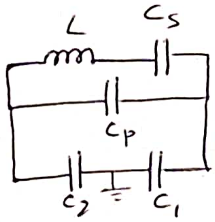


Sol:

∵ we know from colpitts oscillator:

$$w_o = \frac{1}{\sqrt{L\frac{C_1 \cdot C_2}{C_1 + C_2}}}$$

→ where $\frac{C_1 C_2}{C_1}$ ⇒ is the total cap. seen by the inductor.



* similary the total cap. seen by L:

$$\Rightarrow C_T = \frac{C_s\left[C_p + \frac{C_1 C_2}{C_1 + C_2}\right]}{C_s + C_p + \frac{C_1 C_2}{C_1 C_2}}$$

, C_s series with $\left[C_p // \left(\frac{C_1 C_2}{C_1 + 2}\right)\right]$ series comb. of C_1 & C_2.

- for $C_1 = 1pF$

$$\Rightarrow C_{T_1} = \frac{0.012\left(4 + \frac{10 * 1}{10 + 1}\right)}{0.012 + 4 + \frac{10 * 1}{10 + 1}} = 0.01197 pf$$

- for $C_1 = 10pF$:

$$\Rightarrow C_{T_2} = \frac{0.012\left(4 + \frac{10 * 10}{10 + 10}\right)}{0.012 + 4 + \frac{10 * 10}{10 + 10}} = 0.01198 pf$$

$$\Rightarrow f_o = \frac{1}{2\pi\sqrt{L C_T}} \quad \Rightarrow f_{o_1} = \frac{1}{2\pi\sqrt{0.52 * 0.01197 * 10^{-12}}} = 2.0172 \ MHz$$

$$\Rightarrow f_{o_2} = \frac{1}{2\pi\sqrt{0.52 * 0.01198 * 10^{-12}}} = 2.0162 \ MHz$$

$\Rightarrow \Delta f = 1 \ KHZ$ → (very small)

EX: A 2 MHz quartz crystal is specified to have $L = 0.52 H$, $C_s = 0.012 PF$, $C_p = 4 PF$ & $r = 120 \Omega$ find f_s, f_p & Q:

sol:

$$\Rightarrow f_s = \frac{1}{2\pi\sqrt{LC_s}} = \frac{1}{2\pi\sqrt{0.52 * 0.012 * 10^{-12}}}$$

$$= 2.015 \ MHz$$

$$\Rightarrow f_p = \frac{1}{2\pi\sqrt{L\frac{C_sC_p}{C_s+C_p}}} = \frac{1}{2\pi\sqrt{0.52\frac{0.012 * 10^{-12} * 4 * 10^{-12}}{(0.012+4) * 10^{-12}}}}$$

$$= 2.018 \ MHz$$

$$\Rightarrow Q = \frac{\omega_o L}{R} = \frac{2 * 10^6 * 2\pi * 0.52}{120}$$

$$= 54,454$$