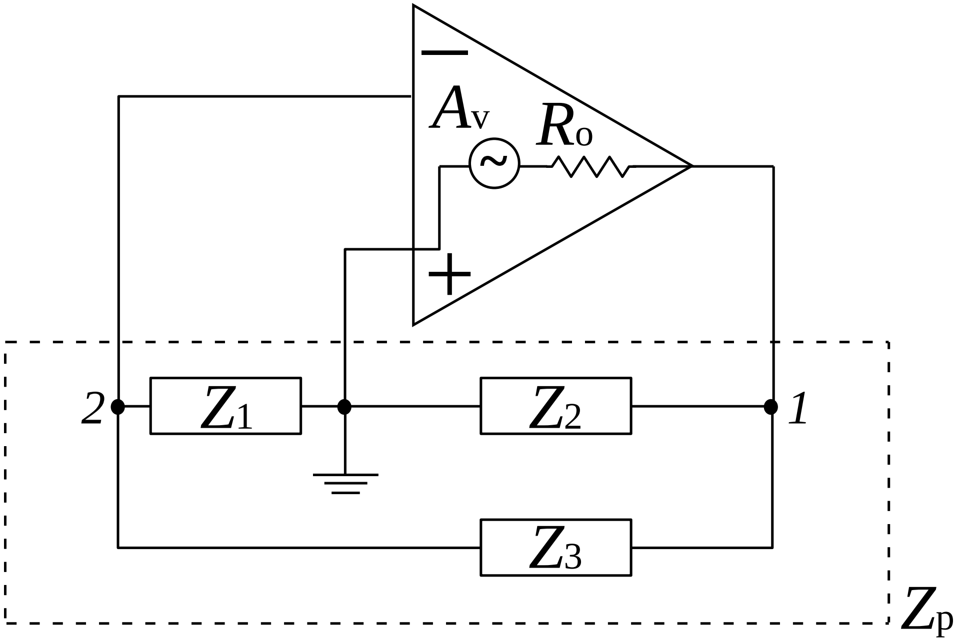
LC Oscillators

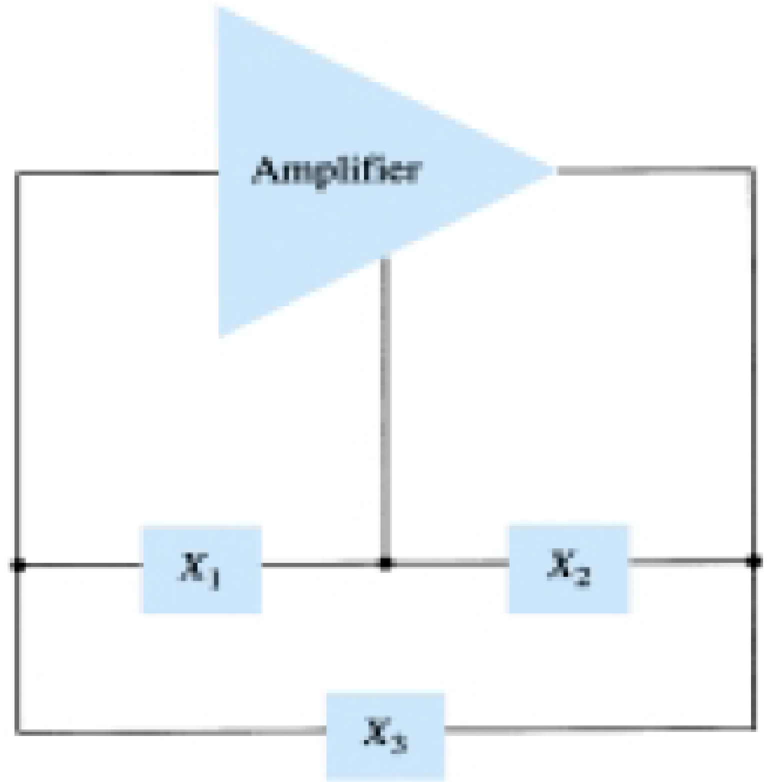
- The frequency selection network (Z_1, Z_2 and Z_3) provides a phase shift of 180^o
- The amplifier provides an addition shift of 180^o



Two well-known Oscillators:

- Colpitts Oscillator
- Harley Oscillator

TUNED OSCILLATOR CIRCUIT



Oscillator Type	Reactance elements in the tank circuit		
	X_1	X_2	X_3
Hartley Oscillator	L	L	C
Colpitts Oscillator	C	C	L

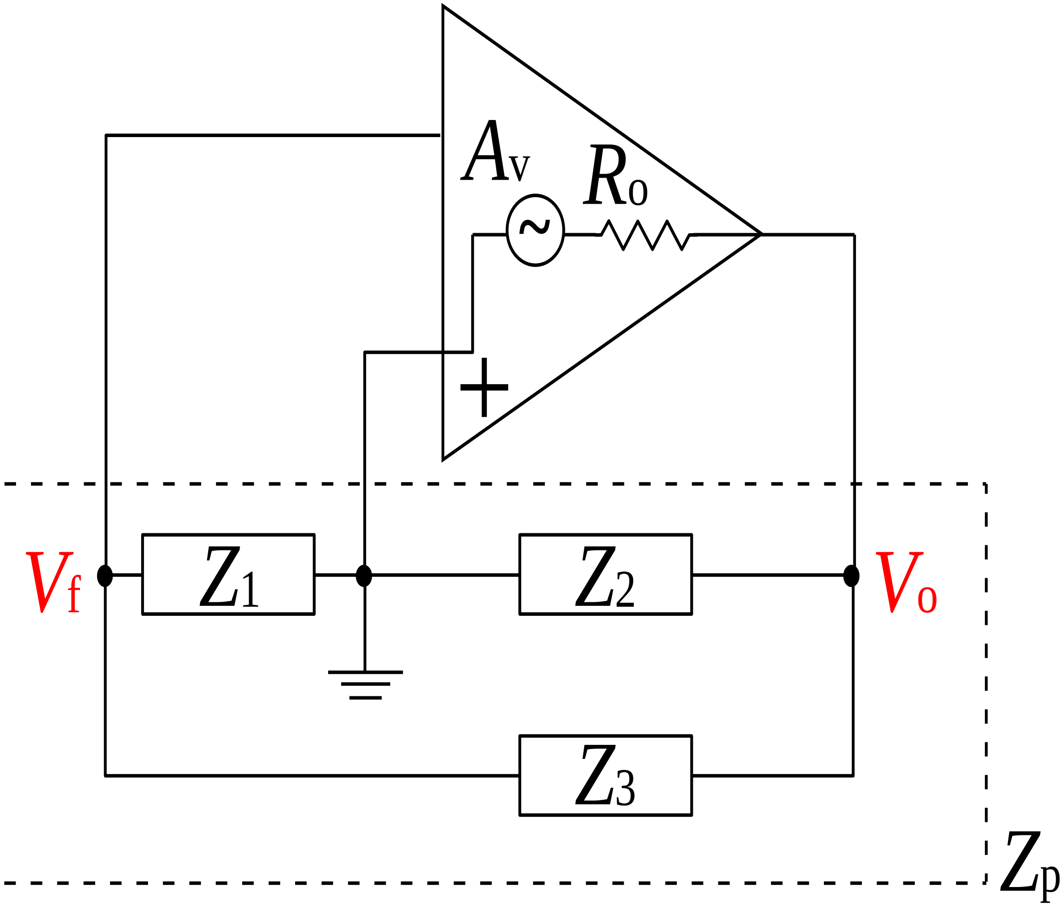
$$\beta = \frac{V_f}{V_o}$$

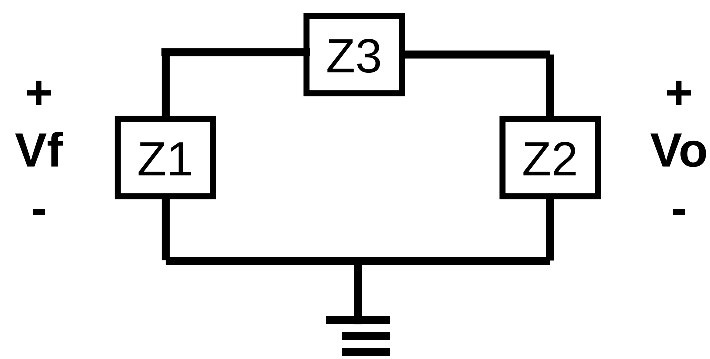
$$V_f = \frac{Z_1}{Z_1 + Z_3} V_o$$

$$\beta = \frac{Z_1}{Z_1 + Z_3}$$

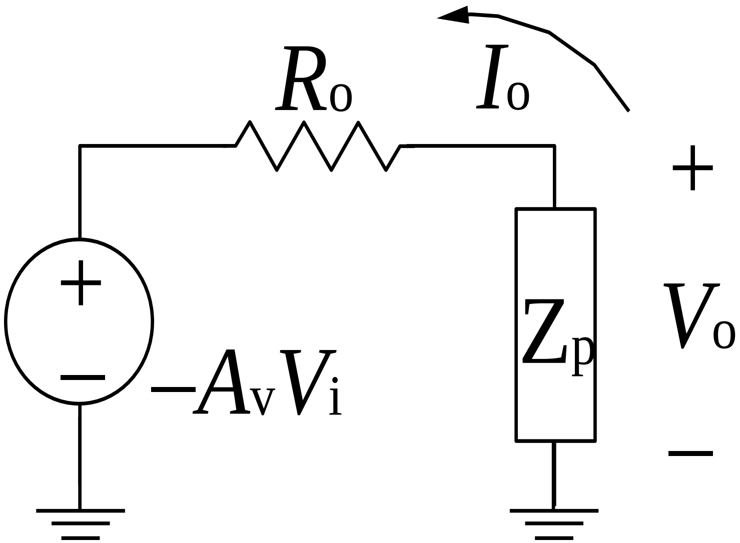
$$Z_p = Z_2 // (Z_1 + Z_3)$$

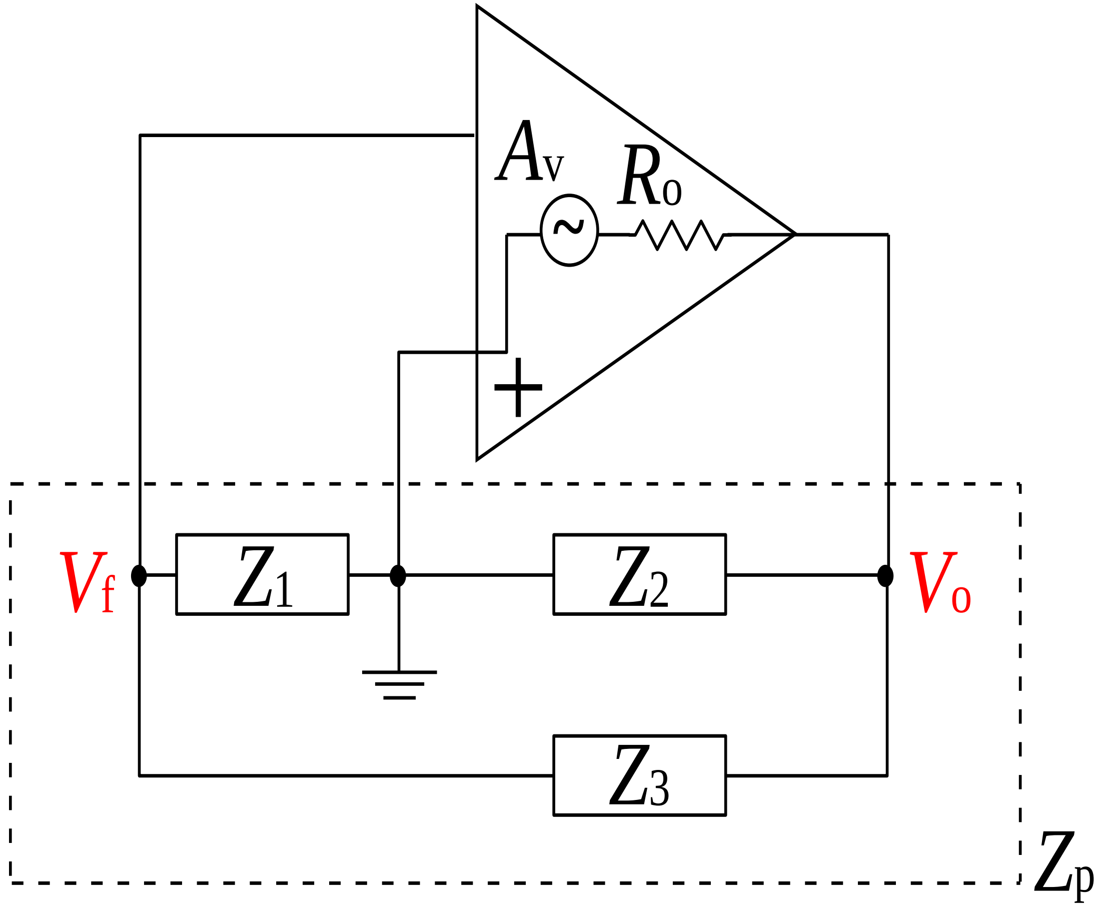
$$= \frac{Z_2 (Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}$$





For the equivalent circuit from the output

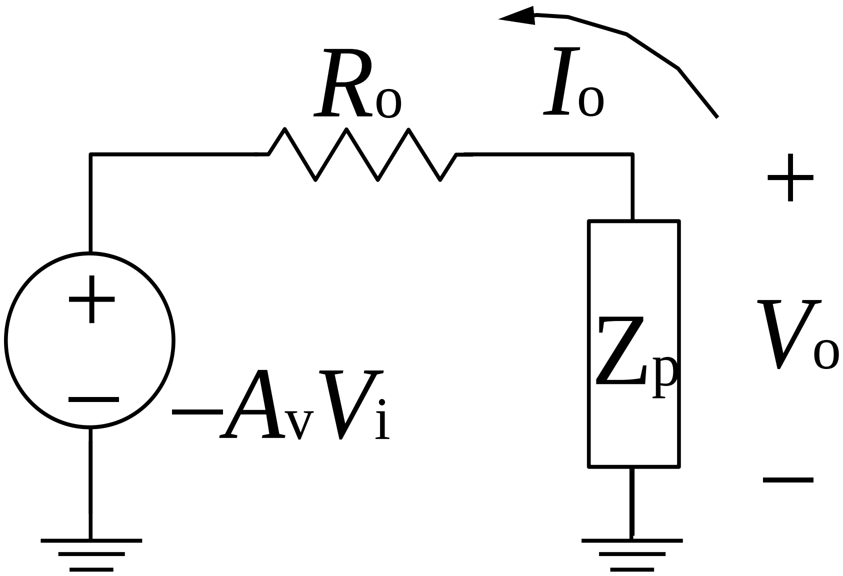




$$I_o = \frac{-A_v V_i}{R_o + Z_p} = \frac{V_o}{Z_p}$$

$$\frac{V_o}{V_i} = \frac{-A_v Z_p}{R_o + Z_p}$$

For the equivalent circuit from the output



$$\frac{-A_v V_i}{R_o + Z_p} = \frac{V_o}{Z_p}$$

$$\frac{V_o}{V_i} = \frac{-A_v Z_p}{R_o + Z_p}$$

Therefore, the amplifier gain is obtained,

$$A = \frac{V_o}{V_i} = \frac{-A_v Z_2 (Z_1 + Z_3)}{R_o (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)}$$

Therefore, the amplifier gain is

$$A = \frac{V_o}{V_i} = \frac{-A_v Z_2(Z_1 + Z_3)}{R_o(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

The Feedback gain is

$$\beta = \frac{Z_1}{Z_1 + Z_3}$$

The loop gain,

$$A\beta = \frac{-A_v Z_1 Z_2}{R_o(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

The loop gain,

$$A\beta = \frac{-A_v Z_1 Z_2}{R_o(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

If the impedance are all pure reactances, i.e.,

$$Z_1 = jX_1, \quad Z_2 = jX_2 \text{ and } \quad Z_3 = jX_3$$

The loop gain becomes,

$$A\beta = \frac{A_v X_1 X_2}{jR_o(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

The loop gain becomes,

$$A\beta = \frac{A_v X_1 X_2}{jR_o(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

$$\angle AB = 0$$

The imaginary part = 0 only when X_1+ X_2+ X_3=0

- It indicates that at least one reactance must be –ve (capacitor)
- X_1 and X_2 must be of same type and X_3 must be of opposite type

The loop gain becomes,

$$A\beta = \frac{A_v X_1 X_2}{jR_o(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

$$\angle AB = 0$$

The imaginary part = 0 only when $X_1 + X_2 + X_3 = 0$

$$R_o(X_1 + X_2 + X_3) = 0$$

$$R_o \neq 0$$

$$(X_1 + X_2 + X_3) = 0$$

$$X_1 + X_2 = -X_3$$

For Colpitts Oscillator

$$X_1 + X_2 = -X_3 \Rightarrow \frac{1}{j\omega c_1} + \frac{1}{j\omega c_2} = -j\omega L$$

$$\frac{j\omega c_2 + j\omega c_1}{j\omega c_1 \times j\omega c_2} = -j\omega L \Rightarrow \frac{j\omega c_2 + j\omega c_1}{j^2\omega^2 c_1 c_2} = -j\omega L$$

$$j^2 = -1 \Rightarrow \frac{j\omega(c_2 + c_1)}{\omega^2 c_1 c_2} = j\omega L \Rightarrow \frac{(c_2 + c_1)}{\omega^2 c_1 c_2} = L$$

$$\frac{(c_2 + c_1)}{L c_1 c_2} = \omega^2$$

$$\frac{1}{L \dfrac{c_1 c_2}{(c_2 + c_1)}} = \omega^2$$

$$\boxed{\omega_o = \frac{1}{\sqrt{L C_T}}} \qquad \boxed{C_T = \frac{C_1 C_2}{C_1 + C_2}}$$

For Hartley Oscillator

$$X_1 + X_2 = -X_3 \Rightarrow j\omega L_1 + j\omega L_2 = -\frac{1}{j\omega c}$$

$$j\omega(L_1 + L_2)\, j\omega c = -1$$

$$j^2 = -1 \Rightarrow \omega^2 (L_1 + L_2) C = 1$$

$$\boxed{\omega_o = \frac{1}{\sqrt{(L_1 + L_2)C}}}$$

The loop gain becomes,

$$A\beta = \frac{A_v X_1 X_2}{jR_o(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

$$\angle AB = 0$$

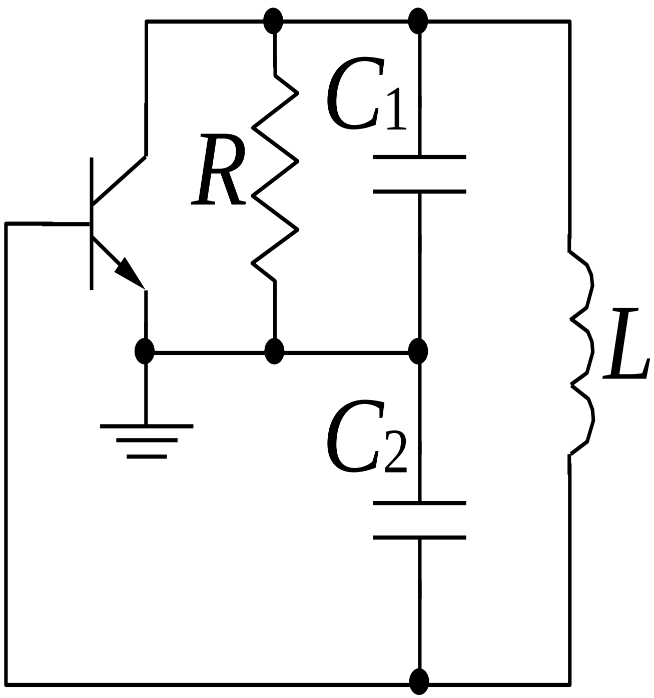
The imaginary part = 0 only when $X_1 + X_2 + X_3 = 0$

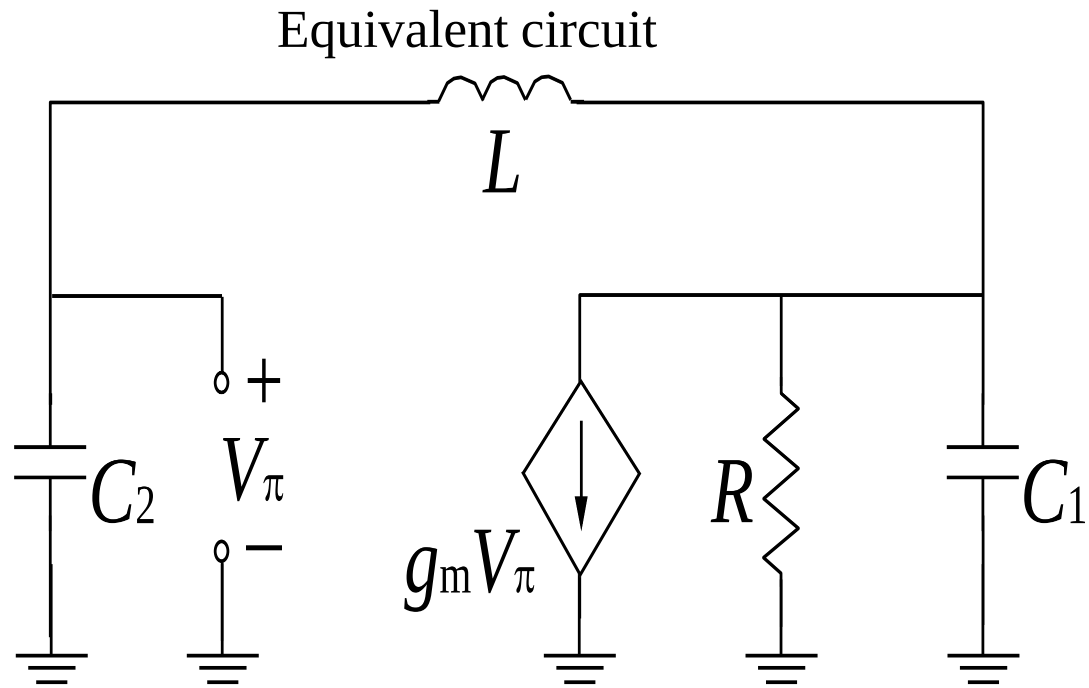
$$A\beta = \frac{-A_v X_1 X_2}{X_2(X_1 + X_3)} = \frac{-A_v X_1}{(X_1 + X_3)} = \frac{-A_v X_1}{-X_2} = \frac{A_v X_1}{X_2}$$

For Unit Gain $A\beta = 1$

$$A\beta = 1 \quad \Rightarrow \quad A_v = \frac{X_2}{X_1} \Rightarrow \beta = \frac{X_1}{X_2}$$

Colpitts Oscillator





In the equivalent circuit, it is assumed that:

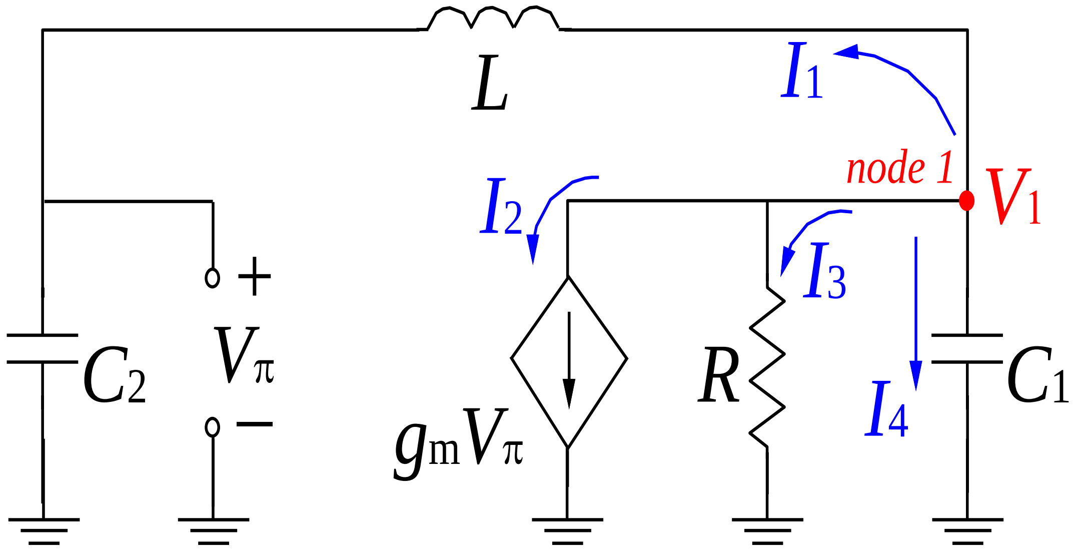
- Linear small signal model of transistor is used
- The transistor capacitances are neglected
- Input resistance of the transistor is large enough

At node 1,

$$i_1 = \frac{V_1 - V_\pi}{j\omega L}$$

$$i_1 = j\omega C_2 V_\pi$$

$$V_1 = V_\pi(1 - \omega^2 L C_2)$$

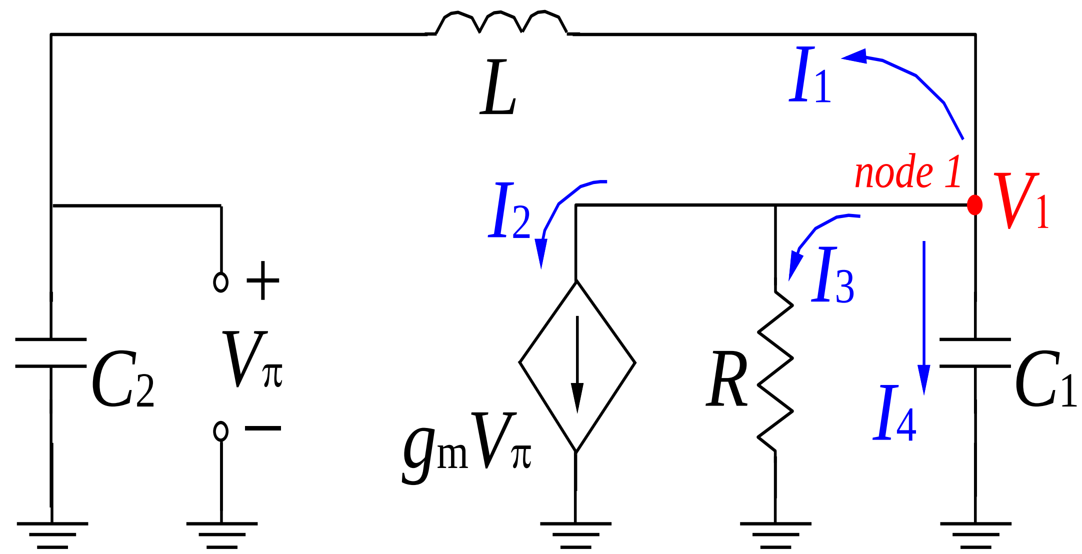


Apply KCL at node 1,

$$\sum I_{in} = \sum I_{out}$$

$$0 = I_1 + I_2 + I_3 + I_4$$

$$j\omega C_2 V_\pi + g_m V_\pi + \frac{V_1}{R} + j\omega C_1 V_1 = 0$$

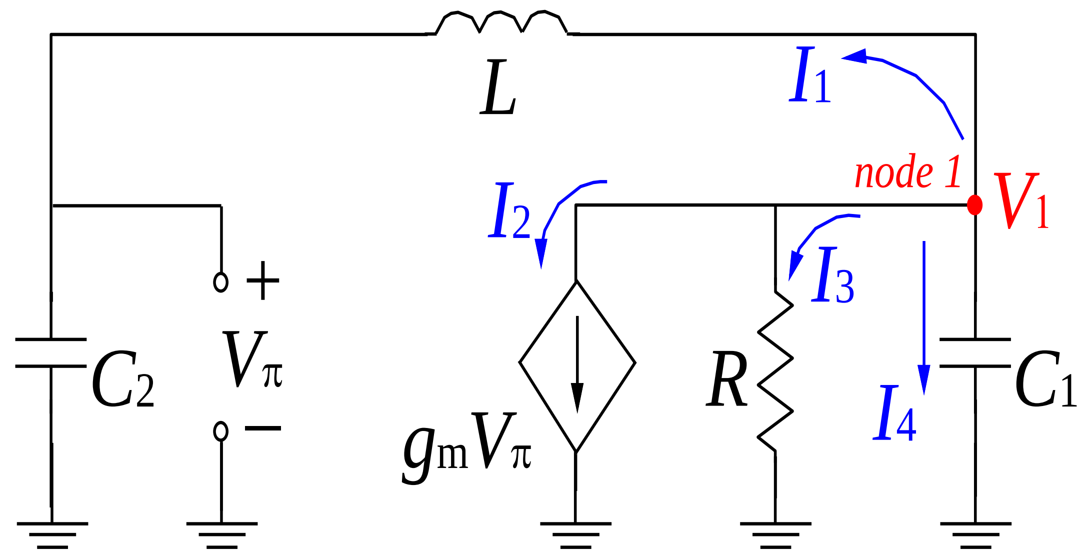


Apply KCL at node 1,

$$j\omega C_2 V_\pi + g_m V_\pi + \frac{V_1}{R} + j\omega C_1 V_1 = 0$$

$$\because V_1 = V_\pi (1 - \omega^2 L C_2)$$

$$j\omega C_2 V_\pi + g_m V_\pi + V_\pi (1 - \omega^2 L C_2)\left(\frac{1}{R} + j\omega C_1\right) = 0$$



$$j\omega C_2 V_\pi + g_m V_\pi + V_\pi (1-\omega^2 LC_2)\left(\frac{1}{R} + j\omega C_1\right) = 0$$

For Oscillator V_π must not be zero, therefore it enforces,

$$\left(g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R}\right) + j\left[\omega(C_1+C_2) - \omega^3 LC_1C_2\right] = 0$$

$$\left(g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R}\right) + j\left[\omega(C_1 + C_2) - \omega^3 LC_1C_2\right] = 0$$

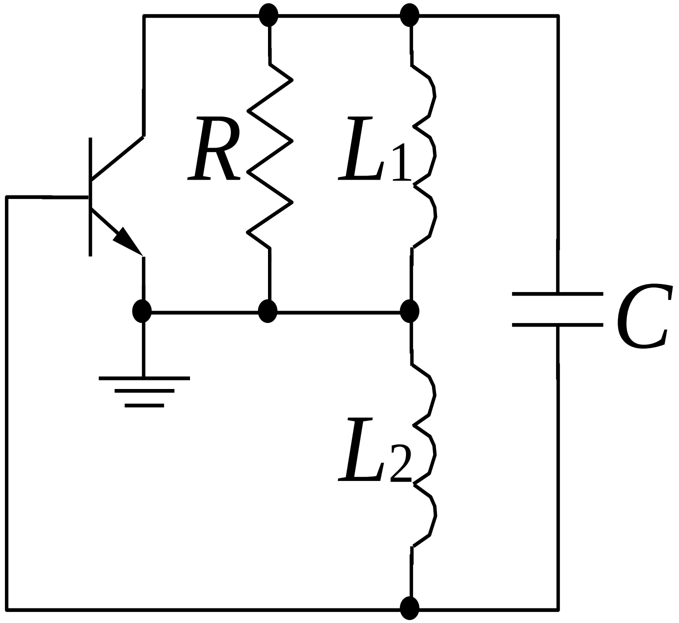
Imaginary part = 0, we have

$$\omega_o = \frac{1}{\sqrt{LC_T}} \qquad C_T = \frac{C_1C_2}{C_1 + C_2}$$

Real part = 0, yields

$$g_m = \frac{C_2}{RC_1}$$

Hartley Oscillator



Find the expression of

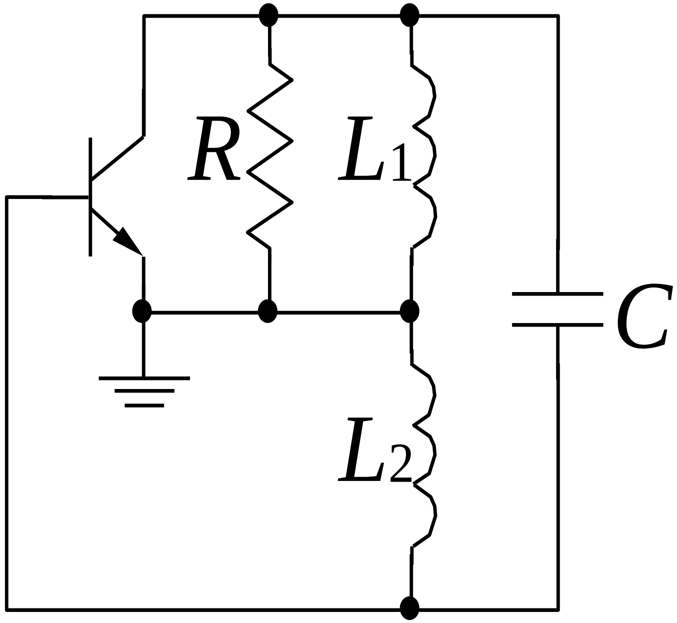
ω_0?

g_m?

Online Report

Due date: 26/10/2016

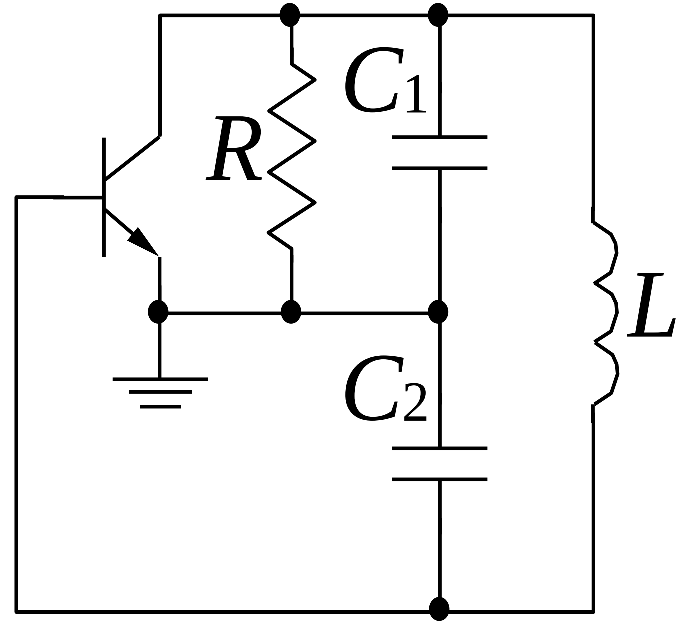
Hartley Oscillator



$$\omega_o = \frac{1}{\sqrt{(L_1 + L_2)C}}$$

$$g_m = \frac{L_1}{RL_2}$$

Colpitts Oscillator



$$\omega_o = \frac{1}{\sqrt{LC_T}} \quad C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$g_m = \frac{C_2}{RC_1}$$