Sheet 3

Active Filters

1. Design KHN filter to realize BPF, $f_o = 10$ KHZ, BW = 200 HZ, choose $C = 10nF$, what is the value of center freq. gain.

sol:

$f_o = 10KHZ$, $BW = 200$ HZ, $C = 10nF$

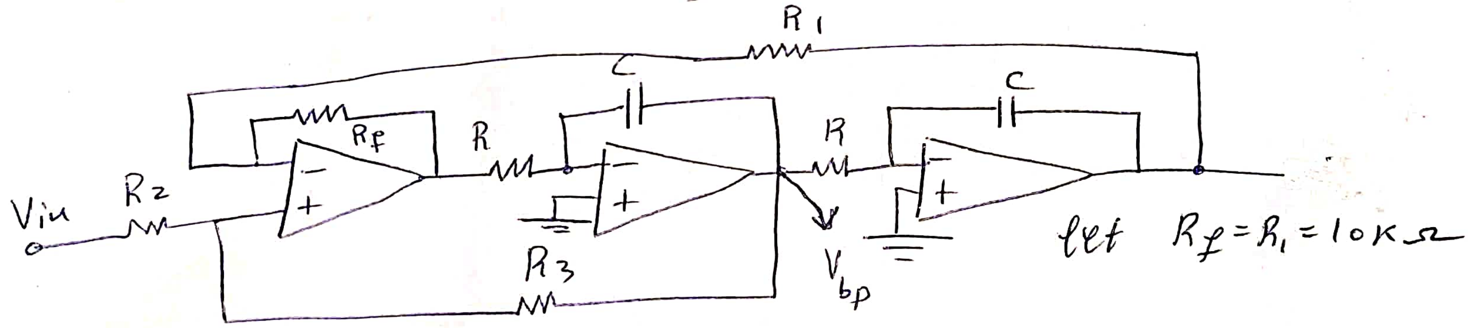
$$\Rightarrow Q = \frac{f_o}{BW} = \frac{10 \times 10^3}{200} = 50$$

$$\because f_o = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi \times 10 \times 10^{-9} \times 10 \times 10^3} = 1.59\ K\Omega$$

$$\Rightarrow K = 2 - \frac{1}{Q} = 2 - \frac{1}{50} = 1.98$$

$$\Rightarrow \frac{R_3}{R_2} = 2Q - 1 \Rightarrow \text{let } R_2 = 1K\Omega$$

$$\Rightarrow R_3 = 2(50-1)R_2 = 98\ K\Omega$$



let $R_f = R_1 = 10K\Omega$

* center frequency:

$$\Rightarrow V_{BP} = -\frac{\omega_o}{s} V_{HP} = \left(-\frac{\omega_o}{s}\right) \frac{Ks^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2} (V_{in})$$

$$\Rightarrow \frac{V_{BP}}{V_{in}} = \frac{-K\omega_o s}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2} = \frac{-jK\omega\omega_o}{-\omega^2 + j\frac{\omega_o}{Q}\cdot\omega + \omega_o^2}$$

$$\Rightarrow A_m = \left|\frac{V_{BP}}{V_{in}}\right|_{\omega=\omega_o} = \frac{K\omega_o\omega_o}{\sqrt{(\omega_o^2 - \omega_o^2)^2 + \left(\frac{\omega_o^2}{Q}\right)^2}} = \frac{K\omega_o^2}{\omega_o^2/Q} = KQ$$

$$\Rightarrow A_m = KQ = (1.98)(50) = 99$$

2. Design KHN filter to realize HPF with $f_o = 10\,KHz$ & $Q = 2$, choose $C = 1nf$. what is the value of high frequency gain obtained? what is the center frequency gain of the BPF that is simultaneously available at the o/p of the 1st integrator.

<u>sol</u>:

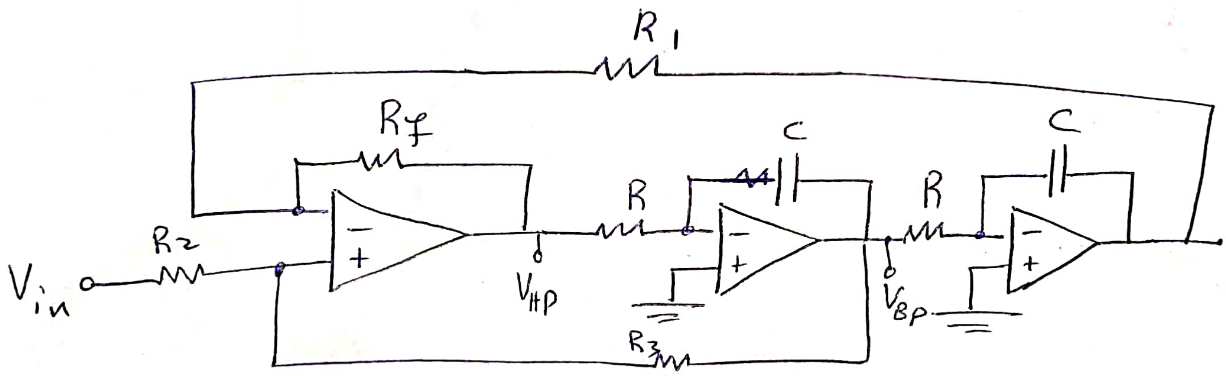
$f_o = 10\,KHz, \quad Q = 2, \quad C = 1nf$

$$\Rightarrow f_o = \frac{1}{2\pi RC} \quad \Rightarrow R = \frac{1}{2\pi(10^{-9})(10\times10^{3})} = 15.9\,K\Omega$$

$$\because K = 2 - \frac{1}{Q} = 2 - \frac{1}{2} = 1.5$$

$$\because \frac{R_3}{R_2} = 2Q - 1 = 0 \quad \Rightarrow R_3 = 0$$

let $R_2 = 10K$



* at High frequency:
$$\frac{V_{HP}}{V_{in}} = \frac{Ks^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

at $s \to \infty$:
$$\left|\frac{V_{HP}}{V_{in}}\right|_{s\to\infty} = \left|\frac{K\cancel{s^2}}{\cancel{s^2} + \frac{\omega_o}{Q}s + \omega_o^2}\right|_{s\to\infty} = K$$

* center freq gain of BPF:
$$\Rightarrow \left|\frac{V_{BP}}{V_{in}}\right| = \left|\frac{-jK\omega\omega_o}{-\omega^2 + j\frac{\omega_o}{Q}\omega + \omega_o^2}\right|$$

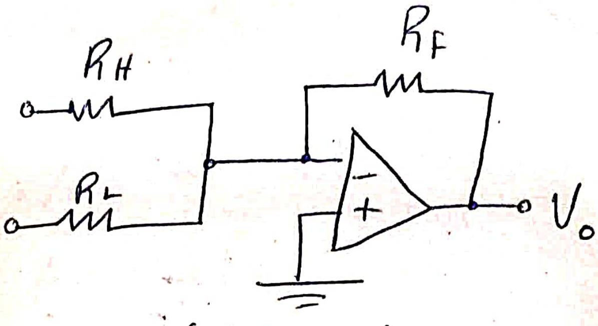
$$\Rightarrow \left|\frac{V_{BP}}{V_{in}}\right|_{\omega=\omega_o} = \frac{K\omega_o^2}{\sqrt{(\omega_o^2-\omega_o^2)^2 + \left(\frac{\omega_o^2}{Q}\right)^2}} = KQ$$
$$= (1.5)(2) = 3$$

3. Use KHN ct. with an o/p of a summing amplifier to design a band-stop notch filter with $f_o = 5\,KHz$, $f_n = 8\,KHz$, $Q = 5$ & dc gain of 3, select $C = 1\,nF$.

Sol

$\because f_o = 5\,KHz$

$f_n = 8\,KHz$



$$\left(\frac{R_H}{R_L}\right) = \left(\frac{\omega_n}{\omega_o}\right)^2$$

$$\Rightarrow \left(\frac{f_n}{f_o}\right)^2 = \frac{R_H}{R_L}$$

let $R_H = 10\,K\Omega \Rightarrow \left(\frac{8}{5}\right)^2 = \frac{10K}{R_L}$

$$\Rightarrow \boxed{R_L = 3.906\,K\Omega}$$

* for the KHN filter :

$$f_o = \frac{1}{2\pi RC} \quad , \quad C = 1\,nF$$

$$\Rightarrow R = \frac{1}{2\pi(5\times10^3)(1\times10^{-9})} = 31.83\,K\Omega$$

$$Q = 5 \quad \Rightarrow K = 2 - \frac{1}{Q} = 2 - \frac{1}{5} = 1.8$$

$$\frac{R_3}{R_2} = 2Q - 1 \Rightarrow \text{let } R_2 = 10\,K\Omega$$

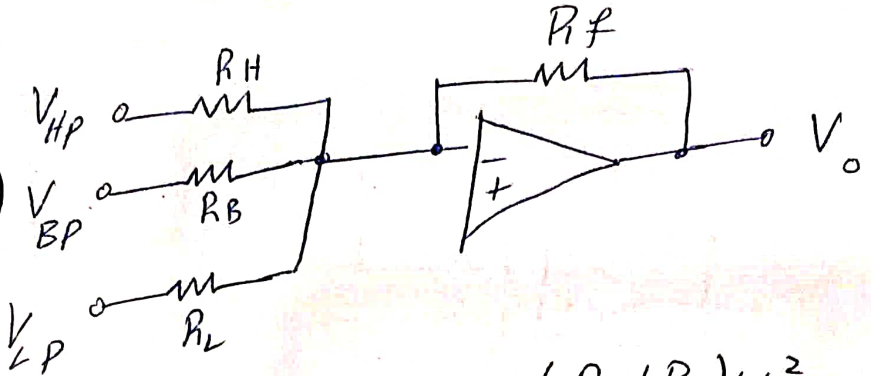
$$R_3 = (10-1)(10\,K\Omega) = 90\,K\Omega$$

$$\Rightarrow \frac{V_o}{V_i} = -K\,\frac{(R_F/R_H)s^2 + (R_F/R_L)\,\omega_o^2}{s^2 + s(\omega_o/Q) + \omega_o^2}$$

at $s \to 0$: $\Rightarrow \left.\frac{V_o}{V_i}\right|_{s\to 0} = -K\left(\frac{R_f}{R_L}\right) \Rightarrow K\,\frac{R_f}{R_L} = \underbrace{3}_{\text{dc gain}} \Rightarrow R_f = 21\,K\Omega$

4. Use the KHN ct. with summing amplifier to get flat gain of all pass filter.

Sol:



$$\Rightarrow V_o = -\left(\frac{R_F}{R_H} V_{HP} + \frac{R_F}{R_B} V_{BP} + \frac{R_F}{R_L} V_{LP}\right)$$

$$\Rightarrow \frac{V_o}{V_i} = -K \frac{(R_F/R_H) s^2 - s(R_F/R_B)\omega_o + (R_F/R_L)\omega_o^2}{s^2 + s(\omega_o/Q) + \omega_o^2}$$

$$= -K\left(\frac{R_F}{R_H}\right) \cdot \frac{s^2 - s\left(\frac{R_H}{R_B}\right)\omega_o + (R_H/R_L)\omega_o^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

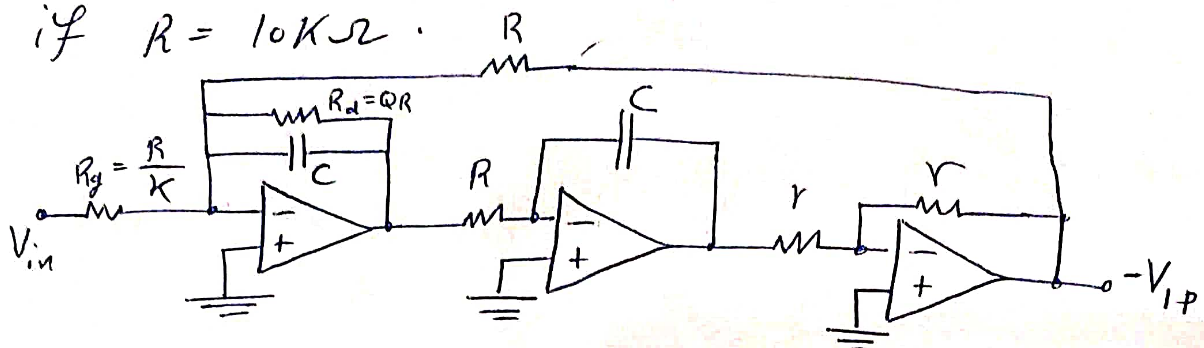
* for an all pass filter :

$$T(s) = \text{gain} * \frac{s^2 - s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$

$$\Rightarrow \boxed{\frac{R_H}{R_B} = \frac{1}{Q}} \quad , \quad \frac{R_H}{R_L} = 1 \Rightarrow \boxed{R_H = R_L}$$

$$\text{Flat gain} = -K\left(\frac{R_F}{R_H}\right)$$

5. Use the Tow-Thomas filter shown to design a 2nd order BPF with $f_0 = 10\,KHz$, $Q = 20$ & unity center gain if $R = 10K\Omega$.



Sol:

$$f_0 = \frac{1}{2\pi C\sqrt{R_2 R_3}} \quad , R_2 = R_3 = R$$

$$\Rightarrow f_0 = \frac{1}{2\pi C R} = \frac{1}{2\pi C(10K)} = 10 * 10^3$$

$$\Rightarrow C = \frac{1}{2\pi * 10 * 10^3 * (10 * 10^3)} = 1.59\,nF$$

$$Q = \frac{R_1}{\sqrt{R\,R}} = \frac{R_d}{R} \Rightarrow R_d = QR = (20)(10K)$$

$$\Rightarrow R_d = 200\,K\Omega$$

$$A_m = \frac{R_d}{R_g} = \frac{200\,K}{R_g} = 1$$

$$\Rightarrow R_g = 200\,K\Omega$$

6. use the Tow-Thomas biquad Filter shown in the previous problem to analyze the TF. of the 2nd order low pass filter & design the filter $f_o = 10\,KHz$ & center frequency gain of 50 : if $R = 10\,K\Omega$, give values of C, R_d & R_g.

Sol:

$$\because V_{LP} = V_{BP}\left(\frac{-1}{sRC}\right)$$

$$\Rightarrow T_{LP} = \left(-\frac{1}{\cancel{s}RC}\right)\frac{-\left(\frac{1}{R_gC}\right)\cancel{s}}{s^2 + \frac{1}{R_dC}s + \frac{1}{R^2C^2}}$$

$$= \frac{\frac{1}{RR_gC^2}}{s^2 + \frac{1}{R_dC}s + \frac{1}{R^2C^2}} = \frac{a_o}{s^2 + \frac{w_o}{Q}s + w_o^2}$$

$$\Rightarrow \boxed{w_o = \frac{1}{RC}}\ , \quad \frac{w_o}{Q} = \frac{1}{R_dC} \Rightarrow Q = \frac{w_o}{R_dC}$$

$$\boxed{Q = \frac{1}{RR_dC^2}}$$

$$\Rightarrow |T_{LP}|_{w=w_o} = \frac{a_o}{\sqrt{(w_o^2-w_o^2)^2 + \left(\frac{w_o^2}{Q}\right)^2}} = \frac{a_o}{w_o^2/Q} = \frac{a_oQ}{w_o^2}$$

$$= \frac{\frac{1}{RR_gC^2}\left(\frac{Q}{w_o}\right)}{w_o}$$

$$= \frac{\frac{R_d\cancel{C}}{\cancel{R}R_g\cancel{C^2}}}{\frac{1}{\cancel{RC}}}$$

$$= \frac{R_d}{R_g}$$