# Electronic Systems

## Active Filters

### Lecture 5

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# The Active Filters Contents:

1. Introduction to Filters.
2. Low Pass Filter.
3. High Pass Filter.
4. Band Pass Filter.
5. Butterworth Filter.
6. Chebyshev Filter.
7. Bessel Filter.
8. **KHN Biquad Filter.**
9. Multiple Feedback Filters.
10. State Variable Filters.

# Biquad Filters

- A **Biquad filter** is a type of linear filter that implements a transfer function that is the ratio of two quadratic functions. The name *Biquad* is short for *biquadratic*. Any second-order filter topology can be referred to as a *biquad*.

# Kerwin–Huelsman–Newcomb (KHN) Biquad filter

- This is a second-order (Biquad) filter that can produce simultaneous low-pass, high-pass, and band-pass outputs from a single input. Its derivation comes from rearranging a high-pass filter's transfer function, which is the ratio of two quadratic functions. By using different states as outputs, different kinds of filters can be produced.

# (KHN) Biquad filter

- **General Filter Transfer Function (Second Order):**

Mathematically, filters are commonly described using transfer functions. The general expression for the second order or biquadratic transfer function is usually expressed in the standard form as:

- $H(s) = \frac{V_o}{V_{in}} = \frac{a_2 S^2 + a_1 S + a_0}{S^2 + \left(\frac{w_o}{Q}\right) S + w_o^2}$

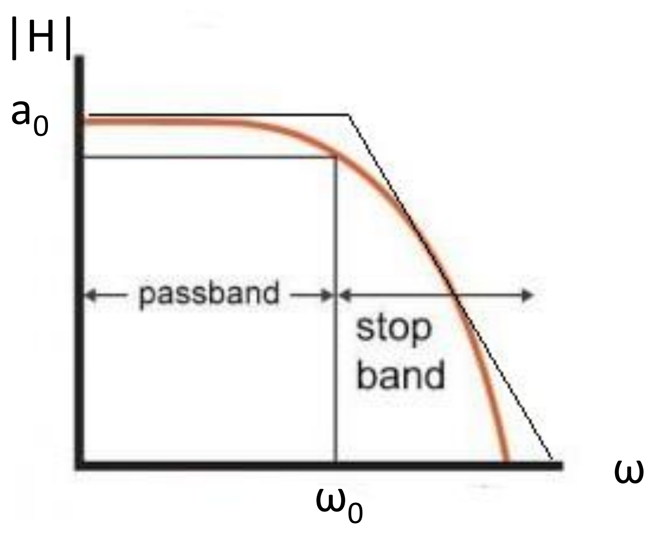
*Where*

- $w_o$ cut-off Frequency(rad/sec)
- $w_o = 2\pi\, f_o$
- Q  pole quality Factor
- $a_2$, $a_1$ and $a_0$ contants

# (KHN) Biquad filter

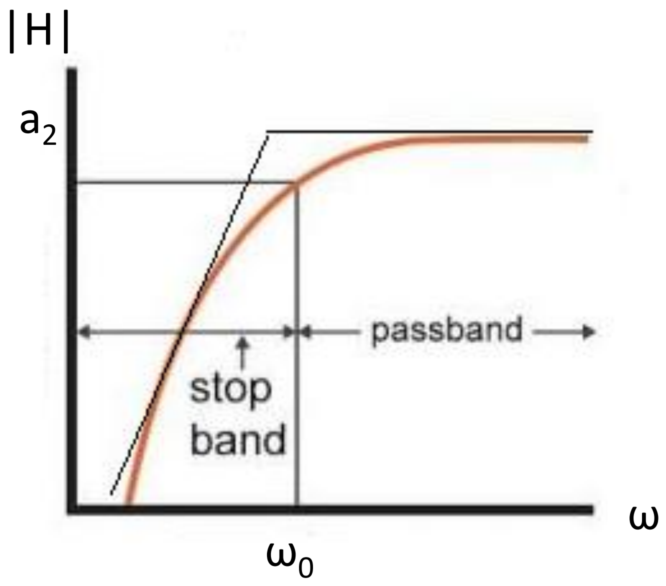
- Low pass Filter (LPF):

$$H_{LP} = \frac{a_o}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$




- High pass Filter (HPF):

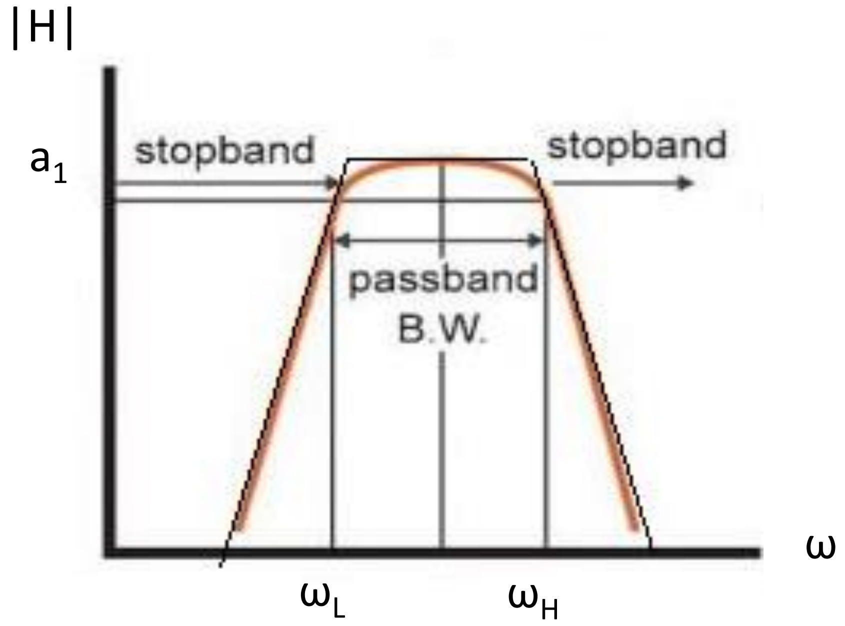
$$H_{HP} = \frac{a_2 s^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$



# (KHN) Biquad filter

- Band pass Filter (BPF):

$$H_{BP} = \frac{a_1 s}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$



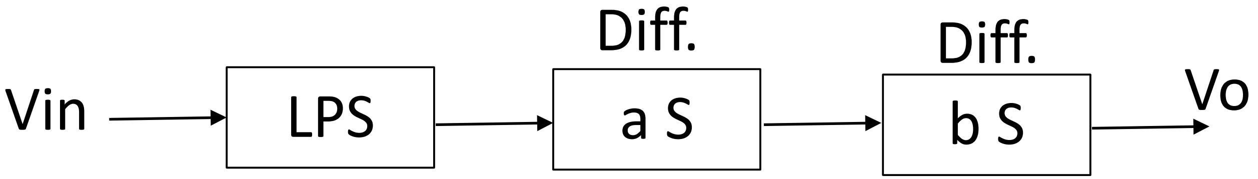

*Where*

- $w_o$ cut-off Frequency(rad/sec)
- $w_L, w_H$= lower and higher cutoff Frequencies.
- $w_o = \sqrt{w_L w_H}$ $\qquad f_o = \sqrt{f_L f_H}$

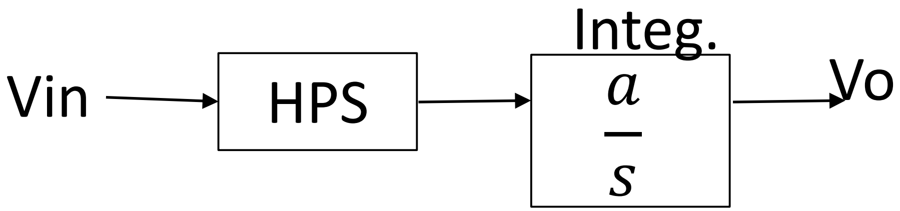
# Frequency Transformation

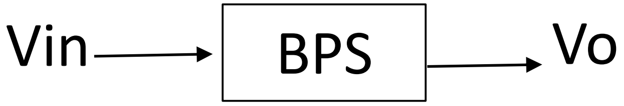
Differentiation ------------ S

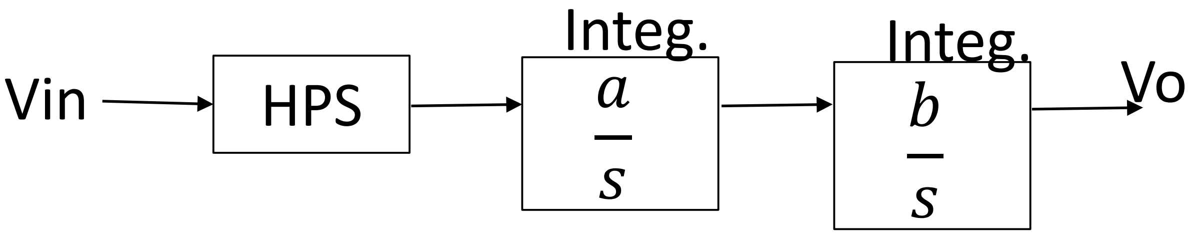
Integration --------------- $\frac{1}{s}$

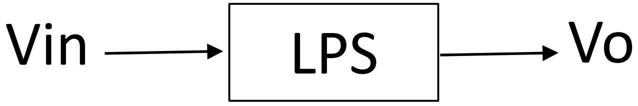






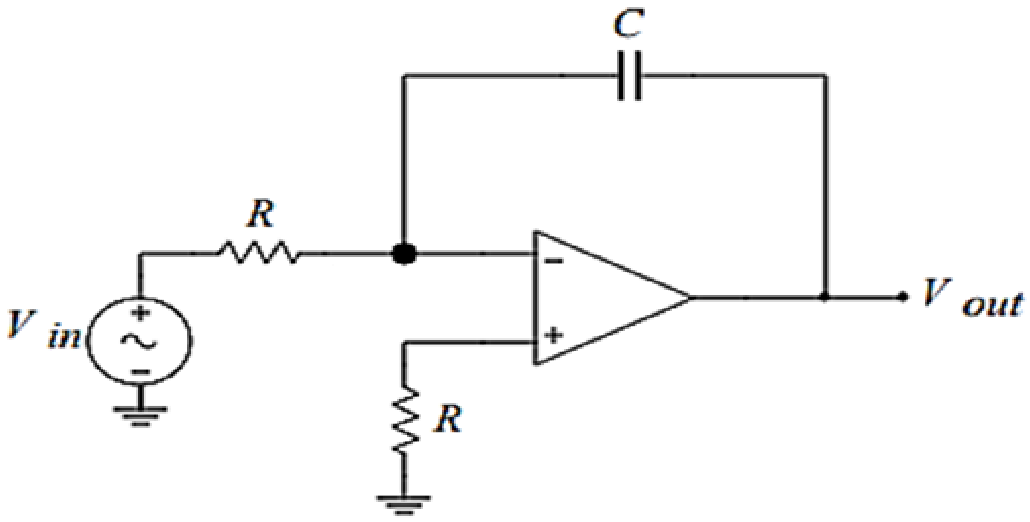



# (KHN) Biquad filter

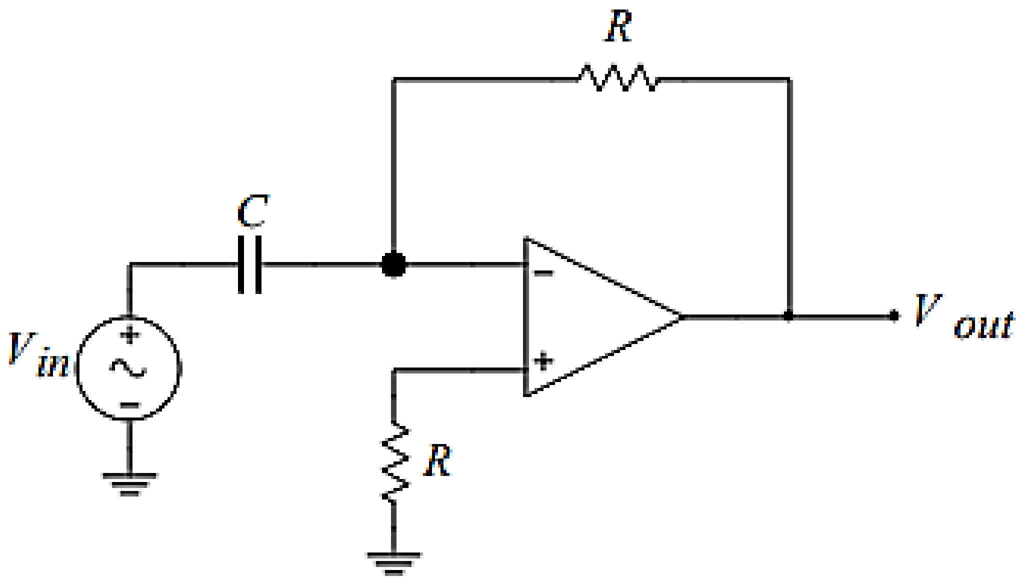
## Op-Amp Integrator




$$\frac{V_o}{V_{in}} = -\frac{{}^{1}/_{sC}}{R} = -\frac{1}{sCR}$$

$$\frac{V_o}{V_{in}} = -\left(\frac{1}{RC}\right)\left(\frac{1}{S}\right) = \frac{a}{s}$$

## Op-Amp Differentiator




$$\frac{V_o}{V_{in}} = -\frac{R}{{}^{1}/_{sC}} = -sRC$$

$$\frac{V_o}{V_{in}} = -RC(S) = \text{a } s$$

# (KHN) Biquad filter

- Consider Second order filter

$$\frac{V_{hp}}{V_{in}} = \frac{K s^2}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2}$$

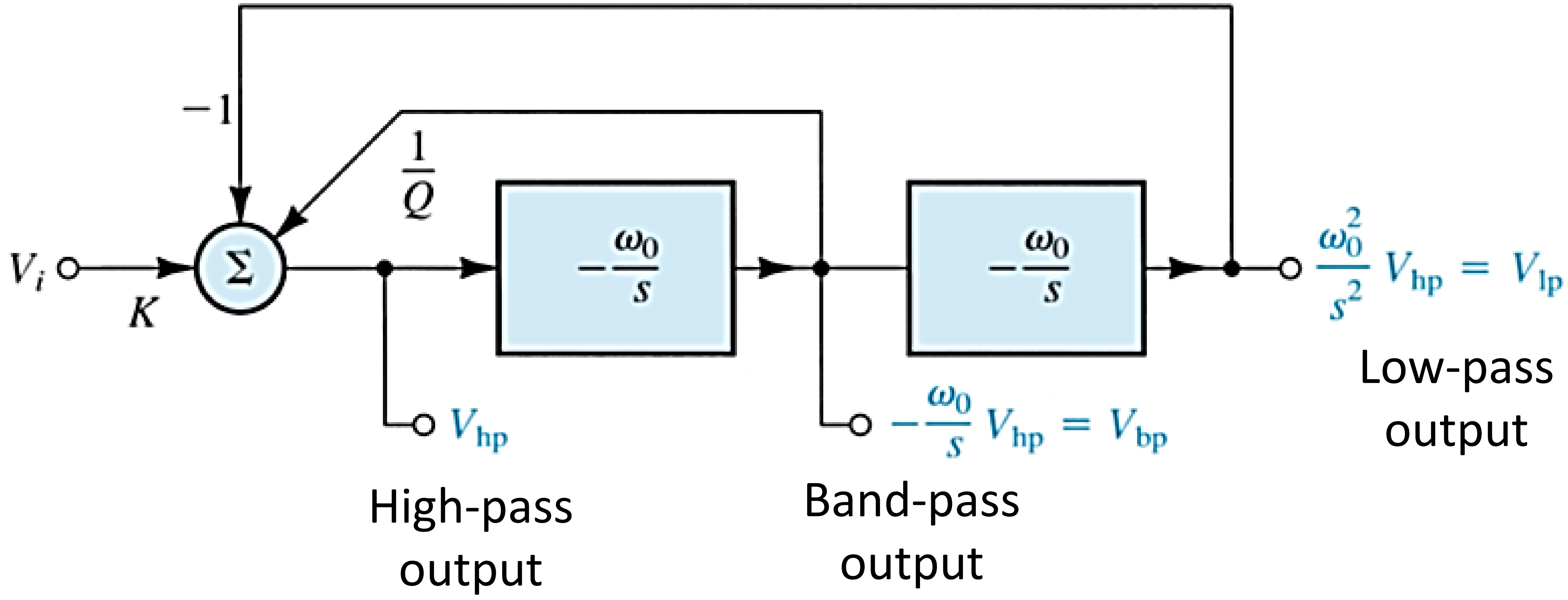
$$\therefore s^2 V_{hp} + \left(\frac{\omega_o}{Q}\right) s V_{hp} + \omega_o^2 V_{hp} = K s^2 V_{in} \qquad \div s^2$$

$$V_{hp} + \left(\frac{\omega_o}{Q}\right) \frac{1}{s} V_{hp} + \frac{\omega_o^2}{s^2} V_{hp} = K V_{in}$$

$$\therefore V_{hp} = K V_{in} - \left(\frac{1}{Q}\right)\left(\frac{\omega_o}{s}\right) V_{hp} - \left(\frac{\omega_o}{s}\right)\left(\frac{\omega_o}{s}\right) V_{hp} \quad \text{I}$$

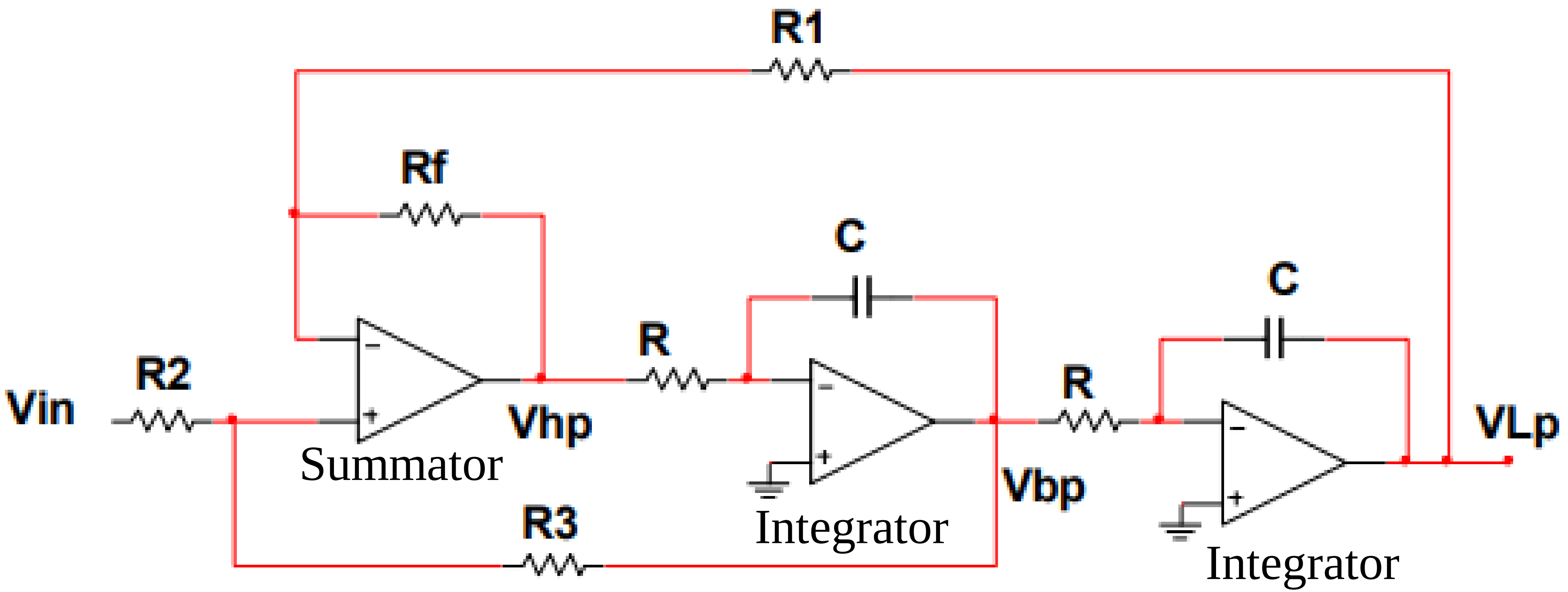
# (KHN) Biquad filter

$$\therefore V_{hp} = K V_{in} - \left(\frac{1}{Q}\right)\left(\frac{\omega_0}{s}\right) V_{hp} - \left(\frac{\omega_0}{s}\right)\left(\frac{\omega_0}{s}\right) V_{hp} \quad \text{I}$$



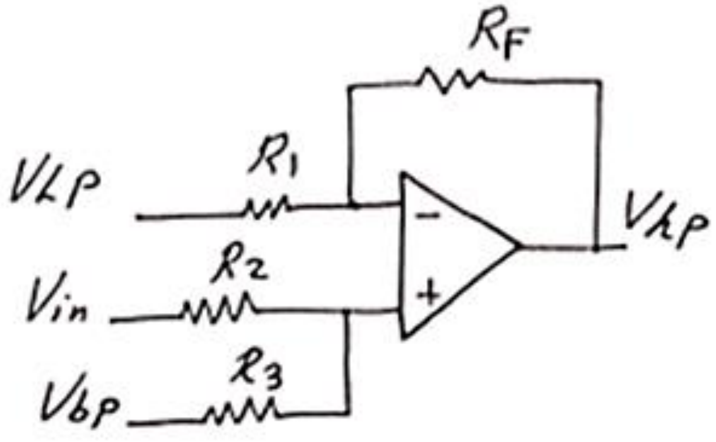
# (KHN) Biquad filter

- Circuit Diagram



# (KHN) Biquad filter

## Summator




$$V_{LP} = \frac{\omega_o^2}{s^2} V_{hp}$$

$$V_{LP} = \left(-\frac{1}{RC}\right)\left(-\frac{1}{RC}\right)\frac{1}{s^2} V_{hp}$$

$$V_{LP} = \frac{1}{s^2 C^2 R^2} V_{hp} = \frac{\omega_o^2}{s^2} V_{hp}$$

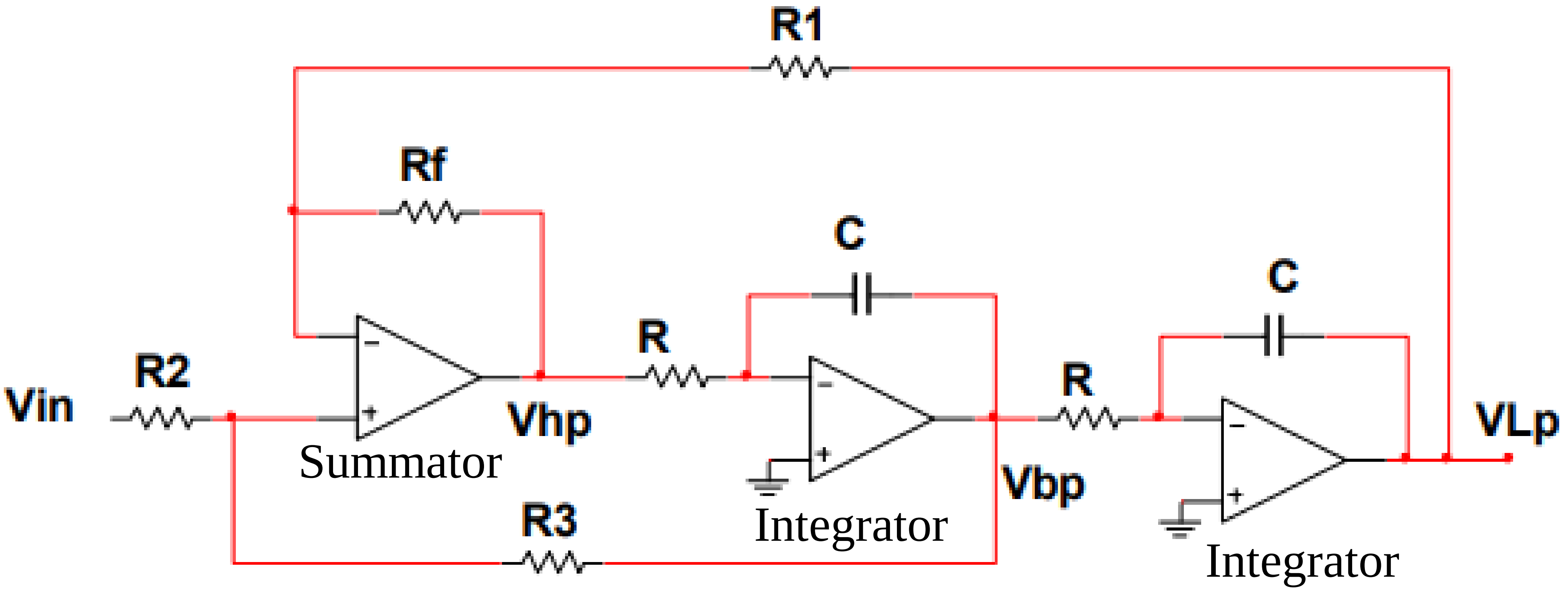
$$\therefore\ \omega_o = \frac{1}{RC}$$

$$V_{bp} = -\frac{1}{RC}\frac{1}{s} V_{hp} \rightarrow V_{bp} = -\frac{1}{sRC} V_{hp} = -\frac{\omega_o}{s} V_{hp}$$

$$V_{hp} = -\frac{R_F}{R_1} V_{LP} + \left(1+\frac{R_F}{R_1}\right)\left[\frac{V_{in} R_3}{R_2+R_3} + \frac{V_{bp} R_2}{R_2+R_3}\right]$$

$$V_{hp} = \left(1+\frac{R_F}{R_1}\right)\frac{R_3}{R_2+R_3} V_{in} + \left(1+\frac{R_F}{R_1}\right)\frac{R_2}{R_2+R_3} V_{bp} - \frac{R_F}{R_1} V_{LP}$$

# (KHN) Biquad filter




$$V_{hP} = \left(1+\frac{R_F}{R_1}\right)\frac{R_3}{R_2+R_3} V_{in} + \left(1+\frac{R_F}{R_1}\right)\frac{R_2}{R_2+R_3} V_{bP} - \frac{R_F}{R_1} V_{LP}$$

# (KHN) biquad filter (summary)

$$V_{hp} = \left(1+\frac{R_F}{R_1}\right)\frac{R_3}{R_2+R_3}V_{in} + \left(1+\frac{R_F}{R_1}\right)\frac{R_2}{R_2+R_3}V_{bp} - \frac{R_F}{R_1}V_{Lp}$$

$$V_{hp} = \left(1+\frac{R_F}{R_1}\right)\frac{R_3}{R_2+R_3}V_{in} + \left(1+\frac{R_F}{R_1}\right)\frac{R_2}{R_2+R_3}\left(-\frac{\omega_0}{s}V_{hp}\right) - \frac{R_F}{R_1}V_{hp}\left(\frac{\omega_0^2}{s^2}\right)$$

$$V_{Lp} = \frac{1}{s^2C^2R^2}V_{hp} = \frac{\omega_0^2}{s^2}V_{hp}$$

$$V_{hp} = K\,V_{in} - \left(\frac{1}{Q}\right)\left(\frac{\omega_0}{s}\right)V_{hp} - \left(\frac{\omega_0}{s}\right)\left(\frac{\omega_0}{s}\right)V_{hp} \quad \text{I}$$

* let $\boxed{R_F = R_1}$ $\rightarrow$ ∴ $\boxed{K = \frac{2R_3}{R_2+R_3}}$

* $\frac{2R_2}{R_2+R_3} = \frac{1}{Q}$ $\rightarrow$ $\boxed{\frac{R_3}{R_2} = 2Q - 1}$

∴ $\boxed{K = 2 - \frac{1}{Q}}$

* $\boxed{f_o = \frac{1}{2\pi RC}}$

* For BPF

$\boxed{Q = \frac{f_o}{B.W}}$

$f_L = f_o - \frac{B.W}{2}$

$f_H = f_o + \frac{B.W}{2}$

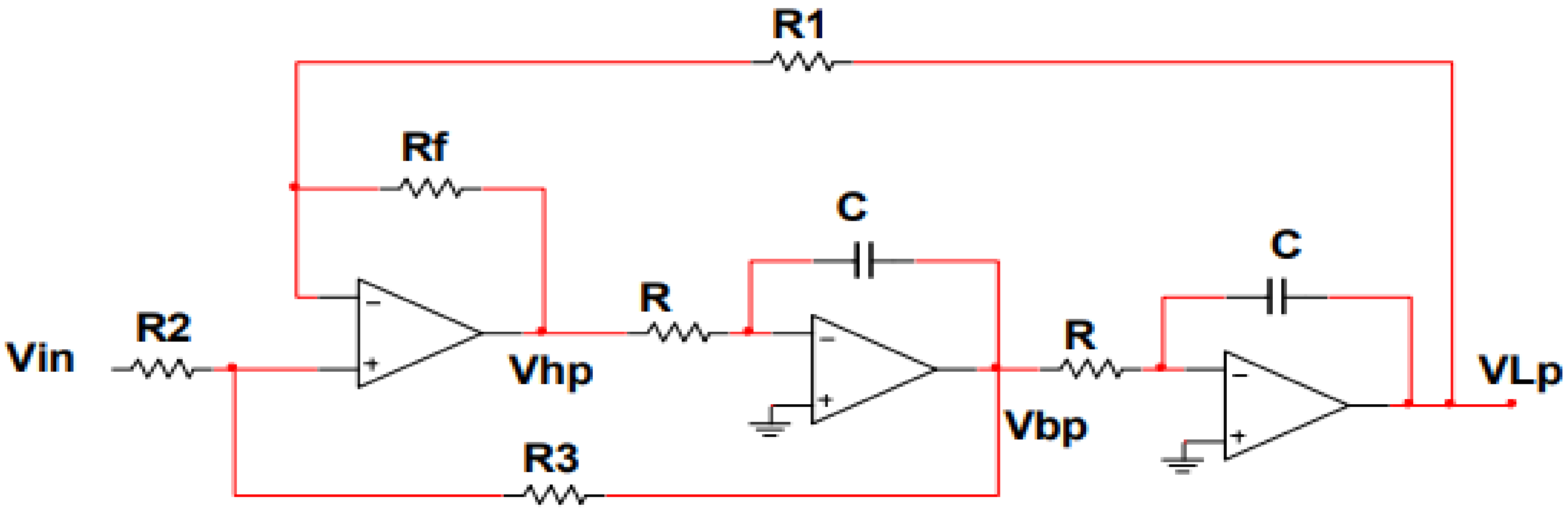
# (KHN) Biquad filter

**Example 1:**

Design a KHN Filter to relize a BPF with center frequency of 10 KHz and bandwidth of 100Hz.

Calculate the center frequency gain (Am). Hint: use 1 nF capacitor.

Solution



# Solution:

* $f_o = 10 \times 10^3 = \dfrac{1}{2\pi RC}$ , $\boxed{C = 1nF}$

$\therefore R = \dfrac{1}{2\pi [10 \times 10^3][1 \times 10^{-9}]} = 15920\,\Omega$

$\boxed{R = 15.92\,k\Omega}$

* $\dfrac{R_F}{R_1} = 1 \rightarrow$ choose $\boxed{R_1 = 10k\Omega \rightarrow \therefore R_F = 10k\Omega}$

* Quality-Factor $Q = \dfrac{f_o}{B.W} = \dfrac{10 \times 10^3}{100} = 100$

$\boxed{Q = 100}$

* $K = 2 - \dfrac{1}{Q} = 2 - \dfrac{1}{100} \rightarrow \therefore \boxed{k = 1.99}$

* $\dfrac{R_3}{R_2} = 2Q - 1 = 2 \times 100 - 1$

$\dfrac{R_3}{R_2} = 199 \rightarrow$ choose $\therefore \boxed{\begin{array}{l} R_2 = 1k\Omega \\ R_3 = 199k\Omega \end{array}}$

# How to get the center Frequency gain (Am)?

$$\frac{V_{hp}}{V_{in}} = \frac{K s^2}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2}$$

$$\therefore V_{hp} = \frac{K s^2}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2} V_{in}$$

but $V_{bp} = -\frac{\omega_0}{s} V_{hp}$

$$\therefore V_{bp} = \frac{K s^2}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2} \left(-\frac{\omega_0}{s}\right) V_{in}$$

$$\therefore \frac{V_{bp}}{V_{in}} = \frac{-K \omega_0 s}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2} = H_{bp} = T_{bp}$$

$$\therefore H_{bp}(j\omega) = \frac{-jK\omega_0 \omega}{(j\omega)^2 + \left(\frac{\omega_0}{Q}\right) j\omega + \omega_0^2}$$

$$T_{bp}(j\omega) = \frac{-jK\omega_0 \omega}{-\omega^2 + j\frac{\omega_0 \omega}{Q} + \omega_0^2}$$

$$T_{bp}(j\omega) = \frac{-jK\omega_0 \omega}{[\omega_0^2 - \omega^2] + j\frac{\omega_0 \omega}{Q}}$$

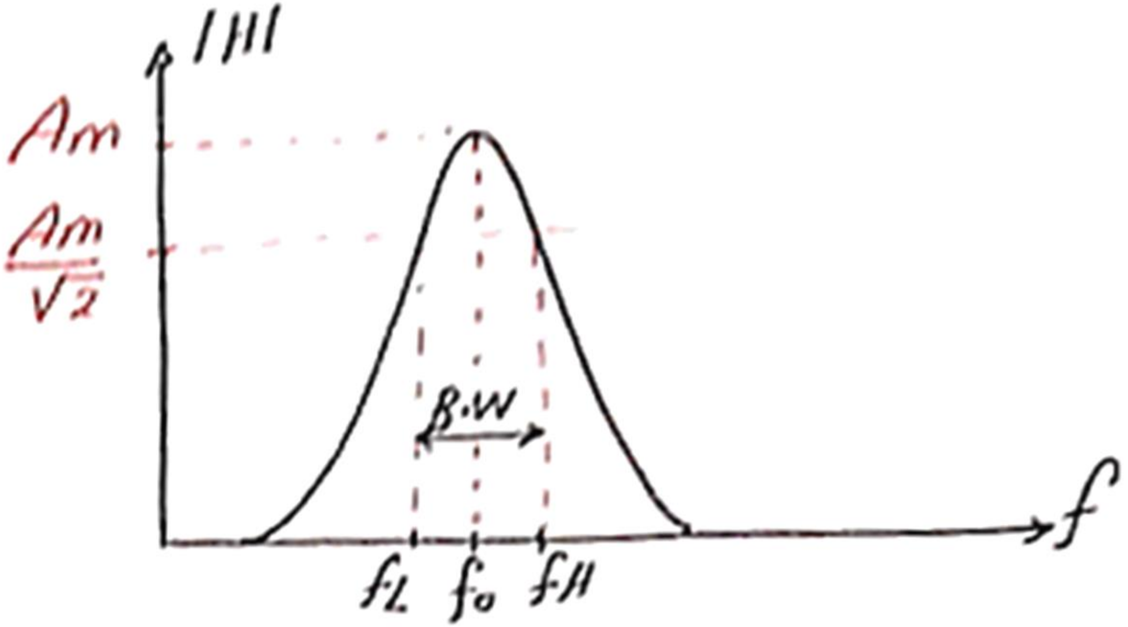
$$|T_{bp}| = \frac{K\omega_0 \omega}{\sqrt{[\omega_0^2 - \omega^2]^2 + \left(\frac{\omega_0 \omega}{Q}\right)^2}}$$

at $\omega = \omega_0$, $|T_{bp}| = A_m$ max. gain

$$\therefore A_m = \frac{K\omega_0 . \omega_0}{\sqrt{\cancel{[\omega_0^2 - \omega_0^2]^2} + \left(\frac{\omega_0 \omega_0}{Q}\right)^2}} = \frac{K\omega_0^2}{\frac{\omega_0^2}{Q}}$$

$$\boxed{A_m = K.Q}$$

- Center Frequency gain(Max. gain)




$$A_m = k \cdot Q = 1.99 \times 10$$

$$\boxed{A_m = 199}$$