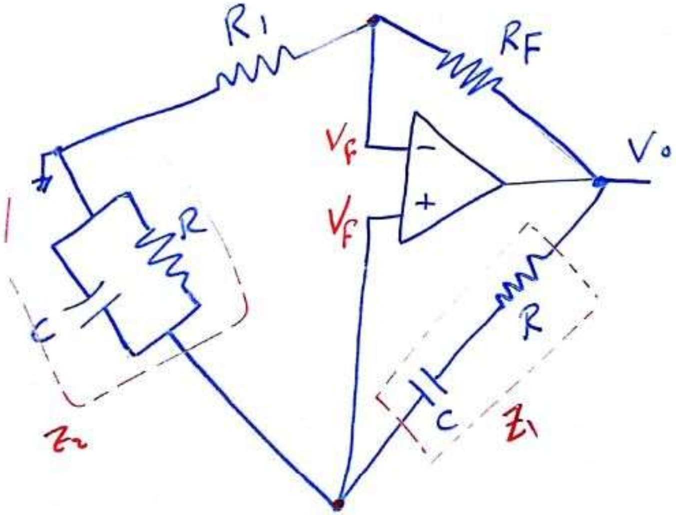
Electronic Circuits Analysis III

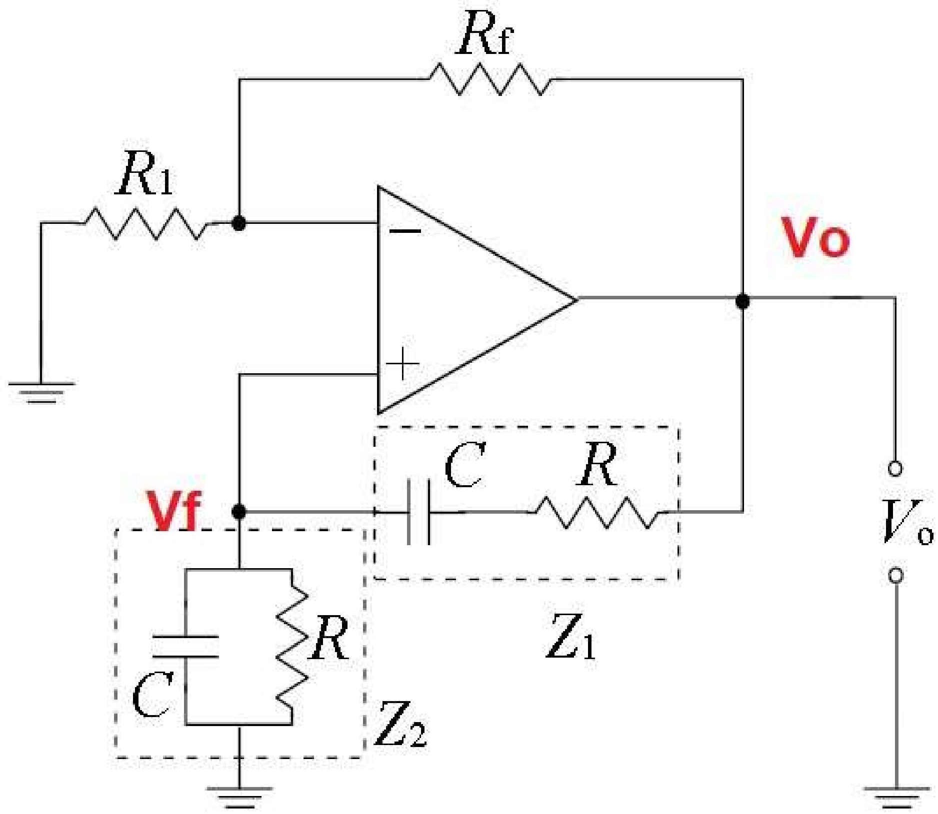
Lecture (2)
Oscillators
Wein-Bridge oscillator
Fall 2020

Wien-Bridge Oscillator



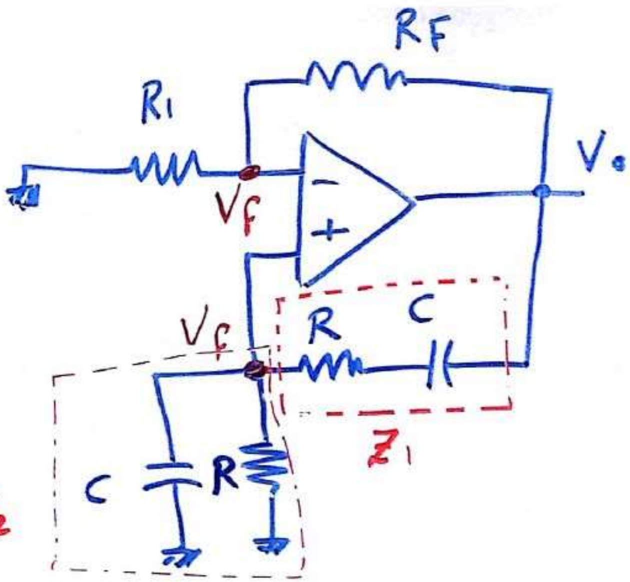
Balanced - Bridge

$$\therefore \frac{R_F}{R_1} = \frac{Z_1}{Z_2}$$



Wien Bridge Oscillator

Proof:

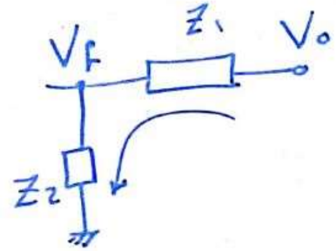


$$A = \frac{V_o}{V_f} \quad , \beta = \frac{V_f}{V_o}$$

$$* \; V_o = \left(1 + \frac{R_F}{R_1}\right) V_f$$

$$\therefore \boxed{A = \frac{V_o}{V_f} = 1 + \frac{R_F}{R_1}}$$

β_ circuit



$$* \; V_f = \frac{V_o}{Z_1 + Z_2} \cdot Z_2$$

$$\therefore \beta = \frac{V_f}{V_o} = \frac{Z_2}{Z_1 + Z_2}$$

For oscillation

$$A = \frac{1}{\beta} = \frac{Z_1 + Z_2}{Z_2} = 1 + \frac{Z_1}{Z_2}$$

$$1 + \frac{R_F}{R_1} = 1 + \frac{Z_1}{Z_2}$$

$$\frac{R_F}{R_1} = \frac{Z_1}{Z_2}$$

$$* \; Z_1 = R + \frac{1}{j\omega C} = R - jX_C$$

$$Z_1 = R + \frac{1}{SC} = \frac{SCR + 1}{SC}$$

$$* \; Z_2 = \frac{R \cdot \frac{1}{SC}}{R + \frac{1}{SC}} = \frac{R}{SCR + 1}$$

$$\frac{R_F}{R_1} = \frac{SCR+1}{SC} \times \frac{(SCR+1)}{R}$$

$$\frac{R_F}{R_1} = \frac{(SCR+1)^2}{SCR}$$

$$\frac{R_F}{R_1} = \frac{S^2C^2R^2+2SCR+1}{SCR}$$

$$\frac{R_F}{R_1} = SCR + 2 + \frac{1}{SCR}$$

$$S = j\omega$$

$$\frac{R_F}{R_1} = j\omega RC + 2 + \frac{1}{j\omega RC}$$

$$\frac{R_F}{R_1} = 2 + j\left[\omega RC - \frac{1}{\omega RC}\right]$$

$$I_m = 0$$

$$\omega RC = \frac{1}{\omega RC}$$

$$\omega^2 = \frac{1}{R^2C^2}$$

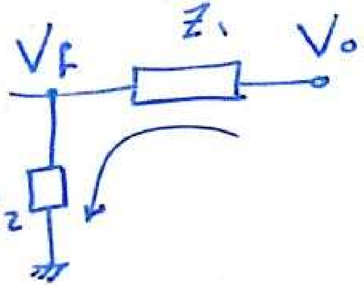
$$\omega = \frac{1}{RC} = 2\pi f_o$$

$$\therefore f_o = \frac{1}{2\pi RC}$$ oscillation Frequency

oscillation Condition

$$\frac{R_F}{R_1} = 2$$

Feedback Factor (β)



* $V_f = \frac{V_o}{Z_1 + Z_2} \cdot Z_2$

$\therefore \beta = \frac{V_f}{V_o} = \frac{Z_2}{Z_1 + Z_2}$

* $Z_2 = \frac{R \cdot \frac{1}{SC}}{R + \frac{1}{SC}} = \frac{R}{SCR + 1}$

* $Z_1 = R + \frac{1}{SC} = \frac{SCR + 1}{SC}$

$$\beta = \frac{\frac{R}{SCR+1}}{\frac{SCR+1}{SC} + \frac{R}{SCR+1}}$$

$$\beta = \frac{R}{\frac{(SCR+1)^2}{SC} + R}$$

$$\beta = \frac{SCR}{(SCR+1)^2 + SCR}$$

$$\beta = \frac{SCR}{S^2C^2R^2 + 2SCR + 1 + SCR}$$

$$\beta = \frac{SCR}{S^2C^2R^2 + 3SRC + 1}$$

Put $S = j\omega$

$$\beta = \frac{j\omega RC}{(j\omega)^2 C^2 R^2 + 3j\omega RC + 1}$$

$$\beta = \frac{j\omega RC}{-\omega^2 C^2 R^2 + 1 + j3\omega RC}$$

$$\beta = \frac{j\omega RC}{[1-\omega^2 C^2 R^2] + j3\omega RC}$$

But $A = 1 + \frac{R_F}{R_1}$ Real,

$A\beta = 1 \therefore \beta$ must be Real

$\beta \rightarrow$ Real

$1 - \omega^2 C^2 R^2 = 0$

$\omega^2 R^2 C^2 = 1 \rightarrow \omega = \frac{1}{RC}$

$$\beta = \frac{\cancel{j}\omega RC}{\cancel{j}3\omega RC} \rightarrow \boxed{\beta = \frac{1}{3}} \therefore \frac{1}{\beta} = 3 = 1 + \frac{R_F}{R_1} \rightarrow \frac{R_F}{R_1} = 2$$

$Im = 0$

$$\omega RC = \frac{1}{\omega RC}$$

$$\omega^2 = \frac{1}{R^2 C^2}$$

$$\omega = \frac{1}{RC} = 2\pi f_o$$

$$\therefore f_o = \frac{1}{2\pi RC}$$ oscillation Frequency

oscillation Condition

$$\frac{R_F}{R_1} = 2$$

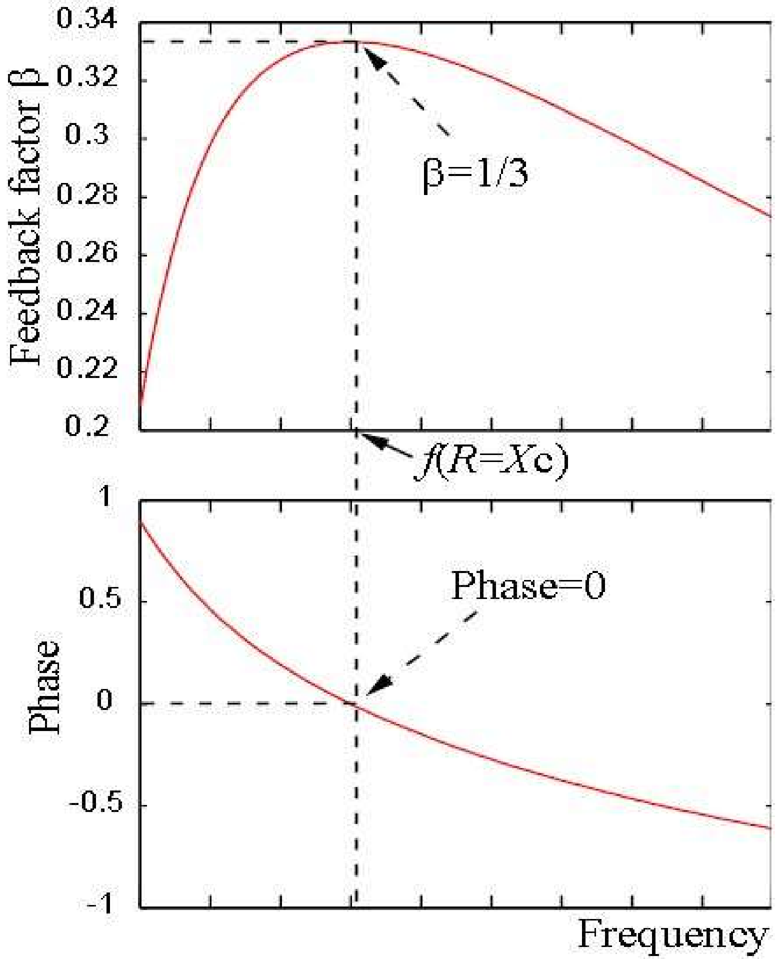
Wien Bridge Oscillator

$$\boxed{\beta = \frac{RX_C}{3RX_C + j(R^2 - X_C^2)}}$$

$$\boxed{\begin{aligned} \angle A &= 0 \\ \angle \beta &= 0 \end{aligned}}$$

Imaginary part of (B) = 0

$$\boxed{\beta = \frac{1}{3}}$$



Wien Bridge Oscillator

$$\boxed{\beta = \frac{1}{3}}$$

Due to ***Barkhausen Criterion***,

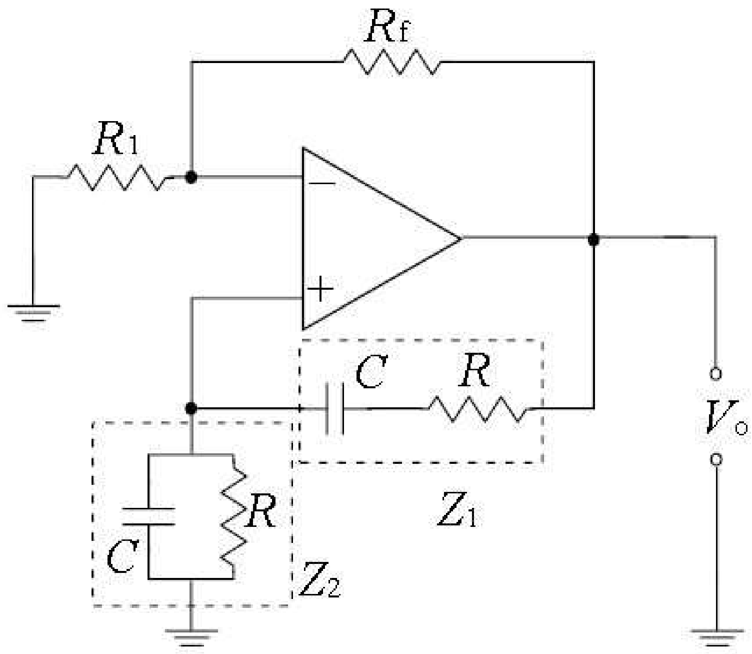
Loop gain $A_v\beta = 1$

where

A_v : Gain of the amplifier

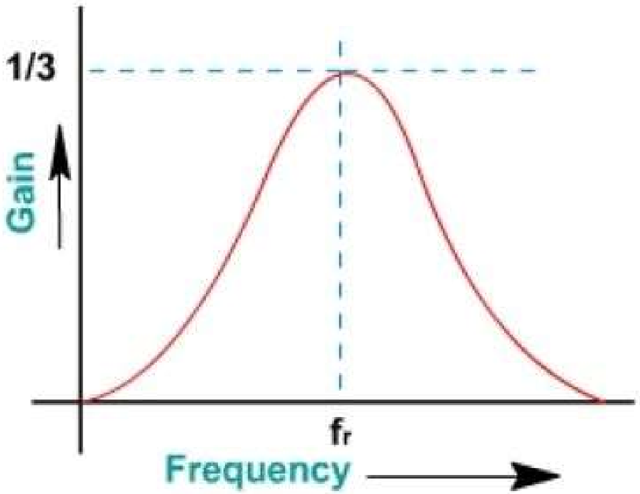
$$A_v\beta = 1 \Rightarrow A_v = 3 = 1 + \frac{R_f}{R_1}$$

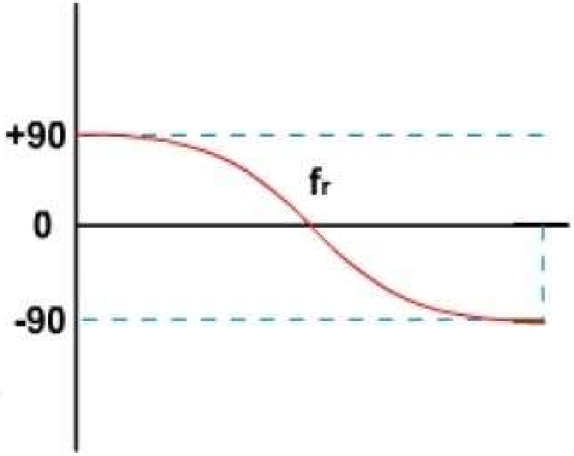
Therefore, $\frac{R_f}{R_1} = 2$



Wien Bridge Oscillator

Wien Bridge Oscillator





$$\boxed{\beta = \frac{1}{3}}$$

$$\omega = 1/\sqrt{R_1 R_2 C_1 C_2}$$

$$\omega = 1/RC \qquad f_r = \frac{1}{2\pi RC}$$

Example

Design a wein-Bridge oscillator of frequency 10 KHZ.

Sol.

$$f_o = \frac{1}{2\pi R C} = 10 \times 10^3$$

let $\boxed{C = 0.01 \mu F}$

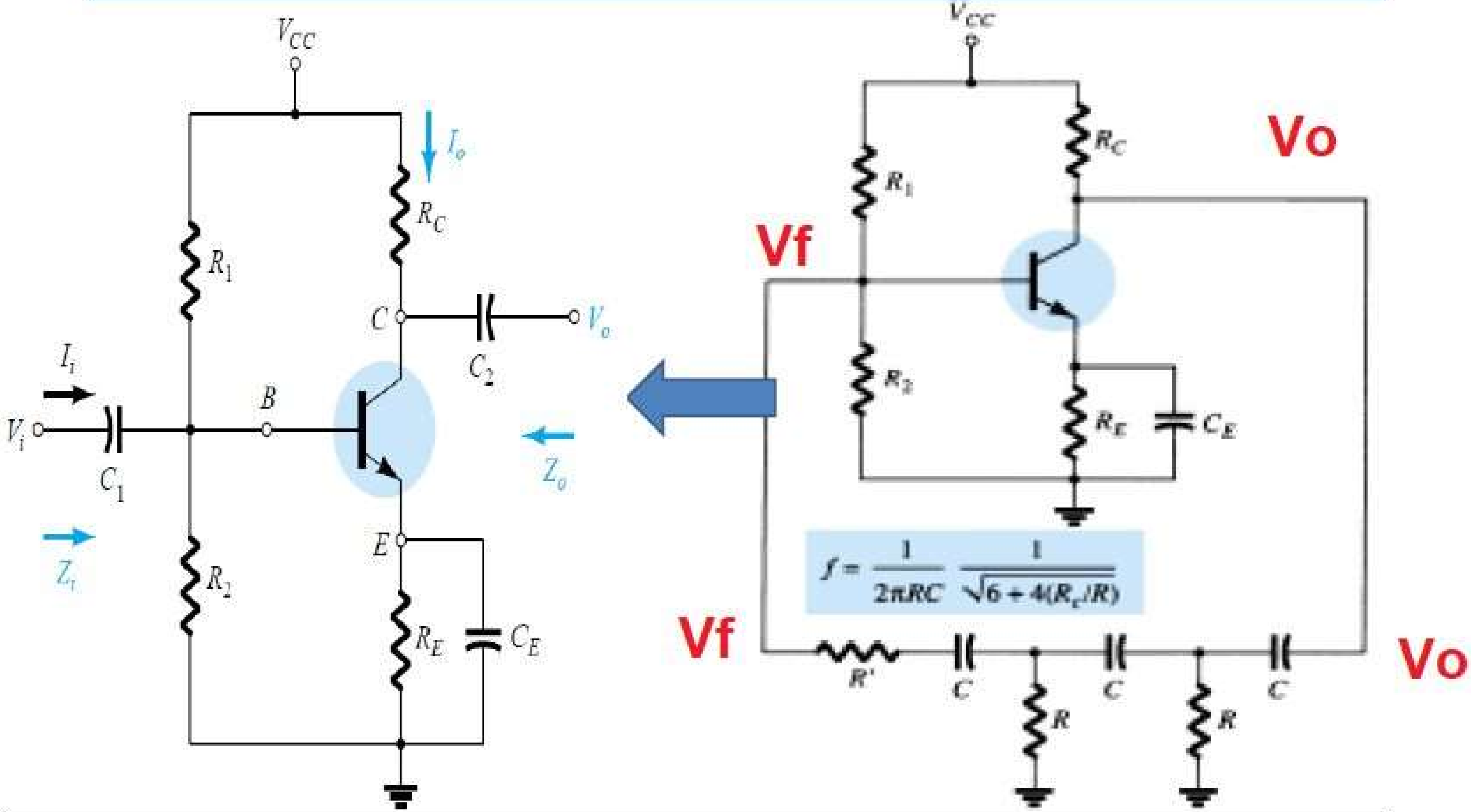
$$10^4 = \frac{1}{2\pi R (0.01 \times 10^{-6})}$$

$$R = 1592 \Omega = 1.592 K\Omega$$

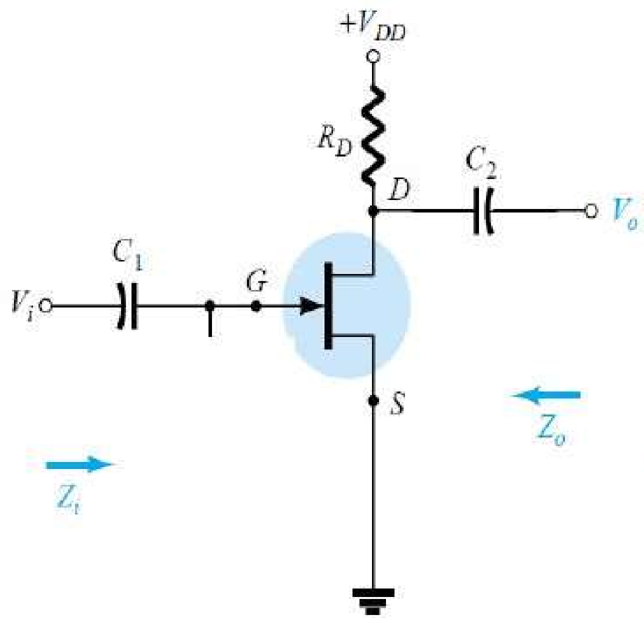
$* \frac{R_F}{R_1} = 2 \rightarrow$ let $\boxed{\begin{array}{l} R_1 = 10 K\Omega \\ R_F = 20 K\Omega \end{array}}$

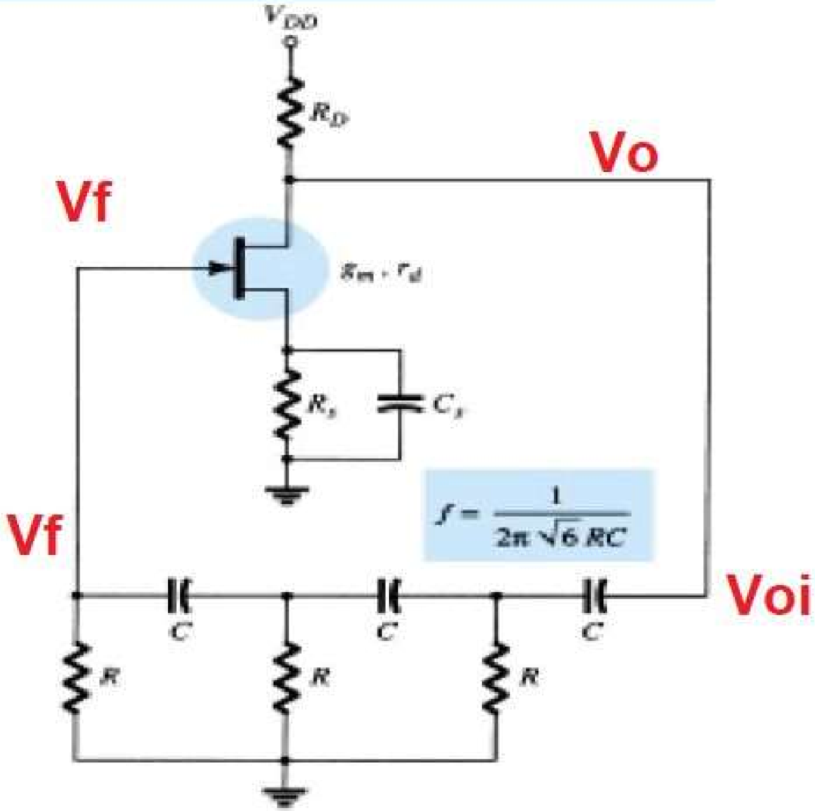
	RC Phase Shift Oscillator	Wien Bridge Oscillator
1.	It is a phase shift oscillator used for low frequency range.	It is also a phase shift oscillator used for low frequency range.
2.	The feedback network is RC network with three RC sections.	The feedback network is lead-lag network which is called Wien bridge circuit.
3.	The feedback network introduces 180° phase shift.	The feedback network does not introduce any phase shift.
4.	Amplifier circuit introduces 180° phase shift.	Amplifier circuit does not introduce any phase shift.
5.	The frequency of oscillations is, $f = \frac{1}{2\pi RC\sqrt{6}}$	The frequency of oscillations is, $f = \frac{1}{2\pi RC}$
6.	The amplifier gain condition is, $\lvert A \rvert \geq 29$	The amplifier gain condition is, $\lvert A \rvert \geq 3$
7.	The frequency variation is difficult.	Mounting the two capacitors on common shaft and varying their values, frequency can be varied.

Transistor Phase-Shift Oscillator



FET Phase-Shift Oscillator

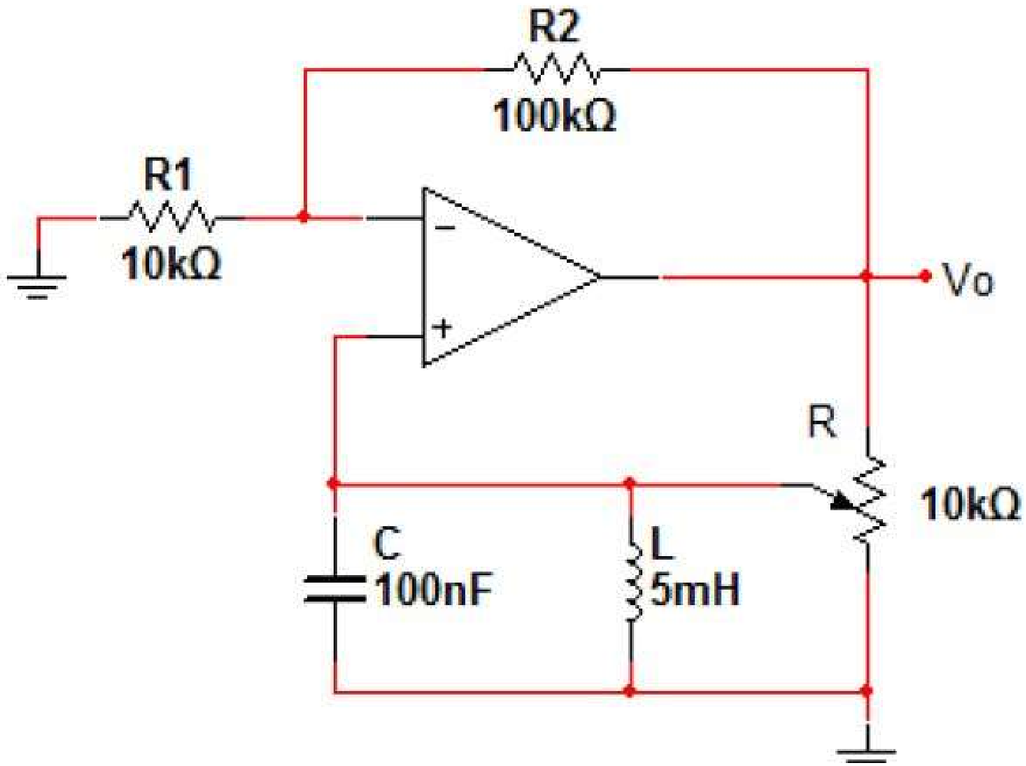




Example

Analyze the oscillator circuit shown in Figure.

1. Derive an expression for the oscillation frequency (f_o).
2. Calculate the minimum value of R for oscillation.
3. Calculate the frequency of oscillation.



Solution

1. Derive an expression for the oscillation frequency (f_o).

* open-Loop gain

$$\boxed{A = 1 + \frac{R_2}{R_1} = \frac{v_o}{v_f}}$$

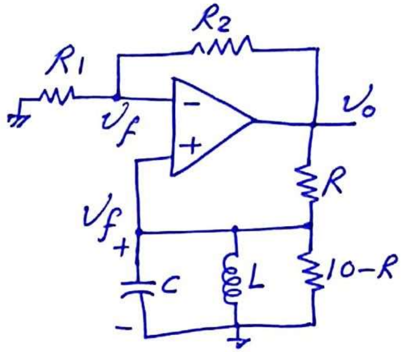
$$* \ \beta = \frac{v_f}{v_o} = \frac{Z}{R+Z}$$

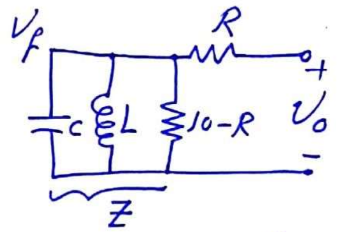
oscillation Condition

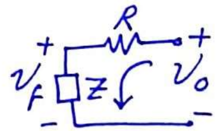
$$A = \frac{1}{\beta} = \frac{R+Z}{Z}$$

$$1 + \frac{R_2}{R_1} = \frac{R}{Z} + 1$$

$$\therefore \boxed{\frac{R}{Z} = \frac{R_2}{R_1}} \quad \boxed{1}$$







$$* \ \frac{1}{Z} = \frac{1}{10-R} + SC + \frac{1}{SL}$$

At oscillation Frequency, $S = j\omega_o$

$$\therefore \frac{1}{Z} = \frac{1}{10-R} + j\omega_o C + \frac{1}{j\omega_o L}$$

$$\therefore \frac{1}{Z} = \frac{1}{10-R} + j\omega_o C - j\frac{1}{\omega_o L}$$

$$\boxed{\frac{1}{Z} = \frac{1}{10-R} + j\left(\omega_o C - \frac{1}{\omega_o L}\right)}$$

sub. into $\boxed{1}$

$$\therefore R\left[\frac{1}{10-R} + j\left(\omega_o C - \frac{1}{\omega_o L}\right)\right] = \frac{R_2}{R_1}$$

$$\frac{R}{10-R} + j\left(\omega_o C - \frac{1}{\omega_o L}\right) = \frac{R_2}{R_1} + j0$$

$$\therefore \omega_o C - \frac{1}{\omega_o L} = 0$$

$$\therefore \omega_o^2 = \frac{1}{L \cdot C} \rightarrow \omega_o = \frac{1}{\sqrt{L \cdot C}} = 2\pi f_o$$

$$\therefore f_o = \frac{1}{2\pi\sqrt{L \cdot C}}$$ oscillation Frequency

Condition $$\boxed{\frac{R}{10-R} = \frac{R_2}{R_1}}$$

2. **Calculate the minimum value of R for oscillation.**

$$\frac{R}{10-R} = \frac{R_2}{R_1} = \frac{100}{10} = 10$$

$$\therefore \quad 10(10-R) = R$$

$$100 - 10R = R$$

$$100 = 11R$$

$$R = \frac{100}{11} = 9.091\ k\Omega \ \#$$

$\Rightarrow$ min. value of R to sustain oscillation.

3. **Calculate the frequency of oscillation.**

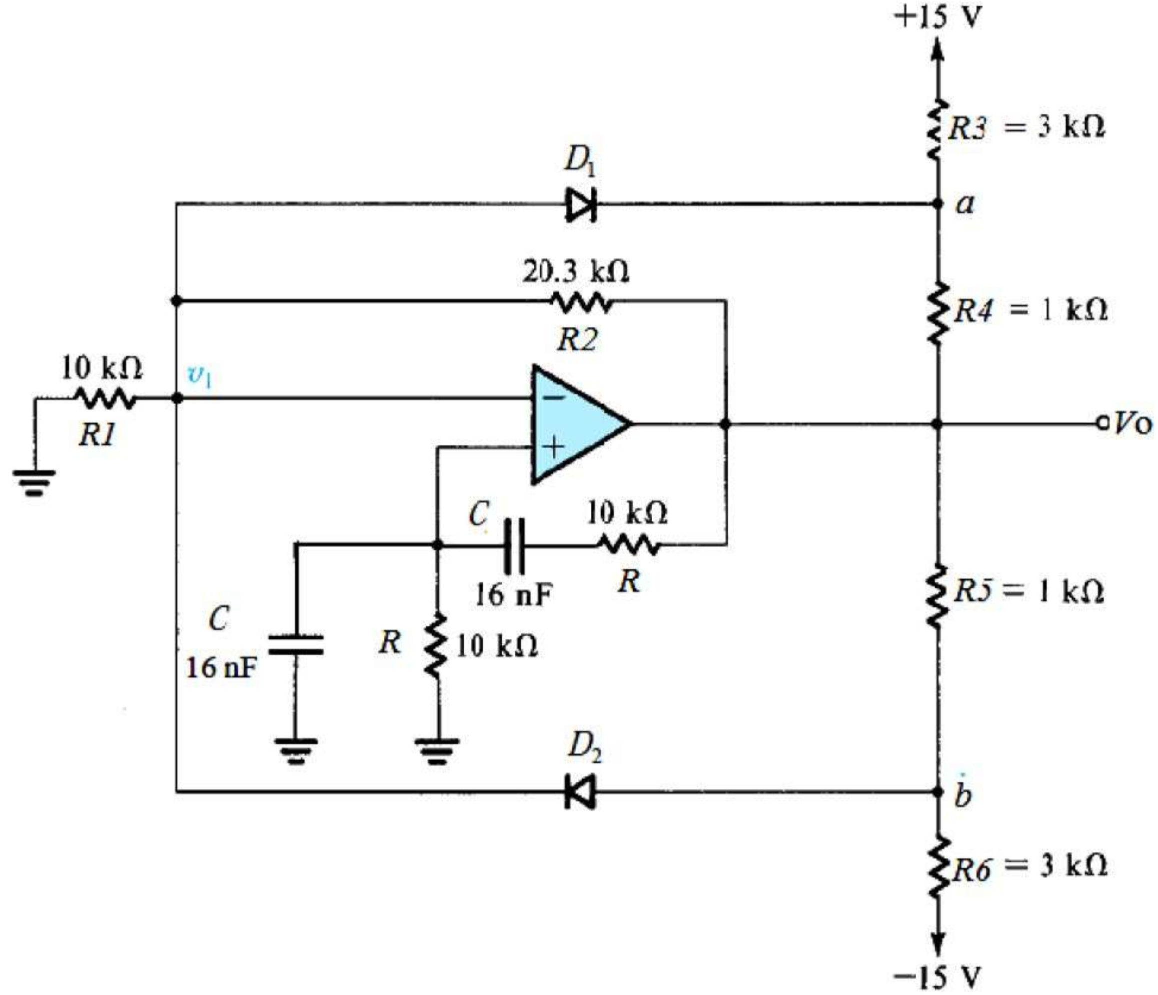
$$f_o = \frac{1}{2\pi\sqrt{L.C}} = \frac{1}{2\pi\sqrt{(5\times10^{-3})(100\times10^{-9})}}$$

$$f_o = 7.11763\ KHz \ \#$$

Example

Analyze the Wien-bridge oscillator with amplitude limiter shown in Figure

1. Derive an expression for the oscillation frequency (f_o).
2. Calculate the frequency of oscillation.
3. Calculate the peak-to-peak output voltage ($V_{o\text{-}pp}$)



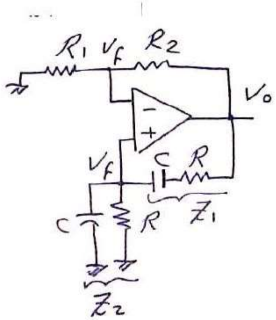
* $\boxed{A = \frac{V_o}{V_f} = 1 + \frac{R_2}{R_1}}$

* $\beta = \frac{V_f}{V_o} = \frac{Z_2}{Z_1 + Z_2}$

* $\frac{1}{\beta} = \frac{Z_1 + Z_2}{Z_2} = 1 + \frac{Z_1}{Z_2}$

$A.\beta = 1 \rightarrow A = \frac{1}{\beta}$

$1 + \frac{R_2}{R_1} = 1 + \frac{Z_1}{Z_2}$



$$\frac{R_2}{R_1} = \frac{Z_1}{Z_2}$$

* $Z_1 = R + \frac{1}{sC} = \frac{sCR+1}{sC}$

* $Z_2 = \frac{R \, \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{R}{sCR+1}$

$\therefore \frac{R_2}{R_1} = \frac{sCR+1}{sC} \cdot \frac{sCR+1}{R}$

$$\frac{R_2}{R_1} = \frac{s^2C^2R^2 + 2sCR + 1}{sCR}$$

$\frac{R_2}{R_1} = sCR + 2 + \frac{1}{sCR}$ put $s = j\omega$

$$\frac{R_2}{R_1} = j\omega_o RC + 2 + \frac{1}{j\omega_o RC}$$

$$\frac{R_2}{R_1} = j\omega_o RC + 2 - j\frac{1}{\omega_o RC}$$

$$\frac{R_2}{R_1} = 2 + j\left[\omega_o RC - \frac{1}{\omega_o RC}\right]$$

$\therefore \boxed{\frac{R_2}{R_1} = 2}$ oscillation condition.

$\therefore \omega_o RC = \frac{1}{\omega_o RC}$

$$\omega_o^2 = \frac{1}{(RC)^2}$$

$$\omega_o = \frac{1}{RC}$$

$$2\pi f_o = \frac{1}{RC}$$

$\therefore f_o = \frac{1}{2\pi RC}$ # oscillation frequency

2. Calculate the frequency of oscillation.

$$f_o = \frac{1}{2\pi (10^4)(16 \times 10^{-9})}$$

$$\boxed{f_o = 994.72 \text{ Hz}}$$

3. Calculate the peak-to-peak output voltage ($V_{o\text{-pp}}$)

* As V_o goes positive

$$V_X = \frac{V_o}{R_1 + R_2} \cdot R_2 = \frac{V_o}{10 + 20.3} \cdot 10 = 0.33 V_o$$

$$\boxed{V_X = 0.33 V_o} \; ① \quad 0.5$$

$$* \; V_b = \frac{-V_{CC} \cdot R_5}{R_5 + R_6} + \frac{V_o R_6}{R_5 + R_6}$$

$$V_b = \frac{-15 \times 1}{1 + 3} + \frac{V_o \times 3}{1 + 3}$$

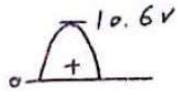
$$\boxed{V_b = -3.75 + 0.75 V_o} \; ② \quad 0.5$$

For D_2 to be ON $\quad V_b - V_X = 0.7 V$

$$(-3.75 + 0.75 V_o) - (0.33 V_o) = 0.7$$

$$0.42 V_o = 4.45$$

$$\boxed{V_o \cong 10.6 V} \; (+\text{ve peak})$$



* As V_o goes negative

$$V_a = \frac{V_{CC} R_4}{R_3 + R_4} + \frac{V_o R_3}{R_3 + R_4}$$

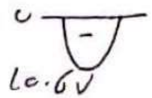
$$V_a = \frac{15 \times 1}{4} + \frac{3 V_o}{4}$$

$$\boxed{V_a = 3.75 + 0.75 V_o} \; ③$$

For D_1 to be ON $\quad V_X - V_a = 0.7$

$$0.33 V_o - (3.75 + 0.75 V_o) = 0.7$$

$$\boxed{V_o = -10.6 V} \; (-\text{ve peak})$$



∴ The peak-to-peak output voltage is

$$V_{opp} = 10.6 - (-10.6)$$

$$\boxed{V_{opp} = 21.2 V} \; \#$$