Electronic Systems

Active Filters

Lecture 8

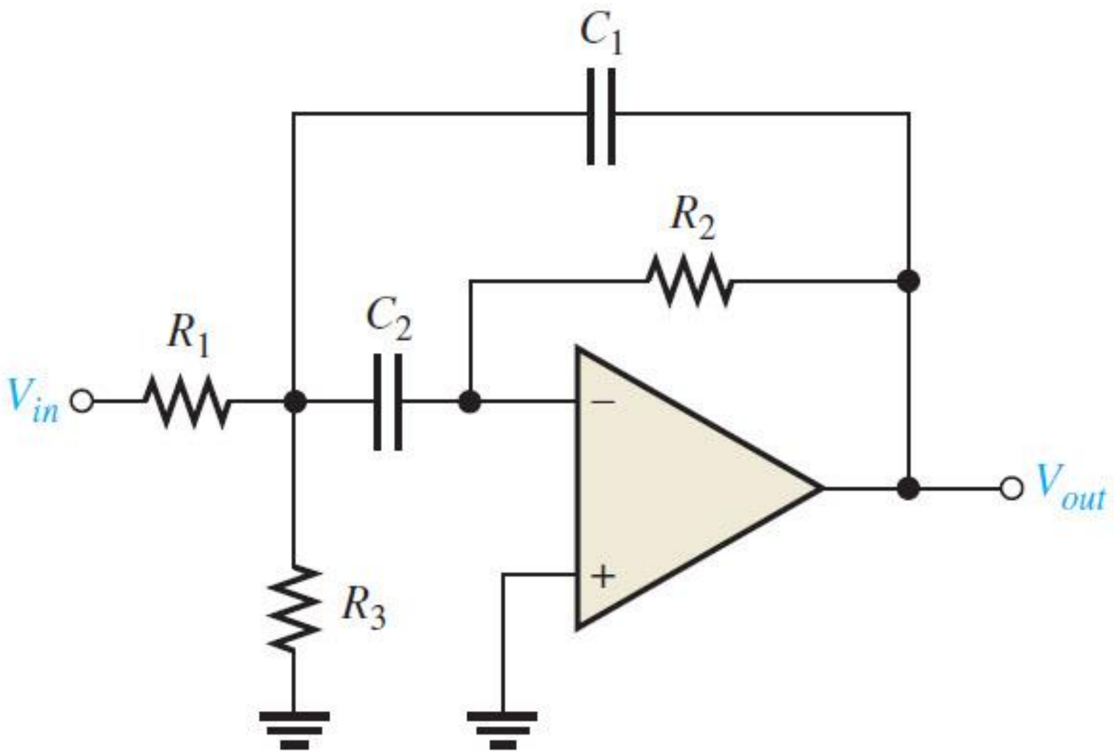
Dr. Roaa Mubarak

The Active Filters Contents:

1. Introduction to Filters.
2. Low Pass Filter.
3. High Pass Filter.
4. Band Pass Filter.
5. Butterworth Filter.
6. Chebyshev Filter.
7. Bessel Filter.
8. KHN Biquad Filter.
9. State Variable Filters.
10. **Multiple Feedback Filters.**

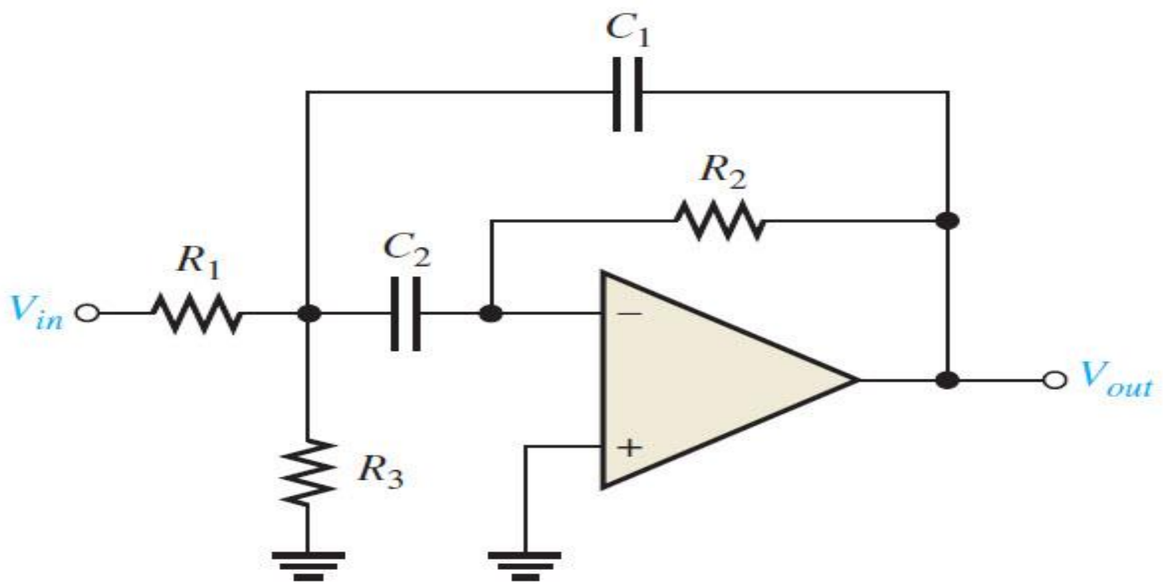
Multiple Feedback single op-amp Filter

Multiple Feedback BPF



- The multiple feedback band-pass filter circuit has two feedback, the path from output via R2 back to input and the path from output via C1 back to the input. That is why it is referred as multiple-feedback. The circuit has both low pass filter and high pass filter. The capacitor C1 and resistor R1 forms the LPF while the capacitor C2 and R2 forms the HPF.

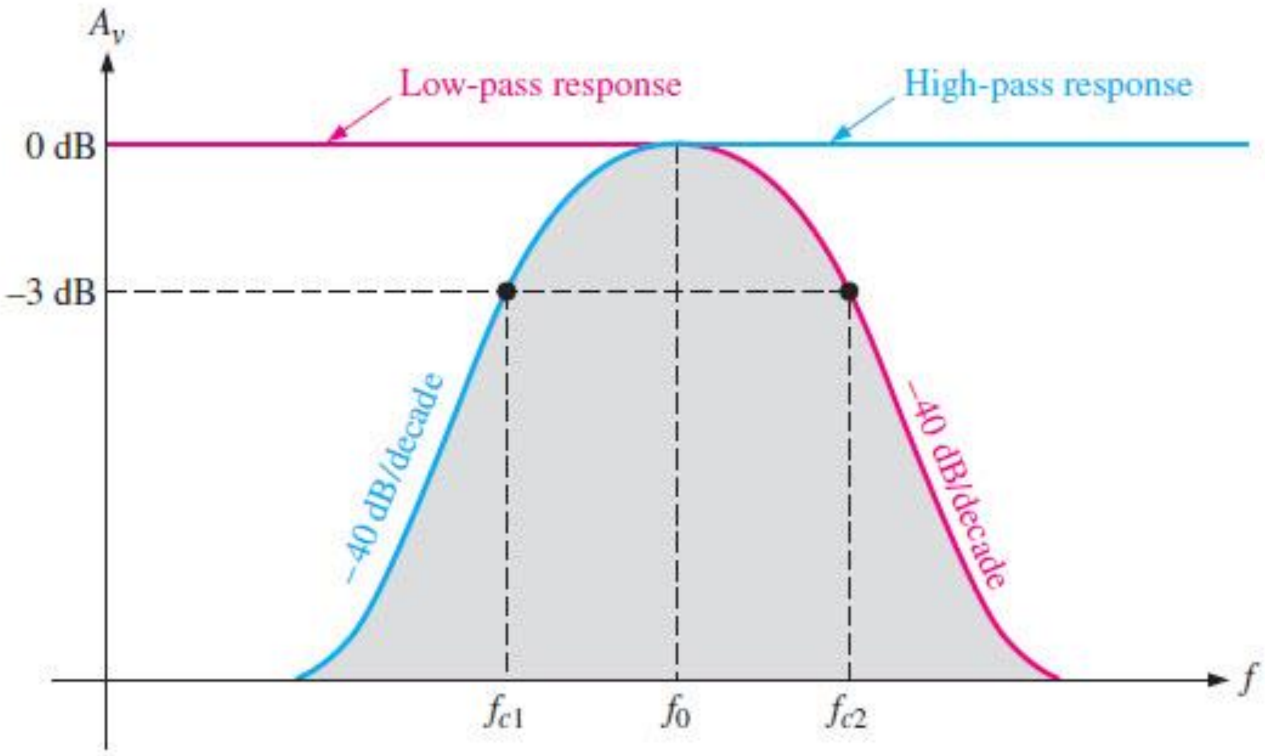
Multiple Feedback BPF



- LPF: R1, C1
- HPF: C2, R2

- The 2 feedback paths moving through the resistance R2 and capacitor C1.
- The elements Resistance R1 and C1 offer the low pass response and resistance R2 and capacitor C2 offer the high pass response.
- The extreme gain value is A0, which exists at the center frequency. Q values of lesser than the ten generally exist in this category of the filter.
- The relation for the mid-frequency is created as regarding the resistance R1 and R3 indicates in parallel combination are shown from the capacitor C1.

Multiple Feedback BPF



- The center frequency fo of the band-pass filter can be expressed as,

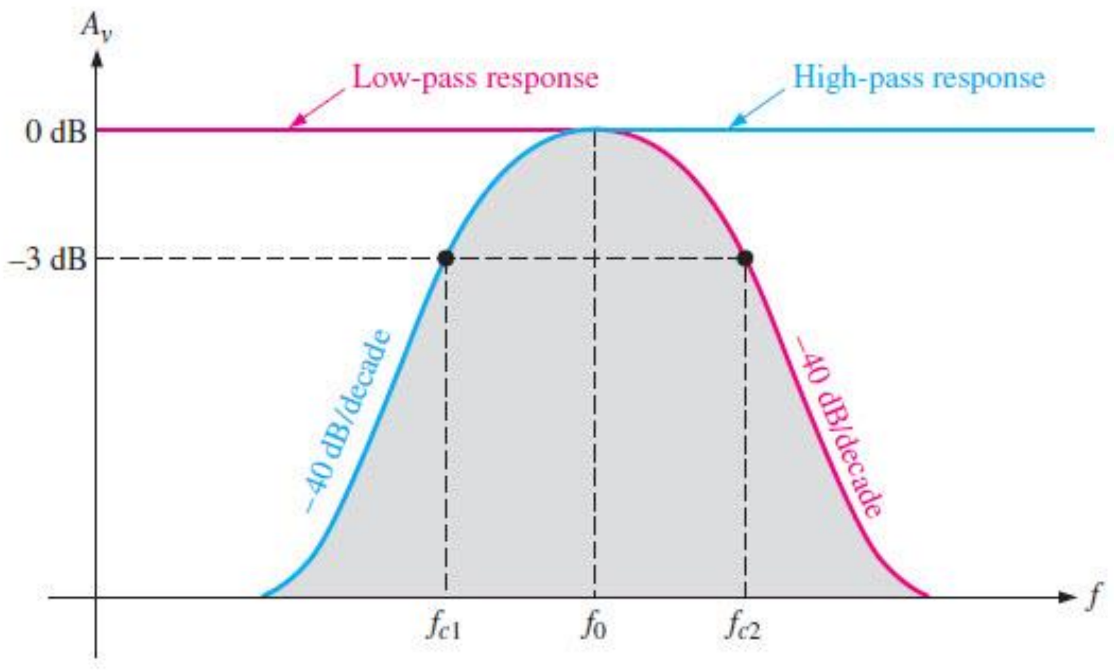
$$f_c = \frac{1}{2\pi\sqrt{(R_1 || R_3)R_2 C_1 C_2}}$$

Multiple Feedback BPF

- if $C_1 = C_2 = C$
- $F_c = \dfrac{1}{2\pi\sqrt{(R_1 \| R_3)R_2C^2}} = \dfrac{1}{2\pi C\sqrt{(R_1 \| R_3)R_2}}$

$$= \frac{1}{2\pi C\sqrt{\frac{(R_1R_3)R_2}{(R_1+R_3)}}}$$

$$f_c = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_3}{R_1R_2R_3}}$$



Multiple Feedback BPF

- The **Quality factor** or **Q-factor** is,

$$Q = \frac{f_0}{BW}$$

$$Q = \frac{f_0}{f_{c2} - f_{c1}}$$

- Without derivation, the value of the three **resistors R1,R2 and R3** are as follows,

$$R_1 = \frac{Q}{2\pi f_0 C A_0}$$

$$R_2 = \frac{Q}{\pi f_0 C}$$

$$R_3 = \frac{Q}{2\pi f_0 C(2Q^2 - A_0)}$$

Multiple Feedback BPF

- For the creation of the gain values, we solve Q in the expression or resistance R1 and R2.

$$Q = 2\pi f_0 A_0 C R_1$$

$$Q = \pi f_0 C R_2$$

so

$$2\pi f_0 A_0 C R_1 = \pi f_0 C R_2$$

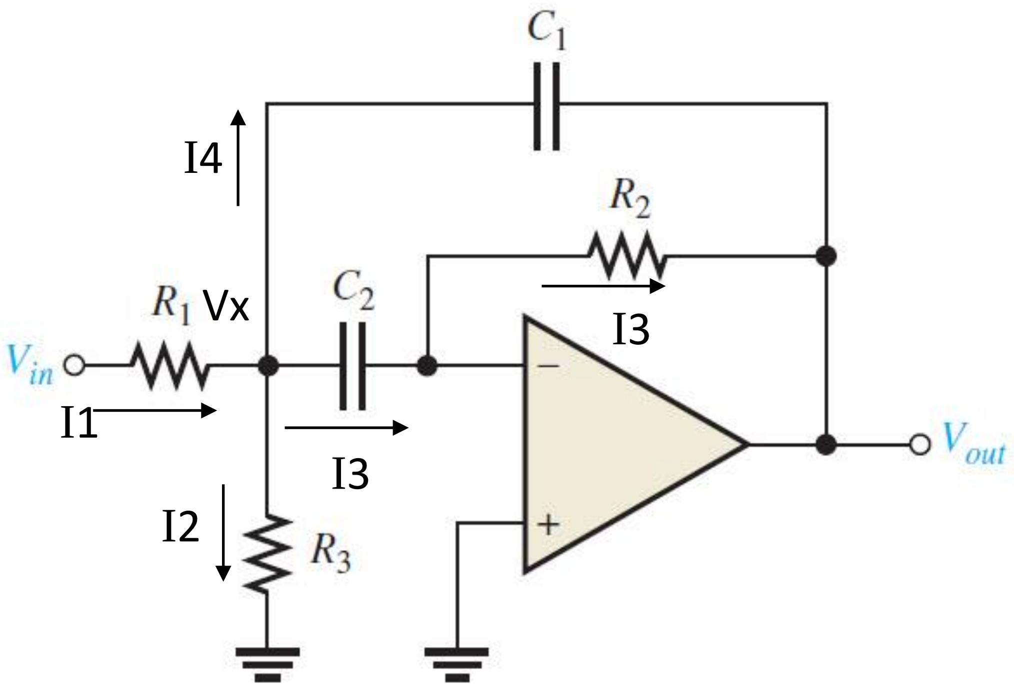
$$2A_0 R_1 = R_2$$

$$A_0 = R_2/2R_1$$

$$A_0 = \frac{R_2}{2R_1}$$

Multiple Feedback BPF

Proof: $$f_c = \frac{1}{2\pi\sqrt{(R_1 \| R_3) R_2 C_1 C_2}}$$



* $$I_3 = \frac{V_x - 0}{1/SC_2} = \frac{0 - V_o}{R_2}$$

$$SC_2 V_x = -\frac{V_o}{R_2}$$

$$\boxed{V_x = -\frac{V_o}{SC_2 R_2}}$$

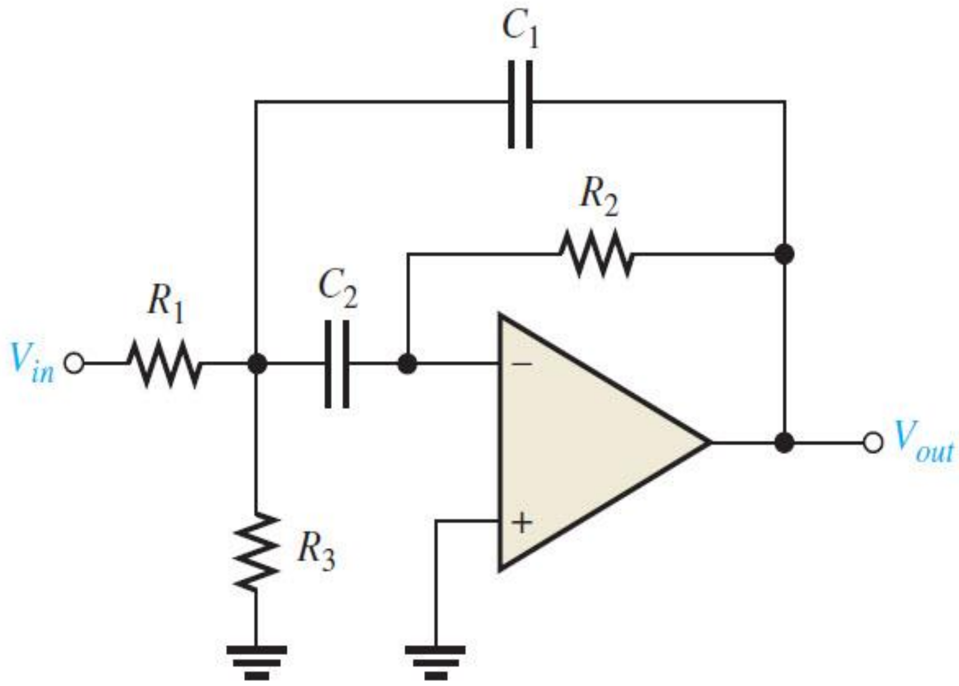
* Node (V_x)

$$I_1 = I_2 + I_3 + I_4$$

$$\frac{V_{in} - V_x}{R_1} = \frac{V_x}{R_3} + \frac{V_x - 0}{1/SC_2} + \frac{V_x - V_o}{1/SC_1}$$

$$\frac{V_{in}}{R_1} - \frac{V_x}{R_1} = \frac{V_x}{R_3} + SC_2 V_x + SC_1 V_x - SC_1 V_o$$

Multiple Feedback BPF



$$\frac{V_{in}}{R_1} = V_x\left[\frac{1}{R_1} + \frac{1}{R_3} + sC_2 + sC_1\right] - sC_1V_o$$

$$V_{in} = V_x\left[\left(1 + \frac{R_1}{R_3}\right) + sC_2R_1 + sC_1R_1\right] - sC_1R_1V_o$$

$$= \frac{-V_o}{sC_2R_2}\left[\left(1 + \frac{R_1}{R_3}\right) + sC_2R_1 + sC_1R_1\right] - sC_1R_1V_o$$

$$-V_o\left[\frac{\left(1+\frac{R_1}{R_3}\right) + (C_1R_1 + C_2R_1)s}{sC_2R_2} + sC_1R_1\right]$$

$$= -V_o\left[\frac{\left(1+\frac{R_1}{R_3}\right) + (C_1R_1+C_2R_1)s + s^2C_1C_2R_1R_2}{sC_2R_2}\right]$$

$$A_v = H(s) = \frac{V_o}{V_{in}} = \frac{-sC_2R_2}{s^2C_1C_2R_1R_2 + (C_1R_1+C_2R_1)s + \left(\frac{R_1+R_3}{R_3}\right)}$$

$$A_v = \frac{-sC_2R_2}{\frac{R_1+R_3}{R_3}\left[s^2\frac{R_3}{R_1+R_3}\cdot C_1C_2R_1R_2 + \frac{C_1R_1+C_2R_1}{R_1+R_3}\cdot R_3 s + 1\right]}$$

Multiple Feedback BPF

$$A_v = \frac{-sC_2 \dfrac{R_2 R_3}{R_1 + R_3}}{s^2 (R_1 \| R_3) C_1 C_2 R_2 + \dfrac{R_1 C_1 + R_1 C_2}{R_1 + R_3} R_3 s + 1}$$

$$A_v = \frac{-a_1 s}{\left(\frac{s}{\omega_o}\right)^2 + 2K\left(\frac{s}{\omega_o}\right) + 1}$$

$$\frac{1}{\omega_o^2} = (R_1 \| R_3) C_1 C_2 R_2$$

$$\omega_o = \frac{1}{\sqrt{(R_1 \| R_3) R_2 C_1 C_2}}$$

$$\therefore f_o = \frac{1}{2\pi \sqrt{(R_1 \| R_3) R_2 C_1 C_2}}$$

Multiple Feedback BPF

Example 1:

Design a multiple-feedback band pass active filter using parameters value as following. For design simplification, assume equal value capacitors are 0.01μF. Illustrate the circuit designed and label all the circuit components.

F_o =25 KHZ, BW = 500, A_o = 3.98

Multiple Feedback BPF

Solution

i) Find Q value,

$$C_1 = C_2 = C = 0.01\mu F$$

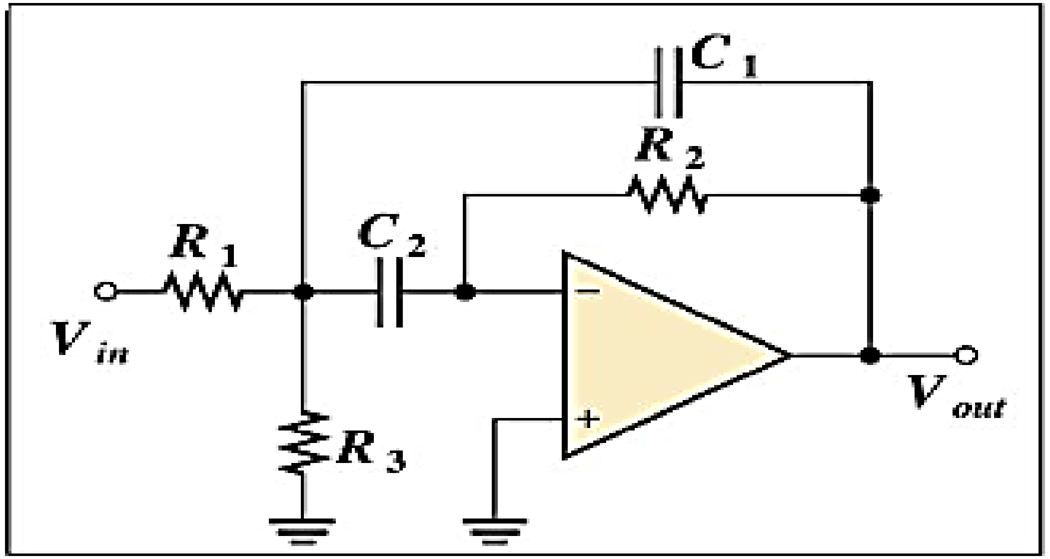
$$Q = \frac{f_o}{BW} = \frac{25k}{500} = 50$$

ii) Find R1, R2 and R3

$$R_1 = \frac{Q}{2\pi f_o C A_o} = 8k\Omega$$

$$R_2 = \frac{Q}{\pi f_o C} = 63.66k\Omega$$

$$R_3 = \frac{Q}{2\pi f_o C\left(2Q^2 - A_o\right)} = 6.37\Omega$$



Thus,

$i)R_1 = 8k\Omega$

$ii)R_2 = 63.66k\Omega$

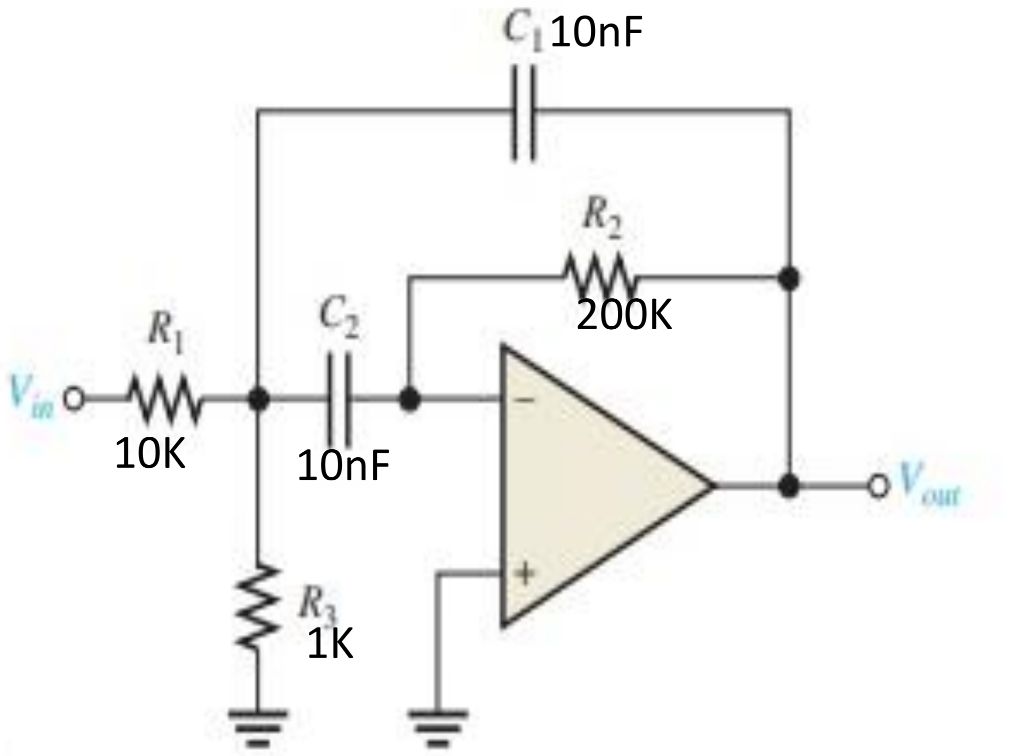
$iii)R_3 = 6.37\Omega$

$iv)C_1 = 0.01\mu F$

$v)C_2 = 0.01\mu F$

Multiple Feedback BPF

Example 2:



- Prove $F_o = \dfrac{1}{2\pi\sqrt{(R1IIR3)R2C1C2}}$
- Calculate:

❑Center Frequency gain (Ao)

❑Quality Factor (Q)

❑Band-width (B.W)

❑Lower and higher cut-off Frequencies (F_1'F_L' &F_2 'F_H')

Solution

[1] Prove ✓

[2] * $A_o = \frac{R_2}{2R_1} = \frac{200}{2\times10}$

$$\boxed{A_o = 10}$$

$$f_o = \frac{1}{2\pi C\sqrt{(R_1 \parallel R_3) R_2}}$$

$$f_o = \frac{1}{2\pi(10\times10^{-9})\sqrt{(\frac{10}{11}\times10^{3})(200\times10^{3})}}$$

$$\boxed{f_o = 1180.3\ Hz}$$

* $Q = \pi f_o R_2 C = \pi(1180.3)(200\times10^{3})(10\times10^{-9})$

$$\boxed{Q = 7.42}$$

* $B.\omega = \frac{f_o}{Q} = \frac{1180.3\ Hz}{7.42}$

$$\boxed{B.\omega = 159.1\ Hz}$$

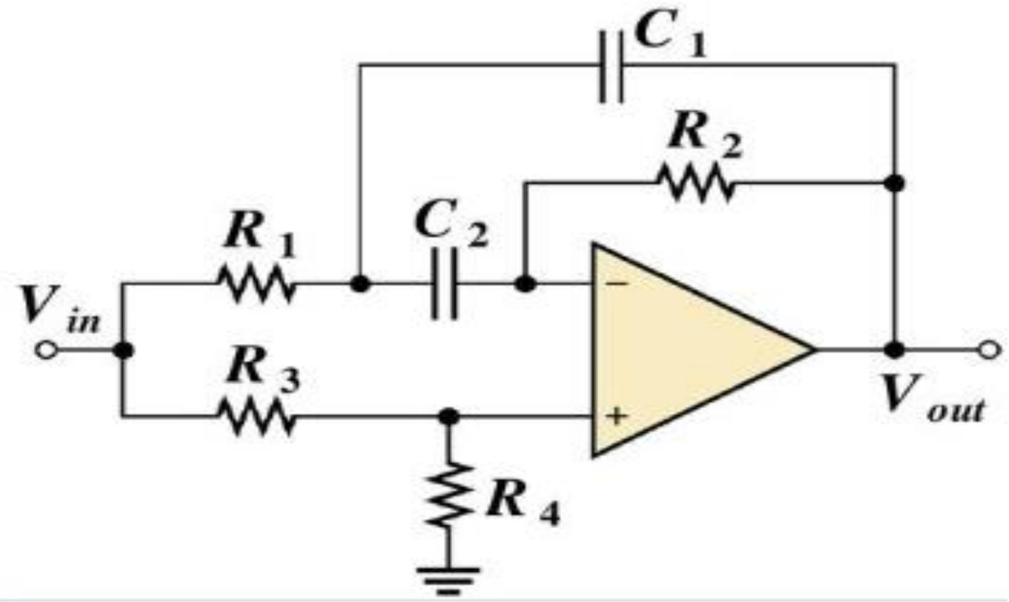
* $f_1 = f_L = f_o - \frac{B.\omega}{2} \cong 1260\ Hz$

* $f_2 = f_H = f_o + \frac{B.\omega}{2} = 1419\ Hz$

Multiple Feedback BSF

$$f_o = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

- LPF: R1, C1
- HPF: C2, R2



- The configuration is similar to the band-pass version BUT R_3 has been moved and R_4 has been added.
- The BSF is opposite of BPF in that it blocks a specific band of frequencies

$$\frac{\omega_o}{Q} = \frac{R_1C_1 + R_1C_2}{R_1R_2C_1C_2}$$

$$Q = \frac{\sqrt{\frac{R_2}{R_1}}}{\sqrt{\frac{C_2}{C_1}} + \sqrt{\frac{C_1}{C_2}}}$$

Multiple Feedback BSF

$$* \; V_1 = I_2 . R_4 = \frac{V_{in}}{R_3 + R_4} R_4$$

$$\boxed{V_1 = \frac{R_4}{R_3 + R_4} . V_{in} = k \; V_{in}} \quad \boxed{1}$$

$$* \; I_3 = \frac{V_2 - V_1}{1/SC_2} = \frac{V_1 - V_o}{R_2}$$

$$S C_2 V_2 - S C_2 V_1 = \frac{V_1 - V_o}{R_2} \qquad \times R_2$$

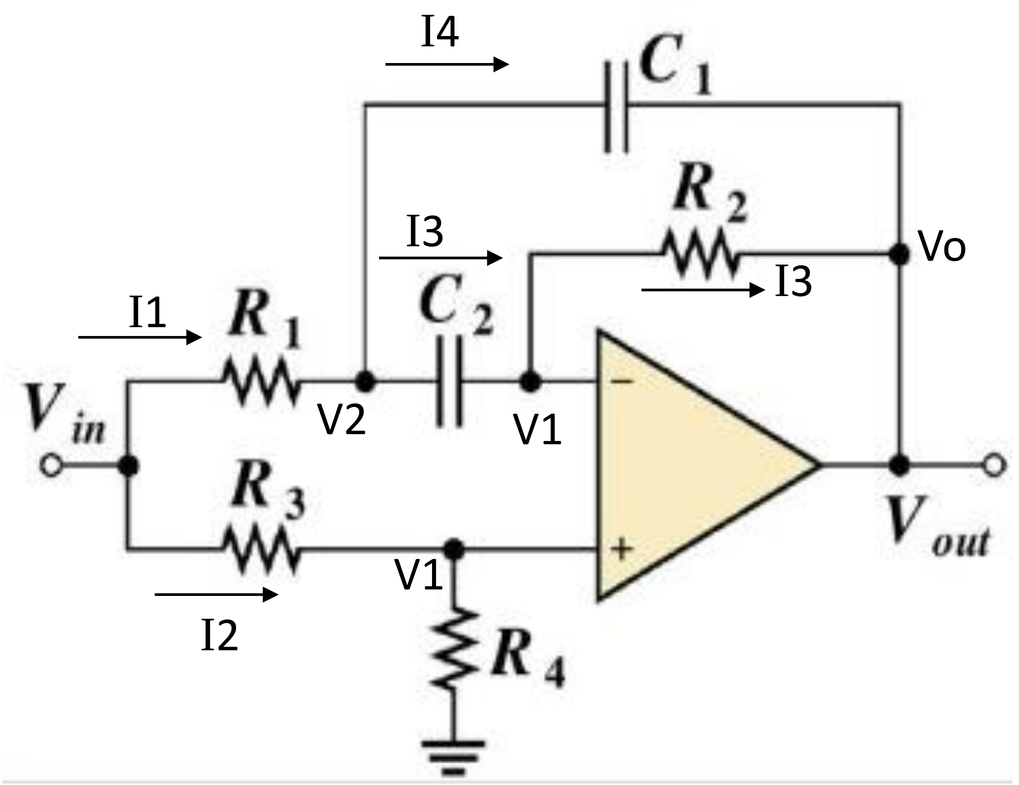
$$S C_2 R_2 V_2 - S C_2 R_2 V_1 = V_1 - V_o$$

$$V_o = [1 + S C_2 R_2] V_1 - S C_2 R_2 V_2$$

$$V_o = [1 + S C_2 R_2] k V_{in} - S C_2 R_2 V_2$$

$$S C_2 R_2 V_2 = [1 + S C_2 R_2] k \; V_{in} - V_o$$

$$\therefore \boxed{V_2 = \frac{k[1 + S C_2 R_2]}{S C_2 R_2} V_{in} - \frac{1}{S C_2 R_2} V_o} \quad \boxed{2}$$



Multiple Feedback BSF

$$I_1 = I_3 + I_4$$

$$\frac{V_{in} - V_2}{R_1} = \frac{V_2 - V_1}{1/sC_2} + \frac{V_2 - V_o}{1/sC_1}$$

$$\frac{V_{in} - V_2}{R_1} = sC_2 V_2 - sC_2 V_1 + sC_1 V_2 - sC_1 V_o$$

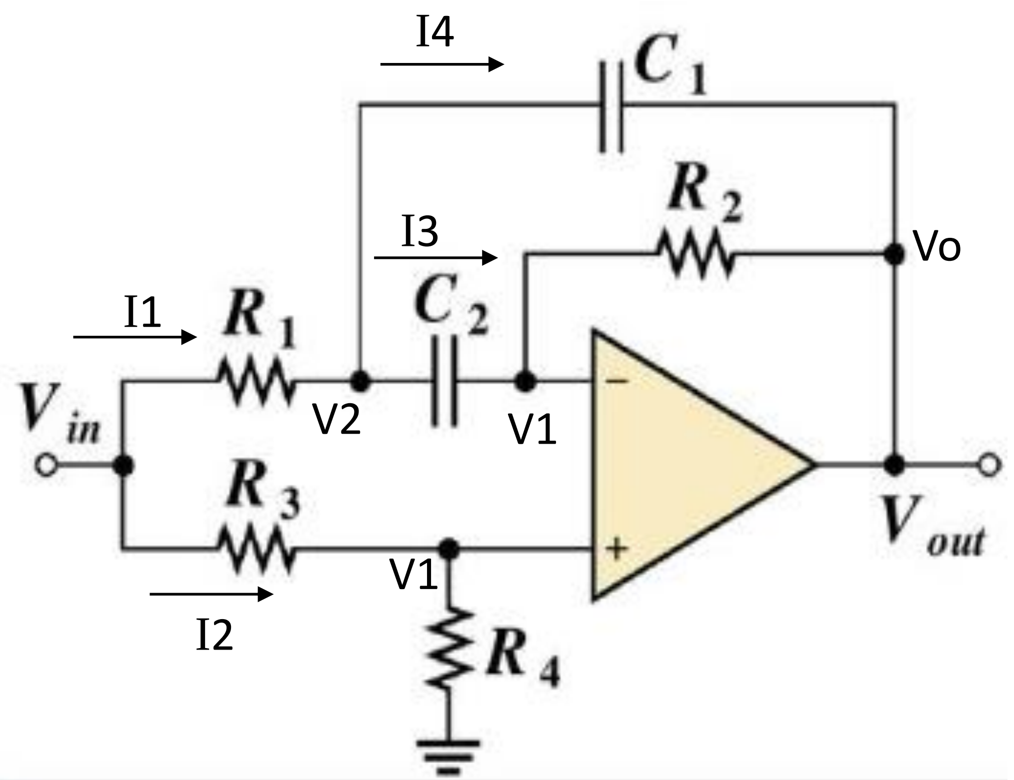
$$V_{in} - V_2 = sC_2 R_1 V_2 - sC_2 R_1 V_1 + sC_1 R_1 V_2 - sC_1 R_1 V_o$$

$$V_{in} = [1 + sC_2 R_1 + sC_1 R_1] V_2 - sC_2 R_1 V_1 - sC_1 R_1 V_o$$

$$V_{in} = [1 + sC_1 R_1 + sC_2 R_1] \left\{ \frac{K(1 + sC_2 R_2)}{sC_2 R_2} V_{in} - \frac{1}{sC_2 R_2} V_o \right\} - sC_2 R_1 V_1 - sC_1 R_1 V_o$$

$$V_{in} = \frac{(1 + sC_1 R_1 + sC_2 R_1)(1 + sC_2 R_2)}{sC_2 R_2} V_{in} k - \frac{1 + sC_1 R_1 + sC_2 R_1}{sC_2 R_2} V_o - sC_2 R_1 V_1 - sC_1 R_1 V_o$$

$$sC_2 R_2 V_{in} = (1 + sC_1 R_1 + sC_2 R_1)(1 + sC_2 R_2) k V_{in} - (1 + sC_1 R_1 + sC_2 R_1) V_o - s^2 C_2^2 R_1 R_2 k V_{in} - s^2 C_1 C_2 R_1 R_2 V_o$$



Multiple Feedback BSF

$$V_o\{1+sC_1R_1+sC_2R_1+s^2R_1R_2C_1C_2\}$$

$$=\{K(1+sC_1R_1+sC_2R_1)(1+sC_2R_1)-sC_2R_2-s^2C_2^2R_1R_2K\}V_{in}$$

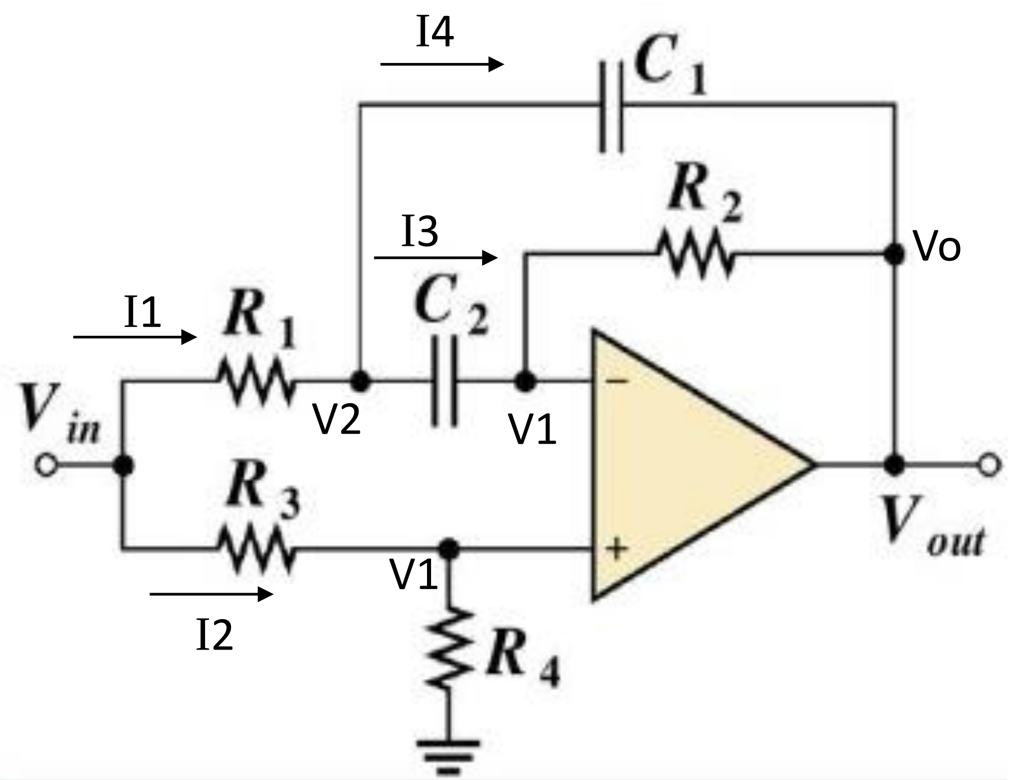
$$\therefore \frac{V_o}{V_{in}}=\frac{K(1+sC_1R_1+sC_2R_1)(1+sC_2R_1)-sC_2R_2-s^2C_2^2R_1R_2K}{s^2R_1R_2C_1C_2+(R_1C_1+R_1C_2)s+1}$$

$$\frac{V_o}{V_{in}}=\frac{K(1+sC_1R_1+sC_2R_1)(1+sC_2R_1)-sC_2R_2-s^2C_2^2R_1R_2K}{R_1R_2C_1C_2\left[s^2+\left(\frac{R_1C_1+R_1C_2}{R_1R_2C_1C_2}\right)s+\frac{1}{R_1R_2C_1C_2}\right]}$$

$$\frac{V_o}{V_{in}}=\frac{K\,f(s)}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}$$

$$\therefore \omega_o^2=\frac{1}{R_1R_2C_1C_2} \rightarrow \omega_o=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\therefore f_o=\frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$



Multiple Feedback BSF

$$f_o = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$

$$\frac{\omega_o}{Q} = \frac{R_1C_1+R_1C_2}{R_1R_2C_1C_2}$$

$$Q = \frac{\sqrt{\frac{R_2}{R_1}}}{\sqrt{\frac{C_2}{C_1}}+\sqrt{\frac{C_1}{C_2}}}$$

$$BW = \frac{f_o}{Q}$$

$$f_l = f_o - \frac{BW}{2}$$

$$f_h = f_o + \frac{BW}{2}$$