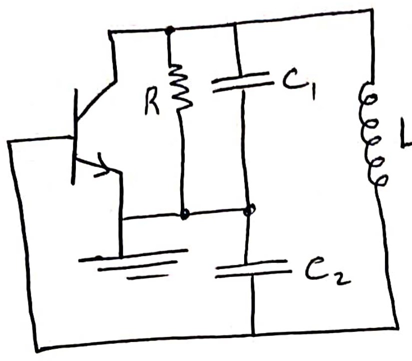
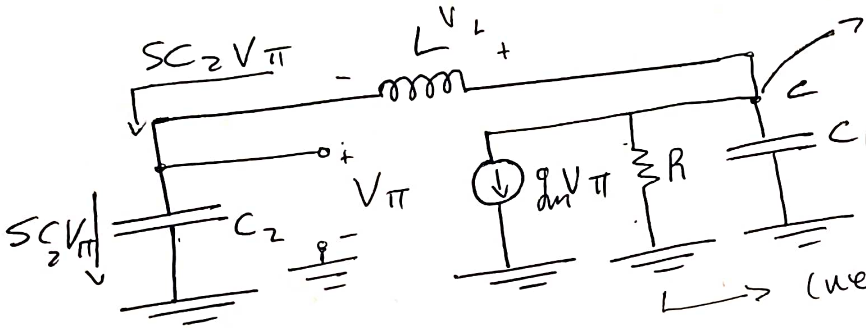
LC oscillators:

1- Colpitts oscillator :





$\to V_C = V_L + V_\pi$

$= (sC_2V_\pi) sL + V_\pi$

$= V_\pi[1 + s^2LC_2]$

$\to$ (neglecting r_π)

* KCL at node C :

$$g_mV_\pi + \frac{V_C}{R} + \frac{V_C}{(\frac{1}{sC_1})} + sC_2V_\pi = 0$$

$$\Rightarrow g_m\cancel{V_\pi} + \cancel{V_\pi}(1+s^2LC_2)(\tfrac{1}{R}+sC_1) + sC_2\cancel{V_\pi} = 0$$

$$\Rightarrow g_m + sC_2 + \frac{1}{R} + sC_1 + s^2\frac{LC_2}{R} + s^3LC_1C_2 = 0$$

$(s = j\omega)$:

$$\Rightarrow \left(g_m + \frac{1}{R} - \frac{\omega^2LC_2}{R}\right) + j\left[\omega C_2 + \omega C_1 - \omega^3LC_1C_2\right] = 0$$

$$\to g_m + \frac{1}{R} = \frac{\omega^2LC_2}{R}$$

$$\Rightarrow g_mR + 1 = \left(\frac{C_1+C_2}{\cancel{L}C_1\cancel{C_2}}\right)(\cancel{LC_2})$$

$$\Rightarrow g_mR = \frac{C_1+C_2}{C_1} - 1 = \frac{\cancel{C_1}+C_2-\cancel{C_1}}{C_1}$$

$$\Rightarrow \boxed{g_mR = \frac{C_2}{C_1}}$$

$\to$ oscillation condition

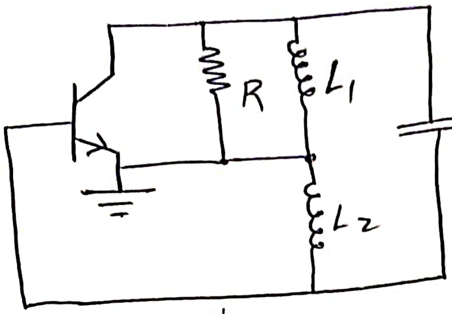
$$\to \cancel{\omega}(C_1+C_2) = \omega^{2}LC_1C_2$$

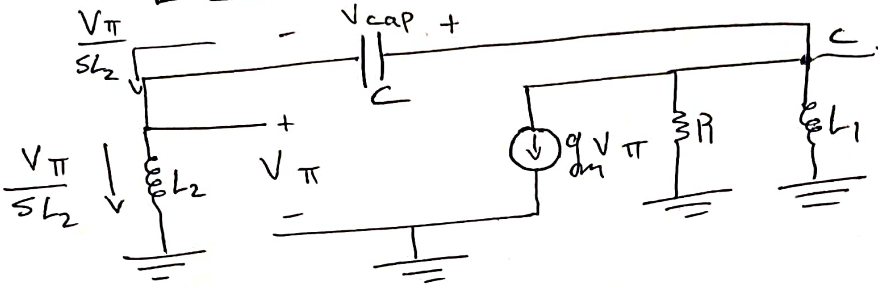
$$\Rightarrow \omega_o^2 = \frac{C_1+C_2}{LC_1C_2}$$

$$\Rightarrow \boxed{\omega_o = \sqrt{\frac{C_1+C_2}{LC_1C_2}}}$$

frequency of oscillation

2- Hartley oscillator :





$$V_C = V_{cap} + V_\pi = \frac{V_\pi}{sL_2}\left(\frac{1}{sC}\right) + V_\pi = V_\pi\left[1+\frac{1}{s^2L_2C}\right]$$

* KCL at node c :

$$\Rightarrow g_m \cancel{V_\pi} + \cancel{V_\pi}\left[1+\frac{1}{s^2L_2C}\right]\left[\frac{1}{R}+\frac{1}{sL_1}\right] + \frac{\cancel{V_\pi}}{sL_2} = 0$$

$$\Rightarrow g_m + \frac{1}{R} + \frac{1}{sL_1} + \frac{1}{s^2RL_2C} + \frac{1}{s^3L_1L_2C} + \frac{1}{sL_2} = 0$$

$\Rightarrow (s = j\omega)$:

$$\Rightarrow \left[g_m + \frac{1}{R} - \frac{1}{\omega^2RL_2C}\right] + j\left[-\frac{1}{\omega L_1} - \frac{1}{\omega L_2} + \frac{1}{\omega^3L_1L_2C}\right] = 0$$

$$g_m + \frac{1}{R} = \frac{1}{\omega_o^2RL_2C}$$

$$\frac{1}{\omega_o L_1} + \frac{1}{\omega_o L_2} = \frac{1}{\omega_o^3L_1L_2C}$$

$$\Rightarrow \omega_o^2\left[\frac{1}{L_1}+\frac{1}{L_2}\right] = \frac{1}{L_1L_2C}$$

$$\Rightarrow \omega_o^2\left[L_2+L_1\right] = \frac{1}{C}$$

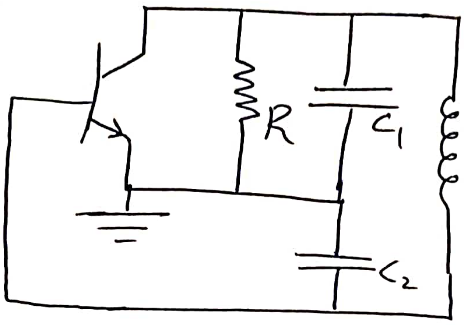
$$\Rightarrow \boxed{\omega_o = \sqrt{\frac{1}{C(L_1+L_2)}}}$$

$$\Rightarrow g_mR + 1 = \frac{\cancel{C}(L_1+L_2)}{L_2\cancel{C}}$$

$$\Rightarrow g_mR = \frac{L_1+\cancel{L_2}-\cancel{L_2}}{L_2}$$

$$\Rightarrow \boxed{g_mR = \frac{L_1}{L_2}}$$

Ex: using a BJT biased at $I_C = 1mA$, design a Colpitts oscillator to operate at $\omega_o = 10^6$ rad/s use $C_1 = 0.01\,\mu F$, $R = 2\,K\Omega$.



Sol:

$$\because\ g_m = \frac{I_C}{V_T} = \frac{1\,mA}{0.025} = 40\ mS$$

$$\Rightarrow \omega_o = \sqrt{\frac{C_1 + C_2}{L C_1 C_2}}$$

$$\Rightarrow L = \frac{C_1 + C_2}{\omega_o^2 C_1 C_2}$$

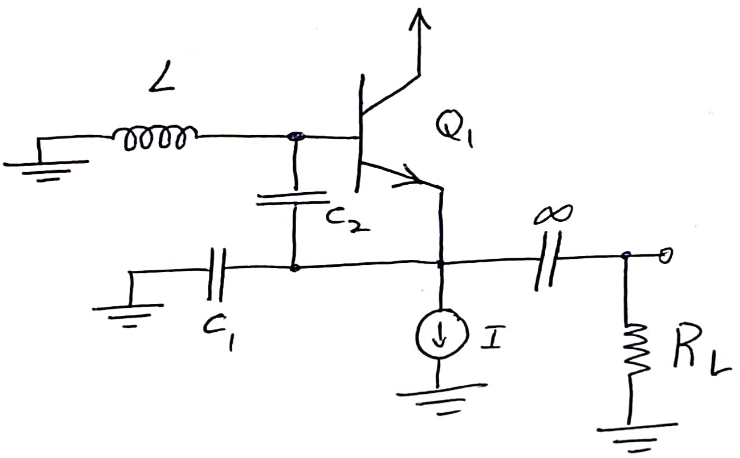
$$\Rightarrow g_m R = \frac{C_2}{C_1}$$

$$\Rightarrow C_2 = (40 \times 10^{-3})(2 \times 10^{3})(0.01 \times 10^{-6})$$
$$= 0.4\ \mu F$$

$$\Rightarrow L = \frac{(0.01 + 0.4) \times 10^{-6}}{(10^6)^2 (0.01 \times 10^{-6})(0.4 \times 10^{-6})} =$$

Q1: For each of the given colpitts oscillator circuits. derive an equation governing the circuit operation, & find the frequency of oscillation & the gain condition that ensures that oscillation will start.

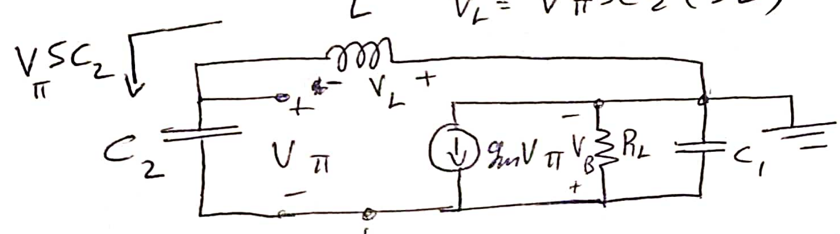
a)



Sol:

- small signal model: (assuming large r_π)

$V_L = V_\pi sC_2 (sL)$



$\hookrightarrow V_B = -V_\pi - V_L = -V_\pi(1+s^2C_2L)$

- KCL at node B:

$$V_\pi sC_2 + g_m V_\pi + \frac{(-V_B)}{R} + \frac{(-V_B)}{(\frac{1}{sC_1})} = 0$$

$$\Rightarrow \cancel{V_\pi} sC_2 + g_m \cancel{V_\pi} + \cancel{V_\pi}(1+s^2C_2L)\left(\frac{1}{R}+sC_1\right) = 0$$

$$\Rightarrow sC_2 + g_m + \frac{1}{R} + sC_1 + s^2\frac{C_2L}{R} + s^3C_1C_2L = 0$$

$$\Rightarrow \left(g_m + \frac{1}{R} - \frac{w^2C_2L}{R}\right) + j\left[w(C_1+C_2) - w^3C_1C_2L\right] = 0$$

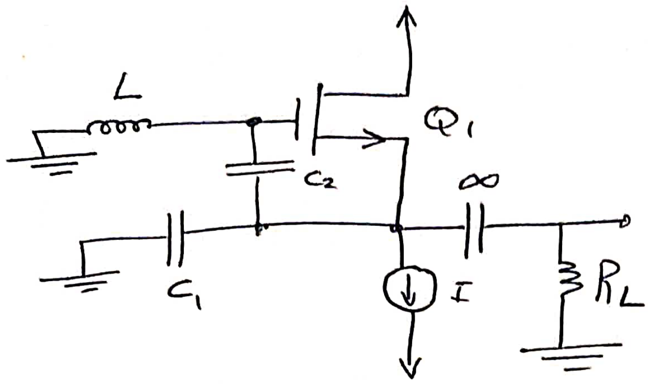
$\hookrightarrow g_m + \frac{1}{R} - \frac{w_o^2C_2L}{R} = 0$

$\hookrightarrow w_o(C_1+C_2) - w_o^3C_1C_2L = 0$

$\Rightarrow \boxed{g_mR = \frac{C_2}{C_1}}$

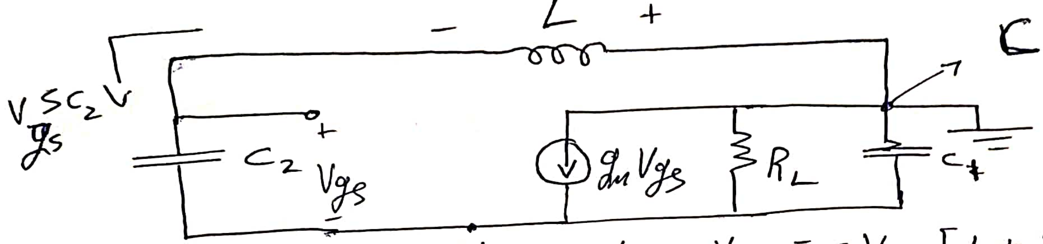
$\Rightarrow \boxed{w_o = \sqrt{\frac{C_1+C_2}{LC_1C_2}}}$

b)



sol:

$V_L = V_{gs} s C_2 (sL)$



$V_b = -V_{gs} - V_L = -V_{gs}[1 + s^2 C_2 L]$

- KCL at node C:

$$\Rightarrow V_{gs} s C_2 + g_m V_{gs} + \left[\frac{0 - V_b}{R_L}\right] + \left[\frac{0 - V_b}{1/sC_1}\right] = 0$$

$$\Rightarrow \cancel{V_{gs}} s C_2 + g_m \cancel{V_{gs}} + \cancel{V_{gs}}[1 + s^2 C_2 L]\left[\frac{1}{R_L} + sC_1\right] = 0$$

$$\Rightarrow sC_2 + g_m + \frac{1}{R_L} + sC_1 + \frac{s^2 C_2 L}{R_L} + s^3 C_1 C_2 L = 0$$

$s = j\omega$:

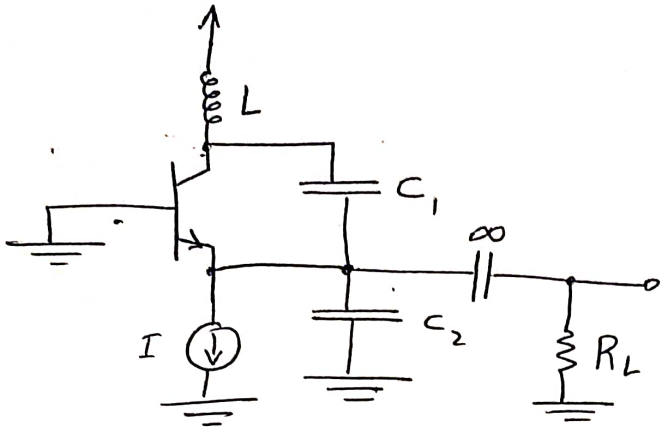
$$\Rightarrow \left[g_m + \frac{1}{R_L} - \frac{\omega^2 C_2 L}{R_L}\right] + j\underline{\left[\omega(C_1 + C_2) - \omega^3 C_1 C_2 L\right]} = 0$$

$$\hookrightarrow \boxed{g_m R_L = \frac{C_2}{C_1}}$$

$$\hookrightarrow \cancel{\omega_0}(C_1 + C_2) - \omega_0^{\cancel{3}\,2} C_1 C_2 L = 0$$

$$\Rightarrow \boxed{\omega_0 = \sqrt{\frac{C_1 + C_2}{L C_1 C_2}}}$$

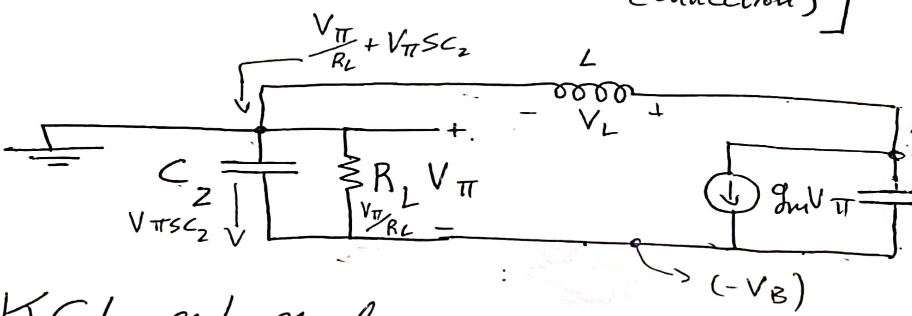
c)



sol:

- small-signal model :

[assuming $r_\pi >>$ (negligeable in parallel connection)]



$V_c = V_L - (-V_B) = \underset{SL(\frac{V_\pi}{R_L} + V_\pi S C_2)}{V_L} + \underset{V_\pi}{V_B}$

$\hookrightarrow V_c = V_\pi + \left(\frac{V_\pi}{R_L} + V_\pi S C_2\right) SL$
$= V_\pi \left[1 + \frac{SL}{R_L} + S^2 L C_2\right]$

- KCL at node C :

$$\Rightarrow g_m \cancel{V_\pi} + \cancel{V_\pi}\left[1 + \frac{SL}{R_L} + S^2 L C_2\right] S C_1 + \left(\frac{\cancel{V_\pi}}{R_L} + \cancel{V_\pi} S C_2\right) = 0$$

$$\Rightarrow g_m + \frac{1}{R_L} + \frac{S^2 L C_1}{R_L} + S C_2 + S C_1 + S^3 L C_1 C_2 = 0$$

$(s = j\omega)$

$$\Rightarrow \underbrace{g_m + \frac{1}{R_L} - \frac{\omega^2 L C_1}{R_L}}_{} + j \underbrace{\left[\omega (C_1 + C_2) - \omega^3 L C_1 C_2\right]}_{} = 0$$

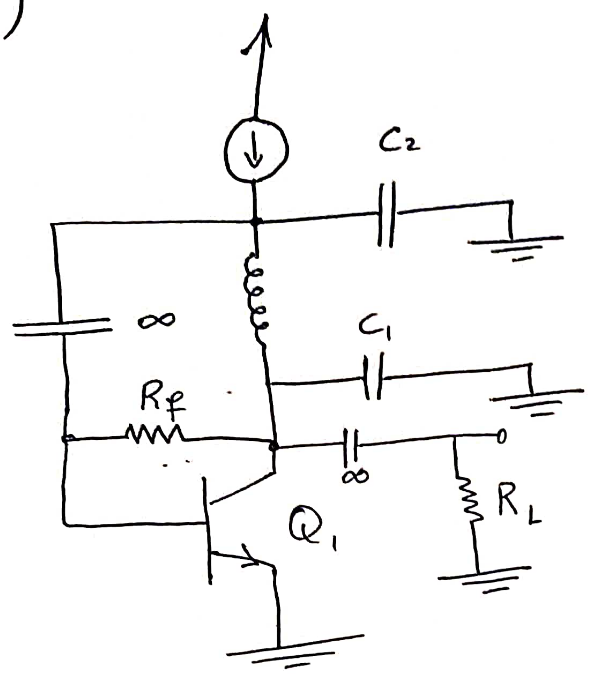
$$\hookrightarrow g_m + \frac{1}{R_L} - \frac{\omega_0^2 L C_1}{R_L} = 0$$

$$\hookrightarrow \cancel{\omega_0} (C_1 + C_2) = \omega_0^{\cancel{3}} L C_1 C_2$$

sub. $\boxed{\omega_0 = \sqrt{\frac{C_1 + C_2}{L C_1 C_2}}}$

$$\Rightarrow \boxed{g_m R_L = \frac{C_2}{C_1}}$$

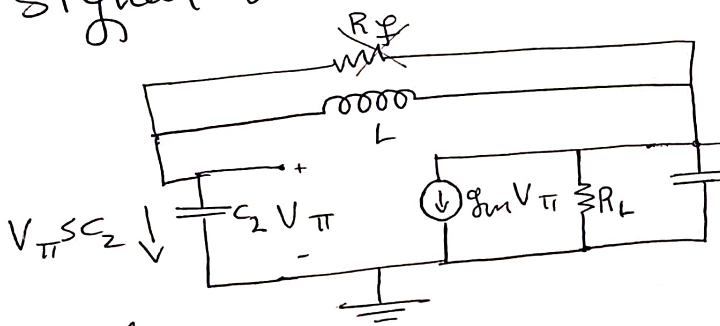
d)



(Assume $R_f >> \omega_o L$)

sol:

- small-signal Model : (assuming $r_\pi >>$)



$\because R_f >> \omega_o L$

$\Rightarrow (R_f // \omega_o L) \simeq \omega_o L$

$\Rightarrow V_c = V_\pi + V_\pi s C_2 (sL)$

- KCL at node c :

$$\Rightarrow g_m \cancel{V_\pi} + \cancel{V_\pi}[1 + s^2 C_2 L]\left[\frac{1}{R_L} + sC_1\right] + \cancel{V_\pi} s C_2 = 0$$

$$\Rightarrow g_m + \frac{1}{R_L} + sC_1 + \frac{s^2 C_2 L}{R_L} + s^3 C_1 C_2 L + s C_2 = 0$$

at $s = j\omega$

$$\Rightarrow \underbrace{g_m + \frac{1}{R_L} - \frac{\omega^2 C_2 L}{R_L}}_{} + j \underbrace{\left[\omega (C_1 + C_2) - \omega^3 C_1 C_2 L\right]}_{} = 0$$

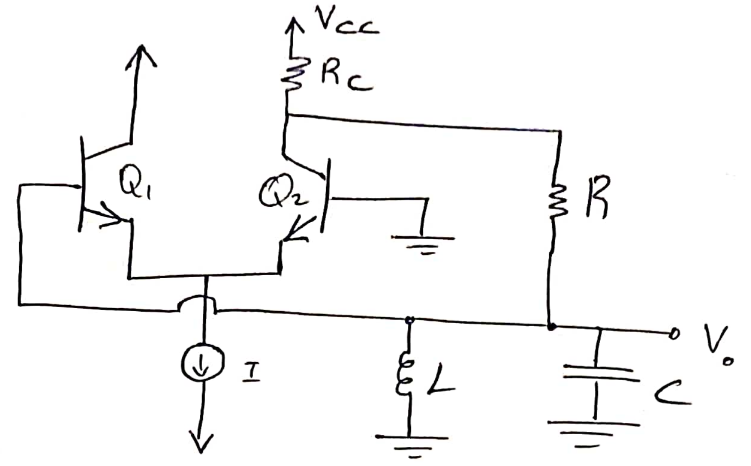
$\hookrightarrow \boxed{g_m R_L = \frac{C_2}{C_1}}$

$\hookrightarrow \cancel{\omega_o}(C_1 + C_2) - \omega_o^{\cancel{3}\,2} C_1 C_2 L = 0$

$\Rightarrow \boxed{\omega_o = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}}$

$\Rightarrow \boxed{\omega_o = \sqrt{\frac{C_1 + C_2}{L C_1 C_2}}}$

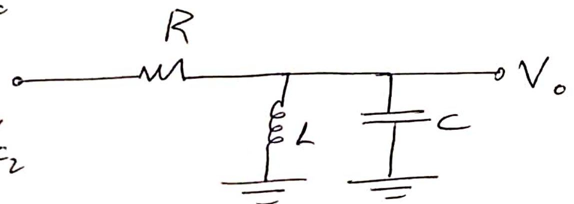
Q_2 consider the oscillator ct. & assume that $\beta = \infty$ for simplicity. $\boxed{8}$



a) Find the frequency of oscillation & min. value of R_C (in terms of bias current I) for oscillation to start.

sol:

* feedback network:



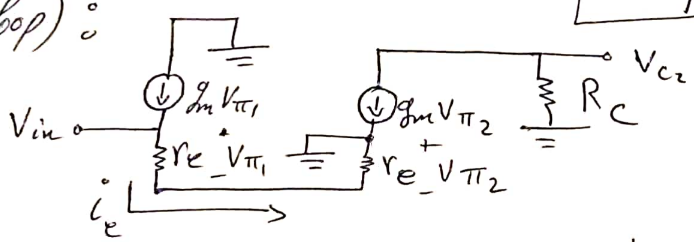
$$\Rightarrow V_o = V_{C_2} \frac{(SL \,//\, \frac{1}{SC})}{R + (SL \,//\, \frac{1}{SC})}$$

$$\Rightarrow \beta = \frac{V_o}{V_{C_2}} = \frac{\frac{SL\left(\frac{1}{SC}\right)}{SL + \frac{1}{SC}}}{R + \left(\frac{SL\left(\frac{1}{SC}\right)}{SL + \frac{1}{SC}}\right)} = \frac{L/C}{R\underbrace{\left(SL + \frac{1}{SC}\right)}_{\text{imaginary} \to 0}+ L/C}$$

$$\omega_o L - \frac{1}{\omega_o C} = 0$$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

* gain (open-loop):



$$\Rightarrow A = \frac{V_{C_2}}{V_{in}}$$

$$\Rightarrow i_e = \frac{V_{in}}{2 r_e}$$

$$\Rightarrow V_{C_2} = -g_m V_{\pi_2} R_C \quad , \quad V_{\pi_2} = -i_e r_e$$

$$\therefore i_e = \frac{V_{in}}{2r_e}$$

$$\Rightarrow V_{C_2} = g_m R_C \cancel{r_e} \left(\frac{V_{in}}{2\cancel{r_e}}\right)$$

$$\Rightarrow A = \frac{V_{C_2}}{V_{in}} = \frac{g_m R_C}{2}$$

$\Rightarrow |A\beta| = 1$ (Magnitude condition)

at $\omega = \omega_0$

$$\Rightarrow \beta = \frac{(L/C)}{(L/C)} = 1$$

$$\Rightarrow \frac{g_m R_C}{2} = 1 \quad , \quad \therefore g_m = \frac{I_C}{V_T} \, , \quad I_C = I/2$$

$$= \frac{I}{2V_T}$$

$$\Rightarrow \frac{I R_C}{4 V_T} \geq 1$$

$$\Rightarrow R_C \geq \frac{4 V_T}{I} \quad , \quad V_T = 0.025$$

$$\Rightarrow \boxed{R_C \geq \frac{0.1}{I}}$$

b) If $R_C = (\frac{1}{I})\,K\Omega$, prove that oscillations will start. if oscillations grow to pt. that V_o is large enough to turn BJTs on & off, show that the voltage at the collector of Q_2 will be a square wave of 1 V peak to peak.

sol:

at $R_C = \frac{1}{I} \Rightarrow A = \frac{I R_C}{4(0.025)} = \frac{(I)(\frac{1}{I})}{0.1} = \underline{\underline{10}}$

$\Rightarrow A\beta = 10$ at $\omega_0 \Rightarrow A\beta \gg 1$

$\Rightarrow$ oscillation will start

for large V_o :

at $V_o \uparrow \Rightarrow Q_1$ has $I_{C_1} = I$, & $I_{C_2} = 0$

$\Rightarrow Q_2 \rightarrow$ cutt off

$\Rightarrow V_{C_2} = V_{CC}$

at $V_o \downarrow \Rightarrow Q_1$ has $I_{C1} = 0 \Rightarrow I_{C_2} = I$

$\Rightarrow Q_2 \rightarrow$ Max current

$\Rightarrow V_{C_2} = V_{CC} - I R_C$

$= V_{CC} - 1 \qquad (\because R_C = \frac{1}{I})$

$\Rightarrow V_{PP} = V_{CC} - (V_{CC} - 1) = 1 \ V_{PP}$
