- what's information Theory

:→ The central problem in information Theory is efficient and reliable Transmission of data from a source To a destination.

→ The father of information Theory «claude shannon».

- shannon's information Theory deals with limits on data Compression «source coding» and reliable data Transmission «channel coding»

- How much can data be compressed? «The best possible compression That can be achieved»

- How fast can data be reliably Transmitted over a noisy channel?

Note: for a noisy channel; The best possible error correction That can be achieved.

⇒ Two basic "point - To - Point" communication Theorems.

- Source coding Theorem: The minimum rate at which data can be compressed losslessly is The entropy rate of The source.

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2. channel coding Theorem : The maximum rate at which data can be reliably Transmitted is The channel capacity of The channel.

Applications:

- Data compression.
- Transmission and modulation.
- Error correcting coding.
- Information security.

⇒ What's information?

. A sequence of symbols That can be interpreted as a message.

. Information resolves uncertainty «عدم التأكد». Information is what you get when your uncertanty about something is reduced

▪ Fair Coin example:

. when you flip a fair coin, you are uncertain of whether it will Land on heads or Tails, you have uncertainty

. when it Lands, your uncertainty is gone.

. Loss of uncertainty = Gain in information.

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. Unfair or biased coin example :

. A coin That always ~~ha~~ Lands on heads and never Lands on Tails.

. when know in advance That it will Lands on heads

. when it Lands on heads, You aren't getting any information because Your uncertainty hasn't been reduced.

- The more uncertain an event, The more information is required To resolve uncertainty of That event.

⇒ Amount of information

. By calculating The amount of ~~the~~ uncertainty we have about an information source, we are also calculating The Amount of information That will receive when we lose That uncertainty.

- Let x be an information source with M possible outcomes. The amount of information $I(x)$ is given by :

$$\boxed{I(x) = \log(M)}$$

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- The information received from an information source equals the logarithm of the number of possibe outcomes

. The unit used to measure $I(x)$ depends on the base of the logarithm.

→ Amount of information:
- The base of logarithm used in this course is base 2
- The amount of information is measured in bits

Example: ~~Fiar Coin~~

"عدد الاحتمالات"

$$I(x) = \log_2 (2) = 1 \text{ bit}$$

base

Note for a fair coin with two possible outcomes, we have one bit of uncertainty. when it Lands we lose that uncertainty and one bit of information.

$$* \ I(x) = \log_2 1 = 0 \text{ bit}$$

- for an unfair coin that always lands on hands, it has only one possible outcome.

→ A dice has six Possible outcomes:

$$\therefore I(x) = \log_2 6 = 2.585 \text{ bit}$$

- Since The last example has more outcomes, The amount of uncirtainty increases, Therefore, The amount of information is bigger Than That of The coin

$$\text{"} \log_2 a = \frac{\log a}{\log 2} = 3.32 \times \log a \text{"}$$

Note → For simplicity, we will use "log" instead of ($\log_2$), while we actually mean ($\log_2$).

- In all The previous examples, The outcomes are equally likely.

⟹ what if The outcomes are not equally likely?

- Let symbols = {$a_1, a_2, \ldots, a_n$} be The possible outcomes of an information source, The probabilities associated with These symbols be {$P_1, P_2, \ldots P_m$}, The amount of information That we receive when you get a specific symbol a_j is:

$$I(a_j) = \log\left(\frac{1}{P_j}\right)$$

Example :

A fair coin with a set of two probabilities {0.5, 0.5} where the first probability is the probability of getting head and the second one is the probability of getting Tail.

$$I(a_1) = \log\left(\frac{1}{P}\right) = \log\left(\frac{1}{0.5}\right) = \underline{\underline{1 \text{ bit}}}$$

Example 2

→ An unfair coin that always comes up as heads, the probabilities are {1, 0}

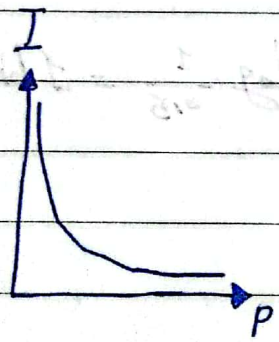
$$I(a_1) = \log\left(\frac{1}{1}\right) = 0 \text{ bit.}$$

Example 3

A biased coin with probabilities of {0.99, 0.01}

$$I(a_1) = \log\left(\frac{1}{0.99}\right) = 0.014 \text{ bit}$$

$$I(a_2) = \log\left(\frac{1}{0.01}\right) = 6.644 \text{ bit}$$



كلما كان الاحتمال أعلى " كلما قلت Information

The outcomes that are less common give us more information and vice versa.

→ **Entropy.**

- what if we want To measure The average uncertanity for an information source ?

• The name for The This measurement is entropy.

. The entropy of an information source (x) is called $H(x)$.

$$H(X) = \sum_{i=1}^{M} P_i \log\left(\frac{1}{P_i}\right) = -\sum_{i=1}^{M} P_i \log(P_i)$$

- Entropy is The average number of bit required To represent an information source.

Example

1- Fair coin with Probabilities of {0,5 , 0,5}

$$H(x) = 0.5 * \log\frac{1}{0.5} + 0.5 * \log\frac{1}{0.5} = 1\ bit$$

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Example

2- Based coin with probabilities of {0,75 , 0,25}

$$H(X) = 0{,}75 * \log\left(\frac{1}{0{,}75}\right) + 0{,}25 * \log\left(\frac{1}{0{,}25}\right) = \underline{\underline{0{,}811}} \text{ bits}$$

- For every Time we flip This based coin, we will get an average of 0,811 bits of information.

3- Unbalanced dice with probabilities of {0,1 , 0,1 , 0,1 , 0,5 , 0,1 , 0,1}

$$H(X) = 5 \times \left(\log\left(\frac{1}{0{,}1}\right) * 0{,}1\right) + 0{,}5 \log\frac{1}{0{,}5} = 2.161 \text{ bi}$$