

Worksheet 4 (Tutorial & Lab)

Exercice 1 *The Law of Large Numbers states that as the sample size increases, the sample mean will get closer to the population mean.*

Write a code in order to visualize this concept.

Exercice 2 *N darts are thrown randomly into a square with sides of length 2, the square centered at O contains a circle with radius 1 (centered at O). This circle is tangent to the four sides of the square. We assume that each dart thrown at random hits a point may be:*

- *Inside the circle, if the distance from the dart to the center O is less than 1,*
 - *Outside the circle, if the distance from the dart to the center O is equal to 1.*
1. *Estimate the value of π using Monte Carlo method, by counting the number n of darts in the circle.*
 2. *Write a program that estimates π by simulation, 10000 random throws are made.*

Exercice 3 *Return to Exercise 3 in the previous series, where we simulated the daily demand for a product over a two-week period.*

To improve the accuracy, use the variance reduction method “antithetic variables method.” Do we really see a reduction in variance?

Exercice 4 *We wish to estimate*

$$I = \int_{-2}^{+\infty} e^{(x-\frac{x^2}{2})} dx.$$

1. a) *Propose an estimator $\hat{I}_1(N)$ of I by Monte Carlo method based on N random variables of distribution $\mathcal{N}(0, 1)$.*
- b) *Write a Python program whose output is a realization of $\hat{I}_1(N)$ and an estimate of the variance of $\hat{I}_1(N)$. Specify the numerical results obtained for $N = 10^6$.*
2. a) *Propose an estimator $\hat{I}_2(N)$ of I by Importance sampling based on N random variables of distribution $\mathcal{N}(1, 1)$.*
- b) *Write a Python program whose output is a realization of $\hat{I}_2(N)$ and an estimate of the variance of $\hat{I}_2(N)$. Specify the numerical results obtained for $N = 10^6$ and compare them with those obtained in question 1.b).*
- c) *Compare the theoretical variances of $\hat{I}_2(N)$ and $\hat{I}_1(N)$. Comment.*

3. a) Express I as a function of

$$J = \int_1^2 e^{-\frac{(x-1)^2}{2}} dx.$$

- b) From the previous question, propose an estimator $\hat{I}_3(N)$ of I using a control variate.
- c) Write a Python program whose output is a realization of $\hat{I}_3(N)$ and an estimate of the variance of $\hat{I}_3(N)$. Specify the results obtained for $N = 10^6$. Comment.
- d) Propose an estimator $\hat{I}_4(N)$ of I by Monte Carlo Method based on N random variables of uniform distribution.
- e) Write a Python program whose output is a realization of $\hat{I}_4(N)$ and an estimate of the variance of $\hat{I}_4(N)$. Specify the results obtained for $N = 10^6$. Comment and compare with previous results.