22_11_2024

SAP // TAS TD

Ex01:

1) $h(n) = TF^{-1}[H(f)]$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} H(f)\, e^{2\pi j f n}\, df$$

$$= \int_{-f_c}^{f_c} e^{2\pi j f n}\, df$$

$$= \left[\frac{e^{2\pi j f n}}{2\pi j n} \right]_{-f_c}^{f_c} = \frac{e^{2\pi j f_c n} - e^{-2\pi j f_c n}}{2\pi j n} = \frac{\sin(2\pi f_c n)}{\pi n}$$

$$= \frac{\sin(B\pi n)}{\pi n}$$

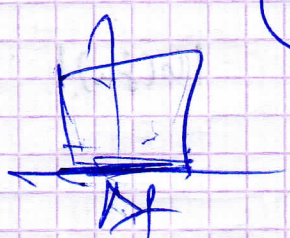
2) a) $\dfrac{\sin(B\pi n)}{\pi n} = 0 \Rightarrow \sin(B\pi n) = 0$

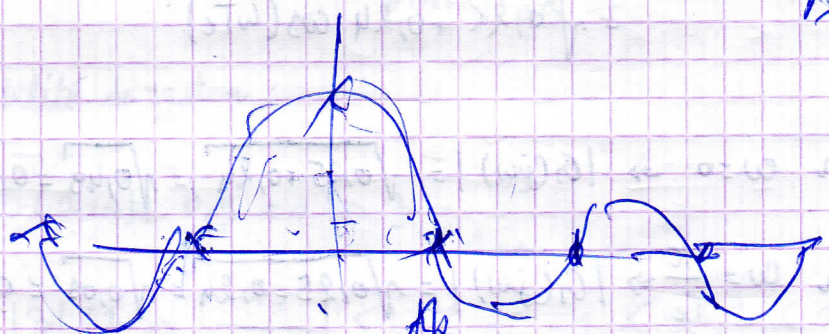
$\Rightarrow B\pi n = k\pi$

$Bn = k = 4$

$n = \dfrac{4}{B} = \dfrac{4}{0{,}125} = 4 \cdot 8 = 32$

$\Rightarrow P = 2n + 1 = 65$





$\Delta f_c = 0{,}2$

$n = \dfrac{k}{B}$

$4 = \Delta k = \dfrac{2}{B} \rightarrow \dfrac{2}{\Delta f} = \Delta f = 4$

28 - 11 - 2024

SAN TD

TD3 . EXO 4 :

[illegible] :

$s(k) = 0,3\ e(k-1) + 0,4\ e(k)$

1) La fonction de transfert :

$S(z) = 0,3\ E(z) z^{-1} + 0,4\ E(z)$

$S(z) = E(z)\ [0,3\ z^{-1} + 0,4]$

$\frac{S(z)}{E(z)} = G(z) = 0,3\ z^{-1} + 0,4$

2) Réponse fréquancielle :

$G(z)|_{z=e^{j\omega Te}} = 0,3\ e^{-j\omega Te} + 0,4$

$= 0,3\ [\cos(\omega Te) - j \sin(\omega Te)] + 0,4$

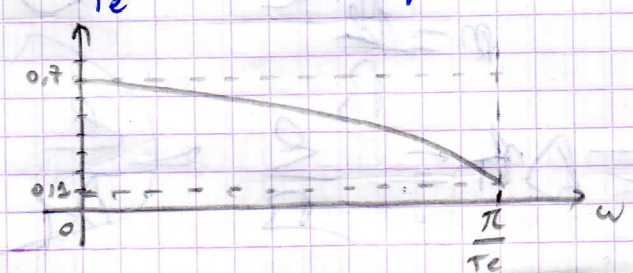
$= 0,3\ [\cos(\omega Te)] + 0,4 - 0,3 j \sin(\omega Te) = G(j\omega)$

$|G(j\omega)| = \sqrt{[0,3\cos(\omega Te) + 0,4]^2 + [0,3 \sin(\omega Te)]}$

$= \sqrt{0,2< + 0,24 \cos(\omega Te)}$

pour $\omega = 0 \Rightarrow |G(j\omega)| = \sqrt{0,25 + 0,24} = \sqrt{0,49} = 0,7$

pour $\omega = \frac{\pi}{Te} \Rightarrow |G(j\omega)| = \sqrt{0,25 - 0,24} = \sqrt{0,01} = 0,1$



TD 3.1 . EXO 1 :

$D_1(z) = z^3 + z^2 + z^1 + 1 = 1z^3 + 1z^2 + 1z^1 + 1z^0$ $\quad a_3 > 0$

- $|a_0| < a_3$ ⟶ $1 < 1$ non satisfaite ⇒ système instable
- $D(1) > 0$
- $D(-1) < 0$
- $a_0^2 - a_3^2 < (a_2 a_0) - (a_1 a_3)$

$D_2(z) = z^2 + z + 0,9$ $\quad a_2 > 0$

- $|a_0| < a_2$ ⟶ $0,9 < 1$
- $D(1) > 0$ ⟶ $2,9 > 0$
- $D(-1) \neq 0$ ⟶ $0,9 > 0$

} ⇒ satisfaite tous les règles donc le système est stable

TD 3.1 . EXO 2 :

$$G(z) = \frac{Az}{z - 0,9}$$

$$FTBF(z) = \frac{G(z)}{1 + G(z)} = \frac{Az}{z - 0,9 + Az}$$

$$FTBF(z) = \frac{Az}{z(1+A) - 0,9}$$

La stabilité de système :

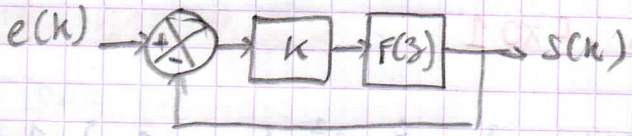
$$z(1+A) - 0,9 = 0 \Rightarrow z = \frac{0,9}{1+A}$$

$$|z| < 1 \Rightarrow \left|\frac{0,9}{1+A}\right| < 1 \quad \Rightarrow -1 < \frac{0,9}{1+A} < 1$$

$$\begin{cases} \frac{0,9}{1+A} > -1 \\ \frac{0,9}{1+A} < 1 \end{cases}$$

on a que $A > 0$ donc le système est stable pour $\forall A > 0$

TD 3.1 : EXO3 :



$$F(3) = \frac{3}{3+0,5^T} \quad \text{avec } T=1s$$

$$FTBF(3) = \frac{k\,F(3)}{1+kF(3)}$$

$$= \frac{k3}{3+0,5+k3} = \frac{k3}{3(k+1)+0,5}$$

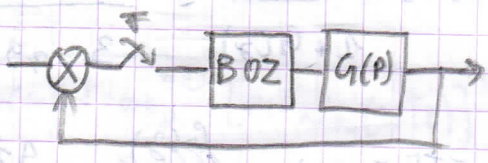
$$\Rightarrow 3(k+1)+0,5=0 \Rightarrow 3 = -\frac{0,5}{k+1} \quad \left\{ |3|<1 \right.$$

$$|3|<1 \quad -1<3<1$$

$$-1 < \frac{-0,5}{k+1} < 1$$

$$\left\{ \begin{array}{l} k > -1,5 \\ k \geq 0,5 \end{array} \right\} \Rightarrow k > -0,5$$

TD 3.1 . EXO 4 : $G(p) = \frac{B}{1+10p}$



1) $FTB_O(3) = ?$

$$\overline{B_O G}(3) = (1-3^{-1})\, 3\left\{\frac{G(p)}{p}\right\}$$

$$= \frac{3-1}{3}\, 3\left\{\frac{B}{p(1+10p)}\right\} = \frac{3-1}{3}\, 3\left\{\frac{\frac{B}{10}}{p(p+\frac{1}{10})}\right\}$$

$$* \quad 3\left\{\frac{a}{p(p+a)}\right\} = \frac{(1-e^{-aT})3}{(3-1)(3-e^{-aT})}$$

$$\Rightarrow \frac{3-1}{3}\, B\, \frac{(1-e^{-aT})3}{(3-1)(3-e^{-aT})} = \frac{B(1-e^{-aT})}{3-e^{-aT}} \qquad a=\frac{1}{10},\ T=1$$

$\Rightarrow \overline{BoG(z)} = \dfrac{0,1B}{z-0,9} = FTBO(z)$

2) FTBF = ?

$$FTBF(z) = \frac{\overline{BoG(z)}}{1+\overline{BoG(z)}} = \boxed{\frac{0,1B}{z+0,1B-0,9}}$$

3) Stabilité

$z + 0,1B - 0,9 = 0 \Rightarrow z = 0,9 - 0,1B$

$|z| < 1 \Rightarrow |0,9 - 0,1B| < 1$

$-1 < 0,9 - 0,1B < 1$

$\boxed{-1 < B < 19}$

4) Réponse indicielle ($B=3$) $e(n) = \varepsilon(n) = u(n) \Rightarrow$

$$FTBF(z) = \frac{0,3}{z-0,6} \Rightarrow S(z) = \frac{0,3}{z-0,6} \cdot \frac{z}{z-1}$$

$$\frac{S(z)}{z} = \frac{0,3z}{(z-0,6)(z-1)} = \frac{A}{z-1} + \frac{B}{z-0,6} \Rightarrow \begin{cases} A = 0,75 \\ B = -0,75 \end{cases}$$

$$S(z) = \frac{0,75z}{z-1} - \frac{0,75z}{z-0,6}$$

$$s(k) = 0,75(1-0,6^k) \quad k \geq 0$$

5) L'erreur statique de Position et de Vitesse

Statique $\rightarrow$ Méthode 1 :

$s(k) = 0,75(1-0,6^k) \quad k \in \mathbb{N}$

$s(\infty) = \lim_{k \to +\infty} 0,75(1-0,6^k) = 0,75$ $\quad e = u$

$\varepsilon(\infty) = E(\infty) - s(\infty) = 1 - 0,75 = 0,25$

→ Méthode 2 :

$$\frac{z-1}{z} = 1 - z^{-1}$$

$$\varepsilon(\infty) = \lim_{k \to +\infty} \varepsilon(k) = \lim_{z \to 1} (1 - z^{-1})\, E(z)$$

$$= \lim_{z \to 1} \frac{(1 - z^{-1})\; E(z)}{1 + FTBO(z)}$$

$$FTBO(z) = \frac{0,3}{z - 0,9} \quad / \quad E(z) = \frac{z}{z-1}$$

$$\varepsilon_p(\infty) = \lim_{z \to 1} \frac{\frac{z-1}{z} \cdot \frac{z}{z-1}}{1 + \frac{0,3}{z-0,9}} = \lim_{z \to 1} \frac{1}{1 + \frac{0,3}{z-0,9}} = \frac{1}{1 + \frac{0,3}{0,1}} = \frac{1}{4} = \boxed{0,2K}$$

vitesse :

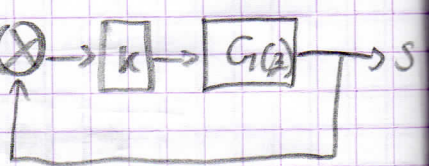
$$\varepsilon_v(\infty) = \lim_{z \to 1} \frac{z-1}{z} \cdot \frac{E(z)}{1 + FTBO(z)} \quad / \quad E(z) = \frac{z}{(z-1)^2}$$

$$= \lim_{z \to 1} \frac{z-1}{z} \cdot \frac{z}{(z-1)^2 \left[1 + \frac{0,3}{z-0,9}\right]} = \lim_{z \to 1} \frac{1}{(z-1)\left[1 + \frac{0,3}{z-0,9}\right]} = \boxed{+\infty}$$

($z \to 1$, $(z-1) \to 0$)

TD 3.2 ; Exo 1 :

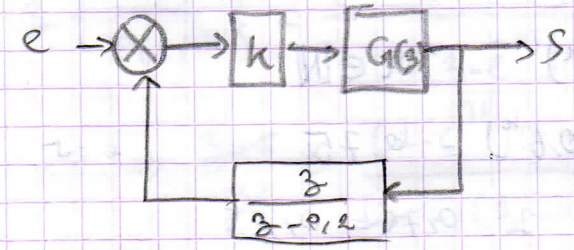
$$G_1(z) = \frac{0,1\,(z + 0,7)}{(z - 0,6)^2} \; ; \quad E(z) = \frac{z}{z-1}$$



$$\varepsilon_p(\infty) = \lim_{z \to 1} \frac{z-1}{z} \cdot \frac{E(z)}{1 + FTBO(z)} = \lim_{z \to 1} \frac{z-1}{z} \cdot \frac{z}{z-1} \cdot \frac{1}{1 + FTBO(z)}$$

$$= \lim_{z \to 1} \frac{1}{1 + \frac{0,1K\,(z+0,7)}{(z-0,6)^2}} = \frac{1}{1 + \frac{0,1K\,(1+0,7)}{(1-0,6)^2}} = \frac{1}{1 + \frac{0,17K}{0,4^2}} = \frac{1}{1 + 2,04K}$$

TD 3.2 Exo 2 :



$$G(z) = \frac{0,1(z+0,7)}{(z-0,6)^2} \; ; \; E(z) = \frac{z}{z-1} \; ; \; F(z) = \frac{1}{z-0,2}$$

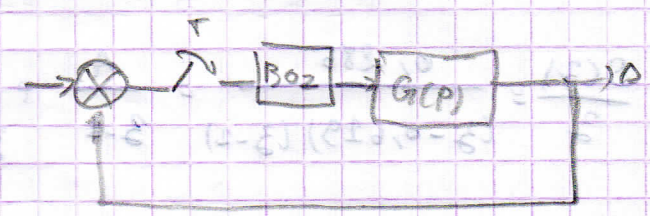
$$FTBO(z) = k\,G(z)\,F(z) = k\,\frac{0,1(z+0,7)}{(z-0,2)(z-0,6)^2}$$

$$\varepsilon(\infty) = \lim_{z\to 1} \frac{z-1}{z}\,\frac{E(z)}{1+FTBO(z)} = \lim_{z\to 1} \frac{1}{1+\frac{0,1(z+0,7)}{(z-0,2)(z-0,6)^2}k} = \frac{1}{1+\frac{0,17}{0,16}k}$$

$$= \frac{1}{1+1,32k}$$

TD3.e . Exo3 :

$$G_1(p) = \frac{B}{1+10p} \; ; T = 1s$$



$$\overline{B_0G}(z) = (1-z^{-1})\,Z\left\{\frac{G_1(p)}{p}\right\}$$

$$= \frac{z-1}{z} Z\left\{\frac{B}{(1+10p)p}\right\} = \frac{z-1}{z} Z\left\{\frac{\frac{1}{10}B}{(\frac{1}{10}+p)p}\right\} \; ; a = \frac{1}{10}$$

$$= \frac{z-1}{z}\,\frac{B(1-e^{-aT})z}{(z-1)(z-e^{-aT})} = \frac{B(1-e^{-aT})}{(z-e^{-aT})} = \frac{B(1-e^{-\frac{1}{10}})}{z-e^{-\frac{1}{10}}}$$

$$= \frac{B(0,095)}{z-0,904}$$

$$FTBF(z) = \frac{B_0G(z)}{1+B_0G(z)} = \frac{0,95B}{z(\underbrace{0,904-0,095B}_{z_0})}$$

- Stabilité :

$$|z_0| < 1$$

$$|0,904 - 0,095B| < 1 \quad \Rightarrow \quad -1 < 0,904 - 0,095B < 1$$

$$< B < 20,$$

→ Réponse indicielle $B = 3$

$$FTBF(z) = \frac{0,285}{z - 0,619} \qquad E(z) = \frac{z}{z-1}$$

$$\frac{S(z)}{E(z)} = \frac{0,285}{z-0,619} \Rightarrow S(z) = \frac{0,285}{z-0,619} E(z)$$

$$= \frac{0,285}{(z-0,619)(z-1)} z$$

$$\frac{S(z)}{z} = \frac{0,285}{(z-0,619)(z-1)} = \frac{A}{z-1} + \frac{B}{z-0,619} \Rightarrow \begin{cases} A = 0,748 \\ B = -0,748 \end{cases}$$

$$\Rightarrow S(z) = 0,748 \frac{z}{z-1} - 0,748 \frac{z}{z-0,619}$$

$$\Rightarrow s(k) = 0,748 (1 - 0,619^k) \quad k \in \mathbb{N}$$

→ Erreur de position

$$\varepsilon_p(\infty) = \lim_{z \to 1} \frac{z-1}{z} \frac{E(z)}{1 + FTBO} = \lim_{z \to 1} \frac{z-1}{z} \frac{z}{z-1} \frac{1}{1 + \frac{0,285}{z-0,9304}} = \boxed{0,252}$$

$$\neq 25,2\ \%$$

$$\Rightarrow \boxed{\varepsilon_v = +\infty}$$