30-10-2024

$$\sum_{n=0}^{P-1} a^n = \frac{1-a^P}{1-a}$$

$$\cancel{A+B-1 = \frac{\sin(\pi f N)}{\sin(\pi f)}}$$

$$\sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} e^{-2\pi j f n} = \underbrace{\sum_{n=-\frac{N-1}{2}}^{0} e^{-2\pi j f n}}_{A} + \underbrace{\sum_{n=0}^{+\frac{N-1}{2}} e^{-2\pi j f n}}_{B} - 1$$

$$A = \sum_{n=-\frac{N-1}{2}}^{0} e^{-2\pi j f n} \qquad m = -n$$

$$= \sum_{m=0}^{\frac{N-1}{2}} e^{2\pi j f m}$$

$$= \sum_{m=0}^{\frac{N-1}{2}} \left(e^{2\pi j f}\right)^m$$

$$= \frac{1 - e^{2\pi f j\left(\frac{N-1}{2}+1\right)}}{1 - e^{-2\pi j f}}$$

$$= \frac{\left(1 - e^{2\pi f j\left(\frac{N+1}{2}\right)}\right) e^{-j\pi f}}{\left(1 - e^{-2\pi j f}\right) e^{-j\pi f}}$$

$$= \frac{e^{-j\pi f} - e^{\pi f j N}}{e^{-j\pi f} - e^{j\pi f}}$$

$$= \frac{-e^{-j\pi f} + e^{\pi f j N}}{e^{j\pi f} - e^{-j\pi f}}$$

$$B = \sum_{n=0}^{+\frac{N-1}{2}} e^{-2\pi j f n}$$

$$= \sum_{n=0}^{+\frac{N-1}{2}} \left(e^{-2\pi j f}\right)^n$$

$$= \frac{1 - e^{-2\pi j f\left(\frac{N-1}{2}+1\right)}}{1 - e^{-2\pi j f}}$$

$$= \frac{1 - e^{-2\pi j f \frac{N+1}{2}}}{1 - e^{-2\pi j f}}$$

$$= \frac{\left(1 - e^{-\pi j f (N+1)}\right) \times e^{j\pi f}}{\left(1 - e^{-2\pi j f}\right) \times e^{j\pi f}}$$

$$= \frac{e^{j\pi f} - e^{-j\pi f N}}{e^{j\pi f} - e^{-j\pi f}}$$

$$\cancel{A+B-1 = \frac{e^{-j\pi f} - e^{\pi f jN} + \dots - e^{-j\pi fN}}{e^{j\pi f} - e^{-j\pi f}} - 1}$$

$$\cancel{= \frac{e^{\pi f jN} - e^{-j\pi fN} - e^{j\pi f} + \dots}{e^{j\pi f} - e^{-j\pi f}}}$$

$$A+B-1 = \frac{-e^{-j\pi f} - e^{\pi f jN} + e^{j\pi f} + e^{\pi f jN}}{e^{j\pi f} - e^{-j\pi f}} - 1$$

$$= \underbrace{\frac{-e^{-j\pi f} + e^{j\pi f}}{e^{j\pi f} - e^{-j\pi f}} - 1}_{\to 0} \qquad \frac{\left(-e^{\pi f jN} + e^{\pi f jN}\right) 2j}{\left(e^{j\pi f} - e^{-j\pi f}\right) 2j}$$

$$\text{Rect}(f) = \boxed{\frac{\sin(\pi f N)}{\sin(\pi f)}}$$

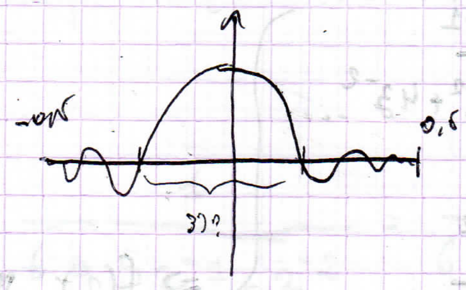
1) Donné l'allure de Rect(f) => sinc(x) تشبه السينك

2) " la valeur maximale de Rect(f) => Rect(0) = P

3) Calculer la largeur du lobe principal => في الدار

sachant que Rect(f) = 0

1) =>



31_10_2024

SAN TD

TD$_2$_Exo1_Suite

- $F(z) = \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})} \Rightarrow \frac{f(z)}{z} = \frac{1-e^{-aT}}{(z-1)(z-e^{-aT})}$

$$\frac{f(z)}{z} = \frac{A}{z-1} + \frac{B}{z-e^{-aT}} = \frac{A(z-e^{-aT})+B(z-1)}{(z-1)(z-e^{-aT})}$$

$$= \frac{z(A+B)-Ae^{-aT}-B}{(z-1)(z-e^{-aT})} \quad \begin{cases} A+B=0 \\ -Ae^{-aT} = -e^{-aT} \\ -B = 1 \end{cases} \Rightarrow \begin{cases} A=-B \\ A=1 \\ B=-1 \end{cases}$$

$$\Rightarrow \frac{f(z)}{z} = \frac{1}{z-1} - \frac{1}{z-e^{-aT}} \Rightarrow f(z) = \frac{z}{z-1} - \frac{z}{z-e^{-aT}}$$

$\Rightarrow f(k) = u(k) - e^{-ankT}$

- $f(z) = \frac{z}{z-2} = \frac{1}{1-2z^{-1}}$

$$\begin{array}{r|l} 1 & 1-2z^{-1} \\ \cline{2-2} -\;1-2z^{-1} & 1+2z^{-1}+4z^{-2}\ldots \\ \hline 0+2z^{-1} & \\ -\;2z^{-1}-4z^{-2} & \\ \hline 0+4z^{-2} & \\ -\;4z^{-2}-8z^{-3} & \\ \hline +8z^{-3} & \\ \vdots & \end{array}$$

$$\Rightarrow f(z) = 1+2z^{-1}+4z^{-2}\ldots = \sum_{k=0}^{+\infty} 2^k z^{-k} = \sum_{k=0}^{+\infty} f(k) z^{-k}$$

$\Rightarrow f(k) = 2^k$

$$f(z) = \frac{3z}{(z-1)(z-0{,}6)(z-0{,}5)} \Rightarrow \frac{f(z)}{z} = \frac{3}{(z-1)(z-0{,}6)(z-0{,}5)}$$

$$= \frac{A}{(z-1)} + \frac{B}{(z-0{,}6)} + \frac{C}{(z-0{,}5)}$$

$$\cancel{\Rightarrow \frac{f(z)}{z} = A(z-0{,}6)(z-0{,}5) + B(z-1)}$$

$$A = \lim_{z \to 1} \cancel{(z-1)} \frac{3}{\cancel{(z-1)}(z-0{,}6)(z-0{,}5)} = \frac{3}{(0{,}4)(0{,}5)} = 15$$

$$B = \lim_{z \to 0{,}6} \cancel{(z-0{,}6)} \frac{3}{(z-1)\cancel{(z-0{,}6)}(z-0{,}5)} = \frac{3}{(-0{,}4)(0{,}1)} = -75$$

$$C = \lim_{z \to 0{,}5} \cancel{(z-0{,}5)} \frac{3}{(z-1)(z-0{,}6)\cancel{(z-0{,}5)}} = \frac{3}{(-0{,}5)(-0{,}1)} = 60$$

$$\Rightarrow f(z) = 15 \frac{z}{(z-1)} - 75 \frac{z}{(z-0{,}6)} + 60 \frac{z}{(z-0{,}5)}$$

$$\Rightarrow f(k) = 15u(k) - 75(0{,}6^k) + 60(0{,}5^k)$$

TD$_2$ - Exo$_2$:

$$y(z) = \frac{1}{6 - 5z^{-1} + z^{-2}} = \frac{z^2}{6z^2 - 5z + 1}$$

$$\frac{y(z)}{z} = \frac{z}{(z-0{,}5)(z-\frac{1}{3})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$$

$$A = \lim_{z \to \frac{1}{2}} \cancel{(z-\tfrac{1}{2})} \frac{z}{\cancel{(z-\frac{1}{2})}(z-\frac{1}{3})} = \frac{\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}} = \frac{\frac{1}{2}}{\frac{4-2}{3}} = 3$$

$$B = \lim_{z \to \frac{1}{3}} \cancel{(z-\tfrac{1}{3})} \frac{z}{(z-\frac{1}{2})\cancel{(z-\frac{1}{3})}} = \frac{\frac{1}{3}}{\frac{1}{3}-\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{2-1}{3}} = -2$$

$$\Rightarrow y(z) = \frac{3z}{z-\frac{1}{2}} - \frac{2z}{z-\frac{1}{3}} \Rightarrow y(k) = 3\left(\frac{1}{2}\right)^k - 2\left(\frac{1}{3}\right)^k$$