```
    if (mode == 1){cpt++;
    else {cpt--;
    delay(500);
    aff.clear()
    aff.print(cpt);
    Serial.println(cpt);
  }
}
```

---

exercice POO

créer un programme qui fait

$$\sum_{m=0}^{N} m$$ après lire N

23-10-2024 TD ~~Cours~~

EXO 1: Calculate the FT of

$$rect(n) = \begin{cases} 1 & ; -\frac{N-1}{2} \leq n \leq \frac{N-1}{2} \\ 0 & ; \text{elsewhere} \end{cases}$$

$N = 21$

$$x(n) \xrightarrow{\mathcal{F}} X(f) = \sum_{n=-\infty}^{+\infty} x(n)\, e^{-2\pi j f n}$$

$$\mathrm{Rect}(f) = \sum_{n=-k}^{k} e^{2j\pi f n} = \sum_{n=-k}^{0} e^{-2j\pi f n} + \sum_{n=0}^{k} e^{-2j\pi f n} - 1$$

$$= \sum_{m=0}^{k} e^{+2j\pi f m} + \sum_{m=0}^{k} e^{-2j\pi f n} - 1$$

$$= A + B - 1$$

$$A = \sum_{m=0}^{k} e^{2j\pi f m} = \sum_{m=0}^{\frac{N-1}{2}} (e^{2\pi j f})^m = \frac{1 - e^{2\pi j f(\frac{N-1}{2}+1)}}{1 - e^{2\pi j f}} = \frac{1 - e^{\pi j f(N+1)}}{1 - e^{2\pi j f}}$$

$$B = \sum_{m=0}^{k} e^{-2j\pi f m} = \sum_{n=0}^{\frac{N-1}{2}} (e^{2\pi j f})^n = \frac{1 - e^{-2\pi j f(\frac{N-1}{2}+1)}}{1 - e^{2\pi j f}} = \frac{1 - e^{-\pi j f(N+1)}}{1 - e^{2\pi j f}}$$

$$A = \frac{(1 - e^{\pi j f(N+1)})(e^{-j\pi f})}{(1 - e^{2\pi j f})(e^{-j\pi f})} = \frac{e^{-j\pi f} - e^{2\pi j f N}}{e^{-j\pi f} - e^{\pi j f}}$$

$$B = \frac{(1 - e^{-\pi j f(N+1)})(e^{j\pi f})}{(1 - e^{-2\pi j f})(e^{j\pi f})} = \frac{e^{j\pi f} - e^{-2\pi j f N}}{e^{j\pi f} - e^{-\pi j f}}$$

continue la solution jusqu'à $\frac{\sin \pi f N}{\sin(\pi f)}$

si

Test après 2 semain de SAN

TD2. EXO4. Suite:

$$y(n) = \left(\frac{1}{2}\right)^{n-2}\left(\frac{1}{3}\right)^{n} u(n)$$

$$= \left(\frac{1}{2}\right)^{-2}\left(\frac{1}{2}\right)^{n}\left(\frac{1}{3}\right)^{n} u(n)$$

$$= 4\left(\frac{1}{2}\right)^{n}\left(\frac{1}{3}\right)^{n} u(n)$$

$$= 4\left(\frac{1}{6}\right)^{n} u(n)$$

$$Y(z) = \sum_{n=0}^{+\infty} y(n)\, z^{-n}$$

$$= \sum_{n=0}^{+\infty} 4\left(\frac{1}{6}\right)^{n} z^{-n} = 4\sum\left(\frac{z^{-1}}{6}\right)^{n} = 4\,\frac{1}{1+\frac{z^{-1}}{6}} = 4\,\frac{z}{z+\frac{1}{6}}$$

$$z(n) = n\left(\frac{1}{3}\right)^{n} u(n) = n\,x(n)$$

$Z(z) = Z[n\,x(n)]$ on a que $Z[nT\,s(n)] = -Tz\,\dfrac{dS(z)}{dz}$

$$\Rightarrow Z(z) = -Tz\,\frac{dX(z)}{dz} \quad ; \quad X(z) = \frac{z}{z-\frac{1}{3}}$$

$$\frac{dX(z)}{dz} = \frac{(1)(z-\frac{1}{3}) - (1)(z)}{(z-\frac{1}{3})^2} = \frac{-\frac{1}{3}}{(z-\frac{1}{3})^2}$$

donc

$$Z(z) = -Tz\,\frac{-\frac{1}{3}}{(z-\frac{1}{3})^2} = Tz\,\frac{1}{3(z-\frac{1}{3})^2}$$

EXO5: تأكد منها في الآلة

$$X(z) = \frac{z}{z-1} - \frac{z - \frac{\sqrt{3}}{2} z^2}{z^2 - \sqrt{3}z + 1}$$

TD$_3$. EXO 1 :

$f(n) = \sum$ résidus de $F(z) \cdot z^{n-1}$

$$F(z) z^{n-1} = \frac{N(z)}{D(z)} = \frac{\Psi(z)}{(z-z_0)^s}$$

résidu en $z = z_0$

$$\text{Res}\left(F(z) z^{n-1}\right) = \frac{1}{(s-1)!} \left. \frac{d^{s-1} \Psi(z)}{dz^{(s-1)}} \right|_{z=z_0}$$

$$F(z) \cdot z^{n-1} = \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})} z^{n-1} = \frac{(1-e^{-aT}) z^{n}}{(z-1)^1 (z-e^{-aT})^1}$$

2 pôles $\Rightarrow$ 2 résidus $\qquad s = 1$

① Résidu en $z = 1$

$$F(z) \cdot z^{n-1} = \frac{(1-e^{-aT}) z^n}{(z-1)(z-e^{-aT})} = \frac{\Psi_1(z)}{z-1}$$

$$\text{Res}\left(F(z) \cdot z^{n-1}\right) = \frac{1}{0!} \left. \frac{d^0 \Psi_1(z)}{dz^0} \right|_{z=1} = \Psi_1(z) \Big|_{z=1}$$

$$\cancel{= \frac{(1-e^{-aT}) z^n}{z-1}} = \frac{(1-e^{-aT})}{1-e^{-aT}} = 1$$

② $z = e^{-aT}$

$$F(z) \cdot z^{n-1} = \frac{(1-e^{-aT}) z^n}{(z-1)(z-e^{-aT})} = \frac{\Psi_2(z)}{z-e^{-aT}}$$

$$\text{Res}\left(F(z) z^{n-1}\right) = \frac{1}{0!} \left. \frac{d^0 \Psi_2(z)}{dt^0} \right|_{z=e^{-aT}}$$

$$= \Psi_2(z) \Big|_{z=e^{-aT}} = \frac{(\cancel{1-e^{-aT}}) e^{-aTn}}{\cancel{1-e^{-aT}}} = e^{-aTn}$$

$\Rightarrow f(n) = 1 + e^{-aTn}$

$$F(z)\,z^{n-1} = \frac{T z^n}{(z-1)^2}$$

$z = 1 \qquad S = 2$

$$F(z)\,z^{n-1} = \frac{\boxed{T z^n}}{(z-1)^2} = \frac{\Psi(z)}{(z-1)^2}$$

$$\mathrm{Res}\left(F(z)\,z^{n-1}\right) = \frac{1}{1!}\,\frac{d\Psi(z)}{dz}\bigg|_{z=1} = \frac{d\Psi(z)}{dz}\bigg|_{z=1}$$

$= nT$

$\Rightarrow f(n) = nT$