

# *Relational Calculus*

## Chapter 4, Part B



# Relational Calculus

- ❖ Comes in two flavours: *Tuple relational calculus* (TRC) and *Domain relational calculus* (DRC).
- ❖ Calculus has *variables, constants, comparison ops, logical connectives* and *quantifiers*.
  - TRC: Variables range over (i.e., get bound to) *tuples*.
  - DRC: Variables range over *domain elements* (= field values).
  - Both TRC and DRC are simple subsets of first-order logic.
- ❖ Expressions in the calculus are called *formulas*. An answer tuple is essentially an assignment of constants to variables that make the formula evaluate to *true*.




# Domain Relational Calculus

- ❖ *Query* has the form:

$$\{ \langle x_1, x_2, \dots, x_n \rangle \mid p(\langle x_1, x_2, \dots, x_n \rangle) \}$$

- ❖ *Answer* includes all tuples  $\langle x_1, x_2, \dots, x_n \rangle$  that make the formula  $p(\langle x_1, x_2, \dots, x_n \rangle)$  be *true*.
- ❖ *Formula* is recursively defined, starting with simple *atomic formulas* (getting tuples from relations or making comparisons of values), and building bigger and better formulas using the *logical connectives*.



# DRC Formulas

❖ *Atomic formula:*

- $\langle x_1, x_2, \dots, x_n \rangle \in Rname$  , or  $X \text{ op } Y$ , or  $X \text{ op constant}$
- $op$  is one of  $<, >, =, \leq, \geq, \neq$

❖ *Formula:*

- an atomic formula, or
  - $\neg p, p \wedge q, p \vee q$ , where  $p$  and  $q$  are formulas, or
  - $\exists X (p(X))$  , where variable  $X$  is *free* in  $p(X)$ , or
  - $\forall X (p(X))$ , where variable  $X$  is *free* in  $p(X)$
- ❖ The use of quantifiers  $\exists X$  and  $\forall X$  is said to bind  $X$ .
- A variable that is not bound is free.



## *Free and Bound Variables*

- ❖ The use of quantifiers  $\exists X$  and  $\forall X$  in a formula is said to bind  $X$ .
  - A variable that is not bound is free.
- ❖ Let us revisit the definition of a query:

$$\left\{ \langle x_1, x_2, \dots, x_n \rangle \mid p(\langle x_1, x_2, \dots, x_n \rangle) \right\}$$


- ❖ There is an important restriction: the variables  $x_1, \dots, x_n$  that appear to the left of `|' must be the *only* free variables in the formula  $p(\dots)$ .



*Find all sailors with a rating above 7*

$$\{ \langle I, N, T, A \rangle \mid \langle I, N, T, A \rangle \in \text{Sailors} \wedge T > 7 \}$$


- ❖ The condition  $\langle I, N, T, A \rangle \in \text{Sailors}$  ensures that the domain variables  $I$ ,  $N$ ,  $T$  and  $A$  are bound to fields of the same Sailors tuple.
- ❖ The term  $\langle I, N, T, A \rangle$  to the left of `|` (which should be read as *such that*) says that every tuple  $\langle I, N, T, A \rangle$  that satisfies  $T > 7$  is in the answer.
- ❖ Modify this query to answer:
  - Find sailors who are older than 18 or have a rating under 9, and are called 'Joe'.



*Find sailors rated > 7 who've reserved boat #103*

$$\{ \langle I, N, T, A \rangle \mid \langle I, N, T, A \rangle \in \text{Sailors} \wedge T > 7 \wedge \\ \exists Ir, Br, D \left( \langle Ir, Br, D \rangle \in \text{Reserves} \wedge Ir = I \wedge Br = 103 \right) \}$$

- ❖ We have used  $\exists Ir, Br, D \dots$  as a shorthand for  $\exists Ir \left( \exists Br \left( \exists D \dots \right) \right)$
- ❖ Note the use of  $\exists$  to find a tuple in Reserves that 'joins with' the Sailors tuple under consideration.



*Find sailors rated > 7 who've reserved a red boat*

$$\{ \langle I, N, T, A \rangle \mid \langle I, N, T, A \rangle \in \text{Sailors} \wedge T > 7 \wedge \\ \exists Ir, Br, D \left( \langle Ir, Br, D \rangle \in \text{Reserves} \wedge Ir = I \wedge \right. \\ \left. \exists B, BN, C \left( \langle B, BN, C \rangle \in \text{Boats} \wedge B = Br \wedge C = 'red' \right) \right) \}$$

- ❖ Observe how the parentheses control the scope of each quantifier's binding.
- ❖ This may look cumbersome, but with a good user interface, it is very intuitive. (Wait for QBE!)





*Find sailors who've reserved all boats*

$$\begin{aligned} & \{ \langle I, N, T, A \rangle \mid \langle I, N, T, A \rangle \in \text{Sailors} \wedge \\ & \quad \forall B, BN, C \left( \neg \left( \langle B, BN, C \rangle \in \text{Boats} \right) \vee \right. \\ & \quad \left. \left( \exists Ir, Br, D \left( \langle Ir, Br, D \rangle \in \text{Reserves} \wedge I = Ir \wedge Br = B \right) \right) \right\} \end{aligned}$$

- ❖ Find all sailors  $I$  such that for each 3-tuple  $\langle B, BN, C \rangle$  either it is not a tuple in Boats or there is a tuple in Reserves showing that sailor  $I$  has reserved it.



*Find sailors who've reserved all boats (again!)*

$$\left\{ \langle I, N, T, A \rangle \mid \langle I, N, T, A \rangle \in \text{Sailors} \wedge \right. \\ \left. \forall \langle B, BN, C \rangle \in \text{Boats} \right. \\ \left. \left( \exists \langle Ir, Br, D \rangle \in \text{Reserves} (I = Ir \wedge Br = B) \right) \right\}$$

- ❖ Simpler notation, same query. (Much clearer!)
- ❖ To find sailors who've reserved all red boats:

$$\text{..... } \left\{ C \neq \text{'red'} \vee \exists \langle Ir, Br, D \rangle \in \text{Reserves} (I = Ir \wedge Br = B) \right\}$$



## *Unsafe Queries, Expressive Power*

- ❖ It is possible to write syntactically correct calculus queries that have an infinite number of answers! Such queries are called unsafe.
  - e.g.,  $\{S \mid \neg (S \in Sailors)\}$
- ❖ It is known that every query that can be expressed in relational algebra can be expressed as a safe query in DRC / TRC; the converse is also true.
- ❖ Relational Completeness: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.



## *Summary*

- ❖ Relational calculus is non-operational, and users define queries in terms of what they want, not in terms of how to compute it. (Declarativeness.)
- ❖ Algebra and safe calculus have same expressive power, leading to the notion of relational completeness.