I Combinatorics and Set Theory

KTH/ICT:IX1500 - Discrete Mathematics, HT25V1

1 Implementation and Assessment

The course includes two to three project tasks on a total of 4 hp. The projects are both assessed in writing and orally. They will be given a summary grade on a scale of A-F. The assessment includes the number of solved project tasks. The first two projects are mandatory for the grade C-E. The third project task is optional and targets the higher grades A-B and requires a grade of C for the first two projects.

In this project task, you will work in a group of two and solve a mathematical task, write a report in Mathematica, and submit it in Canvas.

Carefully read the following information to know which rules apply and what is expected of you.

1.1 Report

The final report should contain:

- title and authors of the report
- email@kth.se
- a summary containing the results of resolved parts
- separate sections for each part containing e.g.
  - mathematical formulas and equations
  - a brief discussion
  - explanatory diagrams
  - your conclusions.
- separate code section (do not mix code and text with conclusions and results).
- Check Canvas for the deadline for submitting the report.

1.2 Presentation

Everyone in the group should be prepared to present the final report. Be prepared to present your solution to the members of the other groups.

1.3 Rules

The task is considered individually and assumes that you have full knowledge of all the material you are presenting. To be approved, you have solved the task and be able to explain the entire task and solution.

To account for the task that you do not have solved is considered cheating. It is also cheating to copy the solution or part of a solution from another. If two solutions are presented as (partially) copies they are rejected both.

If the solution contains information, e.g., paraphrased background material, you must clearly indicate this and properly refer to the source. Any direction quotation should be indicated with quotation marks (" ").

Suspicion of cheating or misleading can be reported to the Disciplinary Board. See also the EECS Code of honour.

1.4 Examination

- Please notice that this project should be reported in groups.
- The report (including code section) should be uploaded according to the information in Canvas.
- Notice that the report shall always be uploaded in time even if you cannot attend at the oral presentation. In case of illness, etc., then the oral presentation may be made during project 3. If you are unable to attend, you must in advance inform the teacher (in writing).
- If you ignore the oral presentation or the uploading of the report without contacting the teacher (in writing) you will have to wait for the next course for a new project exam.

2 Preparations

2.1 Study

Read and study the following texts:

- Chapter 1-2 in the course book (Böiers)
- lecture notes F1-F8

3 Mathematical Task

The project can be solved for pass grade (only problem 3.1) or for higher grade (all of problem 3.1 and 3.2). Keep in mind that it is the mathematical method on the task that is interesting to consider, not only the answer. Furthermore, solve the given task, not a variant or extension.

3.1 For Pass Grade

A particle moves in a the xy plane according to the following moves:

$$U : (m, n) \to (m + 1, n + 1) \tag{1}$$
$$L : (m, n) \to (m + 1, n - 1), \tag{2}$$

where m, n are integers. For example see the following Figure.

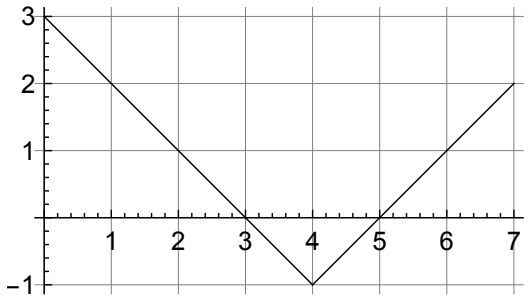

Figure 1:

(a) How many such paths are there from $(0, 4)$ to $(9, 3)$. Explain and print all such paths. For example, the path that corresponds to the Figure 1 is $LLLLUUU$.

(b) How many paths from $(0, 4)$ to $(9, 3)$ touch or cross $x-$axis at least once. Explain and print all such paths.

(c) How many paths from $(0, 4)$ to $(9, 3)$ never touch or cross $x-$axis. Explain and print all such paths.

(d) Enumerate all such paths from $(0, 5)$ to $(11, 4)$ that touch or cross x-axis. Explain your solution.

3.2 For Higher Grade (solve the above problem and the problems in this section)

At a carnival, a competitive game involves two coin-operated cars, denoted by **A** and **B**. 15 children each possess exactly one coin, and each child may contribute their coin to exactly one of the two cars. Every coin allocated to a car advances it forward by one unit. In total, 10 children choose car **A** and 5 children choose car **B**. Determine the number of possible sequences of coin insertions such that, at every point during the sequence, car **A** strictly leads car **B**. Also, calculate the probability that car **A** will be strictly ahead of car **B** throughout the sequence.