THINK FAST

Competitive Programming

From Problem 2 Solution in O(1)

Computational Geometry

Introduction

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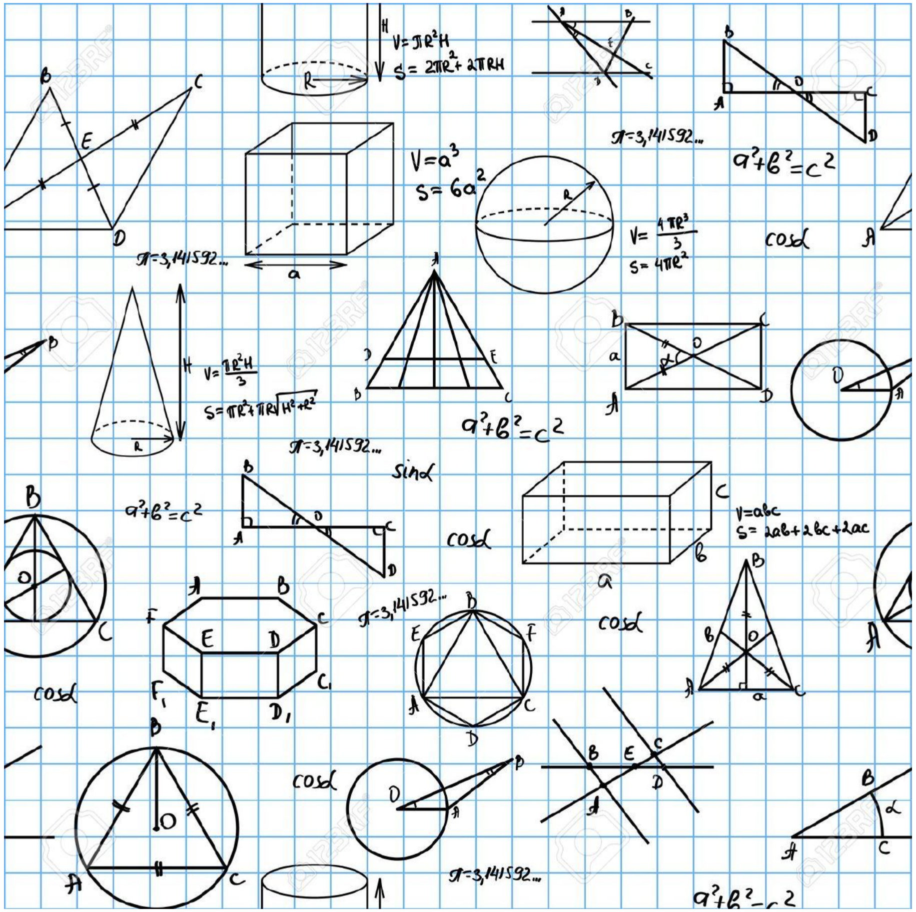
Geometry

About shape, size, relative position of **figures**

Euclid is the father of geometry



Src: http://previews.123rf.com/images/olgamakarova/olgamakarova1202/olgamakarova120200006/12195518-Geometry-seamless-pattern-Tile-for-endless-background-Stock-Vector.jpg

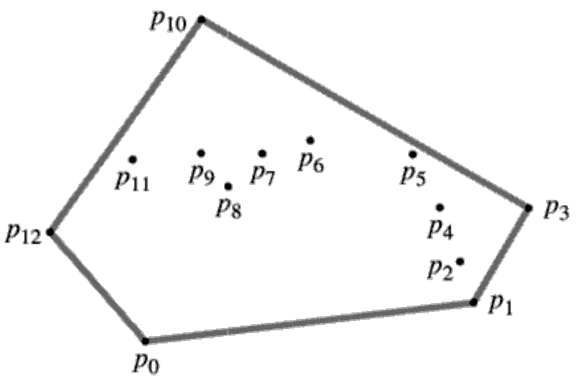
Computational Geometry

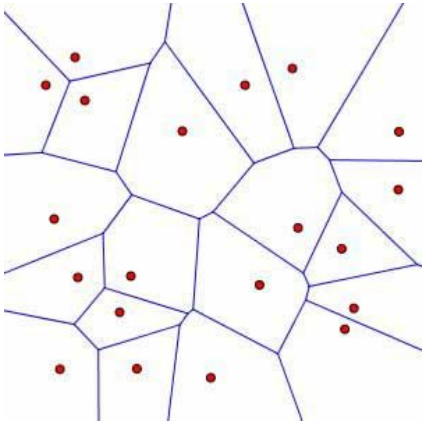
Study of **algorithms** for geometric problems.
Our Major focus on 2D. Few 3D.

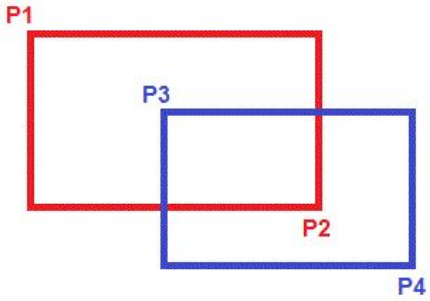
- 3D algorithms may be more complex
- Or much more computations
- So rare in competitions

Real life apps:

- Games
- Graphics and visualization
- Geographic information systems
- More







Src: http://kiharalab.org/3d-surfer/v1.0/convex_hull.gif http://www.ams.org/featurecolumn/images/august2006/diagramintro.1.jpg http://articles.leetcode.com/wp-content/uploads/2011/05/overlap_rect1.gif

Competitions

- Typically 0/1 geometry problem.
- Typically guys avoid it if hard problem
- Corner Cases
  - Lines: Vertical?
  - Points: Collinear?
  - Polyong: Simple? Concave? ..
- Degenerate Cases
  - Line start and end point are same!
- Precision Problems (avoid as possible)
- Lots of new coding? Library copy paste?

Resources

- Books
  - Programming Challenges
  - Competitive Programming
  - Introduction to Algorithms
- http://geomalgorithms.com/algorithms.html
  - Great site: algorithms and codes
- Articles
  - Topcoder: article 1, article 2
- Libraries: lots on web
  - Lib 1, Lib 2
  - Mine will be covered by end of series

Elements

Term	Dimensions	Graphic	Symbol
Point	Zero		$\cdot A$
Line Segment	One		$\overline{AB}$
Ray	One		$\overrightarrow{AB}$
Line	One		$\overleftrightarrow{AB}$
Plane	Two		Plane M

Src: http://geobraniacs.wikispaces.com/file/view/Points-Lines-%26-Planes.gif/90029611/Points-Lines-%26-Planes.gif

Trigonometry

- All about **angles** and their measures
- Angles measure
  - Radians: 0 - 2 π
  - Degrees: 0 - 360
  - Radians is better computationally - so **libraries** use that
- Right angle 90 degree or π/2 radians
- 370 Degree = 10 Degree = 370 % 360

Radians ⇔ Degrees

$$90^\circ = 90^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{2} \text{ radians}$$

$$\pi \text{ radians} = \pi \times \frac{180^\circ}{\pi \text{ radians}} = 180^\circ$$

$$\frac{3\pi}{2} \text{ radians} = \frac{3\pi}{2} \times \frac{180^\circ}{\pi \text{ radians}} = 270^\circ$$

$$2\pi \text{ radians} = 2\pi \times \frac{180^\circ}{\pi \text{ radians}} = 360^\circ$$

$$30^\circ = 30^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{6} \text{ radians}$$

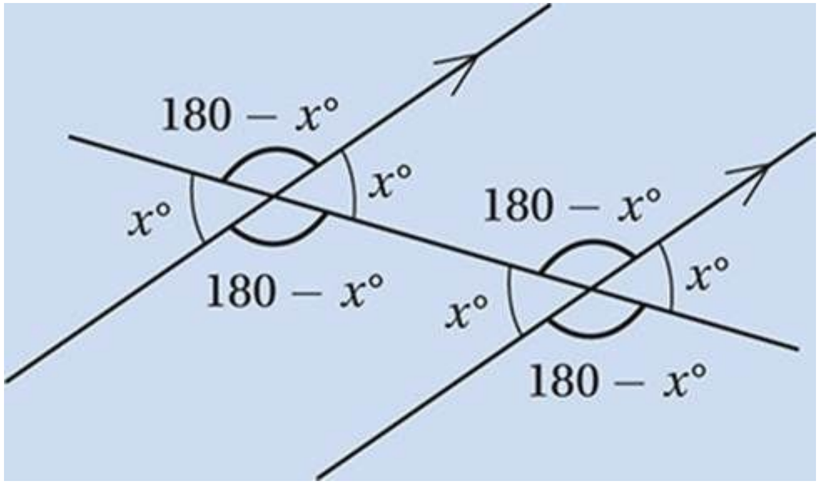
$$45^\circ = 45^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{4} \text{ radians}$$

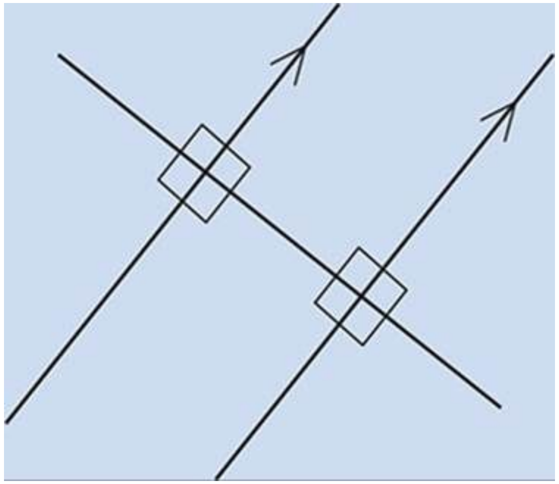
$$60^\circ = 60^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{3} \text{ radians}$$

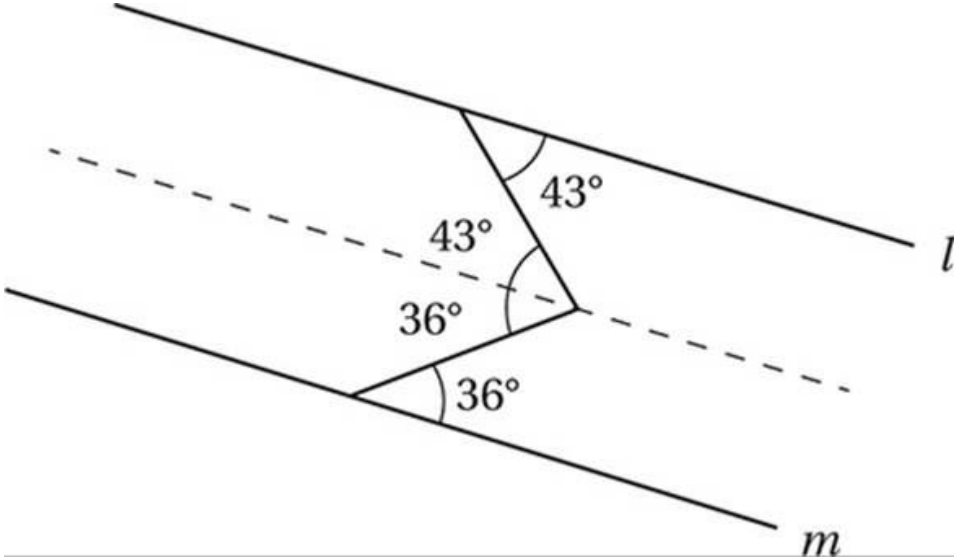
Radians ⇔ Degrees

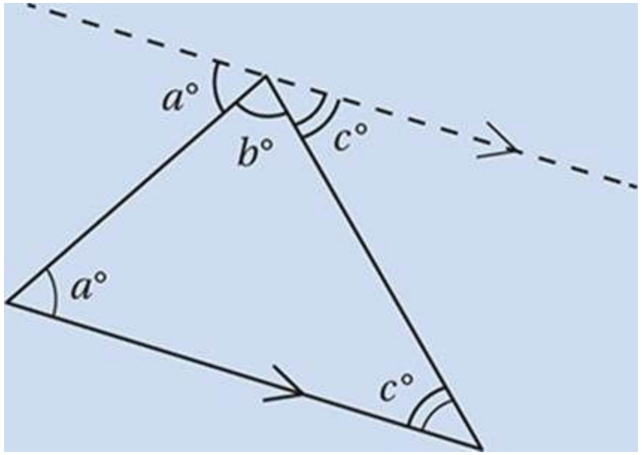
```
const double PI  = acos(-1.0);

double toDegeeFromMinutes(double minutes) {
    return (minutes/60);
}
double toRadians(double degree) {
    return (degree*PI/180.0);
}
double toDegree(double radian) {
    if(radian < 0) radian += 2*PI;
    return (radian*180/PI);
}
```

Angles

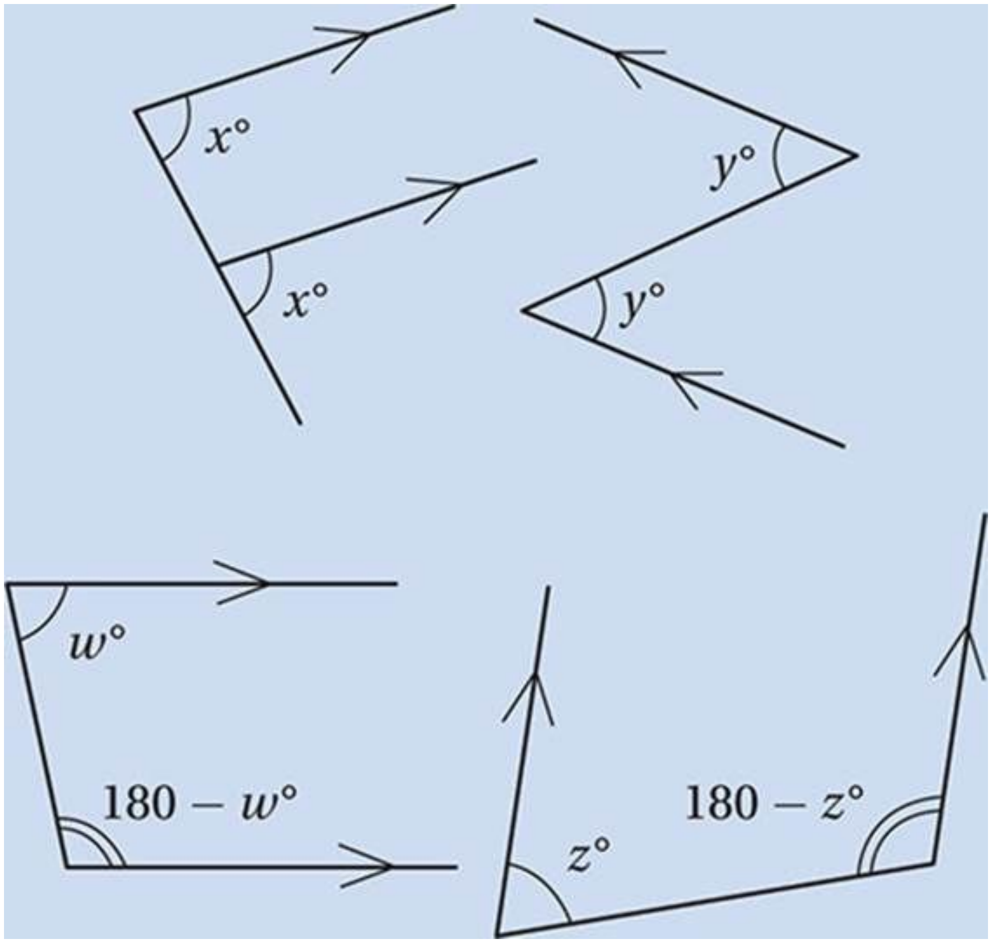


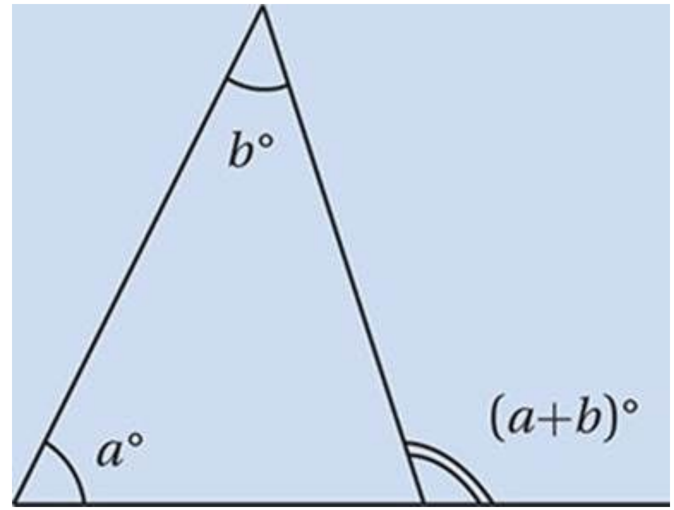


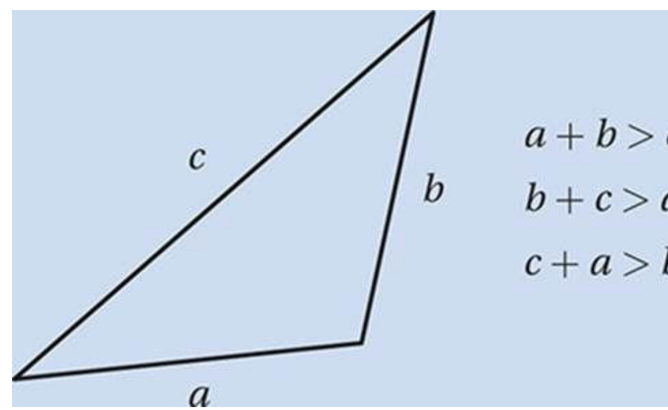




Angles

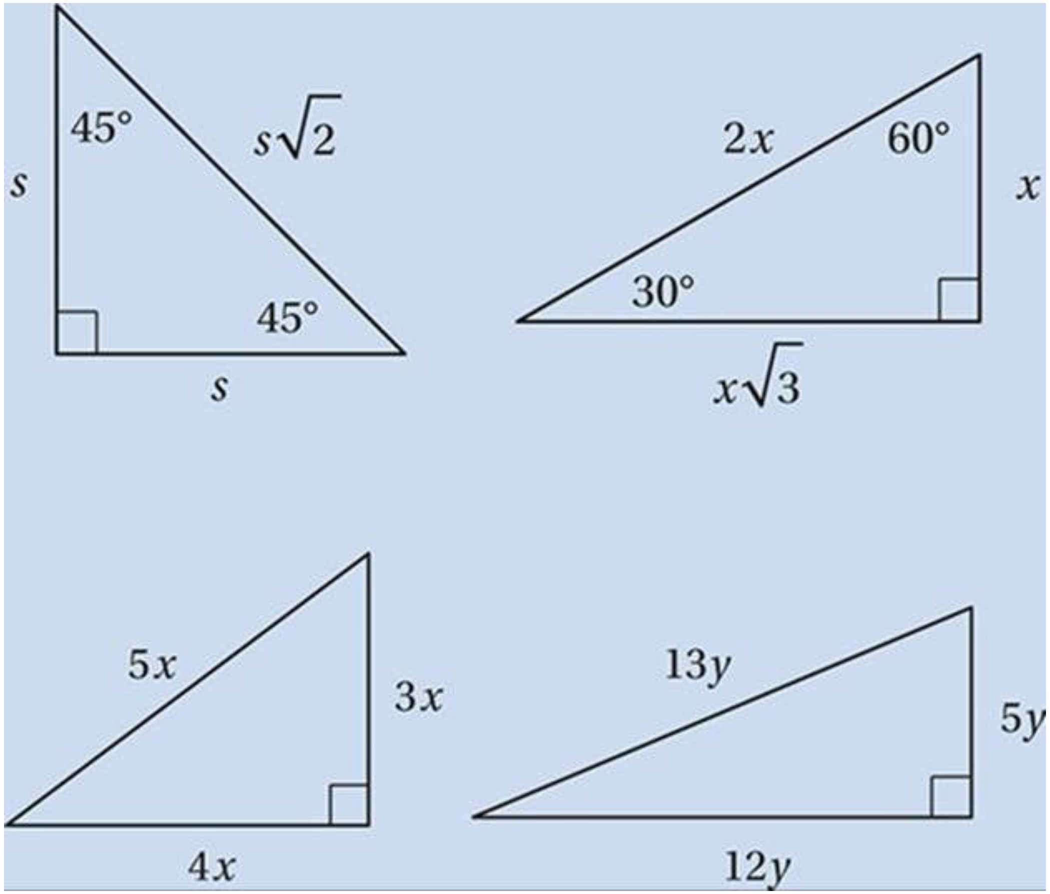






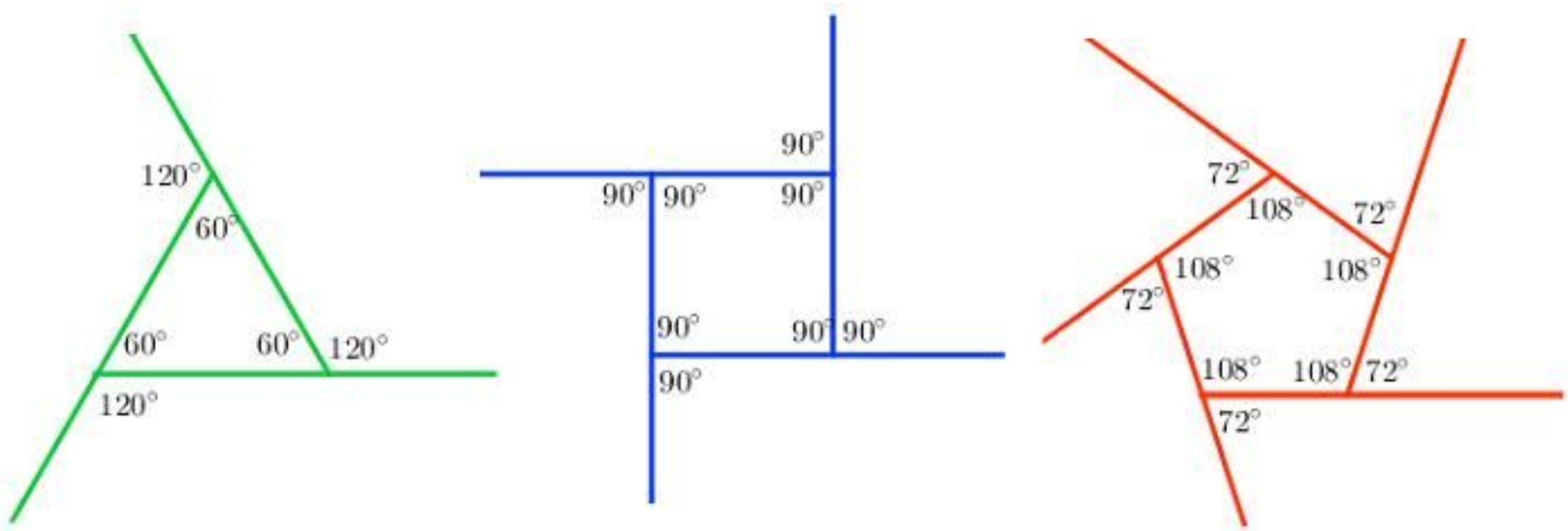
Src: http://schoolbag.info/sat/sat_3/73.html

Angles



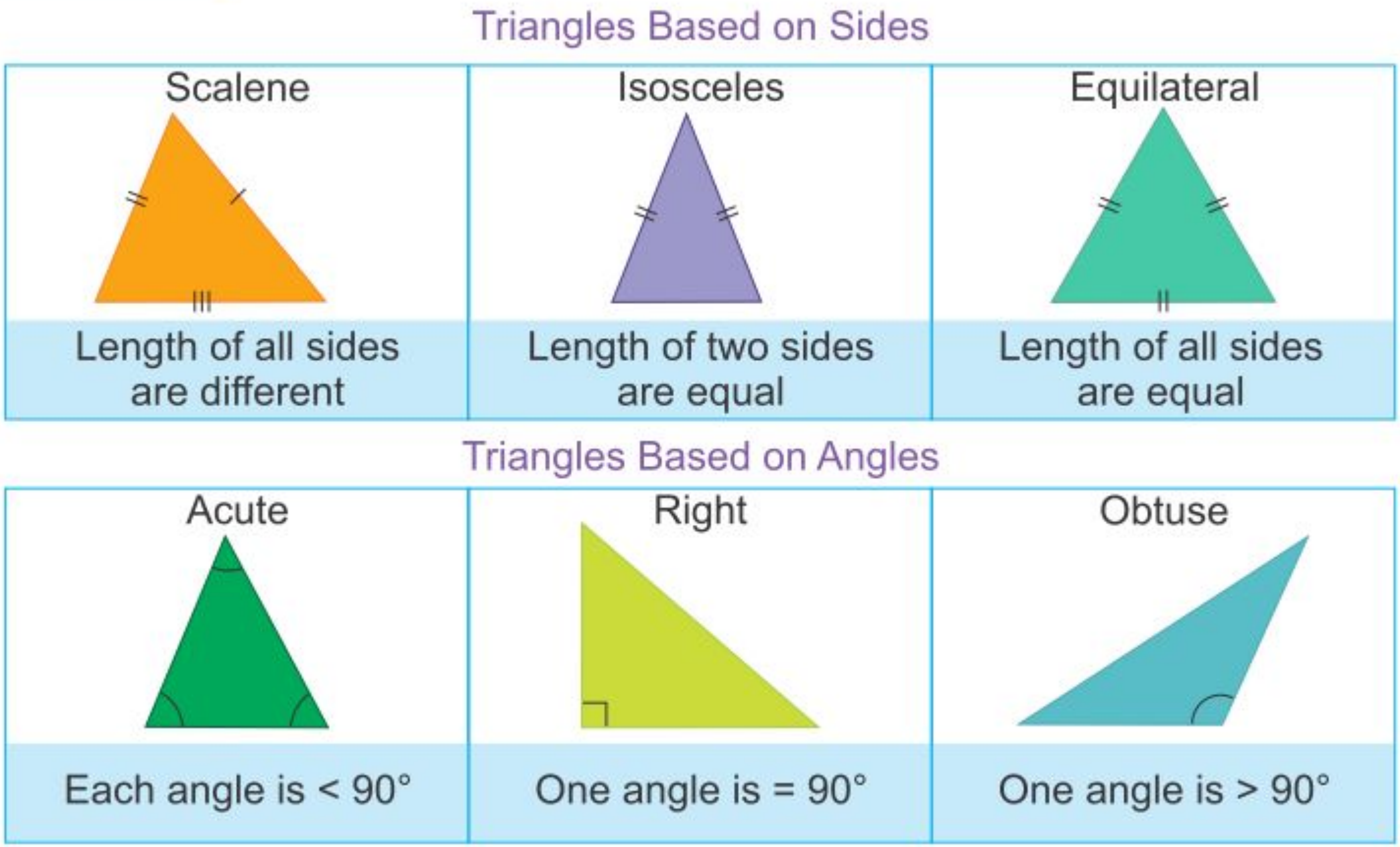
Angles

360 / # sides if all equal



Src: http://www.debate.org/photos/albums/1/6/5244/257096-5244-a4bzy-a.jpg

Triangles Types

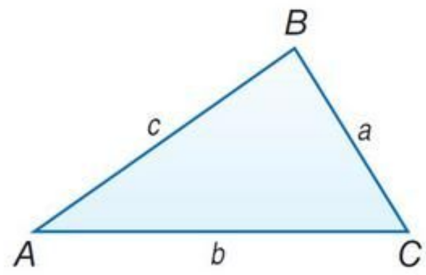


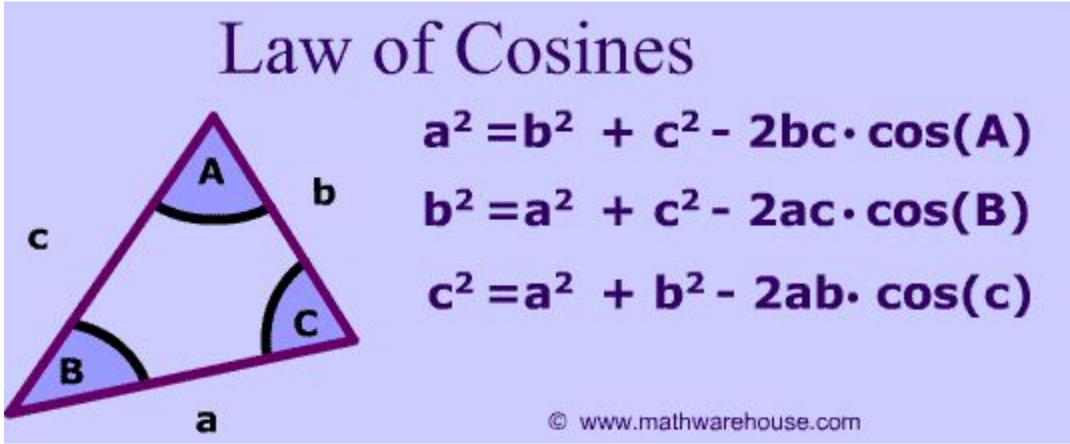
Src: https://s3-ap-southeast-1.amazonaws.com/learnhive/lcards/Types-of-Triangles-521dd275368b9.png

Triangle Laws

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



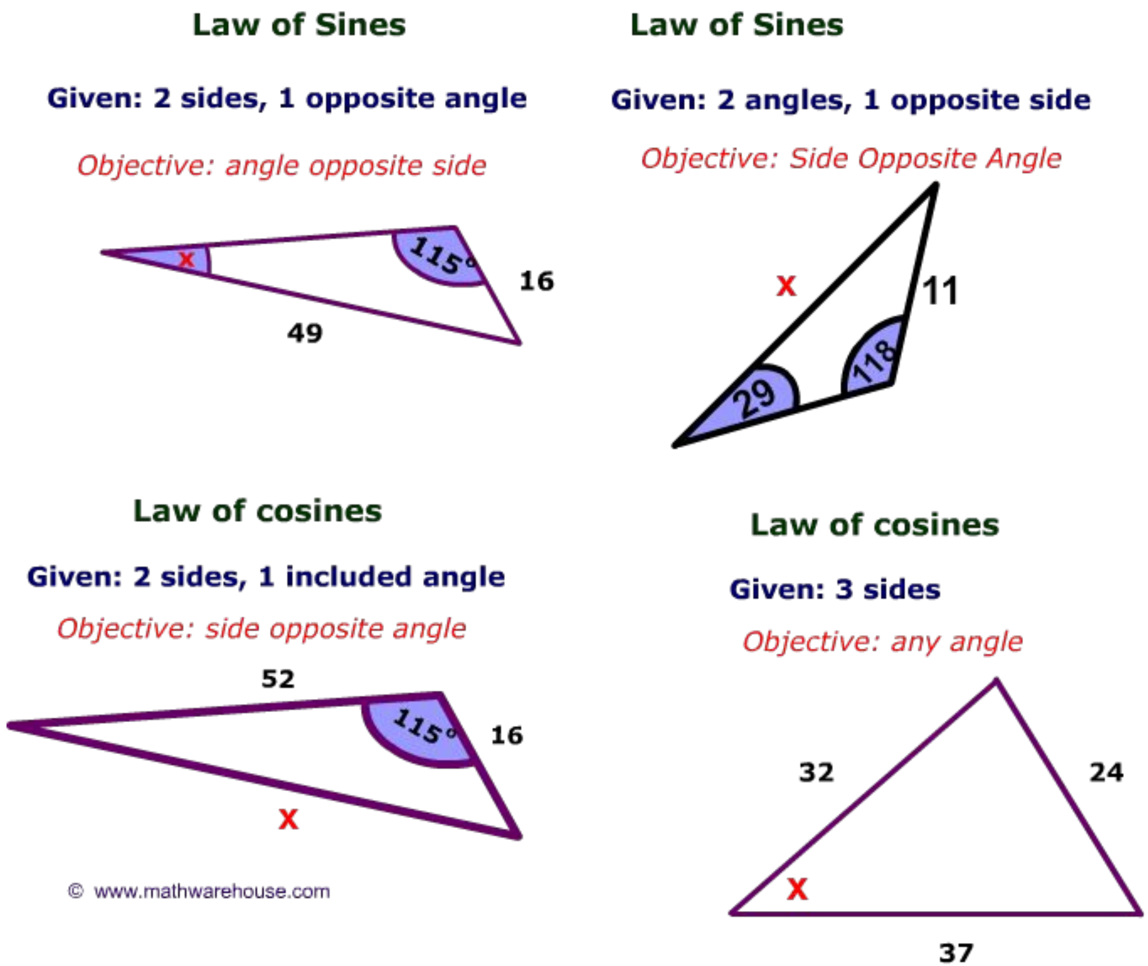


Src: http://www.mathwarehouse.com/trigonometry/images/law-of-cosines/law-of-cosines-formula-and-picture2.png

Solving Triangles

- Given A(angles) or S(sides) of triangle
  - Find other missing values
- 6 different cases!
- AAA, AAS, ASA, SAS, SSA, SSS
- We mainly use the triangle laws
- Homework: Study them and following code

Solving Triangles



Src: http://www.mathwarehouse.com/trigonometry/law-of-sines/images/law-of-sines-and-cosines/law-of-sines-and-cosines-picture.png

Solving Triangles

```
double fixAngle(double A) {
    return A > 1 ? 1 : (A < -1 ? -1 : A);
}

// sin(A)/a = sin(B)/b = sin(C)/c
double getSide_a_bAB(double b, double A, double B) {
    return (sin(A)*b)/sin(B);
}

double getAngle_A_abB(double a, double b, double B) {
    return asin( fixAngle(  (a*sin(B))/b  ) );
}

// a^2 = b^2 + c^2 - 2*b*c*cos(A)
double getAngle_A_abc(double a, double b, double c) {
    return acos( fixAngle(  (b*b+c*c-a*a)/(2*b*c)  ));
}
```

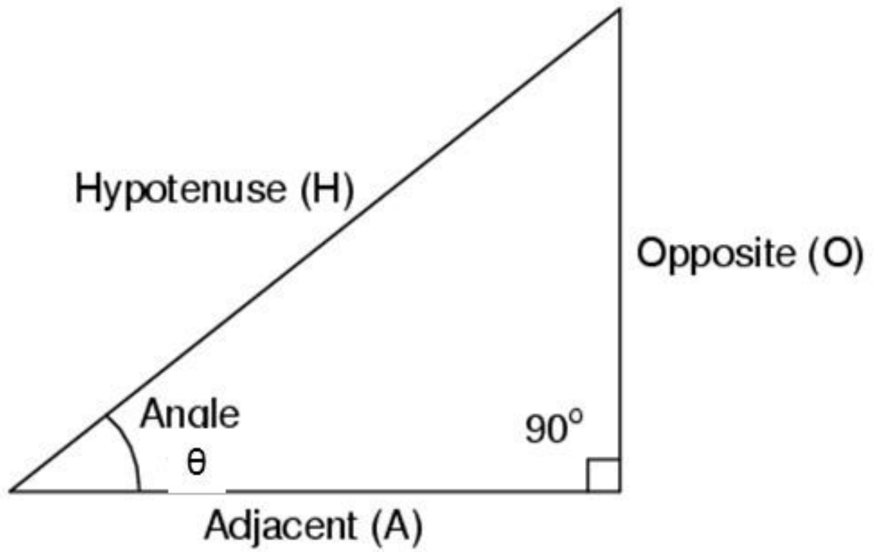
Trigonometric functions

- Sin θ = opposite/hypotenuse — Soh
- Cos θ = adjacent/hypotenuse — Cah
- Tan θ = opposite/adjacent — Toa

$$a^2 + b^2 = c^2$$

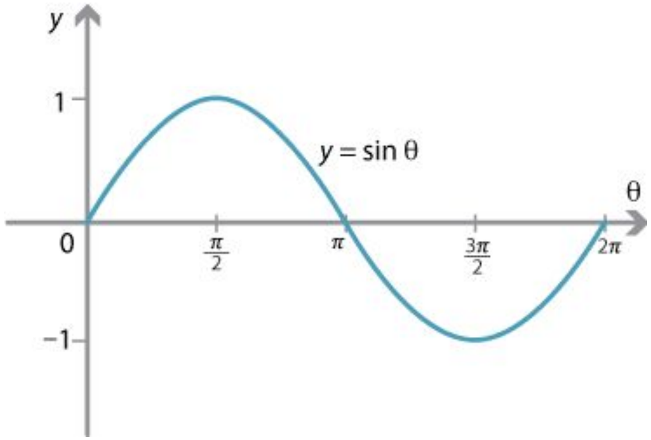
$\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ functions give θ

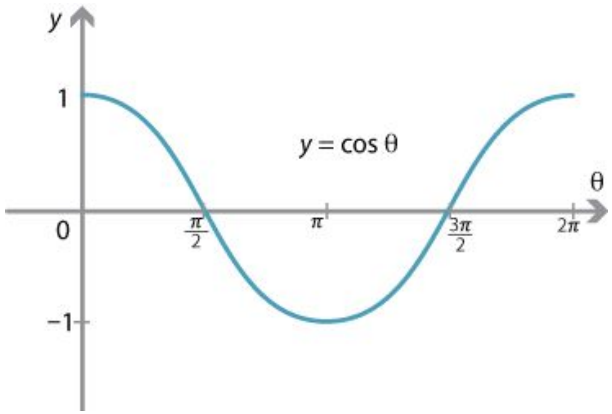
With any 2 values, you can find all sides and all angles

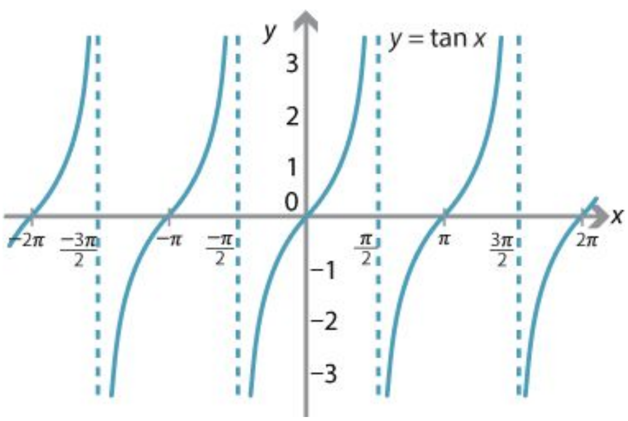


Src: http://images.slideplayer.com/30/9560908/slides/slide_12.jpg

Trigonometric functions







$$\sin\left(\tfrac{\pi}{2} - \theta\right) = +\cos\theta$$
$$\cos\left(\tfrac{\pi}{2} - \theta\right) = +\sin\theta$$
$$\tan\left(\tfrac{\pi}{2} - \theta\right) = +\cot\theta$$
$$\csc\left(\tfrac{\pi}{2} - \theta\right) = +\sec\theta$$
$$\sec\left(\tfrac{\pi}{2} - \theta\right) = +\csc\theta$$
$$\cot\left(\tfrac{\pi}{2} - \theta\right) = +\tan\theta$$

Src: http://amsi.org.au/ESA_Senior_Years/SeniorTopic2/2d/2d_2content_6.html

Trigonometric formula

$\sin(A + B) = \sin A \cos B + \sin B \cos A$

$\sin(A - B) = \sin A \cos B - \sin B \cos A$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Src: http://images.slideplayer.com/2/1035617/slides/slide_1.jpg

Trigonometric functions in C++

- In cmath header .. all in **radians**
  - Please read the 2 tables..see examples
  - Revise input/output ranges...vary much

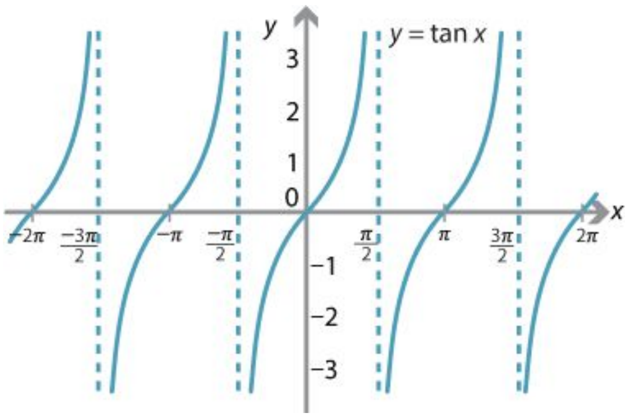
Trigonometric functions

cos	Compute cosine (function)
sin	Compute sine (function)
tan	Compute tangent (function)
acos	Compute arc cosine (function)
asin	Compute arc sine (function)
atan	Compute arc tangent (function)
atan2	Compute arc tangent with two parameters (function)

Hyperbolic functions

cosh	Compute hyperbolic cosine (function)
sinh	Compute hyperbolic sine (function)
tanh	Compute hyperbolic tangent (function)
acosh C++11	Compute area hyperbolic cosine (function)
asinh C++11	Compute area hyperbolic sine (function)
atanh C++11	Compute area hyperbolic tangent (function)

Atan vs Atan 2



Quadrant	Angle	sin	cos	tan
I	0 < α < π/2	> 0	> 0	> 0
II	π/2 < α < π	> 0	< 0	< 0
III	π < α < 3π/2	< 0	< 0	> 0
IV	3π/2 < α < 2π	< 0	> 0	< 0

Atan range is [-PI/2 - PI/2]
Tan of either angles 45 or 135 => positive values?!
How to know the quadrant! We need to use sin/cos too

atan2(y, x) do that for us and return range [-PI, PI]

Src: http://stackoverflow.com/questions/283406/what-is-the-difference-between-atan-and-atan2-in-c

Atan vs Atan 2

$$\operatorname{atan2}(y,x) = \begin{cases} \arctan(\frac{y}{x}) & x > 0 \\ \arctan(\frac{y}{x}) + \pi & y \geq 0\,,\ x < 0 \\ \arctan(\frac{y}{x}) - \pi & y < 0\,,\ x < 0 \\ \frac{\pi}{2} & y > 0\,,\ x = 0 \\ -\frac{\pi}{2} & y < 0\,,\ x = 0 \\ \text{undefined} & y = 0\,,\ x = 0 \end{cases}$$

```
(+1,+1) cartesian is (1.41421,0.785398) polar
(+1,-1) cartesian is (1.41421,2.35619) polar
(-1,-1) cartesian is (1.41421,-2.35619) polar
(-1,1) cartesian is (1.41421,-0.785398) polar
atan2(0, 0) = 0 atan2(0,-0) = 3.14159
atan2(7, 0) = 1.5708 atan2(7,-0) = 1.5708
```

Degree = Radian

----

0 = 0
90 = 1.5708
180 = 3.14159
270 = 4.71239
360 = 6.28319

45 = 0.785398
135 = 2.35619
225 = 3.92699
315 = 5.49779

1.4 = sqrt(2)

Src: http://stackoverflow.com/questions/283406/what-is-the-difference-between-atan-and-atan2-in-c

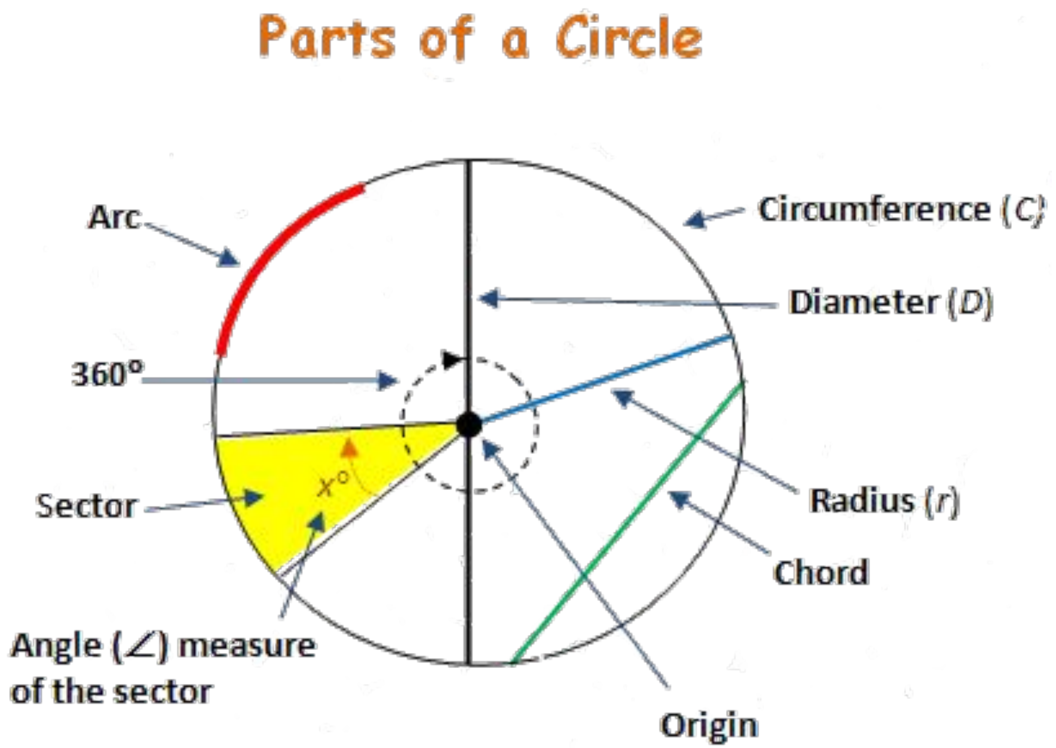
Triangle Area

- Please read triangle article.
  - Ignore hard things
- Homeworks
  - Given 3 sides of triangle, find area?
  - Given the length of three medians of a triangle, find area?
  - Given 3 sides of triangle inside/outside circle? what is circle radius? Totally touching the circle
  - ...

Triangle Area

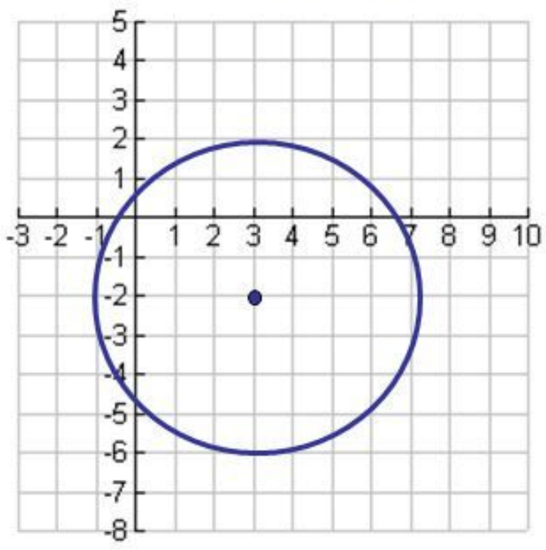
```
double triangleArea( point p0, point p1, point p2 ) {
    double a = length(p0-p1), b = length(p0-p2), c = length(p1-p2);
    double s = (a+b+c)/2;
    return sqrt((s-a)*(s-b)*(s-c)*s); //Heron's formula
    // base=u+v (divided by h) u = (a^2 + b^2 - c^2)/2a,
    // h = sqrt(b^2-u^2)  where base is a.
    // If these 3 points on circle boundry (Trinagle inside circle)
    // double radius1 = (a*b*c)/(4*triangleArea);
    // If circle inside triangle
    // double radius2 = sqrt((s-a)*(s-b)*(s-c)/s);
}

// Given the length of three medians of a triangle, find area
double triangleArea( double m1, double m2, double m3 )
{   // Area of triangle using medians as sides =
    // 3/4 * (area of original triangle)
    if(m1<=0 ||m2<=0 ||m3<=0 )      return -1;  // impossipole
    // For area made by sides as medians
    double s = 0.5 * ( m1 + m2 + m3 );
    double medians_area = (s * ( s - m1 ) * (s - m2) * ( s - m3 ));
    double area = 4.0/3.0 * sqrt(medians_area);
    if(medians_area <= 0.0 || area <= 0) return -1; // impossipole
    return area;
}
```

Circles



Src: http://ssepkowitz.pbworks.com/f/1241790691/SAT_Geometry_Circles1.png

Circles



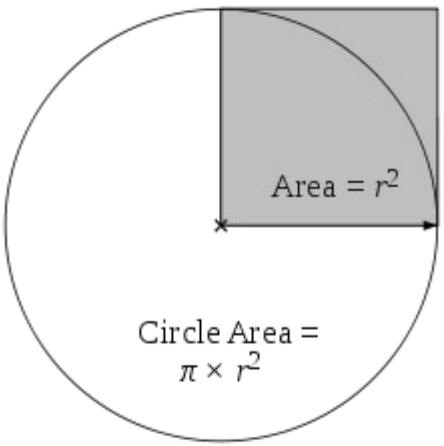
$$(x-h)^2+(y-k)^2=r^2$$

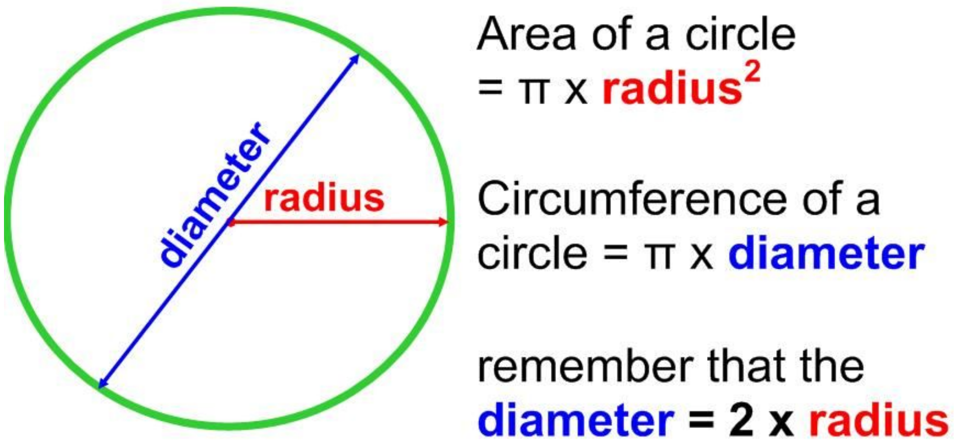
$$(x-3)^2 \;+\; (y-(-2))^2=4^2$$

$$\boxed{(x-3)^2 \;+\; (y+2)^2 = 16}$$

Src: http://images.slideplayer.com/18/6070989/slides/slide_4.jpg

Circles



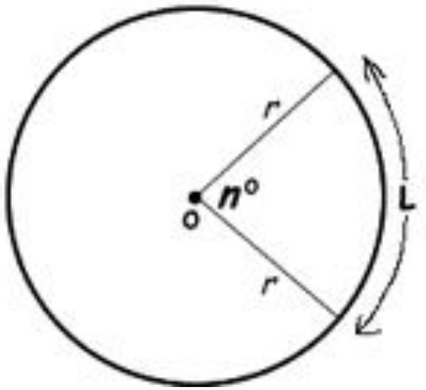


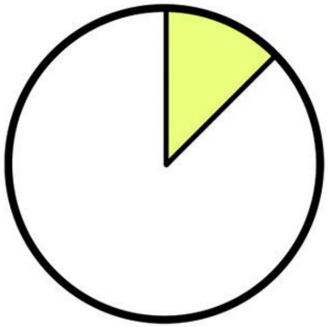
Src: http://img1.wikia.nocookie.net/__cb20140723160517/central/images/4/4d/Circle-formuasl-.jpg https://upload.wikimedia.org/wikipedia/commons/thumb/c/ce/Circle_Area.svg/220px-Circle_Area.svg.png

Circles

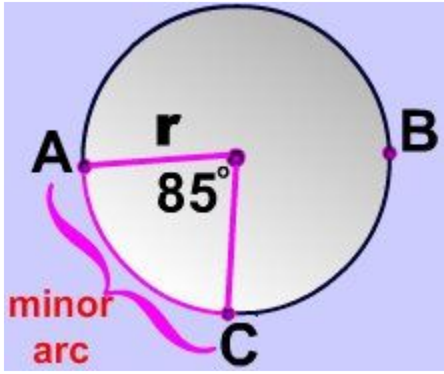
Length of an Arc Formula

$$\text{Length} = \frac{n^\circ}{360^\circ} \times 2\pi r$$





$$\text{sector area} = \frac{n}{360^\circ} \times (\pi \times r^2)$$

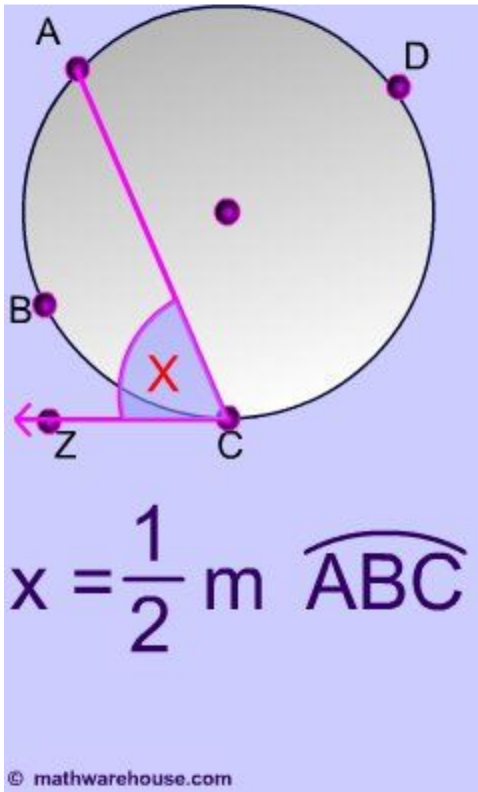


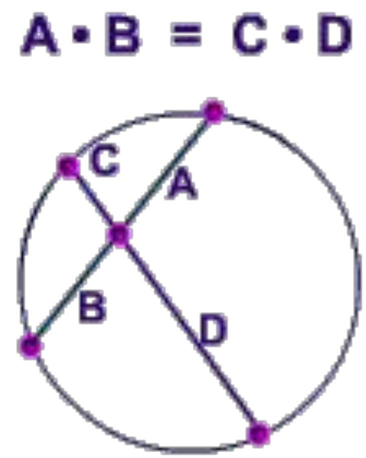
$\overset{\frown}{ABC}$ is the major arc

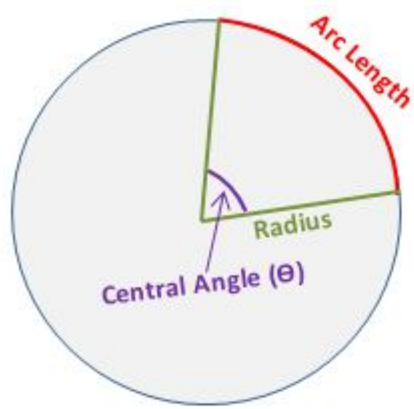
$\overset{\frown}{AC}$ is the minor arc

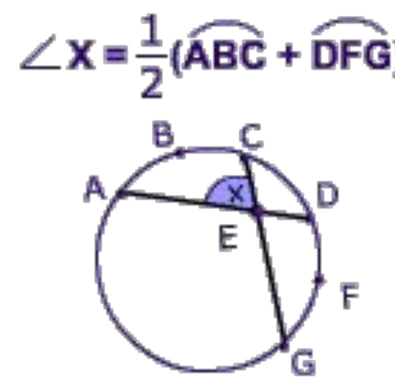
Src: http://thearclengthofacircle.weebly.com/uploads/1/2/3/3/12338358/3055696.gif?160

Circles

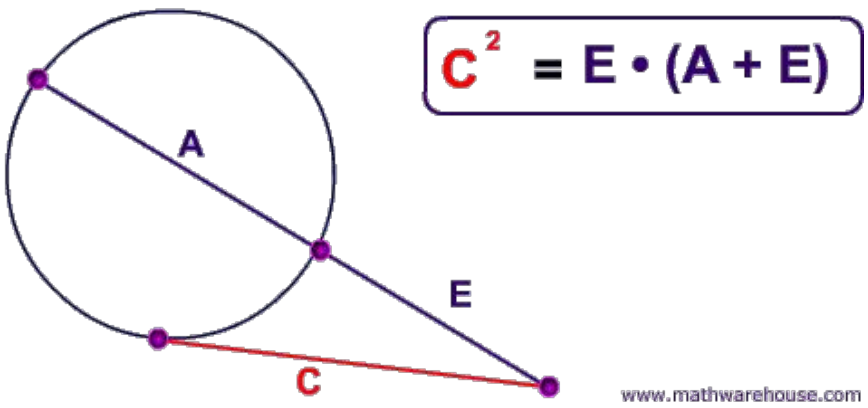

A
D
B
X
Z
C
x = 1/2 m ABC
© mathwarehouse.com

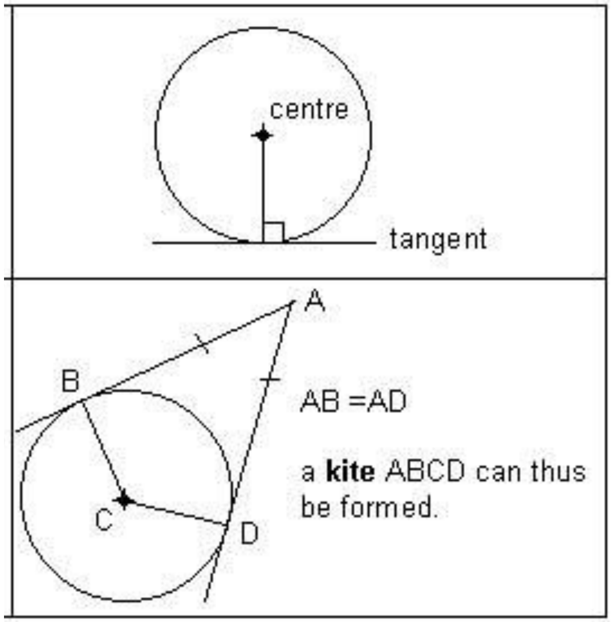

A • B = C • D
C
A
B
D

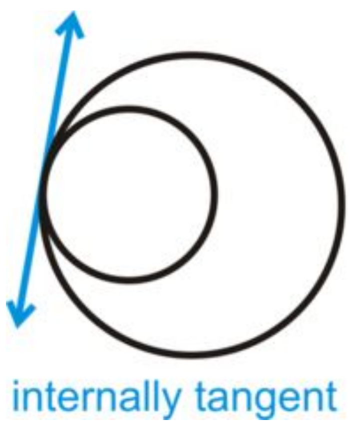

Arc Length
Radius
Central Angle (Θ)

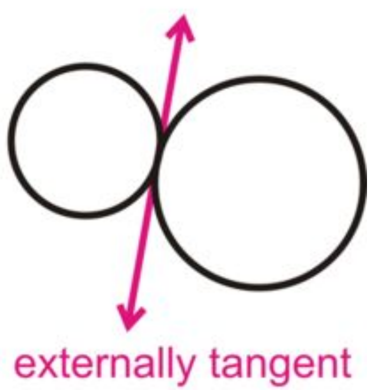

∠X = 1/2(ABC + DFG)
B
C
A
X
D
E
F
G

Circles









Src: http://www.funmaths.com/math_tutorials/images/tutorial_geometry6_clip_image002.jpg http://www.mathwarehouse.com/geometry/circle/images/secant-tangent-sides/secant-tangent-side-picture.gif

تم بحمد الله

علمكم الله ما ينفعكم

ونفعكم بما تعلمتم

وزادكم علماً